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ARITHMETIC

WITH

NUMEROUS EXAMPLES

EXERCISES

AND

EXAMINATION PAPERS

ARRANGED BY

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PREFACE.

In this work I have endeavoured to provide a convenient and systematic manual of Arithmetic suited to the requirements of the middle and upper forms in schools, of candidates for the various public examinations, and of those preparing for commercial pursuits.

In the treatment of the Theory no Algebraical symbols or formulæ have been used. The four fundamental rules, with which it is assumed that the reader is already familiar, are considered in less detail than the rest. Decimals are treated as independent of Vulgar Fractions; Recurring Decimals are sufficiently isolated to allow of their omission, if desired, from a first course; and considerable space is devoted to Approximation and its practical applications. Questions of the kind known as "Rule of Three" are treated, in the first instance, by the Unitary method, but only as a stepping-stone to the Fractional, or Ratio, method.

The order in which the chapters are arranged is not that in which they need necessarily be read. I have preferred rather to collect together, as far as possible, into groups the more closely allied portions of the subject. For instance, all the varieties of Reduction will be found in one chapter, so that any one variety may easily be found when wanted.

I have ventured to introduce a few small innovations;—a new explanation of "borrowing" in subtraction; the uniform use of the word "factor" in place of "measure"; the device of "moving the points" in Contracted Multiplication. I have also inserted a short chapter on the method

of Nine Multiples; and have given greater prominence to practical, than to purely theoretical, Discount.

A special feature of the book is the unusually large number and variety of examples fully worked out and explained in illustration of a rule and its converse; of methods of dealing with problems: of some special device for saving labour, as well as of orderly arrangement of work. In the notes which accompany these examples I have endeavoured to anticipate the difficulties of the student who is without the constant help of an oral teacher.

The Exercises are all collected together at the end of the volume, each set being numbered to match the corresponding chapter of "Bookwork". They have been drawn from so many sources, and so much care has been taken in selecting, adapting, and arranging them, that I trust they will, on trial, not be found wanting as regards either variety or graduation.

In Part I. there are frequent specimens of oral, or mental, questions; such as can, of course, be easily made by the teacher during oral lessons. In Part II. many of the questions on the commercial rules have not been constructed to yield "neat" results, but are of a strictly practical nature.

The Answers have been carefully tested, but among so many (about 8500) some errors have doubtless crept in. I shall be grateful for notification of any that may be discovered, as also for other corrections, or suggestions for the improvement of the book.

A. E. L.

Dec., 1897.

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BOOKWORK.—PART I.

I. NUMERATION, NOTATION.

A **unit** is a *single* thing, or object;
also a *single set* of objects when regarded as a whole.

For instance, *an apple, a sheep; a pair of gloves, a pack of hounds.*

If the unit is of *some particular kind*, mentioned by name, it is called **concrete.**

A **number** states *how many units* there are in some particular group.

For instance, *five* apples, *seven* sheep; *six* pairs of gloves, *three* packs of hounds.

When *no particular kind* of unit is mentioned, (*i.e.* when the unit considered is *abstract*), the number is called an **abstract number.**

For instance, *five, seven, six, three.*

Numeration is the art of expressing numbers in *words.*

As all civilized nations count *by tens*, the method of numeration is, in its main features, the same in most languages. Each of the numbers from one to ten has an independent name, but the names of numbers greater than ten are almost invariably compounds of these ten names.

Thus, in English, "eleven" means *one and ten;* "twelve,"* *two and ten;* "thirteen," *three and ten;* and so on. "Twenty" means *two tens;* "thirty," *three tens;* and so on. Also, "hundred," and "thousand," are probably compounds of words which originally meant *ten tens* and *ten hundreds* respectively. The names "million," "billion," "trillion," &c., are, however, comparatively modern inventions, and are used, in England,† for a *thousand thousands*, a *million millions*, a *million billions*, &c., respectively.

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* The use of "eleven," "twelve," in English, where we might have expected to find *one-teen, two-teen*, is probably due to their derivation from the Gothic *lif* instead of the Anglo-Saxon *ten.*

† In other parts of Europe, and in America, "billion" is used for a *thousand* millions "trillion" for a *thousand* billions, &c.

The method of counting *by tens* is called the **Decimal** system (Latin, *decem*, ten); and the number *ten* is called the **base**, or radix, of the system.

The world-wide use of the number *ten* as base is doubtless due to the primitive custom of counting on the *fingers.*

Any number *might* be used as base.

For instance, if *nine* were base, and "ny" were used for *nine* as we now use "ty" for *ten*, then, just as, with base *ten*, "thir-*ty*-seven" means *three* **tens** *and seven*, so, with base *nine*, "thir-*ny*-seven" would mean *three* **nines** *and seven.*

In some special cases a limited use *is* made of other bases.

For instance, in speaking of "three *score* and eight" sheep, we use *twenty* as base; and in speaking of "five *gross* three *dozen* and seven" buttons, we use *twelve* as base.

Notation is the art of representing numbers by means of symbols, or *figures.*

Various systems of notation have been used by different nations, but, owing to its great superiority, the only one in general use now is the Arabic.*

The ten figures of this system, namely, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9,† are called the **digits** (Latin, *digitus*, a finger).

By means of the ten digits any number whatever can be represented, the need of fresh symbols for large numbers being avoided by making the value of any digit depend upon its *position* as well as upon its *shape.*

For instance, the figure 3, by reason of its shape always represents *three* units of some kind or other, and, if it stand *alone*, these units are *single;* but, if another figure stand to the right of it, it no longer represents three *ones*, but three *sets of ten;* and so on.

Thus, the value of any figure is increased *tenfold* by moving it *one place towards the left.*

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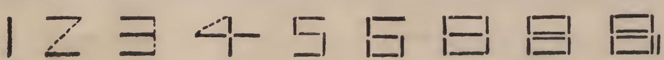
* It acquired the name "Arabic" from having been introduced into Europe by the Moors when they conquered Spain; but the knowledge of it did not begin to spread in Europe till the 12th century, and it was not in general use till much later. The system had its origin in India.

† These symbols were in use in India at least 300 B.C. It has been conjectured that they originated in an arrangement of as many straight strokes as each symbol represents units, thus—



The value which any figure bears *by reason of its shape* is called its *intrinsic*, or *digit* value.

The value which it gains *by position* is called its **local** value (Latin, *locus*, place).

Any number *might* be used as base in a system of notation embodying this principle of local value.

Thus, for instance, just as in the decimal system, with *ten* as base, 342 represents 3 sets of *ten* times *ten*, 4 sets of *ten*, and 2 *single*, objects; so, with *six* as base, 342 would represent 3 sets of *six* times *six*, 4 sets of *six*, and 2 *single*, objects.

The number of digits employed in any such system would be the same as the number chosen as base.

Thus, if *six* were base, the only digits used would be 0, 1, 2, 3, 4, 5; for *six* would then be represented by the figures 10, *seven* by 11; and so on.

The figure 0, (called *nought*, *cipher*, or *zero*,*) has no *intrinsic* value, but is used to fill up a vacant place, and thus give *local* value to another digit.

The figure 1 is often called **unity.**

It is the principle of *local value* that constitutes the great superiority of the Arabic over other systems of notation.

In the Roman system, which was in general use in Europe before the introduction of the Arabic, the symbols are the letters I, V, X, L, C, D, M, representing 1, 5, 10, 50, 100, 500, 1000 respectively, a line placed over a letter increasing its value a thousandfold. Thus, $\overline{V}$ represents 5000, $\overline{X}$, 10000. The symbols IƆ, IƆƆ, CIƆ, CCIƆƆ, were also used for 500, 5000, 1000, 10000 respectively.

The Roman symbols have *no local* value. Thus, X always represents ten *ones* whatever its position, and XX means *ten and ten*, or twenty.

The symbols are arranged as a rule in order of value, the greater to the left, and their separate values are added together; thus CCLXXXVI represents 286. But in order to avoid to some extent repeated use of the same symbol, a symbol of *less* value sometimes stands to the left of one of greater value, in which case its value is *subtracted* from that of the greater; thus IV means *one less than five*, or 4; XL means *ten less than fifty*, or 40; and so on.

Even as a means of merely representing a number the Roman system, as compared with the Arabic, is very clumsy. Thus, for instance, the number MDCCCXCIII, or 1893, requires *ten* symbols instead of *four*. Moreover, owing to the absence of *local value* it is useless for the *operations* of arithmetic; these were performed mechanically, either by means of pebbles† or counters, or by help of an instrument called the abacus.

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* "Cipher" and "zero" are both derived from the same Arabic word, signifying *vacant*.

† Whence the word "calculation," from the Latin *calculus*, a pebble.

To **read**, or write in **words**, a number expressed in *figures*.

If the number consists of not more than four figures, we take each figure separately and mention both its intrinsic and its local value.

For instance, 1372 represents

two *single*,
seven *sets of ten*,
three *sets of ten tens*, or *a hundred*,
and one *set of ten hundreds*, or *a thousand*,
} objects,

which we express shortly, thus:

One *thousand*, **three** *hundred* and **seven**-*ty*-**two**.

Again, 3502786 represents

six *ones*,	or units	of the	1st	order.
eight *tens*,	,,	,,	2nd	,,
seven *hundreds*,	,,	,,	3rd	,,
two *thousands*,	,,	,,	4th	,,
no *ten-thousands*,	,,	,,	5th	,,
five *hundred-thousands*,	,,	,,	6th	,,
and three *millions*,	,,	,,	7th	,,

Here, instead of repeating the word *thousand* three times over, we group together the figures in the local value of which that word occurs, omitting to mention the cipher, and then, beginning with the units of the highest order, express the number thus:

Three *million*, **five-hundred-and-two** *thousand*, **seven** *hundred* and **eigh**-*ty*-**six**.

Similarly in a larger number we group together all the digits representing *millions*, &c.

In order to facilitate the reading of large numbers commas may be inserted after the 3rd, 6th, and 12th figures, counting from right to left; then *the figures between the first and second commas are* **thousands**, and those *between the second and third commas* **millions**; and we read each group separately as a distinct number, adding the local value of the *group*.

For instance, the number 57360942.

Inserting commas between the 9 and 0, and between the 3 and 7, we see that there are 57 *millions*, and 360 *thousands*. 57,360,942

Hence this number is expressed in words thus:

Fifty-seven *million*, **three-hundred-and-sixty** *thousand*, **nine-hundred-and-forty-two**.

Again, to read the number 38764003201435.

Inserting commas between the 4 and 1, the 2 and 3, and the 7 and 8, and remembering that a billion is a million millions, we see that there are 38 *billions*, 764003 *millions*, and 201 *thousands*.

$$\overbrace{38}^{\text{Billions.}},\overbrace{764003}^{\text{Millions.}},\overbrace{201}^{\text{Thousands.}},435$$

Hence we read this number thus:

Thirty-eight *billion*, seven-hundred-and-sixty-four-thousand-and-three *million*, two-hundred-and-one *thousand*, four-hundred-and-thirty-five.

To write in **figures** a number expressed in *words*.

The places of the *units*,* *tens*, *hundreds*, &c., may at first be indicated by little dashes and the commas inserted as before.

For instance, to write in figures the number *Forty-one millions, thirty-five thousand and three.*

First arrange the commas, thus:— ,---,---

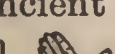
Next write the *millions*, 41, to the left of the left-hand comma;
then the *thousands*, 35, to the left of the right-hand comma;
then the 3 in the *units* place, thus:— 41,-35,--3

Finally, fill up the *vacant* places with *ciphers*, thus:— 41,035,003.

Note.—It should be remembered that *seven* digits are needed to represent a million, *thirteen* to represent a billion. Also, that the *greatest* number consisting of *two* digits is 99, and the *least* is 10; the *greatest* number of *three* digits is 999, and the *least* is 100; and so on.

Numbers taken *in their natural order* are called **consecutive**.

For instance, 17, 18, 19, are consecutive numbers.

Note on the origin of the Roman symbols.—Those denoting the smaller numbers are of very great antiquity, being derived from the hieroglyphics, or picture-writing, of the ancient Egyptians. Thus the symbols I, II, III, IIII had their origin in  rough sketches of one, two, three, four *fingers*; V, of an entire *hand* ; X, of two V's or hands. The symbols for the higher numbers, *e.g.* C, M, are of much later origin, being the initials of the Latin names for the numbers they represent, and L was originally *half* of the letter C.

* When used in connection with abstract numbers the word "units" implies *units of the 1st order*, i.e. *ones*.

II. THE SIMPLE RULES.

The four fundamental operations of arithmetic—Addition, Subtraction, Multiplication, and Division—are called *simple* when the numbers dealt with are *abstract* numbers.*

ADDITION.

Addition is the operation of finding a number which shall contain as many units as there are in two, or more, given numbers together.

The *result* is called the **sum**, the *total*, or the *amount*.

The *sign* of Addition is $+$, and is read "*plus*."

For instance, $3 + 5$ is read "Three *plus* five."

In primitive times the *sum* of numbers would be found by actual *counting;* each unit in each number would be represented perhaps by a pebble, and the whole heap so formed would then be counted.

The use of the Arabic system of notation enables us to obtain a *sum* much more rapidly. We commit to memory the Addition Table (*i.e.* the sum of every pair of the digits) and use its results in the addition of larger numbers. In so doing we employ the following fundamental truth, or *axiom:*—

The sum of numbers is the same as that of all their parts taken in any order.

Hence, regarding each of the numbers to be added as consisting of the separate parts, *units*, *tens*, *hundreds*, &c.; we first collect together all the *units*, then all the *tens*, and so on.

If the resulting number of units exceeds *nine*, we remove *any complete sets of ten* at once and "**carry**" such sets forward to be counted in with the other *tens;* and so on.

Note.—For convenience it is usual to arrange the numbers to be added in a vertical column, so that all the *units* are in one straight line, all the *tens* in another; and so on. It is well, however, to practise adding small numbers *across.*

To test the result of Addition.

Add the numbers in the reverse order, *i.e.* from top to bottom, instead of from bottom to top; or, in the case of "cross" addition, from right to left, instead of from left to right.

---

* Or when they refer to "simple" quantities. See p. 24.

SUBTRACTION.

Subtraction is the operation of finding how many units remain when from a given number a smaller number is taken away.

The *result* is called the **difference**, or the **remainder**.

The *sign* of Subtraction is –, and is read "*minus*."

For instance, 7 – 3 is read "Seven *minus* three."

It follows from the definition that the remainder, if added to the smaller number, will make up the larger number; hence Subtraction is the undoing of the Addition of two numbers; *i.e.* in Subtraction *we have the* **sum** *of two numbers given; we have also* **one** *of those numbers given; and we are required to find the* **other.**

Hence the Addition Table is available for Subtraction.

For instance, as 8 + 7 is 15; conversely, 15 – 7 is 8, and 15 – 8 is 7.

The method used in the subtraction of large numbers corresponds to that used in addition; we take the *units*, *tens*, &c., of the smaller number from the *units*, *tens*, &c., respectively, of the greater. But, as some digit of the smaller number may be greater than the corresponding digit of the larger number, it is often necessary to *rearrange* the larger number.

(i.) For instance, in subtracting 35 from 81, 5 *units* cannot be taken away from 1 *unit*, but we can rearrange the greater number by changing one *ten* into *units;* thus, instead of regarding the number as 8 *tens* and 1 *unit*, we may regard it as 7 *tens* and 11 *units*.

Now, taking 5 *units* from 11 *units*, there remain 6 *units*, and taking 3 *tens* from 7 *tens* there remain 4 *tens;* thus the required difference is 46.

The written work then *might* be arranged thus, the figures 8, 1, being cancelled and replaced by 7 and 11.

$$\begin{array}{cc} 7 & 11 \\ \cancel{8} & \cancel{1} \\ 3 & 5 \\ 4 & 6 \\ \hline \end{array}$$

(ii.) In practice, however, we change a ten into units *mentally*, and remember when we reach the *tens* column, that we must decrease the 8 *tens* by this 1 *ten* as well as by the 3 *tens*.

But to take **1** and **3** *in succession* from 8 is equivalent to taking **4** from 8 *in one operation.*

We therefore "**carry 1**" which we add to the 3 and then take **4** from 8.

$$\begin{array}{r} 81 \\ 35 \\ \hline 46 \\ \hline \end{array}$$

Note.—If a *cipher* occur in the greater number, the mental "rearrangement" required may be more complex. For instance, in taking 267 from 305, as there are *no tens* in 305, we first change 1 *hundred* into 10 *tens*, and then, as before, change *one* of these tens into *units*.

The method (ii.) is based on the following axiom:—

To take several numbers in succession from a given number is equivalent to taking their sum from that given number.

It follows from the foregoing axioms that the *order* in which a succession of additions and subtractions is performed does not affect the result.

For instance, $4 - 3 + 2$ is correctly found, either
by first taking 3 from 4 and afterwards adding 2 to the result;
or, by first adding 2 to 4 and afterwards subtracting 3 from the result.

Again, $7 - 3 + 8 - 5 + 12 - 9$ would involve *five* operations if each addition and subtraction were performed *separately in the order given.*

But as $7 - 3 + 8 - 5 + 12 - 9$ is equivalent to $7 + 8 + 12 - 3 - 5 - 9$, and as to subtract 3, 5, and 9 in succession is equivalent to subtracting their *sum* in *one* operation, the result can be obtained in *three* steps; thus:—

the sum of 7, 8 and 12 is 27;
the sum of 3, 5 and 9 is 17;
and $27 - 17 = 10$.

Also, in the question—*Find the value of* $7 - 13 + 23 - 41 - 19 + 64$, the operations *cannot be performed in the order given.*

We proceed thus:

$$\begin{aligned} 7 - 13 + 23 - 41 - 19 + 64 &= 7 + 23 + 64 - 13 - 41 - 19 \\ &= 94 - 73 \\ &= \underline{21\ \textit{Ans.}} \end{aligned}$$

It is useful to be able to add several numbers together and subtract their sum from another number in *one* operation.

EXAMPLE.—*Take the sum of* 479, 1684, 592, *and* 2348 *from* 8126.

Mental Work.

(Only the figures in thick type are written down.)

1st step.

Beginning at the bottom of the units column, we say (or, *think*)
10, 14, 23 from 26 (changing *two* tens to units) leaves **3**.

2nd step.

Carry 2;
6, 15, 23, 30 from 32 (changing *three* hundreds to tens) leaves **2**.

3rd step.

Carry 3;
6, 11, 17, 21 from 21 (changing *two* thousands to hundreds) leaves **0**.

4th step.

Carry 2;
4, 5 from 8, leaves **3**.

Written Work.

$$\begin{array}{r} 8126 \\ \left.\begin{array}{r} 479 \\ 1684 \\ 592 \\ 2348 \end{array}\right\} \\ \hline \underline{3023\ \textit{Ans.}} \end{array}$$

<u>To test the result of Subtraction.</u>

Add the result to the smaller number; this should give the larger.

MULTIPLICATION.

Multiplication is the operation of finding the *sum* which results when a given number is repeated as many times as there are units in another number.

For instance, to multiply 23 by 4 is to find what 23 repeated *four* times amounts to; *i.e.* 23 multiplied by $4 = 23 + 23 + 23 + 23 = 92$.

The number to be multiplied is called the **multiplicand**; the number by which it is to be multiplied is called the **multiplier**; the *result* is called the **product**.

For instance, in the example above,
23 is the *multiplicand*, 4 is the *multiplier*, and 92 is the *product*.

The *sign* of Multiplication is $\times$, and is read "multiplied by."

For instance, 5×3 is read "Five *multiplied by* three."

It follows from the definition of Multiplication that any product *might* be obtained by Addition.

For instance, 467×93 *might* (at great expense of time and labour) be found by setting down 467 *ninety-three* times and then adding these ninety-three numbers together.

This we avoid by committing to memory the Multiplication Table* (*i.e.* the product of every pair of the digits), and using its results in the multiplication of larger numbers. In so doing we employ the following principles:—

(I.)† *The product is the same, whether we multiply the first of two numbers by the second, or the second by the first.*

For instance, to show that 4 multiplied by 3 is equivalent to 3 multiplied by 4.

We know, from the definition of Multiplication, that

$$\begin{aligned} 4 \times 3 &= 4 + 4 + 4 \\ &= 1+1+1+1+1+1+1+1+1+1+1+1 \\ &= 1+1+1 \quad +1+1+1 \quad +1+1+1 \quad +1+1+1 \\ &= 3 + 3 + 3 + 3 \\ &= 3 \times 4. \end{aligned}$$

Or thus:—

Take 12 counters and arrange them as in the accompanying diagram; then the total number of counters is the same whether we regard them as three *rows of four*, or as four *columns of three*. *i.e.* $4 \times 3 = 3 \times 4$.

```
* * * *
* * * *
* * * *
```

* It is usual to learn the Multiplication Table beyond 9 *times* 9, as far as 12 *times* 12, and it is well to gradually learn it a little further still.

† Sometimes referred to as the *Commutative Law*.

(II.) *The product of two numbers is the same as the sum of the products obtained by multiplying one of them by each of the parts of the other number.*

For instance, considering 5 as made up of the parts 2 and 3, we know by the definition of multiplication, that

$$\begin{aligned} 4 \textit{ multiplied by } 5 &= 4+4+4+4+4 \\ &= 4+4 \qquad\qquad +4+4+4 \\ &= 4 \textit{ multiplied by } 2 + 4 \textit{ multiplied by } 3. \end{aligned}$$

Or thus:—

Take 20 counters and arrange as in the accompanying diagram; then the total number of counters is the same whether we regard them

```
*  *     *  *  *
*  *     *  *  *
*  *     *  *  *
*  *     *  *  *
```

as **five** *columns of four*,

or as **two** *columns of four*, together with **three** more *columns of four*.

i.e., $4 \times 5 = 4 \times 2$ *plus* 4×3.

Note.—It is evident, from the same diagram, that, conversely, 4×5 *minus* $4 \times 3 = 4 \times 2$.

If the product of two numbers be multiplied by a third number the result is called the **continued product** of the *three* numbers; and so on.

Numbers multiplied together are called **factors** of their product.

For instance, the *continued product* of 2, 3, and 5 is 30; and 2, 3, and 5 are *factors* of 30.

(III.) *The order in which the factors are multiplied together does not affect the result.*

For instance, $2 \times 3 \times 5 = 5 \times 3 \times 2 = 3 \times 5 \times 2$.

(IV.) *To multiply in succession by the factors is equivalent to multiplying by their product.*

For instance, $7 \times 2 \times 3 = 7 \times 6$.

To multiply a number by 10.

Affix one cipher to the right of the figure in the units place.

Thus each figure is moved *one place towards the left*, and is consequently increased *tenfold*. Hence, by (II.), the number itself is multiplied by 10.

Similarly, **to multiply by 100, 1000,** &c.

Affix two, three, &c., *ciphers.*

We will now illustrate the application of the foregoing principles:—

EXAMPLE.—*Multiply* 243 *by* 75, *and explain the process.*

Regarding 75 as made up of the parts 5 and 70, we know, by (II.), that we multiply 243 by 75 if we

(i.) multiply 243 by 5,
(ii.) then multiply 243 by 70,
(iii.) and add the results.

(i.) Now 243 is multiplied by 5 if each of its parts be multiplied by 5. Hence, regarding 243 as made up of 2 *hundreds*, 4 *tens*, and 3 *units*, we first multiply the 3 *units* by 5, carrying forward any *complete sets of ten* that result, and set down the remaining *units* in the units place; we next multiply the 4 *tens* by 5, adding in the "carried" *tens;* &c.

Here each small product is known from the Multiplication Table.

We thus obtain 1215 as the product of 243 and 5.

(ii.) Again, we know by (IV.), that we multiply 243 by 70, if we multiply in succession by 7 and 10, the factors of 70.

Hence multiplying 243 by 7 (in the same way as we before multiplied by 5), we obtain the product 1701;

And we multiply 1701 by 10 if we affix a cipher.

Thus we obtain 17010 as the product of 243 and 70.

(iii.) Finally, adding 1215 to 17010 we obtain 18225 as the complete product.

In practice *the cipher due to multiplication by* 10 *is omitted, and its place in the second line is left vacant*, the written work being arranged thus:

```
  243
   75
 ----
 1215
1701
-----
18225
```

Note.—If we multiply 243 by 705, *two* places in the second line, due to multiplication by 100, must be left vacant; and so on.

Hence, *each line of multiplication begins under the figure by which we multiply.*

Again, to multiply 243 by 7500, we first multiply 243 by 75 as before, and then affix two ciphers to the final result, *i.e.* multiply the final result by 100; thus:

```
  243
   7500
 ------
 1215
1701
-------
1822500
```

In the above example we multiplied, as is usual, by the *units* figure *first*. We might equally well have multiplied by the *units* figure *last*.

For instance, the work of 4731 × 524 might stand thus:

```
   4731
    524
  -----
23655
  9462
  18924
-------
2479044
```

Multiplication by one figure may be combined with Addition, or Subtraction, in *one* operation.

For instance, *Multiply* 473 *by* 8, *and subtract the result from* 3951.

Mental Work.

8 times 3 is 24; 24 from 31 leaves **7**.
Carry 3.
8 times 7 is 56; 56 and 3 is 59; 59 from 65 leaves **6**.
Carry 6.
8 times 4 is 32; 32 and 6 is 38; 38 from 39 leaves **1**.

Written Work.

$$\begin{array}{r} 3951 \\ \underline{473 \times 8} \\ \underline{\underline{167}}\ \textit{Ans.} \end{array}$$

The product of *two equal* factors is called the **square** (or *second* power) of one of those factors.

For instance, 7×7, or 49, is the *square* of 7.

The continued product of *three equal* factors is called the **cube** (or *third* power) of one of those factors.

For instance, $5 \times 5 \times 5$, or 125, is the *cube* of 5.

The continued products of *four*, *five*, &c., *equal* factors are called the *fourth*, *fifth*, &c., *powers* of one of those factors respectively.

For instance, $3 \times 3 \times 3 \times 3$, or 81, is the *fourth power* of 3.

A *power* of a number may be indicated by a small figure placed above the number, and called the **index** of the power.

For instance, 7^2 means 7×7, or the *square* of 7, and 10^5 means $10 \times 10 \times 10 \times 10 \times 10$, or the *fifth power* of 10.

To test the result of Multiplication.

Divide the product by one of the factors; then there should be no remainder, and the quotient obtained should be the other factor.

Or, apply the test known as *casting out the nines*.

In this test we first find the remainders when the *sum* of the figures in each of the factors is divided by **9**; we then multiply these remainders together, and divide their product by 9 in order to obtain the remainder (R). We next find the remainder when the *sum* of the figures in the product is divided by 9; this should be the same as the previous remainder (R).

For instance, to show that $5642 \times 347 = 1957774$.

First,

$5+6+4+2$ divided by 9 gives rem^r **8**
and $3+4+7$ „ 9 „ **5**
and $40 \div 9$ gives rem^r **4** (R).

Also,

$1+9+5+7+7+7+4$ divided by 9 gives rem^r **4**, which agrees with (R).

Division.

Division is the operation of finding how many times one number is contained in another.

The *sign* of division is ÷, and is read "*divided by.*"

For instance, 15 ÷ 3 is read "Fifteen *divided by* three."

The number to be divided is called the **dividend**;
the number by which it is to be divided is called the **divisor**;
the result is called the **quotient.**

For instance, in the example above,
15 is the *dividend*, and 3 is the *divisor.*

Also, as to divide 15 by 3 is to find how many *threes* there are in 15, and as we know from the Multiplication Table that 5 *threes* make 15, the *quotient* is 5.

Thus Division is the *undoing* of Multiplication; *i.e.* when *the product of two factors is given, and one of those factors is also given, Division is the operation of finding the other factor.**

Hence the Multiplication Table supplies results in Division.

For instance, as 7 × 8 is 56; conversely 56 ÷ 8 gives quotient 7 and 56 ÷ 7 gives quotient 8.

The *quotient*, (*i.e.* the *number of times* the divisor is contained in the dividend,) *might* always be obtained by *subtracting* the divisor from the dividend as often as possible, and then counting the number of such subtractions.

For instance, if we take 6 from 19, then 6 from the remainder, and so on, we find that after *three* subtractions have been performed the remainder is 1, from which we cannot again take 6.

Thus 19 ÷ 6 gives quotient 3 and remainder 1.

```
19
 6...1st
--
13
 6...2nd
--
 7
 6...3rd
--
 1
```

But we know, that to subtract 6 three times in succession is equivalent to subtracting 6 + 6 + 6, or 18, in one operation; and as we know from the Multiplication Table that *three* times 6 is less, and *four* times 6 is greater than 19, we can judge beforehand how many sixes to subtract, and so by help of the Multiplication Table obtain the result much more concisely thus:

```
19
18...3 sixes.
--
 1
```

* We shall see in Chapter XVIII. that this view also applies to a division in which a final remainder occurs.

A division is called **exact** when *no final remainder* occurs.

In this case, as we have already seen, the divisor and quotient are the factors whose product is the dividend; *i.e.* in *exact* division,

Divisor × Quotient = Dividend .... (i)

And, as we know, from (I.), that in Multiplication the factors are interchangeable, it follows that in Division the divisor and quotient are interchangeable.

Hence, as Dividend ÷ Divisor = Quotient,
so also Dividend ÷ Quotient = Divisor.

Thus Division may be regarded as the operation of finding either
the **number of times** one *given number* is contained in another,
or the **number** which is contained a *given number of times* in another.

A division is called **inexact** when there *is* a final remainder.

Hence, in *inexact* division,

Divisor × Quotient + Remainder = Dividend...(ii)

For instance, in the question *The divisor is* 6, *the quotient* 3, *and the remainder* 1, *find the dividend;* the required dividend $= 6 \times 3 + 1 = 19$.

We will now illustrate the method known as "Long" Division:

EXAMPLE.—*Divide* 18096 *by* 23, *and explain the process.*

It is evident that 23 is contained less than 1000 times and more than 100 times in 18,096; the highest figure in the quotient will therefore represent *hundreds;* we find this figure first.

$$\begin{array}{r|l} 23)\,18096 & 700 \\ \underline{16100} & \;\;80 \\ 1996 & \;\;\;\;6 \\ \underline{1840} & \\ 156 & \\ \underline{138} & \\ 18 & \end{array}$$

1st step.

We find on trial that 7 times 23 is less, but 8 times 23 is greater than 180; *i.e.* 23 is contained more than 700 times, and less than 800 times, in 18096.

Thus the *hundreds* figure in the quotient is 7.

Subtracting 700 times 23 from 18096, we obtain the remainder 1996.

We now proceed to find the next figure in the quotient, namely, that which represents *tens.*

2nd step.

We find on trial that 8 times 23 is less, but 9 times 23 is greater than 199; *i.e.* 23 is contained more than 80 times, and less than 90 times, in 1996.

Thus the *tens* figure in the quotient is 8.

Subtracting 80 times 23 from 1996, we obtain the remainder 156.

3rd step.

As 23 is contained less than 7 times, and more than 6 times in 156, the *units* figure in the quotient is 6.

Subtracting 6 times 23 from 156, we obtain the final remainder 18.

We have now subtracted

first 700 times 23,
then 80 times 23,
and then 6 times 23,

which we know is equivalent to subtracting 23 successively 786 times from 18096. Also 18 remains, from which we cannot again take 23.

Hence the quotient is 786 and the final remainder is 18.

In practice the written work stands thus—*the ciphers at the end of the successive products being omitted and their places being left vacant. Also the figures of the dividend are brought down one by one, as required.*

```
23)18096(786
   161
   ---
    199
    184
    ---
     156
     138
     ---
      18
```

Abridged method of working Long Division.*

This consists in subtracting each figure of the various products *as soon as it is obtained* and writing down the *differences only.*

For instance, the above example by this method is worked thus:

Mental Work.

(Only the figures in thick type are written down.)

1st step.

7 times 3 is 21; 21 from 30 leaves **9**.
Carry 3.
7 times 2 is 14; 14 and 3 is 17; 17 from 18 leaves **1**.

2nd step. Bring down **9**.

8 times 3 is 24; 24 from 29 leaves **5**.
Carry 2.
8 times 2 is 16; 16 and 2 is 18; 18 from 19 leaves **1**.

3rd step. Bring down **6**.

6 times 3 is 18; 18 from 26 leaves **8**.
Carry 2.
6 times 2 is 12; 12 and 2 is 14; 14 from 15 leaves **1**.

Written Work.

```
23)18096(786
    199
     156
      18
```

Note.—The advantages claimed for this method are that it saves time and is very compact in appearance. On the other hand, the greater complexity of the mental process increases the liability to err, and errors if made are also less easy to detect. Moreover when the same figure occurs more than once in the quotient, the labour of multiplying the divisor by that figure has to be repeated.

When the divisor is not greater than 12, the whole of the work is always performed mentally, and the figures of the quotient are written in succession below the corresponding figures of the dividend; this is called "Short" Division.

* Sometimes called the Italian, or Continental, method.

Division by Factors.

It follows conversely from (IV.) that to divide in succession by the factors is equivalent to dividing by their product.

Hence, whenever the divisor is known to be the product of factors not greater than 12, *short* division can be used.

For instance, *to divide* 3621 *by* 24, *i.e.* 4×6.

Dividing first by 4 and then by 6, we obtain the final quotient 150, and the remainders 1 and 5.

We have now to find the complete remainder.

$$24\left\{\begin{array}{l} 4\,\underline{|\,3621} \\ 6\,\underline{|\,905}\ldots 1 \\ \quad\ \underline{150}\ldots 5 \end{array}\right.\left.\begin{array}{l} \\ \\ \\ \end{array}\right\}\ \underline{21\ \textit{rem}^{r}.}$$

When we divide 3621 by 4 we find how many *sets of four* there are in 3621 *units;* namely, 905 *fours* and 1 *unit* over.

When we divide 905 *fours* by 6, we find how many *sets of six-fours,* (*i.e.* how many *twenty-fours,*) there are in 905 *fours;* namely, 150 *twenty-fours* and 5 *fours* over.

Thus the complete remainder is 5 *fours* + 1 *unit*, or 21 *units.*

Hence, in division by *two* factors, to obtain the **complete remainder**: *Multiply the second remainder by the first divisor and add the result to the first remainder.**

Again, *to divide* 6787 *by* $3 \times 5 \times 7$, *i.e. by* 105.

1st step.

6787 *units* contain 2262 *threes* and 1 *unit* over.

2nd step.

2262 *threes* contain 452 *fifteens,* and 2 *threes* over.

$$105\left\{\begin{array}{l} 3\,\underline{|\,6787} \\ 5\,\underline{|\,2262}\ldots 1\ \textit{unit} \\ \quad 7\,\underline{|\,452}\ldots 2\ \textit{threes} \\ \qquad\ \underline{64}\ldots 4\ \textit{fifteens} \end{array}\right.\left.\begin{array}{r} \\ = 1 \\ = 6 \\ = 60 \end{array}\right\}\underline{67\ \textit{rem}^{r}.}$$

3rd step.

452 *fifteens* contain 64 *hundred-and-fives,* and 4 *fifteens* over.

Thus the complete remainder is 4 *fifteens* + 2 *threes* + 1 *unit, i.e.* 67 *units.*

Hence, in the case of *three* factors, to obtain the **final remainder**: *Multiply the third remainder by the product of the first two divisors, the second remainder by the first divisor, and add these results to the first remainder.*

Similarly we might proceed in the case of four, or five, factors.

---

* This rule should be illustrated practically with counters to young beginners, thus:— Take, say, 74 counters and arrange them in *stacks of five;* we thus find that we can make 14 stacks of five and have 4 *single* counters over. Now arrange the stacks in *groups of three stacks;* we thus obtain 4 *groups* and have 2 *stacks* over. Thus 74 divided by the factors of 15 (*i.e.* 5 and 3) yields quotient 4 and remainder 4 *single* counters + 2 *stacks of five* counters, *i.e.* 14 *single* counters.

To divide a number by 10.

Cut off the figure which stands in the units place.

For, by so doing, each of the remaining figures is moved *one place toward the right*, and is, consequently, *decreased tenfold;* hence these remaining figures give the quotient, and the figure cut off, which was originally in the units place, is the remainder.

For instance, 23,4 divided by ten gives quotient 23, remainder 4.

Similarly, **to divide by 100, 1000,** &c.,

Cut off two, three, &c., figures to the right of the number.

For instance, 364,27 divided by 100 gives quotient 364, remainder 27.

Also, **to divide by 20, 200,** &c., or by **30, 300,** &c.,

Cut off one, two, &c., figures, and divide by 2, 3, &c.

For instance, *to divide* 3658 *by* 70.

The remainder 18 results from 1 *ten* remaining from the division of 365 *tens* by 7, together with the 8 *units* remaining from the previous division by 10.

```
7,0 | 365,8
       52...18 remr.
```

Again, *to divide* 38821 *by* 1200.

The remainder 421 results from 4 *hundreds* remaining from the division of 388 *hundreds* by 12, together with 21 *units* remaining from the previous division by 100.

```
12,00 | 388,21
         32...421 remr.
```

Also, *to divide* 305715 *by* 19000.

The remainder 1715, results from 1 *thousand* remaining from the division of 304 *thousands* by 19, together with 715 *units* remaining from the previous division by 1000.

```
19,000)305,715(16
       19
       115
       114
         1715 remr.
```

The following principle is important:

(V.) *The quotient is not altered by multiplying, or dividing, both dividend and divisor by the same number.*

For instance, the quotient of $6 \div 2$ is the same as that of 7 *times* $6 \div 7$ *times* 2.

To test the result of Division.

Multiply the divisor and quotient together, and add the remainder (if any) to the result; this should give the dividend.

Or, subtract the remainder (if any) from the dividend, and then cast out the nines as in proving multiplication.

USE OF SIGNS.

The sign of equality is $=$, *and is read "equals."*
It should, of course, never connect *un*equal amounts.

For instance, in the question *Multiply* 8 *by* 4 *and add* 3 *to the result,* many a beginner will write $8 \times 4 = 32 + 3 = 35$; which is absurd. He *means*, and should be careful to *write*, $8 \times 4 = 32$; $32 + 3 = 35$.

The sign $\therefore$ *is used for the word "therefore."*

A **line*** *placed between two numbers indicates division.*

Thus, $\frac{14}{2}$, and $14/2$, are each equivalent to $14 \div 2$.

The sign which stands to the **left** *of a number indicates the operation to be performed with that number.*

Thus, $7-3$ means that 3 is to be taken from 7. It will *not* do to write $3-7$.

Also, in $8-3+1$, it will *not* do to add the 1 to the 3 and subtract the result from 8, for this is equivalent to *subtracting* the 1. (See p. 8.)

When multiplication, or division, is indicated as well as addition, or subtraction, the multiplication, or division, must be performed **before** *the addition, or subtraction.*

Thus, $4 \times 3 + 2$ means that 4 is to be multiplied by 3 and 2 added to the result; *not* that 2 is to be added to 3 and 4 multiplied by the result.

Similarly, $8 + 12 \div 4$ means that 12 is to be divided by 4 and the result added to 8; *not* that 12 is to be added to 8 and the result divided by 4.

Brackets indicate that their contents are to be regarded as a whole.

Thus, $(14-3) \times 4$ means 11×4, *i.e.* 44.
Similarly, $18 \div (6-4)$ means $18 \div 2$, *i.e.* 9.

When multiplication and division are both indicated and no brackets are used, the operations must be performed in order **from left to right.**

Thus, $30 \div 10 \times 2 = 3 \times 2 = 6$. And $5 \times 8 \div 4 = 40 \div 4 = 10$.

When **no** *sign is placed between a number and a bracket, or when a* **dot** *is placed between two numbers, multiplication is implied.*

Thus, $3(5-1)$ means $3 \times (5-1)$, *i.e.* 3×4, or 12.
And $(7-3)(9-2)$ means $(7-3) \times (9-2)$, *i.e.* 4×7, or 28.
Also, $2 . 3 . 4$ means $2 \times 3 \times 4$.

If one pair of brackets stands within another pair the value of the contents of the inner pair should first be found.

Thus, $4 \div \{7-(8-3)\} = 4 \div \{7-5\} = 4 \div 2 = 2$.

A line placed above (or below) numbers is equivalent to brackets.

Thus, $7 \times \overline{8-5} = 7 \times (8-5) = 7 \times 3 = 21$.

---

* The dots in the sign $\div$ were probably intended to represent the dividend and divisor.

III. MISCELLANEOUS EXAMPLES.

The examples marked (A) which follow show how, in certain cases of Multiplication and Division, labour may be saved by taking advantage of the fact that any number can be multiplied or divided by 10, 100, 1000, &c., *at sight.*

A (i). *Multiply* 3675 *by* 25.

As $100 = 25 \times 4$, if we multiply 3675 by 100 the result will be *four* times as great as that required, which is rectified by dividing this result by 4; thus:—

$$\begin{array}{r|l} 4 & \underline{367500} \\ & \underline{\underline{91875}}\ \textit{Ans.} \end{array}$$

Hence, to multiply by 25, affix *two* ciphers and then divide by **4**.

A (ii). *Multiply* 7398 *by* 125.

As $1000 = 125 \times 8$, if we multiply 7398 by 1000, the result will be *eight* times as great as that required, which is rectified by dividing this result by 8; thus:—

$$\begin{array}{r|l} 8 & \underline{7398000} \\ & \underline{\underline{924750}}\ \textit{Ans.} \end{array}$$

Hence, to multiply by 125, affix *three* ciphers, and then divide by **8**.

Similarly to multiply by 625 affix *four* ciphers and divide the result by **16**.

A (iii). *Divide* 47986 *by* 25.

We know (by V., page 17) that 47986 divided by 25 gives the same quotient as $47986 \times \mathbf{4}$ divided by $25 \times \mathbf{4}$ (*i.e.* 100).

Hence, we multiply by 4 and cut off two figures, obtaining quotient 1919. The two figures cut off, *when divided by* 4, give the rem^r 11.

$$\begin{array}{r} 47986 \\ \underline{4} \\ 1919,44 \end{array}$$

Ans. 1919 + 11 *rem^r*.

Similarly we may deal with divisors 125 and 625.

A (iv). *Multiply* 5876 *by* 11.

As $11 = 10 + 1$, we multiply by **11** if we first *affix* one *cipher* (*i.e.* multiply by 10), *and then add the result to the given number;* thus:—

$$\begin{array}{r} 5876 \\ \underline{58760} \\ \underline{\underline{64636}} \end{array}\ \textit{Ans.}$$

A (v). *Multiply* 8367 *by* 99.

As $99 = 100 - 1$, we multiply by **99** if we *affix* two *ciphers, and then subtract the given number;* thus:

$$\begin{array}{r} 836700 \\ \underline{8367} \\ \underline{\underline{828333}} \end{array}\ \textit{Ans.}$$

A (vi). *Multiply* 3768 *by* 996.

As $996 = 1000 - 4$, we multiply by 996 if we affix *three* ciphers and then subtract *four* times the given number from the result; thus:

3768000
15072
3752928 *Ans.*

Similarly we may deal with such multipliers as 9, 98, 999, &c., and with many multipliers in which the figure 9 occurs; for instance:—

A (vii). *Multiply* 8347 *by* 594.

As $594 = 600 - 6$, we multiply 8347 by 600 and then subtract 6 times 8347 from the result; thus:

8347
600
5008200
50082
4958118 *Ans.*

A (viii). *Divide* 387654 *by* 99.

If we divide 387654 by 100 we obtain quotient 3876 *hundreds* and 54 *units* over. But in 3876 *hundreds* there are 3876 *ninety-nines* and 3876 *units* besides. Hence, we now divide 3876 by 100, obtaining 38 more *ninety-nines* and 38 more *units* over. We now add the three rem^rs^ 54, 76, and 38, obtaining a total rem^r^ 168, which divided by 100, yields 1 more *ninety-nine* and 1 more *unit*. Hence, adding the three quotients, we obtain the complete quotient 3915 and the complete rem^r^ 69.

3876,54
38,76
,38
1,68
1
3915,69

Ans. $3915 + 69$ *rem^r^.*

Similarly we may deal with divisors 999, 9999, &c.

The next two examples illustrate another case in which the work of multiplication may be shortened:—

B (i). *Multiply* 2341 *by* 567.

Observing that 7 is a factor of 56, we regard 567 as $7 + 560$; *i.e.* as $7 + 7 \times 80$.

Hence, if we first multiply 2341 by **7** (i), and then *this* result by **80** (ii), the sum of the two results is the req^d^ product.

2341
567
16387 ... (i)
1310960 ... (ii)
1327347 *Ans.*

B (ii). *Multiply* 325146 *by* 189273 *in three lines.*

Observing that 27 is 3×9, and that 189 is 27×7, we regard 189273 as $3 + 270 + 189000$; *i.e.* as $3 + 3 \times 90 + 270 \times 700$.

Hence, if we multiply 325146 by 3 (i), then this result by 90 (ii), and then the new result by 700 (iii), the sum of all three results is the req^d^ product.

325146
189273
975438 ... (i)
87789420 ... (ii)
61452594000 ... (iii)
61541358858 *Ans.*

The three following are examples of "*unequal*" Division:—

C (i). *Divide* 34722 *into two parts, one of which shall be* 8 *times as great as the other.*

Here we divide by **9**, obtaining 3858 for the smaller part.

$$9 \,\underline{|\, 34722}$$
$$3858$$

Hence, $34722 - 3858 = \underline{30864}$ is the greater.

C (ii). *Divide* 1424 *into* 7 *parts, one of which shall exceed each of the others by* 17.

Here we first *subtract the given excess*, 17, from 1424, and then divide the remainder into 7 *equal* parts, obtaining 201 as one of the six equal parts.

$$\begin{array}{r} 1424 \\ \underline{17} \\ 7\,\underline{|\,1407} \\ 201 \end{array}$$

Hence, $201 + 17 = \underline{218}$ is the other part.

C (iii). *Find three numbers whose sum is* 757, *such that the first exceeds the second by* 12, *and the second exceeds the third by* 17.

It is evident that the 1st exceeds the *3rd* by $12 + 17$, or **29**.

And the 2nd exceeds the 3rd by **17**.

Hence, if we take $29 + 17$, or **46**, from 757, the remainder, 711, is *three* times the 3rd number.

Thus, the required numbers are <u>237, 254, and 266.</u>

$$\begin{array}{r} 757 \\ \underline{46} \\ 3\,\underline{|\,711} \\ 237 \end{array}$$

D. *Find the average of the numbers* 8, 12, 6, 7, 5, 13, 14, 9, *and* 16.

The *average* (or mean) of two or more given numbers is that number which, when repeated as many times as there are numbers given, amounts to their sum.

Hence, to find the average, we *add together all the given numbers and divide their sum by the number of them.*

Here, $8 + 12 + 6 + 7 + 5 + 13 + 14 + 9 + 16 = 90$,
and there are nine numbers.
$\therefore$ the req^{d} average is $90 \div 9$, *i.e.* <u>10 *Ans.*</u>

E. *If* 23 *be added to a certain number, and the sum be divided by* 13, *the double of the quotient, decreased by* 22, *is* 1000: *find the number.*

In questions of this kind where the *result* of a series of operations is given, and it is required to find the original number operated upon, we *must perform upon the given result the* **converse** *of each operation mentioned, and in the* **reverse order.**

Here, the given result is 1000.

Hence, we first *increase* 1000 by 22, obtaining 1022;
then *divide* 1022 by 2, obtaining 511;
next *multiply* 511 by 13, obtaining 6643;
and finally *subtract* 23 from 6643, obtaining <u>6620, the req^{d} number.</u>

F. *If* 4793 *be multiplied by a certain number of two digits the units' figure in the product is* 2, *and the tens' figure is* 5; *find the multiplier.*

The units' figure of the req^d multiplier must be **4**, for this is the only figure by which 3 can be multiplied, so as to give a result ending in 2; hence the first line of multiplication ends thus ... 72.

We have now to find a figure which, added to 7, gives a sum which ends in 5, and this figure is **8**.

Hence the tens' figure in the multiplier must be **6**, for this is the only figure by which 3 can be multiplied so as to give a result ending in 8.

$$\begin{array}{r} 4793 \\ 64 \\ \hline \ldots 72 \\ \ldots 8 \\ \hline \ldots 52 \end{array}$$

∴ the req^d multiplier is 64.

G. *How many four-figure numbers can be represented by the four digits* 1, 2, 3, 0?

There are *six* numbers in which 1 stands first, namely,

1230, 1203, 1320, 1302, 1023, 1032;

and there are *six* numbers in which 2, and *six* in which 3 stands first, namely,

2301, 2310, 2013, 2031, 2130, 2103,

and 3012, 3021, 3120, 3102, 3201, 3210.

But there are *no* numbers in which 0 stands first.
Hence, there are 18 different numbers.

H. *The sum of two numbers is* 473; *their difference is* 87; *find them.*

If we take two numbers (say 8 and 5), and add their difference (3) to the smaller number (5) we obtain the greater (8).

Hence, if we *add the* **difference** *to the* **sum** *of two numbers we obtain the* **double** *of the greater number.*

Here, $473 + 87 = 560$; and $560 \div 2 = 280$, the greater of the required numbers. Hence, $280 - 87 = 193$, the less.

K. *Find the sum of* 14 *consecutive numbers, the least of which is* 173.

The numbers are 173, 174, 175, &c.
i.e. 173, 173 + 1, 173 + 2, &c.

Hence, the req^d sum is

$$173 \times 14 \quad + 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11 + 12 + 13$$
$$= 2422 + 91$$
$$= 2513 \ \textit{Ans.}$$

IV. REDUCTION.

Concrete Quantities.

We estimate the size of a group of concrete units which are all alike and *separate* (such, for instance, as *sheep* or *shillings*) by counting them.

This process we extend to magnitudes (*i.e.* things possessing *size*) which are *not* composed of separate units, but which are *all in one piece*, or continuous. We choose another thing of the same kind (as large or as small as we find convenient) which we regard as the *unit*, and we then compare the thing we wish to estimate with this unit in order to discover *how many such units it would, if cut up, yield.* This is called **measuring** it; and the number of such units which the magnitude measured is found to contain is called its **quantity.**

It almost always happens that, whatever the size of the unit of measurement we choose, there remains, after we have counted off as many such units as possible, a portion of the thing we measure too small to yield another complete unit. This portion may, in its turn, be measured in terms of a *smaller* unit; and so on, to any degree of accuracy which (if the means at our disposal permit) the case requires.

For instance, suppose we wish to discover which of two maps is the longer, and suppose that they are fixed on opposite sides of a room so that we cannot bring them together in order to compare them. We *measure* them in terms of any convenient unit—say the length of this book. Suppose, now, that after marking off the length of this book six times along the side of each map there remains a portion of the length of each too small to allow of this being done a seventh time. Then the question is, as yet, unsolved, and we proceed to measure these two remaining portions in terms of some smaller unit—say the length of a postage-stamp. If we find, on trial, that the first of these portions contain the length of a stamp 5 times, but not 6 times, and that the second portion contain it 4 times, but not 5 times, we need proceed no further, as we now know that the first of the two maps is the longer, the measure of its length, expressed in terms of the units selected,

being 6 *book-lengths* + 5 *stamp-lengths*,
and that of the second 6 *book-lengths* + 4 *stamp-lengths*.

It should be noticed that in neither case does the measure express the *exact* length of the map, for we neglected, as inconsiderable, the very small portions remaining over after the second pair of measurements.

The unit of measurement must always be of like *kind* with the thing measured.

For instance, any solid substance can be measured by *weight* (*i.e.* by discovering how many times as heavy it is as a fixed amount of some particular substance); but we cannot measure *time* by the *pound*, nor *distance* by the *gallon*.

A quantity expressed in terms of *one unit only* is called a **simple quantity.**

For instance, the length 23 *feet.*

A quantity expressed in terms of *more than one unit* is called a **compound quantity.**

For instance, the length 5 *yards*, 2 *feet*, 7 *inches.*

Reduction is the operation of *changing the unit*, or *units*, in which the measure of a quantity is expressed.

The *name* of the unit in terms of which a quantity is measured is called its **denomination.**

A quantity is said to be reduced to a *higher* or *lower denomination* according as the change is made from a smaller to a larger unit, or *vice versa.*

For instance, when we change 2 *feet* into 24 *inches*, we reduce this length to a *lower* denomination.

The connections which exist between various units are called **Tables.**

The following are the chief Tables in use in the British Isles:*

MONEY.

4 farthings (*f.*) = **1 penny** (*d.*).

12 pence = **1 shilling** (*s.*).

20 shillings, or **240 pence** = **1 pound** (£).

Beginners should be practised in reducing small numbers of *pence* to *shillings*, and of *shillings* to *pounds*, mentally until great speed and accuracy is acquired. Hence it is well to *commit to memory* the following Pence Table:—

20 *pence* = 1*s.* 8*d.*	50 *pence* = 4*s.* 2*d.*	80 *pence* = 6*s.* 8*d.*
30 ,, = 2*s.* 6*d.*	60 ,, = 5*s.*	90 ,, = 7*s.* 6*d.*
40 ,, = 3*s.* 4*d.*	70 ,, = 5*s.* 10*d.*	100 ,, = 8*s.* 4*d.*

The following equivalents are also noteworthy:—

1 shilling = 3 *fourpences*, 4 *threepences*, 6 *twopences*, 8 *three-halfpences*, 24 *halfpence*, or 48 *farthings.*

1 half-crown = 5 *sixpences*, 10 *threepences*, or 30 *pence.*

1 pound = 4 *crowns*, 8 half-crowns, 10 *florins*, 40 *sixpences*, 60 *fourpences*, 80 *threepences*, 120 *twopences*, 480 *halfpence*, or 960 *farthings.*

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* For Tables of Foreign Money and the Metric System, see Part II.

The English coins now *current* (*i.e.* in common use) are:—

The *sovereign* (20*s.*) and *half-sovereign*, in gold.

The *crown* (5*s.*), *four-shilling piece*, *half-crown*, *florin* (2*s.*), *shilling*, *sixpence*, and *threepence* } in silver.

The *penny*, *halfpenny*, and *farthing*,* in copper.

The smallest piece of "paper money" used in England is the *five-pound note*. In Scotland and Ireland one-pound notes are current.

A sum of money is still often expressed by the name of coins now no longer used, namely, the *guinea* (21*s.*) and the *half-guinea*.

The letters £, *s*, *d* are the initials of the Latin names *libra*, *solidus*, *denarius* respectively. The use of the word *pound* is due to the primitive custom of *weighing*, not coining, money. A "pound" originally meant 1 lb. weight of silver.

The notation of Fractions is used in writing farthings: thus, 1 farthing and 3 farthings are written $\frac{1}{4}d.$ and $\frac{3}{4}d.$; 2 farthings, or a halfpenny, thus, $\frac{1}{2}d.$ The word *farthing* (*i.e.* "fourthing") means a fourth part.

We estimate the value of all articles of commerce in comparison with gold. Consequently while their market value rises or falls according as the supply is less or more plentiful, that of gold remains constant.

The gold of which a sovereign is composed is not absolutely pure, but consists of 22 parts by weight of pure gold mixed with 2 parts of *alloy*. This is the *standard of value*. The alloy is a mixture of silver and copper used to make the coins harder.†

One ounce of *standard gold* is always worth £3, 17*s.* $10\frac{1}{2}d.$, and gold coins are worth their weight.‡

Silver coins contain 37 parts of pure silver mixed with 3 parts of alloy. Copper coins contain 95 parts of copper, 4 parts of tin, and 1 of zinc. Silver and copper coins are *tokens*, *i.e.* they represent certain values, but do not contain that worth of metal; for instance, the silver obtained from 20 shillings melted down would not be worth £1. On this account silver is not "legal tender" for payment of a debt of more than £2, nor copper for more than 1*s.*

Precious metal, (*i.e.* gold or silver,) when uncoined is called *bullion;* when coined *specie.*

The government establishment for coining is called the *Mint*.

The word *sterling*‖ applied to a sum of money signifies that it is of *standard* value.

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* Only a limited use is made of farthings, namely, for small ready-money payments.

† Australian sovereigns are lighter in colour than English owing to the alloy used containing less copper.

‡ Jeweller's gold contains much more alloy; thus 18 *carat* gold (the finest used by jewellers) contains but 18 parts out of 24 of pure gold.

‖ Sterling is a corruption of *Easterlings* (*i.e.* men from the East), a name given to the German merchants in England in the time of Edward III., and to money coined by them.

Time.

60 **seconds** (*sec.*) = 1 **minute** (*min.*).
60 **minutes** = 1 **hour** (*hr.*).
24 **hours** = 1 **day**.
7 **days** = 1 **week** (*wk.*).
365 **days** = 1 (common) **year** (*yr.*).
366 **days** = 1 **leap year**.
100 **years** = 1 **century**.

Hence, 52 **weeks** + 1 **day** = 1 (common) **year**.

The year is divided into **12 Calendar* Months**, of which **February** contains { in *common* years, **28 days**, in *leap* years, **29 days**. }

Of the others "**Thirty** days hath **September**,†
April, **June**, and **November**,"

and the remaining months each contain **31 days**.

For legal purposes the year is divided into 4 quarters. The English Quarter Days, on which rents, &c., for the preceding period, are due, being:—

March 25th (Lady Day);
June 24th (Midsummer);
September 29th (Michaelmas);
December 25th (Christmas).

Leap year occurs (as a rule) once every *four* years, those years being counted as leap years the **dates** of which can be **divided by 4 without remainder.**

For instance, 1892 was a leap year, for 1892 ÷ 4 yields *no remainder.*

If, however, the year completes a century it is *not* counted as a leap year, *unless the* **number of centuries** *can be divided by* 4 *without remainder.*

For instance, 1900 will *not* be a leap year, for 19 ÷ 4 yields a remainder; but the year 2000 *will be* a leap year, for 20 ÷ 4 yields *no* remainder.

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* So called to distinguish them from **Lunar Months**. A *Lunar* month (Latin *luna*, the moon), the time the moon takes to make its journey round the earth, consists of about 4 weeks.

† The Roman year began in March, hence the names September, October, November, December, derived from the Latin numerals, *septem*, *octo*, *novem*, *decem*, mean 7th, 8th, 9th, 10th month, respectively.

This complex arrangement of the calendar is owing to the fact that the earth does not complete its annual journey round the sun in an *exact* number of days.

The average length of the solar* year is 365 days 5 hours 48 minutes 49 seconds. Thus the common (or civil) year of 365 days is *very nearly* $\frac{1}{4}$ *of a day too short*, and so, if every year were taken as 365 days, the error would in time mount up to such an extent that the months would cease to correspond with their seasons, and *January*, for instance, would gradually travel round into the *summer*.

In order to correct this, Julius Cæsar, in the year 46 B.C., enacted that *every fourth* year should contain 366 days; an arrangement called the Julian Calendar, or **Old Style.**

In the Julian Calendar, then, the average length of the *civil* year is 365¼ days, or 365 days 6 hours, while that of the *solar* year is

365 days 5 hours 48 min. 49 secs.

The civil year was thus made a little *too long* (*i.e.* the calendar, which before had *gained*, was made *to lose* time slowly).

And this loss of 11 min. 11 secs. a year amounts in 400 years to a very little more than 3 days.

As a further correction Pope Gregory, in 1582, ordered that *in every 400 years three of the leap years* in the Julian Calendar should be replaced by *common* years, namely, those which contained an exact number of centuries, unless that number of centuries could be divided by 4 without remainder; an arrangement called the Gregorian Calendar, or **New Style.**

Thus in the New Style there are 97 leap years in every 400 years.

The New Style was immediately adopted in all Roman Catholic countries, but not till much later in others. In England the change was not made till September 2nd, 1752, when (as the Julian Calendar had by that date become 11 days behind time) *eleven days were omitted* from that month, and the day following Sept. 2nd was reckoned as Sept. 14th.

Traces of the Old Style may still be seen in the names Old Lady Day, Old Michaelmas Day, &c.; and in the date April 5th (*i.e.* Old Lady Day) on Government tax-papers, &c., as the last day of the Financial year.

In Russia the old style is still in use. Hence, as the Old Style Calendar is now 12 days behind the New, Christmas Day is kept nearly a fortnight later in Russia than in England.

The Gregorian Calendar is not absolutely *perfect*, but the error is very small, amounting only to 1 day in about 5000 years.

The *standard unit of measurement* of time is the **day**, the average interval between two successive transits of the sun across the meridian of any place; *i.e.* between two *noons*.

A day is considered as beginning at midnight.

* Latin *sol*, the sun.

Avoirdupois Weight.

16 drams (*dr.*) = 1 ounce (*oz.*).
7000 grains, or 16 ounces = 1 pound (*lb.*).
14 pounds = 1 stone (*st.*).
28 pounds, or 2 stones = 1 quarter (*qr.*).
112 pounds, 8 stones, or 4 quarters = 1 hundredweight (*cwt.*).
20 hundredweights = 1 ton.

Avoirdupois* weight is used for *all common substances* subject to waste, such as coal, meat, butter, tea, &c.

In addition may be mentioned—

a *firkin* of butter = 56 *lbs.*
a *pack* of wool = 240 *lbs.*
a *quartern* loaf = 4 *lbs.*
a *sack* of coal = 2 *cwt.*

The abbreviation "*cwt.*" for hundredweight consists of the Roman numeral C (100), with the first and last letters of the word "weight."

Troy Weight.

24 grains (*gr.*) = 1 pennyweight (*dwt.*).
480 grains, or 20 dwts = 1 ounce Troy (*oz. Tr.*).
12 ounces Troy = 1 pound Troy (*lb. Tr.*).

Troy† weight is only used for gold, silver, and precious stones.‡

The pound Troy is no longer a "legal" measure of weight.

The ounce and pound Avoirdupois differ from the ounce and pound Troy; *the only connection is through the grain.*

Thus 1 lb. Avoirdupois contains 7000 grains,
but 1 lb. Troy $= 24 \times 20 \times 12$, or 5760 grains.

Apothecaries' Weight
(used for drugs sold by retail).

20 *grains* = 1 *scruple* (℈); 3 *scruples* = 1 *dram* (ʒ); 8 drams = 1 oz. (℥).

The *standard unit of weight* is the **Imperial pound** Avoirdupois, a piece of platinum very carefully preserved in the wall of the Houses of Parliament.

* French *avoir du pois,* to have some weight. † From the town Troyes in France.
‡ The *carat* used in weighing diamonds = $3\frac{1}{6}$ grs. ‖ Troy.

Length.

12 **inches** (*in.*) = 1 **foot** (*ft.*).
36 **inches**, or 3 **feet** = 1 **yard** (*yd.*).
1760 **yards** = 1 **mile** (*mi.*).*

$5\frac{1}{2}$ **yards**, or 11 **half-yards** = 1 **pole** (*po.*), rod, or perch (*per.*).
220 **yards**, or 40 **poles** = 1 **furlong** (*fur.*).
8 **furlongs** = 1 **mile**.†

The chain used in land-surveying is 22 yards long and contains 100 links.

Hence, **100 links**, or **22 yards** = **1 chain** (*ch.*),
25 *links* = 1 *pole*,
10 *chains* = 1 *furlong*,
80 *chains* = 1 *mile*.

In addition to the above may be mentioned—

the *fathom* = 6 *feet*, used for soundings at sea.
the *nautical mile*, or *knot* = 6080 *feet* „ the rate of ships.
the *cable's length* = 120 *fathoms*.
the *hand* = 4 *inches* used for measuring horses.
the Irish *pole* = 7 *yards*.

Also, 5 *feet* = 1 *pace*, | 9 *inches* = 1 *span*,
3 *miles* = 1 *league*. | 18 *inches* = 1 *cubit*.

Cloth Measure.

$2\frac{1}{4}$ *inches* = 1 *nail*,
9 *inches*, or 4 *nails* = 1 *quarter*,
4 *quarters* = 1 *yard*,
5 *quarters* = 1 *ell*.

Our units of measurement originated in those provided by nature, as we see in the names "hand," "foot," "span," &c.

Also, "yard" means the length of the arm; "furlong," furrow-length; "cubit" is from the Latin *cubitus*, the fore-arm; and "mile" from the Latin *mille*, a thousand (paces).

The *standard unit of length* is the **Imperial yard**, being the distance between two gold pegs in a bronze bar which is kept in the Houses of Parliament with other "standards."

---

* The second part of this Table and exercises upon it are often deferred until Fractions have been learned.

† Often called a "statute" mile, *i.e.* of the length fixed by *act* of Parliament.

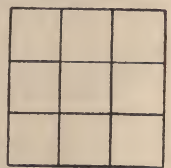
Square Measure.

144 square inches (*sq. in.*) = 1 square foot (*sq. ft.*).
9 square feet = 1 square yard (*sq. yd.*).
$30\frac{1}{4}$ square yards, or 121 square quarter-yards = 1 square pole (*sq. po.*, or P.).
40 square poles = 1 rood (*ro.*, or R.).
4840 square yards, or 4 roods = 1 acre (*ac.*, or A.).
640 acres = 1 square mile (*sq. mi.*).

Square measure is used in measuring surfaces or areas, *e.g.* fields, floors, &c.

The first part of the table may be found by *squaring* Long Measure, as may be seen from the accompanying diagram:

If we take nine squares, having their sides each a foot long, we can arrange them so as to form one larger square, having each of its sides 3 feet, or 1 yard long, thus:—



Hence, 3×3, or 9, square feet make 1 square yard.

Similarly, 12×12, or 144, square inches make 1 square foot.

In this way, then, we can supplement the table of Square Measure from the table of Long Measure.

For instance, as 22 yards = 1 chain, $\therefore 22 \times 22$, or 484, sq. yds. = 1 sq. chain.
And, as 100 links = 1 chain, $\therefore 100 \times 100$, or 10000, sq. links = 1 sq. chain.
Also, as 1760 yards = 1 mile, $\therefore 1760 \times 1760$ sq. yds. = 1 sq. mile.

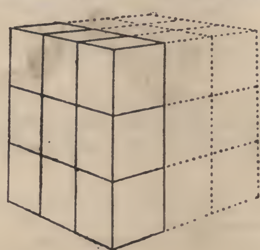
Cubic Measure.

1728 cubic inches (*cub. in.*) = 1 cubic foot (*cub. ft.*).
27 cubic feet = 1 cubic yard (*cub. yd.*).

Cubic Measure is used for measuring space; also the volume of certain solids; *e.g.* stone, timber, &c.

This table is formed by *cubing* the corresponding part of Long Measure.

For instance, if *nine* cubes (blocks measuring a foot each way) were arranged, as in the diagram, in a stack 3 feet long, 3 feet high, and 1 foot wide; and if two more such stacks were placed alongside, as indicated by the dotted lines; the whole pile so formed would contain 3×9, or 27, blocks, and would measure 3 feet, or 1 yard, each way.



Hence, $3 \times 3 \times 3$, or 27, cubic feet make 1 cubic yard.

Measures of Capacity.

2 **pints** (*pt.*) = 1 **quart*** (*qt.*). 8 **pints**, or 4 **quarts** = 1 **gallon** (*gal.*).	**Liquids** (*e.g.* milk).
2 **gallons** = 1 **peck** (*pk.*). 8 **gallons**, or 4 **pecks** = 1 **bushel** (*bus.*). 8 **bushels** = 1 **quarter** (*qr.*).	**Dry goods** (*e.g.* corn).

In addition may be mentioned—

the *firkin* = 9 *gallons*, *kilderkin* = 18 *gallons*, *barrel* = 36 *gallons*, *hogshead* = 54 *gallons*, *butt* = 2 *hogsheads*, *tun* = 2 *butts*,	Beer.
the *hogshead* = 63 *gallons*. *pipe* = 2 *hogsheads*,	Wine.

Also, 4 *gills* = 1 *pint*.
2 *bushels* = 1 *strike*.
4 *bushels* = 1 *sack*.
5 *quarters* = 1 *load*.

A bushel of English wheat = 63 lbs. weight. The hogshead, pipe and butt vary for some particular kinds of wine.

The *standard* unit of capacity is the **Imperial gallon**, the volume of 10 lbs. of pure water.

Hence the rhyme, "A pint of pure water, weighs a pound and a quarter."

Also, as a gallon occupies about $277\frac{1}{4}$ cubic inches of space,
1 cubic foot of water weighs 1000 ounces, nearly.

Miscellaneous.

Number.

12 *units* = 1 *dozen*.
12 *dozens* = 1 *gross*.
20 *units* = 1 *score*.

Paper.

24 *sheets* = 1 *quire*.
480 *sheets*, or 20 *quires* = 1 *ream*.

A sheet of "Foolscap" is $16\frac{1}{2}$ inches long and $13\frac{1}{2}$ inches wide, which is a size often used in the printing of books. Thus in a volume of the size called Quarto (4*to*) a sheet is folded into *four* leaves; in Octavo (8*vo*), into 8 leaves; in Duodecimo (12*mo*), into 12 leaves.

Apothecaries Fluid Measure.

60 *minims* (M) = 1 *fluid drachm* (f. ʒ).
8 *drachms* = 1 *fluid ounce* (f. ℥).
20 *ounces* = 1 *pint* (O).

Angular Measure.

60 *seconds* (60″) = 1 *minute* (1′).
60 *minutes* = 1 *degree* (1°).
90 *degrees* = 1 *right angle*.

The distance on the surface of the globe corresponding to one *degree of latitude* = 60 *nautical* miles, or nearly 70 *statute miles*.

A *ton of shipping* is not a weight but a measure of volume, namely, 40 *cubic feet* of space.

---

* Often called *Imperial* pint and quart to distinguish them from the *Reputed* pint and quart measures commonly used for bottled wines. 6 *reputed quarts* = 1 *gallon*.

The following examples illustrate the process of Reduction:—

EXAMPLE i.—*Reduce £17, 5s. 3d. to pence.*

Explanation.

As in £1 there are 20*s.*,
∴ in £17 there are 17 *times* 20*s.* = 340*s.*
∴ in £17, 5*s.* there are 340 + 5 = 345*s.*

As in 1*s.* there are 12*d.*,
∴ in 345*s.* there are 345 × 12 = 4140*d.*
∴ in 345*s.* 3*d.* there are 4140 + 3 = 4143*d.*

Written Work.

$$\begin{array}{r}
\text{£} \quad s. \quad d. \\
17 \;.\; 5 \;.\; 3 \\
20 \\
\hline
345 \\
12 \\
\hline
4143
\end{array}$$

Ans. 4143*d.*

EXAMPLE ii.—*Reduce £312, 12s. 6d. to half-crowns.*

As 8 half-crowns = £1, we multiply 312 by 8, adding in **5**, *the number of half-crowns in* 12*s.* 6*d.*

$$\begin{array}{r}
\text{£} \quad s. \quad d. \\
312 \;.\; \underbrace{12 \;.\; 6}_{5\ hf.\text{-}cr.} \\
8 \\
\hline
2501
\end{array}$$

Ans. 2501 *hf-cr.*

EXAMPLE iii.—*Reduce* 47135 *farthings to £ s. d.*

As 4 farthings = 1 penny, the number of *farthings* in any sum of money must be *four* times the number of *pence* in that same sum.

We therefore divide the number of farthings by 4, obtaining 11783 *sets of four farthings, i.e. pence,* with 3 *farthings* over. And so on.

$$\begin{array}{r|l}
 & f. \\
4 & 47135 \\
12 & 11783 \ldots\ldots 3f. \\
2{,}0 & 98{,}1 \ldots\ldots 11d. \\
 & 49 \ \ldots\ldots\ 1s.
\end{array}$$

Ans. £49, 1*s.* $11\frac{3}{4}d.$

EXAMPLE iv.—*Reduce* 392 *half-guineas to half-crowns.*

When we cannot proceed directly from one of the given denominations to the other, we must first find some *common* denomination to which both can easily be reduced, taking care that this is not *lower* than it need be.

In this case we choose *sixpences.* Hence, multiplying 392 by 21 (the number of sixpences in half-a-guinea) we obtain 8232 as the number of sixpences in 392 half-guineas. This we divide by 5 (the number of sixpences in half-a-crown), thus obtaining 1646 as the required number of half-crowns with 2 *sixpences, i.e.* 1 *shilling,* over.

$$\begin{array}{r|l}
 & hf\text{-}gs. \\
 & 392 \\
 & 21 \\
\hline
 & 392 \\
 & 784 \\
\hline
5 & 8232 \\
 & 1646 \ldots\ldots 2\ sixpences.
\end{array}$$

Ans. 1646 *hf-cr.* + 1*s.*

EXAMPLE v.—*Reduce* 475302 *seconds to days, &c.*

```
        6,0 | 47530,2
         6,0 | 792,1  .... 42 secs.
  24 { 4 | 132  ......  1 min.
     {  6 | 33...0 }
             5...3 }  12 hrs.
```

Ans. 5 *days* 12 *hrs.* 1 *min.* 42 *secs.*

EXAMPLE vi.—*Reduce* 7 *tons* 13 *cwts.* 23 *lbs.* 9 *ozs. to ounces.*

1st Method.

```
tons.  cwts.  qrs.  lbs.  ozs.
  7  .  13  .  0  .  23  .  9
 20
 153
   4
 612
  28
4919
1224
17159
   16
102963
17159
274553
```

Ans. 274553 *ozs.*

2nd Method.

```
tons.  cwts.  lbs.  ozs.
  7  .  13  .  23  .  9
 20
 153
 112
 329
153
153
17159
   16
274553
```

Here, as no *quarters* are mentioned, we step directly from *cwts.* to *lbs.*, multiplying by 112, instead of by 4 and 28 in succession. Also, we multiply by 16 in *one* line.*

EXAMPLE vii.—*Reduce* 3754219 *ozs. to tons, cwts., &c.*

```
 16 { 4 | 3754219
    {  4 | 938554...... 3 }
 28 { 4 | 234638...... 2 }  11 ozs.
    {  7 | 58659...... 2 }
        4 | 8379...... 6 }  26 lbs.
      2,0 | 209,4 ..........  3 qrs.
            104............ 14 cwt.
```

Ans. 104 *tons* 14 *cwt.* 3 *qrs.* 26 *lbs.* 11 *ozs.*

N.B.—Here short *divisions should always be used.*

* This may be done either by committing to memory the Multiplication Table as far as 16 times 9; or by multiplying by 6 and adding the result, figure by figure, to 171590 (*i.e.* 10 times 17159). See page 12.

EXAMPLE viii.—*Reduce 5 mi. 3 fur. 24 po. 4 yds. to yds.*

1st Method.

mi.	fur.	po.	yds.
5 .	3 .	24 .	4

8
43
40
2 | 1744
5
8724
872
9596 *Ans.* 9596 *yds.*

As $5\frac{1}{2}$ yds. make 1 pole, and, as $1744 \times 5\frac{1}{2} = 5$ times $1744 +$ half of 1744, we may multiply 1744 by 5, then divide 1744 by 2, and add these results.

2nd Method.

mi.	fur.	po.	yds.
5 .	3 .	24 .	4

8
43
40
1744
17448
2 | 19192
9596 *Ans.* 9596 *yds.*

As 11 *half-yards* make 1 pole, we multiply 1744 by 11, adding in 8 *half-yards*, and then divide the result by 2. In this case the multiplier 11 is not written. See A. (iv) p. 19.

EXAMPLE ix.—*Reduce 3 mi. 5 fur. 17 po. 3 yds. 2 ft. 7 in. to inches.*

1st Method.

mi.	fur.	po.	yds.	ft.	in.
3 .	5 .	17 .	3 .	2 .	7
				1 .	6

8
29
40
2 | 1177
5
5888
$588\frac{1}{2}$
6476
3
19431
12
233185

Ans. 233185 *inches.*

Here, in dividing 1177 by 2 we obtain quotient 588 and 1 *half-yard* over; this we replace by 1 ft. 6 in., which we set beside the 2 ft. 7 in. and add in with them when we come to multiply by 3 and 12.

2nd Method.

mi.	fur.	po.	yds.	ft.	in.
3 .	5 .	17 .	3 .	2 .	7

$\underbrace{2 . 7}_{31\ ins.}$

8
29
40
1177
11776
12953
18
103655
12953
233185

Ans. 233185 *inches.*

Here, multiplying 1177, the number of poles, by **11**, and adding in **6** half-yards, we obtain 12953 as the number of *half-yards.* We then multiply by **18**, *the number of inches in half-a-yard,* and add in 2 ft. 7 in. as **31** inches.

EXAMPLE x.—*Reduce* 12 *mi.* 2 *fur.* 135 *yds. to yards.*

Here, as no *poles* are given, we can avoid multiplying by the troublesome $5\frac{1}{2}$.

1st Method.

```
mi.   fur.   yds.
12  .  2  .  135
 8
98
220
1960
196
 135
21695      Ans. 21695 yds.
```

2nd Method.

```
mi.    fur.   yds.
12     2      135
1760          440
21695
```

Here we convert 2 *furs.* to *yards*, and then multiply 1760 by 12 (the multiplier standing *above* the multiplicand), adding in 135 + 440 yards.

EXAMPLE xi.—*Reduce* 1345732 *inches to miles, furlongs, poles, &c.*

As 11 *half-yards* make 1 pole, we first reduce the given inches to *half-yards, i.e.* we divide by 18 (3×6), obtaining 74762 as the number of *half-yards* and 16 *inches* over. We next divide 74762 by 11, obtaining 6796 as the number of *poles* and 6 *half-yards* over.

```
             in.
18 { 3 | 1345732
   { 6 | 448577......1 }
    11 | 74762......5  } 16 in. = 1 ft. 4 in
     40 | 6796......6 half-yds. = 3 yds.
      8 | 169  .....36 poles.
          21   ......1 fur.
```

Ans. 21 *mi.* 1 *fur.* 36 *po.* 3 *yd.* 1 *ft.* 4 *in.*

N.B.—In writing down the answer the beginner must be careful *not to forget to* change *half-yards* to *yards.*

EXAMPLE xii.—*Reduce* 457115 *feet to miles, furlongs, poles, &c.*

Dividing by 3 we obtain 152371 as the number of *yards*. We now multiply by 2, obtaining 304742 as the number of *half-yards*. We next divide by 11 (the number of *half-yards* in 1 *pole*), when the 9 *half-yards* over yield $4\frac{1}{2}$ *yards.*

```
         ft.
  3 | 457115
      152371 ....................2 ft.
           2
 11 | 304742
   40 | 27703...9 half-yds. = 4½ yds.
    8 | 692  ...................23 po.
        86   ................... 4 fur.
```

Thus 457115 *feet* = 86 *mi.* 4 *fur.* 23 *po.* $4\frac{1}{2}$ *yds.* 2 *ft.;*

or, taking *half a yard* from the 2 *ft.* and adding it to the $4\frac{1}{2}$ *yards*, we have the neater form of *Ans.* 86 *mi.* 4 *fur.* 23 *po.* 5 *yds.* 0 *ft.* 6 *in.*

EXAMPLE xiii.—*Reduce* 5 *ac.* 3 *ro.* 31 *po.* 14 *sq. yds.* 7 *sq. ft. to square feet.*

To find the number of sq. yds. equivalent to 951 poles we multiply 951 by 30, divide 951 by 4, and add the results.

The $\frac{3}{4}$ sq. yd. remaining over from this division yields $\frac{27}{4}$ sq. ft. or $6\frac{3}{4}$ sq. ft., which we place against the 7 sq. ft. and add in with it when we multiply by 9.

$$\begin{array}{rrrrr}
\text{ac.} & \text{ro.} & \text{po.} & \text{sq. yd.} & \text{sq. ft.} \\
5\ . & 3\ . & 31\ . & 14\ . & 7 \\
4 & & & & 6\frac{3}{4} \\
\hline
23 & & & & \\
40 & & & & \\
\hline
4\,|\,951 & & & & \\
30 & & & & \\
\hline
28544 & & & & \\
237\frac{3}{4} & & & & \\
\hline
28781 & & & & \\
9 & & & & \\
\hline
259042\frac{3}{4} & & & &
\end{array}$$

Ans. $259042\frac{3}{4}$ *sq. ft.*

[This process corresponds to the 1st method of Ex. ix. We should gain nothing here by adopting the 2nd method, as that would involve multiplying 951 by 121 and then dividing the result by 4.]

EXAMPLE xiv.—*Reduce* 17 *ac* 1 *ro. to square yards.*

Here, as no *poles* are given, we avoid multiplying by the troublesome $30\frac{1}{4}$, and step directly from acres to yards, remembering that 4840 sq. yds. make 1 ac., and that, consequently, 1210 sq. yds. make 1 rood.

(Here, also, the multiplier, 17, stands *above* the multiplicand.)

$$\begin{array}{rr}
\text{ac.} & \text{ro.} \\
17\ . & 1 \\
4840 & \\
\hline
33880 & \\
4840 & \\
1210 & \\
\hline
83490 &
\end{array}$$

Ans. 83490 *sq. yds.*

EXAMPLE xv.—*Reduce* 14170 *sq. yds. to acres, roods, poles, &c.*

We first change 14170 sq. yds. to *sq. quarter-yds.*; then, as 121 *sq. quarter-yds.* make 1 *sq. pole*, we divide by 11×11, obtaining 468 poles and 52 *sq. quarter-yds.*, *i.e.* 13 *sq. yds.* over, &c.

$$\begin{array}{rl}
& \text{sq. yd.} \\
& 14170 \\
& 4 \\
\hline
121 \left\{ 11 \right. & |\,56680 \\
11 & |\,5152 \ldots 8 \;\; \} \; 52 \text{ sq. qr.-yds.} = 13 \text{ sq. yds.} \\
4,0 & |\,46,8 \ldots 4 \\
4 & |\,11 \ldots\ldots\ldots\ldots 28 \text{ po.} \\
& 2 \ldots\ldots\ldots\ldots 3 \text{ ro.}
\end{array}$$

Ans. 2 *ac.* 3 *ro.* 28 *po.* 13 *sq. yds.*

Example xvi.—*Reduce* 1470358 *sq. in. to poles, &c.*

$$
\begin{array}{l}
\qquad\qquad\text{sq. in.}\\
144\left\{\begin{array}{r|l} 12 & 1470358 \\ 12 & 122529\ldots 10 \end{array}\right.\\
\qquad\quad 9 \mid 10210\ldots\ 9 \quad \Big\}\ 118\ \textit{sq. in.}\\
\qquad\qquad 1134\ldots\ldots\ldots\ 4\ \textit{sq. ft.}\\
\qquad\qquad\quad 4\\
121\left\{\begin{array}{r|l} 11 & 4536 \\ 11 & 412\ldots 4 \end{array}\right.\\
\qquad\qquad 37\ldots 5 \quad \Big\}\ 59\ \textit{sq. quarter-yds.} = 14\tfrac{3}{4}\ \textit{sq. yds.}
\end{array}
$$

Hence 1470358 *sq. in* = 37 *po.* $14\frac{3}{4}$ *sq. yds.* 4 *sq. ft.* 118 *sq. in.*;

or, as $\frac{1}{4}$ of a sq. yd. = $\frac{1}{4}$ of 9 sq. ft. = $2\frac{1}{4}$ sq. ft. = 2 sq. ft. 36 sq. in., if we take 2 sq. ft. 36 sq. in. from the 4 sq. ft. 118 sq. in., and add it to the $14\frac{3}{4}$ sq. yds., we obtain in improved form the *Ans.* 37 *po.* 15 *sq. yds.* 2 *sq. ft.* 82 *sq. in.*

Example xvii.—*Reduce* 3754813 *cub. in. to cubic yards.*

We first reduce the cub. in. to cub. ft., dividing by 12 × 12 × 12 (*i.e.* 1728), and obtaining the remainder, 1597 cub. in. (See p. 16.) We then divide the 2172 cub. ft. by 3 × 9 (*i.e.* 27), &c.

$$
\begin{array}{l}
\qquad\qquad\text{cub. in.}\\
1728\left\{\begin{array}{r|l} 12 & 3754813 \\ 12 & 312901\ldots\ 1 \qquad\quad = \qquad 1 \\ 12 & 26075\ldots\ 1\times 12 \ = \quad 12 \end{array}\right\}\ 1597\ \textit{cub. in.}\\
27\left\{\begin{array}{r|l} 3 & 2172\ldots 11\times 144 = 1584 \\ 9 & 724\ldots\ldots\ldots\ldots\ 0 \end{array}\right.\\
\qquad\quad 80\ldots\ldots\ldots\ldots\ldots\ 4 \quad\Big\}\ 12\ \textit{cub. ft.}
\end{array}
$$

Ans. 80 *cub. yds.* 12 *cub. ft.* 1597 *cub. in.*

Example xviii.—*Express* 13 *lbs.* 4 *ozs. Avoirdupois in grains.*

As 7000 *grains* make 1 *lb.* Avoirdupois, and as 4 *ozs.* is $\frac{1}{4}$ of a *lb.* Av., there are 1750 *grains* in 4 *ozs.* Hence we multiply the 13 by 7000 and add in 1750, obtaining 92750 as the number of *grains.*

$$
\begin{array}{rr}
\text{lbs.} & \text{ozs.}\\
13\ . & 4\\
7000 & \\
\hline
91000 & \\
1750 & \\
\hline
92750 &
\end{array}
$$

Ans. 92750 *grs.*

To test the work of Reduction.

Reduce the answer back to the form given in the question.

V. THE COMPOUND RULES.

The principles involved in the addition, subtraction, multiplication, and division of *compound quantities* are those explained in Chapter II. The examples which follow illustrate the methods used.

Compound Addition.

It is evident that the quantities to be added must be of the *same kind.*

For instance, 3 *acres* + 4 *acres* make 7 *acres;*
but 3 *acres* + 4 *tons* make neither 7 *acres* nor 7 *tons.*

Also, that when the quantities *are* of the same kind, each *denomination* must be dealt with separately.

For instance, 3 *pence* + 4 *pence* make 7 *pence;*
but 3 *shillings* + 4 *pence* make neither 7 *shillings* nor 7 *pence.*

Hence, we arrange the quantities in columns so that all the quantities in the same column are of the same denomination; we then add together the numbers in the column of lowest denomination, and carry forward to the next column as many complete units of that next higher denomination as the sum yields, setting the remainder below the column added; and so on.

The following example will serve to illustrate the process with which, at least in the case of *money*, we assume that the student is already familiar.*

Example i.—*Add together* 3 *mi.* 1 *fur.* 17 *po.* 3 *yds.;* 1 *mi.* 7 *fur.* 23 *po.* 4 *yds.;* 11 *mi.* 6 *fur.* 10 *po.* 1 *yd.;* 8 *mi.* 2 *fur.* 35 *po.* 5 *yds.;* 19 *mi.* 0 *fur.* 7 *po.* 2 *yds.; and* 2 *mi.* 4 *fur.* 13 *po.* 4 *yds.*

mi.	fur.	po.	yds.
3	1	17	3
1	7	23	4
11	6	10	1
8	2	35	5
19	0	7	2
2	4	13	4
46	6	28	$2\frac{1}{2}$ *Ans.*

Adding the numbers in the column headed "yards" we obtain the sum 19 yds. We now remove from the 19 *yards* as many complete *poles* as we can (in this case 3, for 3 times $5\frac{1}{2}$ is $16\frac{1}{2}$), and set down the remaining $2\frac{1}{2}$ yds., carrying forward 3 *poles.*

We next add the numbers in the column headed "poles," adding in the 3 carried, and divide the sum, 108 *poles,* by 40, obtaining 2 *furlongs* to carry forward, and 28 *poles* remaining, which we set down.

And so on.

* Addition and Subtraction of money may be illustrated *practically* to young beginners, by using counters of different colours (say yellow, white, and brown) to represent pounds, shillings, and pence.

Compound Subtraction.

EXAMPLE ii.—*From* 14*s.* 3*d.* *take* 7*s.* 10*d.*, *and explain the process.*

s.	d.	
14	3	
7	10	
6	5	*Ans.*

The Pence.

We cannot take 10 *pence* from 3 *pence*, so we change one of the 14 *shillings* into 12 *pence*, which we mentally add to the 3 *pence*, thus making up 15 *pence*.

Then 10 *pence* from 15 *pence* leaves **5** *pence.*

The Shillings.

Remembering that one of the 14 shillings has been changed into pence, as we have now to take away 7 other shillings, it is evident that

$14 - 1 - 7$ will be the number of shillings left.

But *to take* 1 *and* 7 *in succession from* 14 *is equivalent to taking* **8** *from* 14 *in* one *operation.*

So when we pass from the pence to the shillings column, we "carry" **1**, which we *add to the* 7 *shillings.*

Then 8 *shillings* from 14 *shillings* leaves **6** *shillings.*

Addition and Subtraction (as was shown with abstract numbers, on page 8), can often be conveniently combined in *one* operation.

EXAMPLE iii.—*Take the sum of* £2, 15*s.* $7\frac{1}{2}$*d.*; £16, 9*s.* $0\frac{1}{4}$*d.*; £13, 18*s.* $9\frac{1}{4}$*d.*; £5, 13*s.* $2\frac{3}{4}$*d.*; *and* £1, 3*s.* $7\frac{3}{4}$*d.* *from* £45, 13*s.* $5\frac{1}{4}$*d.*

£	s.	d.	
45	13	$5\frac{1}{4}$	
2	15	$7\frac{1}{2}$	
16	9	$0\frac{1}{4}$	
13	18	$9\frac{1}{4}$	
5	13	$2\frac{3}{4}$	
1	3	$7\frac{3}{4}$	
5	13	$1\frac{3}{4}$	*Ans.*

1st step.

Adding the farthings in the five lowest lines, we obtain the sum $2\frac{1}{2}$*d.*; and $2\frac{1}{2}$*d.* from $3\frac{1}{4}$*d.* leaves $\frac{3}{4}$d.
Carry 3.

2nd step.

Adding the pence in the same five lines and including the 3 carried, we obtain the sum 2*s.* 4*d.*; and 2*s.* 4*d.* from 2*s.* 5*d.* leaves **1d.**
Carry 2.

3rd step.

Adding the *units* column of the shillings, and including the carried 2, we obtain the sum 30, and 30 from 33 leaves **3**. Carry 3.

Then adding the *tens* column, and including the carried 3, we obtain the sum 6; and 6 from 7 (changing £3) leaves **1**.

Carry 3.

4th step.

Adding the *units* column of the pounds, and including the carried 3, we obtain the sum 20; and 20 from 25 leaves **5**. Carry 2.

Then adding the *tens* column of the pounds, and including the carried 2, we obtain the sum 4; and 4 from 4 leaves 0.

Compound Multiplication.

We know from the definition of Multiplication (see page 9) that the multiplier is the *number of times* the multiplicand is repeated.

Thus **the multiplier can never be concrete, but is always an abstract number.**

Hence, in Compound Multiplication the *multiplicand* is a *compound quantity;* the *multiplier* is an *abstract number;* and the *product* a quantity of the same kind as the multiplicand.

For instance,

7 tons 2 *cwt.* × 3 = 7 *tons* 2 *cwt.* + 7 *tons* 2 *cwt.* + 7 *tons* 2 *cwt.* = 21 *tons* 6 *cwt.*

Again, in the question, *If* 1 lb. *of tea cost* 2*s.*, *what will* 7 lbs. *cost?* The answer, 14*s.* is the result of the following *mental* process:—

"As 7 lbs. will cost 7 *times as much* as 1 lb., the cost of 1 lb. must be repeated 7 times." We do *not* multiply 2*s.* by 7 *lbs.*, but by 7.

The following examples illustrate various cases that occur:—

(1) When the multiplier is not greater than one of the factors of the Multiplication Table.

Example iv.—*Multiply* 3 *fur.* 29 *po.* 4 *yds.* 1 *ft. by* 7.

1st step.

7 times 1 *ft.* = 7 *ft.* = 2 *yds.* 1 *ft.*
Set down 1, and carry 2.

2nd step.

7 times 4 *yds.* = 28 *yds.*, 28 + 2 (carried) = 30 *yds.* = 5 *po.* 2½ *yds.*
Set down 2½, and carry 5.
And so on.

mi.	fur.	po.	yds.	ft.
	3 .	29 .	4 .	1
				7
3 .	2 .	8 .	2½ .	1

Finally, converting the *half-yard* in the result into 1 *ft.* 6 *in.* we write the answer thus:—

Ans. 3 *mi.* 2 *fur.* 8 *po.* 2 *yds.* 2 *ft.* 6 *in.*

(2) When the factors of the multiplier are known from the Multiplication Table.

Example v.—*Multiply* 2 *tons*, 13 *cwts.* 1 *qr.* 17 *lbs. by* 63.

As 63 = 7 × 9, we multiply first by 7, and then multiply the result by 9 (or *vice versa*, first by 9 and then the result by 7), thus obtaining the required result. See (IV.), page 10.

tons.	cwts.	qrs.	lbs.	
2 .	13 .	1 .	17	
			7	
18 .	13 .	3 .	7	
			9	
168 .	4 .	1 .	7	*Ans.*

(3) When the multiplier is rather less, or rather greater, than the product of some two factors of the Multiplication Table.

EXAMPLE vi.—*Multiply* £1, 5*s*. 7*d*. *by* 93.

Here, as the Multiplication Table provides no factors which yield a product nearer to 93 than 9×10, we multiply in succession by 9 and 10, thus obtaining 90 times £1, 5*s*. 7*d*. (i).

We next multiply £1, 5*s*. 7*d*. by 3, setting the result (ii) below (i).

Finally, we add the results (i) and (ii), thus obtaining 93 times £1, 5*s*. 7*d*.

£	s.	d.	
1	5	7	×3
		9	
11	10	3	
		10	
115	2	6	...(i).
3	16	9	...(ii).
£118	19	3	*Ans.*

EXAMPLE vii.—*Multiply* £3, 17*s*. 5*d*. *by* 39.

The factors in the Multiplication Table which *most nearly* produce 39 are 4×10.

We therefore multiply in succession by 4 and 10, thus obtaining 40 times £3, 17*s*. 5*d*. (i).

We then subtract £3, 17*s*. 5*d*. from (i), thus obtaining 39 times £3, 17*s*. 5*d*.

£	s.	d.	
3	17	5	
		4	
15	9	8	
		10	
154	16	8	...(i).
3	17	5	
£150	19	3	*Ans.*

(4) When the multiplier exceeds the limit of the Multiplication Table.

In such cases we may regard the multiplier as composed of the parts *units*, *tens*, *hundreds*, &c., multiply by each of these parts separately, and then add the results. See (II.) on page 10.

EXAMPLE viii.—*Multiply* £1, 5*s*. 7*d*. *by* 283.

Here, $283 = 200 + 80 + 3$, so we first multiply in succession by 10, 10, and 2, obtaining 200 times £1, 5*s*. 7*d*. (i).

We next multiply the second line by 8, obtaining 80 times £1, 5*s*. 7*d*., and set the result (ii) below (i).

We then multiply the top line by 3, obtaining 3 times £1, 5*s*. 7*d*. (iii), and set the result below (ii).

Finally, we add these three results together, thus obtaining 283 times £1, 5*s*. 7*d*.

£	s.	d.	
1	5	7	×3
		10	
12	15	10	×8
		10	
127	18	4	
		2	
255	16	8	...(i).
102	6	8	...(ii).
3	16	9	...(iii).
£362	0	1	*Ans.*

The following method may be used instead of that of Example viii, and, when the multiplier is *very large*, is to be preferred:—

EXAMPLE ix.—*Multiply* £11, 13*s.* $3\frac{1}{4}$*d. by* 4957.

1st step.
Multiply 1 *farthing* by 4957, reducing the result to *pence* (i).

2nd step.
Multiply 3 *pence* by 4957, set the result under (i) and add it to (i), reducing the sum to *shillings* (ii).

3rd step.
Multiply 13 *shillings* by 4957, set the result under (ii), and add it to (ii), reducing the sum to *pounds* (iii).

4th step.
Multiply £11 by 4957, set the result under (iii), and then add it to (iii).

Written Work.

$$\begin{array}{rl}
4\,| & 4957 \\
 & 1239\frac{1}{4} \ldots\ldots\ldots\ldots \text{(i)} \\
 & 14871 \\
12\,| & 16110\frac{1}{4} \\
 & 1342 \ldots\ldots\ldots 6\frac{1}{4} \ldots\text{(ii)} \\
 & 64441 \\
2,0\,| & 6578,3 \ldots\ldots\ldots 6\frac{1}{4} \\
 & 3289 \;.\; 3 \;.\; 6\frac{1}{4} \ldots\text{(iii)} \\
 & 54527 \\
 & \pounds 57816 \;.\; 3 \;.\; 6\frac{1}{4} \;\textit{Ans.}
\end{array}$$

We have supposed that all the work, except what is given on the right, has been performed mentally.

Note.—The multiplication of compound quantities by very large numbers is of no great importance, as such products can be more concisely obtained, as we shall see later, by the method of *Practice.*

The method of the following example is important, and may often be employed with advantage:—

EXAMPLE x.—*Multiply* £2, 19*s.* $7\frac{1}{2}$*d. by* 1203.

$$\begin{array}{rl}
 & 1203 \\
 & 9 \\
2\,| & 10827 \\
12\,| & 5413 \ldots\ldots \tfrac{1}{2}d. \\
2,0\,| & 45,1 \ldots\ldots 1d. \\
 & 22 \ldots\ldots\ldots 11s.
\end{array}$$

£	s.	d.	
1203 .	0 .	0	
		3	
3609 .	0 .	0	
22 .	11 .	$1\frac{1}{2}$	
3586 .	8 .	$10\frac{1}{2}$	*Ans.*

Here, observing that £2, 19*s.* $7\frac{1}{2}$*d.* only differs from £3 by $4\frac{1}{2}$*d.*,
we first multiply **£3** by 1203,
we then multiply **$4\frac{1}{2}$d.** by 1203 (*i.e.* multiply 1203 by 9, and reduce this number of *halfpence* to £ *s. d.*).

Finally we **subtract** these results.

Note.—If we *add* the two results in this example we obtain 1203 times **£3**, 0*s.* $4\frac{1}{2}$*d.*

COMPOUND DIVISION.

We know (see page 13) that Division, being the converse of Multiplication, may be regarded either as the operation of finding

(i) the *multiplicand* when product and multiplier are given;
or (ii) the *multiplier* when product and multiplicand are given.

Also (see page 40), if the multiplicand be a *concrete* quantity, the product is also a concrete quantity of the same kind; but the *multiplier* must always be an *abstract* number.

For instance, (i) *Divide* 77 *feet into* 11 *equal parts.*

Here we have given the *product* 77 *feet,* and the *multiplier* 11, and are asked to find the **multiplicand 7** *feet.*

(ii) *How many times is* 3 *shillings contained in* 93 *shillings?*

Here we have given the *product* 93 *shillings*, and the *multiplicand* 3 *shillings,* and are asked to find the **abstract multiplier 31.**

Hence, in Compound Division, the dividend is a concrete quantity; and either

(i) the *divisor* an *abstract number*, and the quotient a concrete quantity of the same kind as the dividend;

or (ii) the *divisor a concrete quantity* of the same kind as the dividend, and the quotient an abstract number.

It is very important to notice that the *divisor* and the *quotient cannot* both *be concrete* together.

Note.—Division of the first of these two classes is sometimes called *Partition,* or *Sharing;* division of the second kind, *Quotition,* or *Measuring.*

CLASS I. (*Abstract Divisor*).

Short Division.

EXAMPLE xi.—*Divide* 9 *po.* 16 *sq. yds.* 7 *sq. ft.* 6 *sq. in. by* 6.

1st step.
6 in 9, **1**, and 3 *po., i.e.* $90\frac{3}{4}$ *sq. yds.* over;
$90\frac{3}{4} + 16 = 106\frac{3}{4}$ *sq. yds.*

2nd step.
6 in $106\frac{3}{4}$, **17**, and $4\frac{3}{4}$ *sq. yds. i.e.* $42\frac{3}{4}$ *sq. ft.* over; $42\frac{3}{4} + 7 = 49\frac{3}{4}$ *sq. ft.*

	po.	sq. yds.	sq. ft.	sq. in.	
6)	9 .	16 .	7 .	6	
	1 .	17 .	8 .	43	*Ans.*

3rd step.
6 in $49\frac{3}{4}$, **8**, and $1\frac{3}{4}$ *sq. ft.* over.
Now, $1\frac{3}{4}$ *sq. ft.* $= 1\frac{3}{4} \times 144$ *sq. in.* $= 144 + 108 = 252$ *sq. in.*;
$252 + 6 = 258$ *sq. in.*

4th step.
6 in 258, **43**.

When the factors of the divisor are known from the Multiplication Table "Short" Division may be used:

EXAMPLE xii.—*Divide* £213, 16*s.* $8\frac{1}{2}$*d. by* 42.

Here, as $42 = 6 \times 7$, we may divide in succession by 6 and 7, using "short" division.

The first rem^r is 4*f.*, and the second rem^r is also 4*f.*

Hence, the complete rem^r (see p. 16) is $4 \times 6 + 4 = 28f. = 7d.$

$$
\begin{array}{ll}
 & \quad\;\; \text{£} \qquad\; s. \qquad d. \\
42 \left\{ \begin{array}{l} 6 \\ 7 \end{array} \right. & \begin{array}{|l} 213 \;.\; 16 \;.\; 8\frac{1}{2} \\ \hline 35 \;.\; 12 \;.\; 9\frac{1}{4} \ldots 4f. \\ \hline \end{array} \\
 & \;\; 5 \;.\; \;\;1 \;.\; 9\frac{3}{4} \ldots 4f.
\end{array}
\left. \begin{array}{l} \\ \\ \end{array} \right\} \underline{28f.}
$$

Ans. £5, 1*s.* $9\frac{3}{4}$*d.* + 7*d. rem^r.*

Or we may proceed by the method of "Long" Division, thus:

Long Division.

EXAMPLE xiii.—*Divide* £213, 16*s.* $8\frac{1}{2}$*d. by* 42.

1st step.

Dividing £213 by 42 we obtain quotient £5 and rem^r £3.

Reducing the rem^r, £3, to *shillings*, and adding in 16*s.*, we obtain 76*s.*

2nd step.

Dividing 76*s.* by 42 we obtain quotient 1*s.* and rem^r 34*s.*

Reducing the rem^r 34*s.* to *pence*, and adding in 8*d.*, we obtain 416*d.*

3rd step.

Dividing 416*d.* by 42 we obtain quotient 9*d.* and rem^r 38*d.*

Reducing the rem^r 38*d.* to farthings, and adding in 2*f.*, we obtain 154*f.*

4th step.

Dividing 154*f.* by 42 we obtain quotient 3*f.* and rem^r 28*f.*, *i.e.* 7*d.*

```
        £      s.      d.  £
    42)213  .  16  .  8½(5
       210
         3
        20     s.
    42)76(1
       42
       34
       12      d.
    42)416(9
       378
        38
         4     f.
    42)154(3
       126
        28f. = 7d.
```

Ans. £5, 1*s.* $9\frac{3}{4}$*d.* + 7*d. rem^r.*

Note.—In Compound Long Division the *written* work may be considerably curtailed by using the abridged method described on page 15.

EXAMPLE xiv.—*Divide* 81 *cwts.* 1 *qr.* 7 *lbs.* 5 *ozs. by* 53.

1st Method.

```
      cwt.  qr.  lb.  ozs.  dr. cwt.
53)81 . 1 . 7 . 5 . 0(1
   53
   28
    4      qr.
53)113(2
   106
     7
    28     lb.
53)203(3
   159
    44
    16
   269
    44     oz.
53)709(13
    53
   179
   159
    20
    16     dr.
53)320(6
   318
     2
```

2nd Method.

```
      cwt.  qr.  lb.  oz.  dr. cwt.
53)81 . 1 . 7 . 5 . 0(1
   28       qr.
   113(2
     7      lb.
   203(3
    44      oz.
   709(13
   179
    20      dr.
   320(6
     2
```

In the *1st method* the mental work corresponds to that described in Ex. xiii, except that the Table of Avoirdupois Weight is used in place of the Money Table.

In the 2*nd method* abridged division is used. Also in the successive reductions the multipliers are omitted.

Ans. 1 *cwt.* 2 *qr.* 3 *lb.* 13 *oz.* 6 *dr.*+2 *dr. rem*r.

Division by 100.

EXAMPLE xv.—*Divide* £322, 12*s.* 1*d. by* 100.

In this case it is *not necessary to write down the divisor*, for we know (see p. 17), that division by 100 is performed at once, by striking off *two* figures on the right of the number divided.

1st step.

Cutting off *two* figures, on the right, from £322, we obtain quotient £**3** and rem^{r} £22. Reducing £22 to shillings, and adding in 12*s.*, we obtain 452*s.*

2nd step.

Cutting off *two* figures, on the right, from 452*s.*, we obtain quotient **4***s.* and rem^{r} 52*s.* Reducing 52*s.* to pence, and adding in 1*d.*, we obtain 625*d.*

And so on.

```
  £      s.   d.
3,22 . 12 . 1
  20
4,52
  12
6,25
   4
1,00
```

Ans. **£3**, 4*s.* 6¼*d.*

Similarly we proceed in the case of divisors 1000, 10000, &c., cutting off, at each stage, *three*, *four*, &c., figures respectively.

CLASS II. (*Abstract Quotient*).

When the divisor and the dividend are both *compound* quantities, we must, before the division can be performed, reduce them both to the *same denomination.*

In choosing this common denomination, we take care, if we would avoid unnecessary labour, that it is as *high* as possible.

EXAMPLE xvi.—*How many times is* 7 *lbs.* 4 *ozs. contained in* 4 *cwts.* 2 *qrs.* 10 *lbs.* 12 *ozs.?*

In this case we reduce both quantities to *quarter-pounds.*

Multiplying 7 by 4 and adding in the 4 ozs. as 1 *quarter-lb.* we obtain 29 *quarter-lbs.*

Reducing 4 cwt. 2 qrs. 10 lbs. 12 ozs. also to *quarter-lbs.* and adding in 12 *ozs.* as 3 *quarter-lbs.* we obtain 2059 *quarter-lbs.*

Dividing 2059 by 29 we find that 29 *qr.-lbs.* is contained 71 times in 2059 *qr.-lbs.*

```
lbs. ozs.      cwts. qrs. lbs. ozs.
 7 . 4          4 . 2 . 10 . 12
 4              4
29             18
               28
              154
              36
              514
                4
          29)2059(71
             203
               29
               29
```

Ans. 71 *times.*

EXAMPLE xvii.—*How many men could each receive* 14*s.* 7*d. from a fund of* £6, 18*s.?*

Reducing both quantities to the common denomination *pence*, we obtain 175*d.* and 1656*d.*

Dividing 1656 by 175 we find that 175*d.* is contained 9 *times* in 1656*d.*, and that 81*d.*, *i.e.* 6*s.* 7*d.* is left over.

Note.—Here, again, the quotient 9 is an *abstract number.* The answer 9 *men* results from the following *mental* process:—

"As 175 *pence* is contained 9 *times* in 1656 *pence*, therefore 9 *men* could each receive that sum."

```
 s.  d.        £   s.  d.
14 . 7         6 . 18 . 0
12            20
175           138
               12
          175)1656(9
              1575
                81d. = 6s. 7d.
```

Ans. 9 men, and 6*s.* 7*d. over.*

VI. MISCELLANEOUS EXAMPLES.

A. *If* 123 *articles, bought at* £2, 17*s.* 9*d. each, are sold at* £3, 2*s.* 3*d. each, what profit is made?*

We first subtract £2, 17*s.* 9*d.* from £3, 2*s.* 3*d.* to find the profit on *one* article, namely, 4*s.* 6*d.*

Hence, the total profit is 123 times 4*s.* 6*d.* = £27, 13*s.* 6*d.* *Ans.*

Note.—Much labour would be wasted if we multiplied £2, 17*s.* 9*d.* and £3, 2*s.* 3*d.* *separately* by 123 and *afterwards* subtracted the results.

B. *A merchant bought* 28 *gallons of spirits at* 36*s. a gallon, diluted it with water, and then sold the mixture at* 32*s. a gallon, making a profit of* £4. *How much water did he add?*

The total cost was 36 × 28 = 1008*s.*
The total profit was £4 = 80*s.*
∴ the whole mixture was sold for 1088*s.*
But one gallon was sold for 32*s.*,
And 32*s.* is contained 34 times in 1088*s.*

i.e. 34 gallons of *mixture* were sold, of which 28 was *spirit.*
∴ 34 − 28 = 6 gallons of water must have been added.

C. (i). *Divide* £16, 8*s.* 6*d. between A and B so that for every half-crown A has B may have a florin.*

Each time A receives 5 *sixpences* B is to receive 4 *sixpences;*
∴ with 9 sixpences this could be done once.
But in £16, 8*s.* 6*d.* there are 657 sixpences.
And 9 *sixpences* is contained **73 times** in 657 *sixpences.*

Hence, A's share is 73 *half-crowns* = £9, 2*s.* 6*d.*
and B's „ 73 *florins* = £7, 6*s.* 0*d.* } *Ans.*

C. (ii). *Divide* £1, 16*s.* 2*d. among* 7 *men,* 12 *women, and* 17 *boys, so that each woman may have twice as much, and each man three times as much, as each boy.*

Here the 7 men	receive as much as		21 boys,
„ 12 women	„	„	24 boys,
„ 17 boys	„	„	17 boys;
∴ all the persons	„	„	62 boys.

Hence, as all the persons receive £1, 16*s.* 2*d.* among them, a boy's share is found by dividing this sum by 62, whence we obtain quotient 7*d.*

Thus a boy's share is 7*d.*, a woman's 14*d.*, and a man's 21*d.* *Ans.*

C. (iii). *Divide £10, 15s. 6d. among 8 men, 9 women, and 10 boys, giving each woman 2s. less than each man, and 3s. more than each boy.*

Here each *man* has 5*s.* *more* than each *boy.*
Also ,, *woman* ,, 3*s.* ,, ,, *boy.*

∴ 40*s.* would supply the extra payment for 8 men, and 27*s.* would supply the extra payment for 9 women.

Now 40*s.* + 27*s.* = 67*s.* = £3, 7*s.*

Hence we subtract £3, 7*s.* from £10, 15*s.* 6*d.*, and divide the remainder *equally* among the total number of persons.

Thus a boy's share is 5*s.* 6*d.*,
Hence a woman's ,, 8*s.* 6*d.*,
and a man's ,, 10*s.* 6*d.* } *Ans.*

£	s.	d.
10	15	6
3	7	0
7	8	6

20

27) 148 (5 *s.*
135
13
12

27) 162 (6 *d.*
162

D. *A sum of £26, 9s. was made up of a certain number of half-sovereigns, twice as many shillings, three times as many half-crowns, and seven times as many sixpences; how many coins were there altogether?*

Here 1 *half-sovereign* + 2 *shillings* + 3 *half-crowns* + 7 *sixpences*
= (20 + 4 + 15 + 7) *sixpences*
= 46 *sixpences.*

Also, £26, 9*s.* = 1058 *sixpences.*
And 46 *sixpences* is contained 23 times in 1058 *sixpences.*

£	s.
26	9

40

46) 1058 (23
92
138
138

∴ the total number of half-sovereigns	was	23
,, ,, shillings	,,	46
,, ,, half-crowns	,,	69
,, ,, sixpences	,,	161
∴ the total number of coins	,,	299 *Ans.*

E. *A sum of £4, 5s. was distributed among 101 school-children so that each boy had 9d. and each girl 1s. How many boys were there?*

We first consider how much money would be used in giving 9*d.* to *every child.*

This would take 101 × 9 = 909*d.* = 75*s.* 9*d.*

There would now remain the difference between £4, 5*s.* and 75*s.* 9*d.*; that is, 9*s.* 3*d.*

And this, by the question, was sufficient to provide an extra 3*d.* for each *girl.*

Hence, as 9*s.* 3*d.* = 37 *threepences*, there must have been 37 *girls.*

∴ the number of *boys* was 101 − 37 = 64 *Ans.*

F. *A and B have* 8s. *between them; B and C have* 11s.*; A and C have* 15s. *How much has each?*

Twice A's money + *twice* B's + *twice* C's = 8*s*. + 11*s*. + 15*s*. = 34*s*.

∴ A's money + B's + C's = 17*s*.

But, by the question, B's + C's = 11*s*.;
∴ A's money is 17*s*. − 11*s*. = 6*s*.

Hence, B's money is 8*s*. − 6*s*. = 2*s*.,
And C's money is 11*s*. − 2*s*. = 9*s*.

G (i). *Find the number of days from September* 7*th*, 1891, *to June* 16*th*, 1892.

		days.
In 1891,	from September 7th to the end of the month	23
,,	in October	31
,,	,, November	30
,,	,, December	31
In 1892,	,, January	31
,,	,, February (leap-year)	29
,,	,, March	31
,,	,, April	30
,,	,, May	31
,,	from June 1st to June 16th	16

∴ the total number of days was ... 283 *Ans.*

N.B.—In calculating the number of days between two given dates the *first of the days mentioned is not counted.*

For instance, from May 5th to May 6th is 1 day;
and from May 23d to May 27th is 4 days.

G (ii). *The "Saturday Review" of October* 12*th*, 1889, *was numbered* 1772. *What was the number on a copy bearing the date December* 31*st*, 1892?

Here we first find the number of *weeks* in the period.

Now, from Oct. 12th, 1889, to Oct. 12th, 1890, there were 365 days.
,, ,, 1890, ,, 1891, ,, 365 ,,
,, ,, 1891, ,, 1892 (leap-year), 366 ,,
and from Oct. 12th, 1892, to Dec. 31st, 1892,
there were 19 + 30 + 31 = 80 ,,

Hence, the total length of the period was 1176 ,,
But 1176 *days* = 168 *weeks*.

∴ the required number is 1772 + 168 = 1940 *Ans.*

G (iii). *August 8th, 1892, fell on a Monday; what day of the week was April 14th, 1891?*

As $365 \div 7$ gives quotient 52 and rem^r 1, it follows that if any particular date in one year fall on a *Monday* it will fall on a *Tuesday* in the next year, unless that be leap-year, in which case it will fall on a *Wednesday;* and so on.

Now, 1892 was leap-year.

Hence, counting *back* two days, as Aug. 8th, 1892, was a Monday, Aug. 8th, 1891, was a *Saturday.*

Again, the number of days between April 14th and Aug. 8th in 1891, was $16 + 31 + 30 + 31 + 8 = 116$ days,

And $116 \div 7$ gives quotient 16 and rem^r 4.

i.e., there were 16 complete weeks and 4 days in this period.

Hence, counting *back* four days from Saturday, we find that April 14th, 1891. fell on a Tuesday.

G (iv). *On September 13th, 1891, a man had lived 14000 days; find the date of his birth.*

$$\begin{array}{r} 365\,)\,14000\,(\,38 \\ \underline{1095} \\ 3050 \\ \underline{2920} \\ 130 \\ \underline{9} \\ 121 \end{array}$$

Dividing 14000 by 365 we obtain quotient 38 and rem^r 130.

Thus the period contains 38 common years + 130 days.

But in 38 years there were 9 leap-years, which would account for 9 extra days.

Subtracting 9 from 130 we obtain rem^r 121.

And $1891 - 38 = 1853$.

Hence, the man was born 121 days *before* Sept. 13th, 1853.

Now, counting back from Sept. 13th,

there were in Sept., 1853 (*up to the 13th*),	13	days.
" " Aug., ...	31	"
" " July, ...	31	"
" " June, ...	30	"
Total	105	"

And $121 - 105 = 16$ days.

Thus the man was born 16 days *before* June 1st, 1853.

i.e., he lived 16 days in May, 1853,

∴ the date of his birth was May 15th, 1853.

VII. FACTORS, MULTIPLES, PRIMES.

A **Factor** of a number *divides* that number *exactly* (*i.e.* without remainder).

For instance, 7 is a *factor* of 35; 13 is a *factor* of 91; and 2, 3, 8, 12, are all *factors* of 24.

A **Multiple** of a number *contains* that number *exactly*.

For instance, 35 is a *multiple* of 7; 91 is a *multiple* of 13; and 24 is a *multiple* of each of the numbers 2, 3, 8, 12.

Note.—As every number is exactly divisible by itself and by 1, *every* number has, strictly speaking, at least two factors; hence when a number is spoken of, shortly, as having *no* factors, the words "except itself and unity" are implied.

Also the words *divisible* and *divisor* are commonly used with the sense "exactly divisible" and "exact divisor."

A **Prime number** (or *prime*) has *no factors*, except itself and unity.

For instance, 5, 13, 29, 53, are *primes.*

Hence, the *prime factors* of a number are those *prime* numbers which exactly divide that number.

For instance, 5 and 7 are *prime* factors of 35; but 4 and 6, though *factors*, are not prime *factors* of 12.

A number which *has* factors, other than itself and unity, is called *composite.*

A **common factor** of two, or more, numbers *divides each* of them exactly.

For instance, 7 is a *common* factor of 63 and 35; and 4 is a *common* factor of 36, 84, and 100.

A **common multiple** of two, or more, numbers *contains each* of them exactly.

For instance, 48 is a *common* multiple of 6, 8, and 12.

Two numbers are *prime to each other*, when they have *no common* factor, except unity.

For instance, 7 is prime to 11; 8 is prime to 13; 12 is prime to 25.

Hence two *composite* numbers *may* be, but two *prime* numbers *must* be, prime to each other.

A number is called **even** or **odd** according as it is, or is not, exactly divisible by 2.

Thus, 0, 2, 4, 6, &c. are *even*, and 1, 3, 5, 7, &c. are *odd*.

Tests of Divisibility.*

The following tests enable us to discover at once, without the labour of a trial division, whether a given number is, or is not, exactly divisible by any number less than 12 (except 7).

Also, to obtain the remainder (if any) without actually dividing.

(I.) *A number is divisible by 2, if the right-hand digit is* **even**, *and not unless.*

Thus, we know at a glance, that 75403729**8** is, and that 3840228**3** is *not*, divisible by 2.

(II.) *A number is divisible by 5, if the right-hand digit is either* **0** *or* **5**, *and not unless.*

Thus, we know at a glance, that 145631**0** and 784636**5** *are* divisible by 5; and that 370525**4** is *not* divisible by 5.

(III.) *A number is divisible by 4, or by 25, if the number formed by the* **two** *last digits is divisible by 4, or by 25, and not unless.*†

Thus, by this test we quickly discover that 3876453**72** *is* divisible by 4, and that 7608893**26** is *not* divisible by 4.

Also, that 24763**75** *is*, and that 38673**85** is *not*, divisible by 25.

(IV.) *A number is divisible by 3, or by 9, if the* **sum of its digits** *is divisible by 3, or by 9, and not unless.*

For instance, 725634 *is* divisible by 9, for the *sum of its digits* is 27; and 137625 is *not* divisible by 9, but *is* divisible by 3, for the *sum of its digits* is 24.

N.B.—In practice it is not necessary actually to add *all* the digits of the number together, we may "cast out" the nines one by one as they are obtained, since all we require to know is the remainder, if any.

Thus we can see at once that 7 2 5 6 3 4 *is* divisible by 9, since its digits can be arranged in pairs, each of which pairs makes up nine.

Also, we can see that 3472856 *is not* divisible by 9 (nor by 3), for, after discarding three pairs each amounting to 9, the digit 8 is left alone.

Hence also we know, without actually dividing, that 3472856÷9 yields *remainder* 8.

Again, if we "cast out" two *threes* from this rem^r 8, we also know that 3472856÷3 yields rem^r 2.

---

* For *proofs* of these tests see Appendix.

† Similar tests applied to the *three* last figures hold for 8 and 125.

The test for 6 is evidently that the number be divisible by both 2 and 3, *i.e.* that it is *even*, and that the *sum of its digits* is divisible by 3.

(V.) *A number is divisible by* 11 *if the difference between the sums of the digits in the units', hundreds', &c. places, and those in the alternate tens', thousands', &c. places, is divisible by* 11; *and not unless.*

For instance, in the number 745432809

as $9+8+3+5+7=32$; and as $0+2+4+4=10$;
and as $32-10=\mathbf{22}$, which is divisible by 11,

$\therefore$ 745432809 *is* divisible by 11.

EXAMPLE i.—*Resolve* 42840 *into its prime factors.*

1st Method.

Here we divide in succession by 2 as long as the quotients are *even;* then by 5 because the quotient ends in 5; then by 3 as long as the *sums of the digits* of the quotients are divisible by 3. We then find *by trial* that 7 is a factor and yields quotient 17, a prime number.

$$\begin{array}{r|l} 2 & 42840 \\ 2 & 21420 \\ 2 & 10710 \\ 5 & 5355 \\ 3 & 1071 \\ 3 & 357 \\ 7 & 119 \\ & 17 \end{array}$$

$\therefore$ $42840 = \underline{2\times2\times2\times5\times3\times3\times7\times17}$ *Ans.*

2nd Method.

1st step. Cut off the 0 (*i.e.* divide by 10).
2nd step. Divide by 4, since **84** is divisible by 4.
3rd step. Divide by 9, since the *sum of the digits* of 1071 is divisible by 9.
4th step. Obtain the factor 7 by trial.

$$\begin{array}{r|l} 4 & 4284,0 \\ 9 & 1071 \\ 7 & 119 \\ & 17 \end{array}$$

$\therefore$ $42840 = 10\times4\times9\times7\times17 = 2\times5\times2\times2\times3\times3\times7\times17.$

Or, with the index notation (see p. 12), $42840 = \underline{2^3\times3^2\times5\times7\times17}$ *Ans.*

EXAMPLE ii.—*Is* 823 *a prime number?*

We know at once that 823 is not divisible by either 2, 3, 5 or 11.

We now try the other primes, namely, 7, 13, 17, &c., in succession, up to 29, and obtain a remainder in each case.

Also, when we divide by 29, we again obtain a remainder. And the quotient in this case is **28**, a number *less than the trial divisor* 29. We need therefore proceed no further, for if any number greater than 29 were a factor of 823 the quotient would be one of the numbers already tried and discarded.

$$\begin{array}{r} 29)823(28 \\ 58 \\ \hline 243 \\ 232 \\ \hline 11 \end{array}$$

$\therefore$ 823 *is* a prime.

VIII. GREATEST COMMON FACTOR.*

The **Greatest Common Factor** (G.C.F.) of two or more numbers is the *greatest* number which *divides* each of them exactly.

When the prime factors of the numbers are known their G.C.F. is evident.

For instance, as $35 = 5 \times 7$,
and $77 = 11 \times 7$,
7 is the *only* common factor of 35 and 77.
$\therefore$ 7 is their *greatest* common factor.

Again, as $36 = 2 \times 2 \times 3 \times 3$,
$40 = 2 \times 2 \times 2 \times 5$,
and $56 = 2 \times 2 \times 2 \times 7$,

and as the number 2 occurs as a common factor twice over, but no other prime factor is common to all three numbers,

$\therefore$ the G.C.F. of 36, 40, and 56 is 2×2, or **4**.

Note.—After a little practice the G.C.F. of small numbers such as the above can easily be found *mentally*.

EXAMPLE i.—*Write down the G.C.F. of* 21×11 *and* 35×11.

Ans. 7×11, or 77.

EXAMPLE ii.—*Find, by resolving the numbers to their prime factors, the G.C.F. of* 126, 210, *and* 231.

Here we observe that there are two, and only two, factors *common to all three* of the given numbers, namely, 3 and 7.

$\therefore$ the G.C.F. is 3×7, or 21.

$$\begin{array}{r|l} 2 & 126 \\ 3 & 63 \\ 3 & 21 \\ & 7 \end{array} \qquad \begin{array}{r|l} 2 & 210 \\ 5 & 105 \\ 3 & 21 \\ & 7 \end{array} \qquad \begin{array}{r|l} 3 & 231 \\ 7 & 77 \\ & 11 \end{array}$$

$3 \times 7 = 21$ *Ans.*

The labour of resolving large numbers to their prime factors in order to discover their G.C.F. is avoided in practice by adopting the following method, or some modification of it.

* The word *measure* is often used instead of *factor*; and Greatest Common Measure (G.C.M.); or Highest Common Divisor (H.C.D.); instead of Greatest Common Factor (G.C.F.).

EXAMPLE iii.—*Find the G.C.F. of* 1517 *and* 5986.

1st step.
Dividing the greater number by the less we obtain the rem^r 1435.

2nd step.
Dividing the previous divisor 1517 by 1435 we obtain the rem^r 82.

3rd step.
Dividing the previous divisor 1435 by 82 we obtain the rem^r 41.

4th step.
Dividing the previous divisor 82 by 41, we obtain *no* rem^r. The *last divisor*, 41, is the G.C.F.

```
1517)5986(3
     4551
     ----
     1435)1517(1
          1435
          ----
            82)1435(17
                 82
                ---
                615
                574
                ---
                 41)82(2
                    82
                    ==
```

Ans. 41.

The above method depends upon the following principles:—

(I.) *Every common factor of two numbers is a factor of the* **sum** *of* **any multiples** *of those numbers.*

For instance, as 7 is a common factor of 14 and 21,
$\therefore$ 7 is a factor of $14 \times \mathbf{4} + 21 \times \mathbf{3}$, *i.e.* of 119;
also 7 is a factor of $14 \times \mathbf{5} + 21 \times \mathbf{1}$, *i.e.* of 91; and so on.

(II.) *Every common factor of two numbers is a factor of the* **difference** *of* **any multiples** *of those numbers.*

For instance, as 7 is a common factor of 14 and 49,
$\therefore$ 7 is a factor of $49 \times \mathbf{3} - 14 \times \mathbf{5}$, *i.e.* of 77;
also 7 is a factor of $49 \times \mathbf{1} - 14 \times \mathbf{2}$, *i.e.* of 21; and so on.

Hence, in Example iii., it follows from (II.) that every common factor of 5986 and 1517
is a factor of $5986 \times 1 - 1517 \times 3$, *i.e.* of 1435.

And from (I.) that every common factor of 1517 and 1435 is a factor of $1517 \times 3 + 1435 \times 1$, *i.e.* of 5986.

$\therefore$ the G.C.F. of 1517 and 5986 is also the G.C.F. of 1435 and 1517.

In other words, the *second* pair (divisor and dividend) have the same G.C.F. as the *first* pair. Similarly for the *third* pair, &c.

Hence, finally, we conclude that the G.C.F. of 1517 and 5986 is *the same* as that of 41 and 82, which is evidently **41**.

In the same way the G.C.F. of *three*, or more, numbers may be obtained by first finding the G.C.F. of two of them; then the G.C.F. of *the first result* and the *third number;* and so on.

EXAMPLE iv.—*Find the G.C.F. of* 3157, 3731, *and* 4715.

```
3157)3731(1                           287)4715(16
     3157                                 287
      574)3157(5                          1845
          2870                            1722
1st result, 287)574(2                      123)287(2
                574                            246
                                                41)123(3
                                 Ans. 41.          123
```

Note.—If the abridged method of division (see page 15) be employed, Example iii. would be worked thus:—

Mental Work.

1st step.
3 times 7 is 21, 21 from 26, 5.
Carry 2.
3 times 1 is 3, 3 and 2 is 5, 5 from 8, 3.
3 times 5 is 15, 15 from 19, 4.
Carry 1.
3 times 1 is 3, 3 and 1 is 4, 4 from 5, 1.
&c.

Written Work.

```
1517)5986(3
   1435)1517(1
         82)1435(17
             615
      Ans. 41)82(2
```

The work of finding a G.C.F. may often be much condensed by a judicious use of the principles (I.) and (II.) on page 55, combined with a knowledge of the Tests of Divisibility on page 52.

Thus (i) Any factors which *evidently* divide *all* the given numbers may be first removed and *reserved* as factors of the G.C.F.

(ii) Any factor which evidently divides *one* or more of the given numbers, but does not divide *all* of them, may be *rejected* at any stage of the work, as it cannot be a factor of the G.C.F.

(iii) As the remainder resulting from any of the subtractions contains the G.C.F., our object is to make this remainder as *small* as we can. Hence it is often convenient to *reverse* the order in such subtractions, when by so doing the difference which results is smaller than would be obtained by following the ordinary process of division.

The examples which follow illustrate such modifications of the process.*

---

* It is best for young beginners to confine their attention to the methods of Examples i. ii. iii. and iv.

EXAMPLE v.—*Find the G.C.F. of* 9063 *and* 14787.

Here we observe that 9 is a factor of both the numbers (see page 52). We therefore first divide by 9 and reserve 9 as a factor of the G.C.F.

We next operate upon 1007 and 1643.

1st step.

Double 1007 and subtract (reverse way) 1643 from it, thus obtaining a difference, 371, less than what would have been obtained by taking 1007 from 1643.

2nd step.

Treble 371 and subtract (reverse way) 1007 from it, obtaining the difference 106.

3rd step.

Treble 106 and subtract from 371, obtaining the difference 53.

4th step.

Double 53 and subtract from 106, when no remainder occurs.

9	9063	14787
	1007	1643
		2014
	1113	371
	106	318
	106	53

$9 \times 53 = 477$ *Ans.*

Hence the G.C.F. of 1007 and 1643 is 53,

$\therefore$ the G.C.F. of 9063 and 14787 is 9×53, *i.e.* 477.

EXAMPLE vi.—*Find the G.C.F. of* 40457 *and* 420325.

Here we first reject 25 from 420325, for we see that 5 cannot be a factor of the G.C.F.

Dividing 420325 by 25; *i.e.* multiplying 420325 by 4 and cutting off the last two figures (see p. 19), we obtain quotient 16813.

We next operate upon 16813 and 40457.

1st step.

Double 16813, and subtract from 40457, obtaining remr 6831.

2nd step.

Treble 6831 and subtract (reverse way) 16813 from the result, obtaining remr 3680.

3rd step.

Reject in succession the factors 10, 4, 4, none of which contain a factor of the G.C.F. (for the two given numbers are *odd*, and do not both end in 5); we now know that the reqd G.C.F. is either 23 or 1, for 23 is a *prime.*

4th step.

Divide 6831 by 23, when no remainder occurs. $\therefore$ 23 is the reqd G.C.F.

	420325	
	4	
	16813,00	40457
	20493	33626
4	368,0	6831(297
4	92	46
	23	223
		207
		161
		161

Ans. 23.

Note.—For the G.C.F. of compound quantities see Chapter X.

IX. LEAST COMMON MULTIPLE.

The **Least Common Multiple** (L.C.M.) of two or more numbers is the *least* number which *contains* each of them exactly.

When the prime factors of the numbers are known those of their L.C.M. can be written down at once.

For instance, as $35 = 5 \times 7$, and $77 = 7 \times 11$, it is evident that *every* common multiple of 35 and 77 must contain each of the primes 5, 7, and 11.

$\therefore$ the *least* common multiple of 35 and 77 is $5 \times 7 \times 11$, or 385.

$$\begin{aligned} \text{Again, as } 36 &= 2 \times 2 \times 3 \times 3, \\ 40 &= 2 \times 2 \times 2 \times 5, \\ \text{and } 56 &= 2 \times 2 \times 2 \times 7, \end{aligned}$$

it is evident that in *every* common multiple of 36, 40, and 56
the prime **2** must occur as a factor *not less than* **three times** over,
" **3** " " " **twice** over,
and **5** and **7** " " " **once.**

$\therefore$ the *least* common multiple is $2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 7$, or 2520.

Hence all the different prime factors to be found in the numbers must appear in their L.C.M.

Also, if any particular prime factor occurs *twice over* in one or more of the numbers but does not occur *more than twice* over in any one of them, it *must occur twice*, but *must* **not** *occur more than twice* over in their L.C.M. And so on.

EXAMPLE i.—*Find, by resolving each of the numbers to its prime factors, the L.C.M. of* 126, 210, 231.

Here we observe that 3 is the only factor which occurs twice over in any one of the given numbers, and that no prime occurs more than twice over in any of the numbers.

$$\begin{array}{r|l} 2 & 126 \\ \hline 3 & 63 \\ \hline 3 & 21 \\ \hline & 7 \end{array} \qquad \begin{array}{r|l} 2 & 210 \\ \hline 5 & 105 \\ \hline 3 & 21 \\ \hline & 7 \end{array} \qquad \begin{array}{r|l} 3 & 231 \\ \hline 7 & 77 \\ \hline & 11 \end{array}$$

Hence the L.C.M. is $2 \times 3 \times 3 \times 7 \times 5 \times 11 = \underline{6930}$ *Ans.*

It is not always necessary to resolve each of the numbers *separately* into its prime factors, as it often happens that one or more of the numbers whose L.C.M. is required is contained in another of them.

For instance, in order to find the L.C.M. of 21, 35, 63, and 105, it is not necessary to consider 21 and 35, for these are both contained in 105, and,

consequently, *any number which contains* 105 *must also contain* 21 and 35. Here then we need only consider 63 and 105.

Now $63 = 7 \times 3 \times 3$, and $105 = 5 \times 3 \times 7$.

Hence the L.C.M. of 21, 35, 63, and 105 is $7 \times 3 \times 3 \times 5$, or 315.

In the case of numbers which can easily be resolved into their prime factors, the process is commonly arranged as follows:—

EXAMPLE ii.—*Find the L.C.M. of* 7, 8, 12, 14, 16, 24, 30, *and* 35.

1st line.

Cut out 7, 8, 12 as they are contained in 14, 16, 24, respectively.

Divide by 2, *i.e.* by any *prime* which is contained in *two at least* of the remaining numbers.

Bring down 35, which is not exactly divisible by 2, to the next line.

$$
\begin{array}{r|l}
2 & \cancel{7},\ \cancel{8},\ \cancel{12},\ 14,\ 16,\ 24,\ 30,\ 35 \\
\hline
2 & \cancel{7},\ 8,\ 12,\ 15,\ 35 \\
\hline
2 & 4,\ 6,\ 15,\ 35 \\
\hline
5 & 2,\ \cancel{3},\ 15,\ 35 \\
\hline
 & 2,\ 3,\ 7
\end{array}
$$

L.C.M. is $2 \times 2 \times 2 \times 5 \times 2 \times 3 \times 7$

$= 80 \times 21 = \underline{1680\ Ans.}$

2nd line.

Cut out 7, as it is contained in 35.
Divide again by 2.
Bring down 15 and 35 to the next line.

Continue this process until a line is reached in which *no two* numbers have *any common factor*.

Finally, multiply together all the divisors and the numbers in this last line, and the result is the L.C.M. required.

In many cases the above process may be dispensed with, for the factors of the L.C.M. can easily be found *by inspection*, and be written down one by one as follows:—

EXAMPLE iii.—*Find the L.C.M. of* 15, 21, 27, 30, 35, *and* 42.

Written Work.

$3 \times 5 \times 7 \times 3 \times 3 \times 2 = 21 \times 90 = \underline{1890\ Ans.}$

Mental Work.

1st step. As $15 = 3 \times 5$, we write down $\mathbf{3 \times 5}$, as two factors of the L.C.M.

2nd step. As $21 = 3 \times 7$, and we already have a 3 set down, we append **7** only.

3rd step. As $27 = 3 \times 3 \times 3$, and we already have one 3 set down, we now need but *two* more, and so append $\mathbf{3 \times 3}$.

4th step. As $35 = 5 \times 7$, and as we already have both these factors written down, we append no fresh factor at this stage.

5th step. As $42 = 7 \times 3 \times 2$, we need only to append the fresh factor **2**.

Finally, $3 \times 5 \times 7 \times 3 \times 3 \times 2 = 21 \times 9 \times 10 = 1890$.

When the numbers whose L.C.M. is required are not easily resolved into factors *at sight*, we first find their G.C.F., by help of which the factors of their L.C.M. are found.

For instance, in order to find the L.C.M. of 391 and 437, we first find their G.C.F., namely, 23; we then obtain by division, $391 = 23 \times 17$ and $437 = 23 \times 19$.

Thus the required L.C.M. is $23 \times 19 \times 17 = 437 \times 17 = 7429$.

It follows from the above that *the L.C.M. of* **two** *numbers is equal to their product divided by their G.C.F.*

For the L.C.M. of 391 and 437 is

$$\begin{aligned} & 437 \times 17 \\ = {} & 437 \times 391 \div 23 \\ = {} & \text{product} \div \text{G.C.F.} \end{aligned}$$

Hence, to find the L.C.M. of **two** large numbers we first find their G.C.F., then *divide one of the given numbers by their G.C.F., and multiply the quotient obtained by the other given number.*

EXAMPLE iv.—*Find the L.C.M. of* 1333 *and* 1767.

```
1333)1767(1
     1333
      434)1333(3
          1302
    G.C.F. 31)434(14
              31
              124
              124
```

```
31)1333(43
   124
    93
    93

  1767
    43
  5301
 7068
L.C.M. = 75981 Ans.
```

Here we first find the G.C.F., namely, 31; we then divide 1333 by 31, obtaining quotient 43; finally, we multiply 1767 by 43, obtaining the L.C.M. 75981.

EXAMPLE v.—*Find the L.C.M. of* 1353, 1517, *and* 2829.

Here the G.C.F. is found to be 41.
Dividing each of the given numbers by 41, we obtain

$$\begin{aligned} 1353 &= 41 \times 33 = \mathbf{41} \times 11 \times 3 \\ 1517 &= \mathbf{41} \times 37 \\ 2829 &= 41 \times 69 = \mathbf{41} \times 23 \times 3 \end{aligned}$$

Hence the L.C.M. is $\mathbf{41} \times 37 \times 23 \times 11 \times 3$
$= 1517 \times 23 \times 33 = \underline{1151403}$ *Ans.*

Note.—In this example, in order to save space, the work done in finding the G.C.F., and in the succeeding divisions and multiplications, is not given. The student, however, should *in all cases show all the written work.*

X. MISCELLANEOUS EXAMPLES.

In order to find the G.C.F., or L.C.M., of *compound quantities*, we must express them all in *one single denomination*, and then find the G.C.F., or L.C.M., in that denomination.

A (i). *Find the largest sum of money which is exactly contained in both* £23, 3*s.* 6*d.* *and* £14, 3*s.* 3*d.*

In other words, find the G.C.F. of £23, 3*s.* 6*d.* and £14, 3*s.* 3*d.*

Here we choose the common denomination *threepences* in which to express both quantities.

We find that £23, 3*s.* 6*d.* = 1854 *threepences*,
and £14, 3*s.* 3*d.* = 1133 *threepences.*

We next find the G.C.F. of 1854 and 1133, namely, 103.

Hence the G.C.F. of 1854 *threepences* and 1133 *threepences* is 103 *threepences* = £1, 5*s.* 9*d.* *Ans.*

A (ii). *Find the smallest debt which could be exactly discharged either in dollars* (4*s.* 2*d.*), *half-crowns, florins, shillings, francs* (10*d.*), *or threepences.*

The reqd sum of money is the L.C.M. of a dollar, a half-crown, a florin, a shilling, a franc, and a threepence;

i.e. the L.C.M. of 50*d.*, 30*d.*, 24*d.*, 12*d.*, 10*d.*, and 3*d.*
$= 5 \times 5 \times 2 \times 3 \times 2 \times 2$ *pence* = 50*s.* = £2, 10*s.* *Ans.*

B (i). *Find the greatest number which will divide both* 4659 *and* 1068 *and leave the remainder* 4 *in each case.*

We first subtract 4 from each number, obtaining 4655 and 1064.
We then find the G.C.F. of 4655 and 1064, namely, 133.
i.e. 133 is the greatest number which divides 4655 and 1064 *without remr.*
∴ 133 is the greatest number which divides both 4659 and 1068, *with remr* 4.

B (ii). *Find the least number which, when divided by* 35 *yields remainder* 34, *when divided by* 42 *yields remainder* 41, *and when divided by* 56 *yields remainder* 55.

The L.C.M. of 35, 42, and 56 is $5 \times 7 \times 3 \times 2 \times 4 = 70 \times 12 = 840$;
i.e. 840 is *exactly* divisible by each of the numbers 35, 42, 56.

Hence the reqd number is $840 - 1 = 839$ *Ans.*

C. *Prove that* 421 *and* 536 *are prime to each other.*

In other words, show that their G.C.F. is *unity.*

D. *Find* (i) *the multiple of* 67 *which is nearest to* 10000. *Also* (ii) *the multiple of* 67 *which is nearest to* 1000.

(i) Dividing 10000 by 67 we obtain quotient 149 and remr 17. Now 17 is *less than half* of 67.
Hence, the reqd multiple is $10000 - 17$, or 9983 *Ans.* (i).

$$
\begin{array}{r}
67\,)\,10000\,(\,149 \\
\underline{67} \\
330 \\
\underline{268} \\
620 \\
\underline{603} \\
17
\end{array}
$$

(ii) Again, from the above division, we see that $1000 \div 67$ gives quotient 14, and a remr 62, which is *more than half* of 67. (*i.e.* 1000 falls short of containing 67, just fifteen times by 5).

Hence, in this case, the reqd multiple is $1000 + 5 =$ 1005 *Ans.* (ii).

When the sum, or difference, of products or quotients is required, we may often save labour if we detect the presence of a *common factor.* For instance:—

E (i). *Find the sum of* 87×23 *and* 29×37.

Here, instead of multiplying 87 by 23 and 29 by 37, and then adding the results, we observe that 29 is a common factor of 87 and 29. We, therefore, *first add* 3×23 to 37, and *afterwards multiply* the result by 29. (See Theorem II. on p. 10.)

Thus, $87 \times 23 + 29 \times 37 = 29 \times (69 + 37) = 29 \times 106 =$ 3074 *Ans.*

E (ii). *Find the difference between* $24552 \div 62$ *and* $11563 \div 31$.

Here, as 31 is a common factor of both divisors, we first find the difference between $24552 \div 2$ and 11563, and *afterwards divide* the result by 31.

Thus, $24552 \div 62 - 11563 \div 31 = (12276 - 11563) \div 31$
$= 713 \div 31 =$ 23 *Ans.*

F (i). *Find all the divisors of* 1155.

Resolve 1155 into prime factors: then, if we include 1 and 1155, we see that there are, in all, *sixteen* divisors, namely:—

1, 5, 3, 7, 11,
5×3, 5×7, 5×11, 3×7, 3×11, 7×11,
$5 \times 3 \times 7$, $5 \times 3 \times 11$, $5 \times 7 \times 11$, $3 \times 7 \times 11$,
and $5 \times 3 \times 7 \times 11$.

$$
\begin{array}{r|l}
5 & 1155 \\ \hline
3 & 231 \\ \hline
7 & 77 \\ \hline
 & 11
\end{array}
$$

i.e. 1, 3, 5, 7, 11; 15, 35, 55, 21, 33, 77; 105, 165, 231, 385; 1155.

F (ii). *Find all the common factors of* 3003 *and* 1287.

Resolve into prime factors.

Hence *all* the factors common to both numbers are

3, 11, 13;

3×11, 3×13, 11×13;

and $3 \times 11 \times 13$.

$$\begin{array}{r|l} 3 & 3003 \\ \hline 7 & 1001 \\ \hline 11 & 143 \\ \hline & 13 \end{array} \qquad \begin{array}{r|l} 3 & 1287 \\ \hline 3 & 429 \\ \hline 11 & 143 \\ \hline & 13 \end{array}$$

i.e. 3, 11, 13, 33, 39, 143, and 429.

G. *The G.C.F. of two numbers is* 440; *their L.C.M. is* 19360; *one of the numbers is* 1760; *find the other.*

The L.C.M. of two numbers = their product ÷ their G.C.F. (See p. 60.)

∴ the product of the numbers = their L.C.M. × their G.C.F.

Hence the product of the req^d numbers = 19360×440.

But one of the numbers is 1760;

$$\begin{aligned} \therefore \text{ the other is } & (19360 \times 440) \div 1760 \\ &= (19360 \times 440) \div (4 \times 440) \\ &= 19360 \div 4^* \\ &= \underline{4840 \textit{ Ans.}} \end{aligned}$$

H. *The continued product of three consecutive numbers is* 35904; *find them.*

Resolving into factors we find that

$$\begin{aligned} 35904 &= 4 \times 4 \times 4 \times 3 \times 11 \times 17 \\ &= 4 \times 4 \times 2 \times 3 \times 11 \times 17 \times 2 \\ &= (4 \times 4 \times 2) \times (3 \times 11) \times (2 \times 17) \\ &= \underline{32 \times 33 \times 34 \textit{ Ans.}} \end{aligned}$$

$$\begin{array}{r|l} 4 & 35904 \\ \hline 4 & 8976 \\ \hline 4 & 2244 \\ \hline 3 & 561 \\ \hline 11 & 187 \\ \hline & 17 \end{array}$$

K. *By what number must* 23958 *be multiplied that the product may be a perfect cube?*

[A *perfect* cube is a number which can be expressed as the continued product of *three equal* factors. See page 12.]

Resolving into prime factors we find that

$$23958 = 2 \times 3^2 \times 11^3.$$

Hence, $23958 \times \mathbf{2^2 \times 3} = 2^3 \times 3^3 \times 11^3$,

$$= (66)^3.$$

∴ the req^d multiplier is $2^2 \times 3$, or $\underline{12 \textit{ Ans.}}$

$$\begin{array}{r|l} 2 & 23958 \\ \hline 9 & 11979 \\ \hline 11 & 1331 \\ \hline & 121 \end{array}$$

---

* It would be a waste of labour to multiply 19360 by 440 and then divide the result by 1760; for 1760 is 440×4, and to multiply by 440 and afterwards to divide by 440×4 is equivalent to simply dividing by 4.

XI. FRACTIONS.

NOTATION.

If any *unit* (or *quantity*) be divided into a number of *equal parts*, one or more of those parts is called a **Fraction** of that unit (or quantity).

If the whole be divided into

two	equal parts,	the parts are	called	*halves;*
three	,,	,,	,,	*thirds;*
four	,,	,,	,,	*fourths*, or *quarters;*
five	,,	,,	,,	*fifths;*
six	,,	,,	,,	*sixths;*

and so on.

For instance, the abstract fraction **Three-eighths** indicates that, *unity* being divided into *eight* equal parts, *three* of those parts are taken; and **Eleven-sixteenths** indicates that, *unity* being divided into *sixteen* equal parts, *eleven* of those parts are taken.

Again, **Seven-twentieths of £1** indicates that, the concrete unit £1 being divided into *twenty* equal parts, *seven* of those parts are taken;

hence, as £1 = 20 shillings,
∴ *one*-twentieth of £1 is 1 shilling,
and *seven*-twentieths of £1 is 7 shillings.

Also, **Nine-tenths of 4 tons** indicates that, the quantity 4 tons being divided into *ten* equal parts, *nine* of those parts are taken;

hence, as 4 tons = 80 cwts.,
∴ *one*-tenth of 4 tons is 8 cwts.,
and *nine*-tenths of 4 tons is 72 cwts., or 3 tons 12 cwts.

Any fraction may be represented in figures by two numbers, one of which is written above the other, with a line between them:

The *lower* number is called the **Denominator** (*i.e.* the *Namer*);

for it shows into *how many equal parts the unit is divided; i.e.* it *names* the kind of part, whether *fifths, twelfths,* &c.

The *upper* number is called the **Numerator** (*i.e.* the *Counter*); for it shows how many of these parts there are in the fraction; *i.e.* it *counts* the number of *fifths*, *twelfths*, &c., as the case may be, which form the fraction.

For instance, *Three-eighths* is written thus: $\frac{3}{8}$; *Eleven-sixteenths*, thus: $\frac{11}{16}$.

Here 8 and 16 are *denominators*, showing that unity is divided into *eight* and *sixteen* equal parts respectively;

and 3 and 11 are *numerators*, showing that the fractions consist of *three* and of *eleven* parts of such size as their respective denominators indicate.

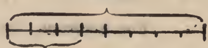
Fractions represented in the manner described above are called **Vulgar** (Latin *vulgus*, common) Fractions.*

A number regarded as made up of units *each of which units is unbroken* and entire is called a *whole* number, or **Integer**; (Latin *integer*, whole), as distinguished from a Fraction (Latin *fractus*, broken).

Note.—A convenient way of illustrating the definition of a fraction is to take a unit of *length.*

For instance, if *one inch* be regarded as the unit, then $\frac{3}{8}$ of an inch indicates the length shown in the accompanying diagram.

1 inch.



$\frac{3}{8}$ of an inch.

Another practical illustration, which has also the advantage of allowing each pupil to experiment for himself, is to provide a number of sheets of paper of the same size and shape. These are the units, and each pupil may easily, by folding, divide a sheet into *fourths*, and be then called upon to exhibit, first one fourth, then three fourths. Then from another set of sheets *eighths*, or *sixteenths*, may be obtained.

The way in which the sheets are divided may also be varied; on one occasion each sheet being divided into the same number of equal squares, on another occasion, into the same number of equal strips. Also the sheets need not be actually separated into parts; the division can be indicated by the creases made in folding them, so that *five-eighths* (for instance) may be exhibited as a single piece of paper.

In the same way, by dividing several sheets each into the same number of equal parts, simple experimental proofs of the processes of Chapter XII. may easily be devised.

---

* In contradistinction to *Decimal* fractions, which, as we shall see later, are represented in another manner.

XII. FRACTIONS.

Proper and Improper Fractions; Mixed Numbers.

It is evident that, if we divide a unit into any number of equal parts, **all** these parts together make up the complete unit.

For instance, Seven *sevenths* is unity, *i.e.* $\frac{7}{7} = 1$.

Similarly, $\frac{10}{10} = 1$; $\frac{19}{19} = 1$; $\frac{31}{31} = 1$; and so on.

Also, if we divide *two, three,* &c., units each into the same number of equal parts, **all** these parts together make up *two, three,* &c., complete units respectively.

For instance, $\frac{14}{7} = 2$; $\frac{20}{10} = 2$; $\frac{38}{19} = 2$; and so on.

Also, $\frac{21}{7} = 3$; $\frac{30}{10} = 3$; $\frac{57}{19} = 3$; and so on.

Hence, a fraction whose numerator exactly contains its denominator represents an *integer*, and this integer is the quotient obtained by dividing the numerator by the denominator.

Again, if the *numerator* of a fraction be *less* than the *denominator*, the fraction must be *less than unity:*

For instance, just as 5 *shillings* is less than 7 *shillings*,
so 5 *sevenths* is less than 7 *sevenths*, or unity,

i.e. $\frac{5}{7}$ is less than $\frac{7}{7}$, or 1.

But if the *numerator* be *greater* than the *denominator*, the fraction must be *greater than unity.*

For instance, 9 *sevenths* is greater than 7 *sevenths*, or unity,

i.e. $\frac{9}{7}$ is greater than $\frac{7}{7}$, or 1.

A fraction whose numerator is *less* than its denominator is called a **proper fraction.**

For instance, $\frac{1}{5}$, $\frac{2}{3}$, $\frac{7}{11}$ and $\frac{19}{20}$ are all *proper* fractions.

A fraction whose numerator is *greater* than its denominator is called an **improper fraction.**

For instance, $\frac{7}{5}$, $\frac{4}{3}$, $\frac{16}{11}$ and $\frac{21}{20}$ are all *improper* fractions.

Hence a *proper* fraction is *less*, and an *improper* fraction is *greater*, than unity.

A number which is partly integral and partly fractional is called a **mixed** number.

For instance, $2\frac{4}{7}$ (read "Two and four-sevenths") is a *mixed* number.

A mixed number can always be expressed as an improper fraction:

For instance, $2\frac{4}{7} = 2$ units $+ 4$ sevenths of a unit;
and as each unit contains 7 *sevenths*,
$\therefore$ 2 *units* + 4 *sevenths* = 14 *sevenths* + 4 *sevenths* = 18 *sevenths*;

$$i.e.\ 2\tfrac{4}{7} = \tfrac{18}{7}.$$

Hence, to express a mixed number as an improper fraction, *we multiply the integer by the denominator, and add the product to the numerator, of the fractional part of the mixed number;* the result is the numerator of the improper fraction, and its denominator is the same as that of the fractional part of the mixed number.

Conversely, an improper fraction can always be expressed as a mixed number:

For instance, $\frac{11}{5}$ is 11 *fifths* = 10 *fifths* + 1 *fifth;*
and as 10 *fifths* is 2 complete units,
$\therefore$ 10 *fifths* + 1 *fifth* = 2 *units* + 1 *fifth;* *i.e.* $\frac{11}{5} = 2\frac{1}{5}$.

Hence, to express an improper fraction as a mixed number, *we divide the numerator by the denominator;* the *quotient* obtained is the *integer*, the *remainder* is the *numerator* of the *fractional* part of the mixed number, and its denominator is the same as that of the improper fraction.

EXAMPLE i.—*Express* $17\frac{5}{23}$ *as an improper fraction.*

The product of 17 and 23 is 391.

$$\text{Hence } 17\tfrac{5}{23} = \frac{391+5}{23} = \frac{396}{23} \quad \underline{Ans.}$$

```
 23
 17
---
161
23
---
391
```

EXAMPLE ii.—*Express* $\frac{684}{53}$ *as a mixed number.*

Dividing 684 by 53 we obtain quotient 12 and rem^r 48.

$$\text{Hence } \frac{684}{53} = 12\tfrac{48}{53} \quad \underline{Ans.}$$

```
53)684(12
   53
   ---
   154
   106
   ---
    48
```

XIII. FRACTIONS.

REDUCTION.

Whatever be the number of equal parts into which we divide the unit, *each part must be twice as large* as it would have been if the unit had been divided into *twice that number of equal parts.*

For instance, 4 *fifths* = twice 4 *tenths* = 8 *tenths;*

$$i.e.\ \frac{4}{5} = \frac{4 \times 2}{5 \times 2} = \frac{8}{10}.$$

Also, *each part must be three times as large* as it would have been if the unit had been divided into *three times that number of parts.*

For instance, 4 *fifths* = three times 4 *fifteenths* = 12 *fifteenths.*

$$i.e.\ \frac{4}{5} = \frac{4 \times 3}{5 \times 3} = \frac{12}{15}.$$

And so on.

Hence the following important principles:—

(I.) *The value of a fraction is not altered by* **multiplying** *both its numerator and denominator by the same number.*

$$\text{For instance, } \frac{3}{7} = \frac{3 \times 5}{7 \times 5} = \frac{15}{35};$$

$$\text{Similarly, } \frac{3}{7} = \frac{3 \times 11}{7 \times 11} = \frac{33}{77};$$

$$\text{also } \frac{3}{7} = \frac{3 \times 29}{7 \times 29} = \frac{87}{203}; \text{ and so on.}$$

i.e. the fractions $\frac{3}{7}$, $\frac{15}{35}$, $\frac{33}{77}$, $\frac{87}{203}$ are all equal to one another.

And conversely,

(II.) *The value of a fraction is not altered by* **dividing** *both its numerator and denominator by the same number.*

$$\text{For instance, since } \frac{3}{7} = \frac{3 \times 5}{7 \times 5} = \frac{15}{35};$$

$$\therefore \text{, conversely, } \frac{15}{35} = \frac{15 \div 5}{35 \div 5} = \frac{3}{7};$$

$$\text{Similarly, } \frac{33}{77} = \frac{33 \div 11}{77 \div 11} = \frac{3}{7};$$

$$\text{also } \frac{87}{203} = \frac{87 \div 29}{203 \div 29} = \frac{3}{7}; \text{ and so on.}$$

Thus we see that the *same portion* of a unit may be represented in innumerable ways.

The process of changing the denominator is called *Reduction* of Fractions.

EXAMPLE i.—*Express* 7 *with denominator* 4.

As $1 = 4$ *fourths*

$\therefore 7 = 7$ times 4 *fourths* $= 28$ *fourths.*

i.e. $7 = \frac{28}{4}$ *Ans.*

EXAMPLE ii.—*Express* $\frac{7}{13}$ *with the denominator* 481.

Dividing 481 by 13 we find that 13 is contained exactly 37 times in 481.

$$\begin{array}{r} 13)481(37 \\ \underline{39} \\ 91 \\ \underline{91} \end{array}$$

Hence $\frac{7}{13} = \frac{7 \times 37}{13 \times 37} = \frac{259}{481}$ *Ans.*

EXAMPLE iii.—*Express* $\frac{7}{9}$ *with numerator* 602.

Dividing 602 by 7 we find that 7 is contained exactly 86 times in 602.

Hence $\frac{7}{9} = \frac{7 \times 86}{9 \times 86} = \frac{602}{774}$ *Ans.*

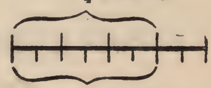
EXAMPLE iv.—*Find the fraction with denominator* 7 *which is equal to* $\frac{129}{301}$.

Dividing 301 by 7 we find that 7 is contained exactly 43 times in 301.

Hence $\frac{129}{301} = \frac{129 \div 43}{301 \div 43} = \frac{3}{7}$ *Ans.*

Note.—The equality of two fractions may be illustrated *visibly* in various ways.

$\frac{3}{4}$ of an inch.



$\frac{6}{8}$ of an inch.

For instance, if we take an *inch* as unit, it is evident from the accompanying diagram that $\frac{3}{4}$ of an inch, and $\frac{6}{8}$ of an inch, both represent the same length.

Or, if we take several sheets of paper of the same size and shape and regard these as units, they may easily, by folding, be divided into *fourths, eighths, sixteenths,* &c. respectively, and the equality of two such fractions as $\frac{3}{4}$ and $\frac{12}{16}$, or of $\frac{5}{8}$ and $\frac{10}{16}$, &c., may then be visibly demonstrated by superposition.

XIV. FRACTIONS.

Reduction to Lowest Terms.

The numerator and denominator of a fraction are called its *Terms.*

To reduce a fraction to its *lowest* terms is to represent the fraction by means of as small a numerator and denominator as possible.

Now we know, from Chapter XIII., that if we divide both numerator and denominator of a fraction by the same number, the value of the fraction remains unaltered.

Hence, so long as the numerator and denominator of the fraction have a *common factor* we can, by dividing both of them by that factor, reduce the fraction to *lower* terms; and the fraction will be in its *lowest* terms when *all* such common factors have been removed.

Thus a fraction is in its **lowest terms** when its numerator and denominator have **no common factor.**

Hence, to reduce a fraction to its lowest terms, *we divide both its numerator and denominator*

either (i) *by their G.C.F.;*
or (ii) *by the factors of their G.C.F. in succession.*

We adopt the latter method when common factors of the numerator and denominator can be easily found by inspection.

For instance, the fraction $\frac{35}{80}$ is evidently not in its lowest terms, for 35 and 80 are both divisible by 5.

$$\text{Hence } \frac{35}{80} = \frac{35 \div 5}{80 \div 5} = \frac{7}{16}.$$

And as 7 and 16 have no common factor, the fraction is now in its lowest terms.

Example i.—*Reduce $\frac{72}{1368}$ to its lowest terms.*

Mental Work.

1st step.

As 9 is evidently a common factor of 72 and 1368 (see p. 52), we divide both by 9, obtaining quotients 8 and 152 respectively.

2nd step.

As 8 is evidently a common factor of 8 and 152, we divide both by 8, obtaining quotients 1 and 19 respectively.

Written Work.

$$\frac{72}{1368} = \frac{8}{152}$$

$$= \frac{1}{19} \; \underline{Ans.}$$

EXAMPLE ii.—*Express the fraction* $\frac{7254}{9126}$ *in its lowest terms.*

Written Work.

$$\frac{7254}{9126} = \frac{3627}{4563} = \frac{403}{507} = \frac{31}{39} \text{ \underline{Ans.}}$$

Mental Work.

1st step.

As 7254 and 9126 are even, we divide both by 2, obtaining quotients 3627 and 4563.

2nd step.

As 3627 and 4563 are evidently both divisible by 9, we divide both by 9, obtaining quotients 403 and 507.

3rd step.

We know [by (ii), page 55] that if 403 and 507 have a common factor that common factor divides their difference 104. Now $104 = 8 \times 13$. Hence as 403 and 507 are both *odd*, we know that if they have any common factor it must be 13.

We find, on trial, that 13 *is* a factor of both numbers, and on dividing 403 and 507 by 13 we obtain quotients 31 and 39.

Also, as 31 is a prime, the fraction must now be in its lowest terms.

Note.—In such cases as this the beginner must be careful not to fall into the common error of *assuming that a pair of odd numbers*, such as 403 and 507, *are necessarily prime to each other*, or, though he may succeed in reducing the given fraction to *lower* terms, he will fail to reduce it to its *lowest* terms.

EXAMPLE iii.—*Reduce* $\frac{6157}{10349}$ *to its lowest terms.*

Here no small common factors of numerator and denominator are apparent.

We therefore perform the operation of finding the G.C.F. of 6157 and 10349, namely, 131.

$$\text{Hence } \frac{6157}{10349} = \frac{6157 \div 131}{10349 \div 131} = \frac{47}{79} \text{ \underline{Ans.}}$$

Note.—In order to save space, the work done in finding the G.C.F. has been omitted here. The student, however, is reminded that, in an examination, no part of the written work should be omitted.

EXAMPLE iv.—*Simplify* $\frac{7007}{3003}$.

Here it is evident, by inspection, that 1001 is the G.C.F. of 7007 and 3003.

$$\text{Hence } \frac{7007}{3003} = \frac{7}{3} = 2\tfrac{1}{3} \text{ \underline{Ans.}}$$

XV. FRACTIONS.

Reduction to the Least Common Denominator.

We know that if we multiply both numerator and denominator of a fraction by any number the value of the fraction is not altered. Hence, by choosing suitable multipliers, it is possible to reduce several fractions having different denominators to equivalent fractions having the same denominator.

For instance, as $\frac{3}{5} = \frac{3 \times 7}{5 \times 7} = \frac{21}{35}$, and $\frac{4}{7} = \frac{4 \times 5}{7 \times 5} = \frac{20}{35}$
$\therefore \frac{3}{5}$ and $\frac{4}{7}$, when reduced to the *common* denominator 35, become respectively $\frac{21}{35}$ and $\frac{20}{35}$.

It is evident that the *common denominator* must be a *multiple* of each of the denominators of the given fractions; also that if the common denominator is as small as possible it must be the **least** *common multiple* of the denominators of the given fractions.

For instance, the least common denominator for the fractions $\frac{2}{3}$, $\frac{3}{4}$ and $\frac{5}{6}$ is the L.C.M. of 3, 4 and 6, *i.e.* 12.

Now, as we have seen in Chapter XIII., to discover the number by which we must multiply both numerator and denominator in order to reduce the fraction $\frac{2}{3}$ to *twelfths*, we divide the new denominator 12 by the denominator of the given fraction, 3, and the quotient, 4, is the required multiplier.

Thus, $\frac{2}{3} = \frac{2 \times 4}{3 \times 4} = \frac{8}{12}$. Similarly, $\frac{3}{4} = \frac{9}{12}$; and $\frac{5}{6} = \frac{10}{12}$.

The letters L.C.D. are sometimes used for the words Least Common Denominator.

Example i.—*Reduce the fractions* $\frac{3}{8}$, $\frac{7}{16}$ *and* $\frac{11}{24}$ *to their least common denominator.*

The L.C.M. of 8, 16 and 24 is **48.**

Now $48 \div 8 = 6$; hence $\frac{3}{8} = \frac{3 \times 6}{8 \times 6} = \frac{18}{48}$;

Again, $48 \div 16 = 3$; hence $\frac{7}{16} = \frac{7 \times 3}{16 \times 3} = \frac{21}{48}$;

Also $48 \div 24 = 2$; hence $\frac{11}{24} = \frac{11 \times 2}{24 \times 2} = \frac{22}{48}$.

Ans. $\frac{18}{48}$, $\frac{21}{48}$, $\frac{22}{48}$.

EXAMPLE ii.—*Reduce* $\frac{9}{20}$, $\frac{13}{24}$ *and* $\frac{23}{56}$ *to their L.C.D.*

The L.C.M. of 20, 24 and 56 is found to be 840.

$$\left.\begin{array}{l} \text{Now } 840 \div 20 = 42; \text{ hence } \frac{9}{20} = \frac{9 \times 42}{20 \times 42} = \frac{378}{840} \\ \text{Again, } 840 \div 24 = 35; \text{ hence } \frac{13}{24} = \frac{13 \times 35}{24 \times 35} = \frac{455}{840} \\ \text{Also } 840 \div 56 = 15; \text{ hence } \frac{23}{56} = \frac{23 \times 15}{56 \times 15} = \frac{345}{840} \end{array}\right\} \underline{\textit{Ans.}}$$

Note.—Labour is saved by writing the common denominator in factors as in the following Example, for by so doing the quotients resulting from the division of the common denominator by the denominators of the fractions are obtained at *sight*.

EXAMPLE iii.—*Reduce* $\frac{11}{14}$, $\frac{13}{18}$ *and* $\frac{16}{21}$ *to their least common denominator.*

Mental Work.	*Written Work.*
1st step. By inspection, the L.C.M. of 14, 18 and 21 is $\mathbf{7 \times 2 \times 3 \times 3}$.	$\frac{11}{14}, \frac{13}{18}, \frac{16}{21}$
2nd step. $7 \times 2 \times 3 \times 3 \div 14 = 9$ (for $7 \times 2 \times 3 \times 3$ is evidently 14×9, and $9 \times 14 \div 14$ is 9); $11 \times 9 = \mathbf{99}$.	$\frac{99,\ 91,\ 96}{7 \times 2 \times 3 \times 3}$.
3rd step. $7 \times 2 \times 3 \times 3 \div 18 = 7$; $13 \times 7 = \mathbf{91}$.	$\frac{99,\ 91,\ 96}{126}$ <u>*Ans.*</u>
4th step. $7 \times 2 \times 3 \times 3 \div 21 = 6$; $16 \times 6 = \mathbf{96}$.	

We can *compare the size* of fractions if we reduce them to a common denominator.

For instance, $\frac{3}{5} = \frac{21}{35}$, and $\frac{4}{7} = \frac{20}{35}$, $\therefore \frac{3}{5}$ is greater than $\frac{4}{7}$; for 21 *thirty-fifths* is evidently greater than 20 *thirty-fifths.*

EXAMPLE iv.—*Compare* $\frac{5}{7}$, $\frac{16}{21}$ *and* $\frac{23}{30}$.

Mental Work.	*Written Work.*
The L.C.M. of 7, 21, 30 is $\mathbf{7 \times 3 \times 10}$.	$\frac{5}{7}, \quad \frac{16}{21}, \quad \frac{23}{30}$
$7 \times 3 \times 10 \div 7 = 30$; $5 \times 30 = \mathbf{150}$.	$\frac{150,\ 160,\ 161}{7 \times 3 \times 10}$
$7 \times 3 \times 10 \div 21 = 10$; $16 \times 10 = \mathbf{160}$.	
$7 \times 3 \times 10 \div 30 = 7$; $23 \times 7 = \mathbf{161}$.	<u>*Ans.*</u> $\frac{5}{7}$ is least; $\frac{23}{30}$ is greatest.

Note.—Thus the fractions, as they stand in the question, are arranged in *ascending order* of magnitude; *i.e.* the least first, and so on.

XVI. ADDITION OF FRACTIONS.

The principle involved in Addition of Fractions is the same as in all other additions; namely, the numbers added must all bear the same *name.*

For instance, just as, in Simple Addition,
3 *tens* + 4 *hundreds* make neither 7 *tens* nor 7 *hundreds*,
and, in Compound Addition,
3 *shillings* + 4 *pence* make neither 7 *shillings* nor 7 *pence*,
so, in Fractions,
3 *fifths* + 4 *tenths* make neither 7 *fifths* nor 7 *tenths*.
But just as, in Simple Addition, 3 *tens* + 4 *tens* make 7 *tens*,
and, in Compound Addition, 3 *shillings* + 4 *shillings* make 7 *shillings*,
so, in Fractions, 3 *fifths* + 4 *fifths* make 7 *fifths*.

Hence if the fractions to be added have different denominators, *we first reduce them to their* **least common denominator,** and then *add the resulting numerators.*

For instance, 3 *fifths* + 2 *sevenths* = 21 *thirty-fifths* + 10 *thirty-fifths*
= 31 *thirty-fifths*.

i.e. $\frac{3}{5}+\frac{2}{7}=\frac{21}{35}+\frac{10}{35}=\frac{31}{35}$.

EXAMPLE i.—*Add* $\frac{7}{12}$, $\frac{11}{16}$ *and* $\frac{13}{20}$ *together.*

1st step.

The L.C.M. of 12, 16 and 20 is **240.**

$240 \div 12 = 20$; $7 \times 20 = $ **140.**
$240 \div 16 = 15$; $11 \times 15 = $ **165.**
$240 \div 20 = 12$; $13 \times 12 = $ **156.**

2nd step.

Adding across we obtain the sum **461.**

As 461 is evidently not divisible by 2, 5 or 3, the only factors contained in the den^r, we see that the result cannot be reduced to *lower* terms.

3rd step.

Finally we reduce the improper fraction $\frac{461}{240}$ to a mixed number.

Written Work.

$$\frac{7}{12}+\frac{11}{16}+\frac{13}{20}$$
$$=\frac{140+165+156}{240}$$
$$=\frac{461}{240}$$
$$=1\frac{221}{240}\ \underline{\textit{Ans.}}$$

N.B.—The answer should always be expressed in its **lowest terms,** and when an improper fraction results it should be reduced to a **mixed number.**

As was pointed out in the last chapter, we may often save labour by writing the common denominator in *factors.*

EXAMPLE ii.—*Find the sum of* $\frac{17}{28}$, $\frac{5}{12}$ *and* $\frac{11}{42}$.

1st step.

The L.C.M. of 28, 12, 42 is $4 \times 7 \times 3$.

$4 \times 7 \times 3 \div 28 = 3;\ 17 \times 3 = \mathbf{51},$
$4 \times 7 \times 3 \div 12 = 7;\ \ 5 \times 7 = \mathbf{35},$
$4 \times 7 \times 3 \div 42 = 2;\ 11 \times 2 = \mathbf{22}.$

2nd step.

Adding across, we obtain the sum **108**.

3rd step.

Dividing both num^r and den^r of the fraction $\frac{108}{4 \times 7 \times 3}$ by 12, we reduce this fraction to its lowest terms $\frac{9}{7}$, &c.

Written Work.

$$\frac{17}{28} + \frac{5}{12} + \frac{11}{42}$$
$$= \frac{51 + 35 + 22}{4 \times 7 \times 3}$$
$$= \frac{108}{4 \times 7 \times 3}$$
$$= \frac{9}{7}$$
$$= 1\tfrac{2}{7}\ \textit{Ans.}$$

In adding **mixed numbers**, the *integers* and *fractions* should be dealt with *separately*. The mixed numbers should *not* be reduced to improper fractions, for this involves much needless labour. We do not reduce all the *pence* to *farthings* before attempting an addition of money!

EXAMPLE iii.—*Add together* $27\frac{5}{8}$, $4\frac{5}{12}$ *and* $11\frac{13}{24}$.

$$27\tfrac{5}{8} + 4\tfrac{5}{12} + 11\tfrac{13}{24} = 42 + \frac{15 + 10 + 13}{24}$$
$$= 42 + \frac{38}{24}$$
$$= 42 + \frac{19}{12}$$
$$= 42 + 1\tfrac{7}{12} = 43\tfrac{7}{12}\ \textit{Ans.}$$

If any of the fractions to be added are not in lowest terms, or are improper fractions, it is generally best to reduce them *before beginning the addition.*

EXAMPLE iv.—*Add* $3\frac{13}{15}$, $\frac{238}{25}$, $1\frac{49}{175}$, $\frac{104}{80}$.

$$3\tfrac{13}{15} + \frac{238}{25} + 1\tfrac{49}{175} + \frac{104}{80} = 3\tfrac{13}{15} + 9\tfrac{13}{25} + 1\tfrac{7}{25} + 1\tfrac{3}{10}$$
$$= 14\frac{130 + 78 + 42 + 45}{5 \times 3 \times 5 \times 2}$$
$$= 14\frac{295}{5 \times 3 \times 5 \times 2}$$
$$= 14\tfrac{59}{30} = 15\tfrac{29}{30}\ \textit{Ans.}$$

XVII. SUBTRACTION OF FRACTIONS.

As in Addition, so in Subtraction, we first reduce the given fractions to their least common denominator.

EXAMPLE i.—*From* $\frac{5}{7}$ *take* $\frac{3}{5}$.

$$\frac{5}{7} - \frac{3}{5} = \frac{25 - 21}{35}$$
$$= \frac{4}{35} \text{ Ans.}$$

EXAMPLE ii.—*Take* $\frac{3}{8}$ *from* $\frac{7}{12}$.

$$\frac{7}{12} - \frac{3}{8} = \frac{14 - 9}{24}$$
$$= \frac{5}{24} \text{ Ans.}$$

In the subtraction of **mixed numbers** the *integers* and *fractions* should be treated *separately*.

EXAMPLE iii.—*Find the difference between* $17\frac{13}{21}$ *and* $43\frac{17}{24}$.

$$43\tfrac{17}{24} - 17\tfrac{13}{21} = 26\frac{119 - 104}{8 \times 3 \times 7}$$
$$= 26\frac{15}{8 \times 3 \times 7}$$
$$= 26\tfrac{5}{56} \text{ Ans.}$$

In the subtraction of mixed numbers it often happens that the *fractional* part to be subtracted is greater than the fractional part of the mixed number from which it is to be taken: in which case we *change a unit* of the latter into fractional parts—just as, in Compound Subtraction, we change, if need be, a *penny* into *farthings.*

EXAMPLE iv.—*Take* $19\frac{11}{12}$ *from* $25\frac{3}{8}$.

Here, after subtracting the integers and reducing the fractions to their L.C.D. we obtain the result $6\frac{9-22}{24}$.

Now 22 *twenty-fourths* cannot be taken from 9 *twenty-fourths.* We therefore *change one* of the 6 units into *twenty-fourths.*

Note.—After a little practice the step marked with an asterisk may be performed mentally, and would then be omitted from the written work.

Written Work.

$$25\tfrac{3}{8} - 19\tfrac{11}{12}$$
$$= 6\frac{9 - 22}{24}$$
$$= 5\frac{24 + 9 - 22}{24} *$$
$$= 5\tfrac{11}{24} \text{ Ans.}$$

EXAMPLE v.—*From* 17 *take* $12\frac{3}{11}$.

Mental Work.

3 *elevenths* from 11 *elevenths* (*i.e.* unity) leaves 8 *elevenths.*

12 from 16 leaves 4.

Written Work.

$$17 - 12\tfrac{3}{11}$$
$$= 4\tfrac{8}{11} \text{ Ans.}$$

EXAMPLE vi.—*Take* $\frac{1881}{8181}$ *from* $\frac{6703}{101}$.

$$\frac{6703}{101}-\frac{1881}{8181}=66\frac{37}{101}-\frac{209}{909}$$
$$=66\frac{333-209}{909}$$
$$=66\frac{124}{909}\ \textit{Ans.}$$

$$\begin{array}{r} 101)6703(66 \\ 606 \\ \hline 643 \\ 606 \\ \hline 37 \end{array}$$

When Addition and Subtraction are both involved in the same question the method of procedure is similar to that explained on page 8.

EXAMPLE vii.—*Simplify* $7\frac{3}{5}-2\frac{1}{2}+5\frac{7}{10}-8\frac{11}{20}$.

$$7\frac{3}{5}-2\frac{1}{2}+5\frac{7}{10}-8\frac{11}{20}=2\frac{12-10+14-11}{20}$$
$$=2\frac{5}{20}$$
$$=2\frac{1}{4}\ \textit{Ans.}$$

EXAMPLE viii.—*Simplify* $3\frac{1}{4}-7\frac{11}{12}+1\frac{5}{48}-8\frac{15}{32}+16$.

$$3\frac{1}{4}-7\frac{11}{12}+1\frac{5}{48}-8\frac{15}{32}+16=5\frac{24-88+10-45}{96}$$
$$=5\frac{34-133}{96}$$
$$=3\frac{192+34-133}{96}*$$
$$=3\frac{93}{96}$$
$$=3\frac{31}{32}\ \textit{Ans.}$$

The work of Addition may be lessened by utilizing Subtraction, if any of the fractions are *very nearly equal to unity.*

EXAMPLE ix.—*Add* $7\frac{23}{24}$, $11\frac{5}{48}$ *and* $5\frac{15}{16}$.

Here, as $7\frac{23}{24}$ differs from the integer 8 by only $\frac{1}{24}$, we write $8-\frac{1}{24}$ in place of $7\frac{23}{24}$.

Similarly, instead of $5\frac{15}{16}$ we write $6-\frac{1}{16}$.

$$7\frac{23}{24}+11\frac{5}{48}+5\frac{15}{16}$$
$$=8-\frac{1}{24}+11\frac{5}{48}+6-\frac{1}{16}$$
$$=25+\frac{5}{48}-\frac{1}{24}-\frac{1}{16}$$
$$=25\frac{5-2-3}{48}$$
$$=25\ \textit{Ans.}$$

* In this Example it was necessary to change 2 units into fractional parts.

XVIII. FRACTIONS.

Multiplication and Division by an Integer.

From the definition of Multiplication on page 9, it follows that *to multiply a fraction by an* **integer** *is to repeat the fraction as many times as there are units in that integer.*

For instance, 3 times two-*sevenths* is six-*sevenths; i.e.* $\frac{2}{7} \times 3 = \frac{6}{7}$.

Thus a *fraction is* **multiplied** *by an* **integer** *if its* **numerator** *be multiplied by that integer.*

This multiplication we at first merely *indicate by the sign*, and then, *before completing the operation, remove from the resulting numerator and denominator any factors common to both*, in order that the product may be in its lowest terms.

For instance, $\frac{5}{12} \times 9 = \frac{5 \times 9}{12} = \frac{5 \times 3}{4}$ (dividing numr and denr by 3)

$$= \frac{15}{4} = 3\frac{3}{4}.$$

Example i.—*Multiply* $\frac{55}{252}$ by 189.

As 189 and 252 are evidently divisible by 9, we divide both by 9, and then "cancel" them, writing the quotients 21 and 28 above and below them respectively.

Then, as 21 and 28 are both divisible by 7, we cancel them also, and write the quotients 3 and 4 above and below them respectively.

Thus the fraction has been reduced to its lowest terms, and replaced by $\frac{55 \times 3}{4}$, *i.e.* $\frac{165}{4}$, which we reduce to a mixed number.

Written Work.

$$\frac{55}{252} \times 189 = \frac{55 \times \overset{\overset{3}{\cancel{21}}}{\cancel{189}}}{\underset{\underset{4}{\cancel{28}}}{\cancel{252}}}$$

$$= \frac{165}{4}$$

$$= 41\frac{1}{4} \quad \textit{Ans.}$$

The process of thus removing factors common to both numerator and denominator is called **cancelling**.

A *mixed number* may be multiplied by an *integer* without reducing the mixed number to an improper fraction.

Example ii.—*Multiply* $105\frac{6}{13}$ *by* 4.

$$105\frac{6}{13} \times 4 = 105 \times 4 + \frac{6}{13} \times 4$$

$$= 420 + \frac{24}{13}$$

$$= 420 + 1\frac{11}{13} = 421\frac{11}{13} \quad \textit{Ans.}$$

Again, *to divide a fraction by an integer is to separate the fraction into an integral number of equal portions; i.e.* to reverse the process of multiplication.*

For instance, as 2 *sevenths* $\times$ 3 = 6 *sevenths*,

$\therefore$, conversely, 6 *sevenths* $\div$ 3 = 2 *sevenths*; *i.e.* $\frac{6}{7} \div 3 = \frac{2}{7}$.

Thus a fraction is divided by an integer if its numerator be divided by that integer.

This method, however, will only apply to the case of a numerator which *exactly contains* that integer.

For instance, we cannot thus divide $\frac{1}{8}$ by 3, for 3 is not contained in 1.

But as we know that $\frac{1}{8} = \frac{3}{24}$, $\therefore \frac{1}{8} \div 3 = \frac{3}{24} \div 3 = \frac{1}{24}$.

Now $\frac{1}{24} = \frac{1}{8 \times 3}$. Hence the quotient in this case is obtained by *multiplying* the *denominator* of the fraction by 3.

It is evident also that the quotient in the former case can be obtained in the same manner, for $\frac{2}{7} = \frac{2 \times 3}{7 \times 3} = \frac{6}{7 \times 3}$.

Thus *a fraction* is **divided** by an **integer** if its **denominator** *be multiplied by that integer.*

This process we at first *merely indicate by the sign*, and then, before completing the work, remove from the resulting num^r and den^r any factors common to both.

EXAMPLE iii.—*Divide* $\frac{91}{100}$ *by* 26.

Here we first indicate the multiplication of the denominator by 26 by the sign of multiplication.

Then as 91 and 26 are both divisible by 13, we cancel 91 and 26, writing the quotients 7 and 2 above and below them respectively.

Written Work.

$$\frac{91}{100} \div 26 = \frac{\overset{7}{\cancel{91}}}{100 \times \underset{2}{\cancel{26}}} = \frac{7}{200} \quad \underline{\textit{Ans.}}$$

In order to divide a **mixed number** by an integer we reduce the mixed number to an improper fraction.†

EXAMPLE iv.—*Divide* $34\frac{2}{9}$ *by* 352.

$$34\frac{2}{9} \div 352 = \frac{308}{9} \div 352 = \frac{\overset{\overset{7}{\cancel{77}}}{\cancel{308}}}{9 \times \underset{\underset{8}{\cancel{88}}}{\cancel{352}}} = \frac{7}{72} \quad \underline{\textit{Ans.}}$$

Here, in cancelling, we divide both num^r and den^r first by 4, then by 11.

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* Only one of the two ways of regarding Division (see pages 14 and 43) applies in all cases to the division of a *fraction* by an integer; for, if the fraction be less than the integer it evidently does not contain it *any number of times.*

† This is not *always necessary;* for instance, $8\frac{16}{17} \div 4 = 8 \div 4 + \frac{16}{17} \div 4 = 2 + \frac{4}{17} = 2\frac{4}{17}$.

Note.—Hence it follows that

(i) a fraction is *doubled* either by *doubling the num^r*, or by *halving the den^r*, according as the *den^r is odd, or even.*

(ii) a fraction is *halved* either by *halving the num^r*, or by *doubling the den^r*, according as the *num^r is even, or odd.*

For instance (i) *Twice* $\frac{3}{7}$ is $\frac{6}{7}$; *Twice* $\frac{3}{8}$ is $\frac{3}{4}$.

(ii) *Half* of $\frac{4}{7}$ is $\frac{2}{7}$; *Half* of $\frac{5}{7}$ is $\frac{5}{14}$.

If we multiply a mixed number by an integer equal to the denominator, the product which results is an integer.

For instance, $6\frac{4}{11} \times \mathbf{11} = 66 + 4 = \mathbf{70}$

$\therefore$, conversely, $70 \div 11 = 6\frac{4}{11}$;

i.e. $6\frac{4}{11}$ expresses the *complete quotient* resulting from the division.

Hence, in cases which admit of a non-integral quotient, *the remainder in an inexact division may be regarded as part of the quotient.*

For instance, "*Divide* 70 *into* 11 *equal parts,*" admits of the answer, "The size of each part is $6\frac{4}{11}$."

But the question, "*How* many times *does* 70 *contain* 11?" does not admit of a fractional answer.

Or, to take a concrete example, "*Divide* 70 *oranges equally among* 11 *boys,*" admits of the answer, "The share of each boy is $6\frac{4}{11}$ oranges."

But to the question, "*How many boys could each receive* 11 *oranges from a heap of* 70 *oranges?*", the answer is "6 boys, and 4 oranges would be left over;" for the answer "$6\frac{4}{11}$ boys" would be meaningless.

Hence, also, *a fraction may be regarded as indicating the* **division** *of the numerator by the denominator.*

For instance, $\frac{70}{11}$ indicates the division of 70 *units* into 11 *equal parts.*

Similarly, $\frac{3}{4}$ indicates the division of 3 *units* into 4 *equal parts.*

But, by definition of a fraction on page 64, $\frac{3}{4}$ also indicates the division of 1 *unit* into 4 equal parts, of which parts 3 are taken.

Hence, 3 *fourths* of 1 *unit* is equivalent to 1 *fourth* of 3 *units.*
And so on.

Or, to take a concrete example,
as one *tenth* of £1 is 2 *shillings,* $\therefore$ 3 *tenths* of £1 is 6 *shillings.*

But one *tenth* of £3 is also 6 *shillings,*

$$\therefore \frac{3}{10} \text{ of } £1 = \frac{1}{10} \text{ of } £3.$$

XIX. MULTIPLICATION OF FRACTIONS.

A fraction as defined on page 64 is called a *Simple* Fraction: *i.e.* a simple fraction is a fraction of a *unit.*

A Fraction *of a fraction* is called a **Compound Fraction.**

For instance, $\frac{3}{5}$ of $\frac{2}{7}$ is a compound fraction.

Now, one *fifth* of $\frac{2}{7}$ is found by *dividing* $\frac{2}{7}$ into 5 equal parts, and we know by Chapter XVIII., that one of these parts is $\frac{2}{7 \times 5}$.

And as three fifths of $\frac{2}{7}$ must be 3 times as great as one fifth of $\frac{2}{7}$,

$\therefore$ $\frac{3}{5}$ of $\frac{2}{7}$ is found by *multiplying* $\frac{2}{7 \times 5}$ by 3, and this product we know is found by multiplying the numerator of the fraction by 3;

$$i.e.\ \frac{3}{5} \text{ of } \frac{2}{7} = \frac{2 \times 3}{7 \times 5} = \frac{6}{35}.$$

Hence a *compound* fraction is converted into a simple fraction by *multiplying the numerators of the fractions together, and the denominators together.*

It follows, from (I.) on page 9, that the value of a compound fraction is not altered by reversing the places of the fractions.

For instance, $\frac{3}{5}$ of $\frac{2}{7} = \frac{2 \times 3}{7 \times 5} = \frac{3 \times 2}{5 \times 7} = \frac{2}{7}$ of $\frac{3}{5}$.

The operation of finding the value of *a fraction of a fraction* is also indicated by placing the *sign of multiplication* between the fractions, and the result is then called the *product* of the fractions.

For instance, $\frac{2}{7} \times \frac{3}{5}$ means $\frac{2 \times 3}{7 \times 5}$, or $\frac{6}{35}$.

But it should be noticed that the definition of Multiplication on page 9 only applies to the case of an *integral multiplier*, and that "multiplication" **by a fraction** includes *division, as well as multiplication, as defined for integers.**

For instance, to multiply, in the fractional sense, $\frac{1}{3}$ by $\frac{1}{4}$, is equivalent to *dividing* $\frac{1}{3}$ by 4.

Again, to multiply $\frac{2}{7}$ by $\frac{3}{5}$, is equivalent to *dividing* $\frac{2}{7}$ by 5, and then *multiplying* the result by 3.

Consequently the number so "multiplied" is not necessarily *increased*, but is decreased, or increased, according as the fractional *multiplier is a proper, or an improper*, fraction.

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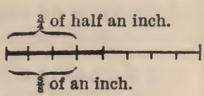
* For a wider definition of Multiplication, applicable alike to fractions as well as integers, see Appendix.

Note.—The accompanying diagram illustrates the rule of this chapter, for each small part is evidently an *eighth* of *a whole inch*, but a *fourth* of *half an inch.*

Hence, 3 *fourths* of *half* an inch = 3 *eighths* of a *whole* inch.



$$i.e.\ \frac{3}{4} \text{ of } \frac{1}{2} = \frac{3 \times 1}{4 \times 2} = \frac{3}{8},$$

when an inch is taken as unit.

EXAMPLE i.—*Simplify* $\frac{15}{28}$ *of* $\frac{16}{25}$.

Here, as before, we do not actually multiply 15 by 16, and 28 by 25, but merely indicate the operation by the sign, until we have, by cancelling, reduced the fraction $\frac{15 \times 16}{28 \times 25}$ to its lowest terms. This consists in dividing 15 and 25 both by 5, and 16 and 28 both by 4, when we obtain the final result $\frac{3 \times 4}{7 \times 5}$; *i.e.* $\frac{12}{35}$.

$$\frac{15}{28} \text{ of } \frac{16}{25}$$

$$= \frac{\overset{3}{\cancel{15}} \times \overset{4}{\cancel{16}}}{\underset{7}{\cancel{28}} \times \underset{5}{\cancel{25}}}$$

$$= \frac{12}{35}\ \underline{\textit{Ans.}}$$

When mixed numbers occur we convert them to improper fractions:—

EXAMPLE ii.—*Multiply* $6\frac{6}{7}$ *by* $2\frac{13}{18}$.

$$6\frac{6}{7} \times 2\frac{13}{18} = \frac{\overset{8}{\cancel{48}}}{\underset{1}{\cancel{7}}} \times \frac{\overset{7}{\cancel{49}}}{\underset{3}{\cancel{18}}} = \frac{56}{3} = 18\frac{2}{3}\ \underline{\textit{Ans.}}$$

Here, having reduced the mixed numbers to improper fractions, the 48 above, and the 18 below, are cancelled and replaced by 8, and 3, respectively. Also the 49 above, and the 7 below, by 7, and 1, respectively.

Similarly, a *fraction* of *a compound fraction* is reduced to a simple fraction.

For instance, $\frac{2}{3}$ of $\frac{2}{5}$ of $\frac{1}{7} = \frac{2}{3}$ of $\frac{2}{35} = \frac{4}{105}$.

$$i.e.\ \frac{2}{3} \text{ of } \frac{2}{5} \text{ of } \frac{1}{7} = \frac{2 \times 2 \times 1}{3 \times 5 \times 7} = \frac{4}{105}.$$

Here, as before, the operation is also indicated by placing signs of multiplication between the fractions, when the result is called the *continued* product of the fractions.

For instance, $\frac{2}{3} \times \frac{2}{5} \times \frac{1}{7} = \frac{2 \times 2 \times 1}{3 \times 5 \times 7} = \frac{4}{105}$.

EXAMPLE iii.—*Simplify* $1\frac{1}{15}$ *of* $3\frac{1}{5}$ *of* $1\frac{7}{8}$ *of* $\frac{25}{36}$.

We first reduce the mixed numbers to improper fractions.

Then the 15 above and the 15 below are cancelled and replaced by 1 and 1; the 16 above and 8 below are replaced by 2 and 1 respectively; the 16 above and 36 below by 4 and 9 respectively; the 25 above and 5 below by 5 and 1 respectively.

Thus the required fraction expressed in lowest terms is $\frac{2\times4\times1\times5}{1\times1\times1\times9}$; *i.e.* $\frac{40}{9}$ or $4\frac{4}{9}$.

$$1\frac{1}{15} \text{ of } 3\frac{1}{5} \text{ of } 1\frac{7}{8} \text{ of } \frac{25}{36}$$

$$= \frac{\overset{2}{\cancel{16}}}{\underset{1}{\cancel{15}}} \times \frac{\overset{4}{\cancel{16}}}{\underset{1}{\cancel{5}}} \times \frac{\overset{1}{\cancel{15}}}{\underset{1}{\cancel{8}}} \times \frac{\overset{5}{\cancel{25}}}{\underset{9}{\cancel{36}}}$$

$$= \frac{40}{9}.$$

$$= 4\frac{4}{9}\ \underline{\textit{Ans.}}$$

EXAMPLE iv.—*Find the continued product of* $\frac{7}{19}$, $\frac{2}{27}$, $2\frac{1}{4}$ *and* $5\frac{3}{7}$.

$$\frac{7}{19} \times \frac{2}{27} \times 2\frac{1}{4} \times 5\frac{3}{7} = \frac{\overset{1}{\cancel{7}}}{\underset{1}{\cancel{19}}} \times \frac{\overset{1}{\cancel{2}}}{\underset{3}{\cancel{27}}} \times \frac{\overset{1}{\cancel{9}}}{\underset{\underset{1}{\cancel{2}}}{\cancel{4}}} \times \frac{\overset{\overset{1}{\cancel{2}}}{\cancel{38}}}{\underset{1}{\cancel{7}}} = \frac{1}{3}\ \underline{\textit{Ans.}}$$

Note.—The beginner is advised not to omit to *write down* "1" when that number replaces a number cancelled, or he may fall into the serious error of omitting it in the answer, and of writing (for instance) "Ans. 3," instead of "Ans. $\frac{1}{3}$."

EXAMPLE v.—*Express* $\frac{171}{187}$ *of* $\frac{682}{868} \times \frac{204}{209}$ *in its simplest form.*

As 682 and 868 are even, we divide them both by 2.

As 204 and 434 are also even, we divide them both by 2.

On inspection we observe that 341 and 209 are both divisible by 11, we ∴ divide both by 11.

Now, as 217 is evidently divisible by 31 we cancel 31 and 217, replacing them by 1 and 7 respectively.

We find, on trial, that 171 is divisible by 19.

Finally, 102 and 187 are both divisible by 17, and the required fraction is $\frac{9\times1\times6}{11\times7\times1}$ or $\frac{54}{77}$.

$$\frac{\overset{9}{\cancel{171}}}{\underset{11}{\cancel{187}}} \times \frac{\overset{\overset{\overset{1}{\cancel{31}}}{\cancel{341}}}{\cancel{682}}}{\underset{\underset{\underset{7}{\cancel{217}}}{\cancel{434}}}{\cancel{868}}} \times \frac{\overset{\overset{6}{\cancel{102}}}{\cancel{204}}}{\underset{\underset{1}{\cancel{19}}}{\cancel{209}}} = \frac{54}{77}\ \underline{\textit{Ans.}}$$

XX. DIVISION OF FRACTIONS.

The *undoing* or reversing, of the process called "Multiplication of Fractions" is called *Division of Fractions.**

For instance, to *divide* $\frac{2}{11}$ by $\frac{5}{7}$, is to find a fraction which when multiplied by $\frac{5}{7}$, produces $\frac{2}{11}$.

i.e. the req^d quotient $\times \frac{5}{7} = \frac{2}{11}$.

But we know, by the last chapter, that $\frac{14}{55} \times \frac{5}{7} = \frac{2}{11}$.

$\therefore$ *the req^d quotient is* $\frac{14}{55}$.

Now $\frac{14}{55} = \frac{2 \times 7}{11 \times 5} = \frac{2}{11} \times \frac{7}{5} =$ *the dividend* $\times$ *the divisor* **inverted.**

Hence, *to divide any number by a fraction,* we **multiply** *the number by the fraction* **inverted.**

EXAMPLE i.—*Divide* $\frac{7}{8}$ *by* $\frac{9}{10}$.

$$\frac{7}{8} \div \frac{9}{10} = \frac{7}{\underset{4}{\cancel{8}}} \times \frac{\overset{5}{\cancel{10}}}{9} = \frac{35}{36} \quad \underline{Ans.}$$

If mixed numbers occur we reduce them to improper fractions.†

EXAMPLE ii.—*Divide* $9\frac{3}{8}$ *by* $2\frac{11}{12}$.

$$9\frac{3}{8} \div 2\frac{11}{12} = \frac{75}{8} \div \frac{35}{12} = \frac{\overset{15}{\cancel{75}}}{\underset{2}{\cancel{8}}} \times \frac{\overset{3}{\cancel{12}}}{\underset{7}{\cancel{35}}} = \frac{45}{14} = 3\frac{3}{14} \quad \underline{Ans.}$$

If *unity* be divided by any number, the quotient is called the **Reciprocal** of that number.

For instance, the *reciprocal* of 7 is $1 \div 7$, *i.e.* $\frac{1}{7}$;

the *reciprocal* of $\frac{4}{5}$ is $1 \div \frac{4}{5}$, *i.e.* $\frac{5}{4}$.

Hence the rule for Division of Fractions may be expressed thus:

Multiply the dividend by the **reciprocal** *of the divisor;*

in which form it includes the rule of Chapter XVIII.

* When we regard Division as the separation of the dividend into *a number of equal parts*, it is evident that we exclude the idea of a *fractional* divisor.

† It is not absolutely necessary always to reduce a mixed number in the *dividend* to an improper fraction: for instance, $16\frac{3}{8} \div \frac{4}{5} = 16\frac{3}{8} \times \frac{5}{4} = 16 \times \frac{5}{4} + \frac{3}{8} \times \frac{5}{4} = 20 + \frac{15}{32} = 20\frac{15}{32}$.

EXAMPLE iii.—*Divide* $3\frac{1}{4}$ *by* 5.

$$3\frac{1}{4} \div 5 = \frac{13}{4} \div 5 = \frac{13}{4} \times \frac{1}{5} = \frac{13}{20} \text{ Ans.}$$

EXAMPLE iv.—*Divide* $2\frac{1}{5}$ *by* $\frac{1}{7}$.

$$2\frac{1}{5} \div \frac{1}{7} = 2\frac{1}{5} \times 7 = 14 + 1\frac{2}{5} = 15\frac{2}{5} \text{ Ans.}$$

A **Complex Fraction** is a fraction the numerator and denominator of which are, one or both of them, *fractions.*

For instance, $\dfrac{\frac{3}{5}}{\frac{7}{8}}$, $\dfrac{5\frac{7}{8}}{9\frac{1}{3}}$, $\dfrac{2}{\frac{8}{11}}$ and $\dfrac{3\frac{1}{4}}{7}$ are *complex* fractions.

Now we know, by Chapter XVIII., that a fraction expresses the quotient of its numerator divided by its denominator.

Hence $\dfrac{\frac{3}{5}}{\frac{7}{8}} = \frac{3}{5} \div \frac{7}{8}$; $\dfrac{2}{\frac{8}{11}} = 2 \div \frac{8}{11}$; and $\dfrac{3\frac{1}{4}}{7} = 3\frac{1}{4} \div 7$.

EXAMPLE v.—*Simplify* $\dfrac{3\frac{2}{5}}{6\frac{3}{8}}$

$$\frac{3\frac{2}{5}}{6\frac{3}{8}} = \frac{\frac{17}{5}}{\frac{51}{8}} = \frac{\overset{1}{\cancel{17}}}{5} \times \frac{8}{\underset{3}{\cancel{51}}} = \frac{8}{15} \text{ Ans.}$$

EXAMPLE vi.—*Simplify* $\dfrac{3}{7\frac{1}{5}}$

$$\frac{3}{7\frac{1}{5}} = \frac{3}{\frac{36}{5}} = \overset{1}{\cancel{3}} \times \frac{5}{\underset{12}{\cancel{36}}} = \frac{5}{12} \text{ Ans.}$$

A complex fraction may also be simplified by *multiplying both its num^r^ and den^r^ by the* L.C.M. *of the den^rs^ of the simple fractions,* thus:—

EXAMPLE vii.—*Simplify* $\dfrac{4\frac{1}{5}}{9\frac{2}{3}}$.

$$\frac{4\frac{1}{5}}{9\frac{2}{3}} = \frac{4\frac{1}{5} \times 15}{9\frac{2}{3} \times 15} = \frac{60+3}{135+10} = \frac{63}{145} \text{ Ans.}$$

EXAMPLE viii.—*Simplify* $\dfrac{7\frac{1}{8}}{38}$.

$$\frac{7\frac{1}{8}}{38} = \frac{7\frac{1}{8} \times 8}{38 \times 8} = \frac{56+1}{38 \times 8} = \frac{\overset{3}{\cancel{57}}}{\underset{2}{\cancel{38}} \times 8} = \frac{3}{16} \text{ Ans.}$$

Note.—In *writing* complex fractions care must be taken to mark the *line of division* clearly, for such a fraction as $\begin{matrix}3\\ \overline{4}\\ 5\end{matrix}$ is ambiguous: we cannot tell whether $3 \div \frac{4}{5}$, or $\frac{3}{4} \div 5$ is intended.

XXI. FRACTIONS.

Simplification of Fractional Expressions.

In order to find correctly the value of an expression consisting of fractions connected in various ways, it is necessary clearly to understand, not only the *meaning* of the *symbols* employed, but also the *order* in which the *operations* indicated are intended to be performed. The work, moreover, should be arranged in an orderly manner. The following are points of special importance:—

(I.) *Fractions connected by the word* **of** *must be regarded as* **one single** *fraction.*

Example i.—*Simplify* $2\frac{3}{4} \div 1\frac{3}{8}$ *of* $\frac{5}{6}$.

$$2\frac{3}{4} \div 1\frac{3}{8} \text{ of } \frac{5}{6} = \frac{11}{4} \div \frac{11}{8} \text{ of } \frac{5}{6} = \frac{\overset{1}{\cancel{11}}}{\underset{1}{\cancel{4}}} \times \frac{\overset{2}{\cancel{8}} \times 6}{\underset{1}{\cancel{11}} \times 5} = \frac{12}{5} = 2\frac{2}{5} \text{ \underline{Ans.}}$$

N.B.—Here we invert *both* $\frac{11}{8}$ and $\frac{5}{6}$, for $\frac{11}{8}$ *of* $\frac{5}{6}$, *i.e.* $\frac{55}{48}$, is *one* fraction.*

Example ii.—*Simplify* $\frac{3}{8} + \frac{5}{6}$ *of* $\frac{3}{10}$.

$$\frac{3}{8} + \frac{\overset{1}{\cancel{5}}}{\underset{2}{\cancel{6}}} \text{ of } \frac{\overset{1}{\cancel{3}}}{\underset{2}{\cancel{10}}} = \frac{3}{8} + \frac{1}{4} = \frac{5}{8} \text{ \underline{Ans.}}$$

N.B.—Here we first find the value of $\frac{5}{6}$ *of* $\frac{3}{10}$, namely $\frac{1}{4}$, and *then* add *this result* to $\frac{3}{8}$. We *must not* add $\frac{5}{6}$ to $\frac{3}{8}$.

Example iii.—*Simplify* $2\frac{2}{3} - \frac{3}{5}$ *of* $\frac{5}{6} + \frac{1}{4}$ *of* $2\frac{1}{3}$.

$$2\frac{2}{3} - \frac{\overset{1}{\cancel{3}}}{\underset{1}{\cancel{5}}} \text{ of } \frac{\overset{1}{\cancel{5}}}{\underset{2}{\cancel{6}}} + \frac{1}{4} \text{ of } \frac{7}{3} = 2\frac{2}{3} - \frac{1}{2} + \frac{7}{12} = 2\frac{8-6+7}{12} = 2\frac{3}{4} \text{ \underline{Ans.}}$$

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* In such cases a distinction is made between "of" and the sign ×; for, if such an expression as $2\frac{3}{4} \div 1\frac{3}{8} \times \frac{5}{6}$ be met with, the operations are intended to be performed *in order from left to right*; *i.e.* $2\frac{3}{4} \div 1\frac{3}{8} \times \frac{5}{6}$ means that $2\frac{3}{4}$ is to be divided by $1\frac{3}{8}$, and *the result* multiplied by $\frac{5}{6}$.

(II.) *Multiplications and Divisions must be performed* **before** *Additions and Subtractions.*

EXAMPLE iv.—*Simplify* $1\frac{1}{3} \times \frac{5}{8} + \frac{3}{4}$.

$$1\frac{1}{3} \times \frac{5}{8} + \frac{3}{4} = \frac{\overset{1}{\cancel{4}}}{3} \times \frac{5}{\underset{2}{\cancel{8}}} + \frac{3}{4} = \frac{5}{6} + \frac{3}{4} = \frac{19}{12} = 1\frac{7}{12} \quad \underline{\underline{Ans.}}$$

N.B.—Here we *first multiply* $\frac{4}{3}$ by $\frac{5}{8}$ and *then add* the result to $\frac{3}{4}$. We *must not add* $\frac{5}{8}$ to $\frac{3}{4}$.

EXAMPLE v.—*Simplify* $5\frac{1}{2} - \frac{3}{16} \div 2\frac{1}{4} + \frac{5}{7} \times 1\frac{1}{6}$.

$$\begin{aligned} 5\frac{1}{2} - \frac{3}{16} \div 2\frac{1}{4} + \frac{5}{7} \times 1\frac{1}{6} &= 5\frac{1}{2} - \frac{\overset{1}{\cancel{3}}}{\underset{4}{\cancel{16}}} \times \frac{\overset{1}{\cancel{4}}}{\underset{3}{\cancel{9}}} + \frac{5}{\underset{1}{\cancel{7}}} \times \frac{\overset{1}{\cancel{7}}}{6} \\ &= 5\frac{1}{2} - \frac{1}{12} + \frac{5}{6} \\ &= 5\frac{6 - 1 + 10}{12} \\ &= 5\frac{15}{12} = 6\frac{1}{4} \quad \underline{\underline{Ans.}} \end{aligned}$$

EXAMPLE vi.—*Simplify* $\dfrac{1\frac{1}{2}}{2\frac{1}{3}} - \dfrac{3\frac{1}{5}}{2\frac{2}{3}} + \dfrac{4\frac{1}{2}}{2\frac{1}{4}}$.

1st Method.

$$\begin{aligned} &\frac{1\frac{1}{2}}{2\frac{1}{3}} - \frac{3\frac{1}{5}}{2\frac{2}{3}} + \frac{4\frac{1}{2}}{2\frac{1}{4}} \\ &= \frac{3}{2} \times \frac{3}{7} - \frac{\overset{2}{\cancel{16}}}{5} \times \frac{3}{\underset{1}{\cancel{8}}} + \frac{\overset{1}{\cancel{9}}}{\underset{1}{\cancel{2}}} \times \frac{\overset{2}{\cancel{4}}}{\underset{1}{\cancel{9}}} \\ &= \frac{9}{14} - 1\frac{1}{5} + 2 \\ &= 1\frac{45 - 14}{70} = 1\frac{31}{70} \quad \underline{\underline{Ans.}} \end{aligned}$$

2nd Method.

$$\begin{aligned} &\frac{1\frac{1}{2}}{2\frac{1}{3}} - \frac{3\frac{1}{5}}{2\frac{2}{3}} + \frac{4\frac{1}{2}}{2\frac{1}{4}} \\ &= \frac{1\frac{1}{2} \times 6}{2\frac{1}{3} \times 6} - \frac{3\frac{1}{5} \times 15}{2\frac{2}{3} \times 15} + \frac{4\frac{1}{2} \times 4}{2\frac{1}{4} \times 4}^{*} \\ &= \frac{9}{14} - \frac{48}{40} + \frac{18}{9} \\ &= \frac{9}{14} - 1\frac{1}{5} + 2 \\ &= 1\frac{45 - 14}{70} = 1\frac{31}{70} \quad \underline{\underline{Ans.}} \end{aligned}$$

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* See page 85, Example vii.

(III.) **Brackets** *indicate that their contents are to be regarded as* **one single** *number.* (See page 18.)

EXAMPLE vii.—*Simplify* $\left(\frac{3}{8}+\frac{5}{6}\right)$ *of* $\frac{3}{10}$.

$$\left(\frac{3}{8}+\frac{5}{6}\right) \text{ of } \frac{3}{10} = \frac{9+20}{24} \text{ of } \frac{3}{10} = \frac{29}{\underset{8}{\cancel{24}}} \text{ of } \frac{\overset{1}{\cancel{3}}}{10} = \frac{29}{80} \quad \underline{\textit{Ans.}}$$

EXAMPLE viii.—*Simplify* $\left(2\frac{1}{4}-1\frac{1}{5}\right)\div\left(5\frac{1}{6}+1\frac{1}{8}\right)$.

$$\left(2\frac{1}{4}-1\frac{1}{5}\right)\div\left(5\frac{1}{6}+1\frac{1}{8}\right) = 1\frac{1}{20}\div 6\frac{4+3}{24} = \frac{21}{\underset{5}{\cancel{20}}}\times\frac{\overset{6}{\cancel{24}}}{151} = \frac{126}{755} \quad \underline{\textit{Ans.}}$$

EXAMPLE ix.—*Simplify* $\frac{117}{285}$ *of* $1\frac{61}{91} - \frac{3\frac{2}{7}}{9\frac{1}{5}} \div \left(\frac{2}{3}+\frac{5}{6}\times\frac{8}{15}\right)+\frac{11}{20}$.

$$\frac{117}{285} \text{ of } 1\frac{61}{91} - \frac{3\frac{2}{7}}{9\frac{1}{5}} \div \left(\frac{2}{3}+\frac{\overset{1}{\cancel{5}}}{\underset{3}{\cancel{6}}}\times\frac{\overset{4}{\cancel{8}}}{\underset{3}{\cancel{15}}}\right)+\frac{11}{20}$$

$$= \frac{\overset{3}{\cancel{\overset{9}{\cancel{117}}}}}{\underset{5}{\cancel{\underset{95}{\cancel{285}}}}}\times\frac{\overset{8}{\cancel{152}}}{\underset{7}{\cancel{91}}} - \frac{\overset{1}{\cancel{23}}}{7}\times\frac{5}{\underset{2}{\cancel{46}}}\div\left(\frac{2}{3}+\frac{4}{9}\right)+\frac{11}{20}$$

$$= \frac{24}{35} - \frac{5}{14}\div\frac{10}{9} + \frac{11}{20}$$

$$= \frac{24}{35} - \frac{\overset{1}{\cancel{5}}}{14}\times\frac{9}{\underset{2}{\cancel{10}}} + \frac{11}{20}$$

$$= \frac{96-45+77}{5\times 7\times 4}$$

$$= \frac{\overset{32}{\cancel{128}}}{5\times 7\times \underset{1}{\cancel{4}}} = \frac{32}{35} \quad \underline{\textit{Ans.}}$$

In the case of expressions arranged in the form of complex fractions the thick line serves the purpose of brackets, indicating that the *value* of the expression which forms the *numerator* is to be *divided* by the *value* of that which forms the *denominator*.

EXAMPLE x.—*Simplify* $\dfrac{2\frac{1}{4}-1\frac{1}{5}}{5\frac{1}{6}+1\frac{1}{8}}$.

$$\frac{2\frac{1}{4}-1\frac{1}{5}}{5\frac{1}{6}+1\frac{1}{8}}=\frac{1\frac{1}{20}}{6\frac{4+3}{24}}=\frac{\frac{21}{20}}{\frac{151}{24}}=\frac{21}{\underset{5}{\cancel{20}}}\times\frac{\overset{6}{\cancel{24}}}{151}=\frac{126}{755}\ \textit{Ans.}$$

Note.—This question is identical with that of Example viii.

EXAMPLE xi.—*Simplify* $\dfrac{2\frac{1}{2}-1\frac{2}{3}}{3\frac{3}{4}\times 1\frac{1}{5}}\div\dfrac{\frac{7}{8}+1\frac{3}{4}}{4\frac{1}{2}\times 2\frac{1}{3}}$.

$$\frac{2\frac{1}{2}-1\frac{2}{3}}{3\frac{3}{4}\times 1\frac{1}{5}}\div\frac{\frac{7}{8}+1\frac{3}{4}}{4\frac{1}{2}\times 2\frac{1}{3}}=\frac{2\frac{1}{2}-1\frac{2}{3}}{3\frac{3}{4}\times 1\frac{1}{5}}\times\frac{4\frac{1}{2}\times 2\frac{1}{3}}{\frac{7}{8}+1\frac{3}{4}}=\frac{\frac{15-10}{6}}{\frac{15}{4}\times\frac{6}{5}}\times\frac{\frac{9}{2}\times\frac{7}{3}}{\frac{7+14}{8}}.$$

Here we begin by inverting the complex divisor bodily. Afterwards, before cancelling, all the three fractions $\frac{15}{4}$, $\frac{6}{5}$, $\frac{21}{8}$, which stand *below the thick lines*, are inverted.

$$=\frac{5}{\underset{3}{\cancel{6}}}\times\frac{\overset{1}{\cancel{\overset{2}{\cancel{4}}}}}{\underset{3}{\cancel{15}}}\times\frac{\overset{1}{\cancel{5}}}{\underset{3}{\cancel{6}}}\times\frac{\overset{1}{\cancel{\overset{3}{\cancel{9}}}}}{\underset{1}{\cancel{2}}}\times\frac{\overset{1}{\cancel{7}}}{\underset{1}{\cancel{3}}}\times\frac{\overset{4}{\cancel{8}}}{\underset{1}{\cancel{\underset{3}{\cancel{21}}}}}=\frac{20}{27}\ \textit{Ans.}$$

A fraction of the form $\dfrac{2}{3+\frac{1}{4}}$ is called a **Continued Fraction.**

In order to simplify such fractions, *we begin with the lowest part, and proceed, step by step, upwards.*

EXAMPLE xii.—*Simplify* $\dfrac{2}{3+\dfrac{1}{5+\frac{3}{4}}}$.

$$\frac{2}{3+\dfrac{1}{5+\frac{3}{4}}}=\frac{2}{3+\dfrac{1}{\frac{23}{4}}}=\frac{2}{3+\frac{4}{23}}=\frac{2}{\frac{73}{23}}=2\times\frac{23}{73}=\frac{46}{73}\ \textit{Ans.}$$

XXII. FRACTIONS.

Value of a Fraction of a Concrete Quantity.

It follows from the definition of a fraction that if we divide the given concrete quantity by the *denominator*, and multiply the result by the *numerator*, we obtain the value of the fraction.

For instance, $\frac{2}{5}$ of £10, 15*s.* is found if we divide £10, 15*s.* into 5 equal parts and multiply one of those parts by 2.

In practice, however, it is generally more convenient to reverse the order of these two operations; *i.e.* to multiply by the numerator first, and afterwards to divide by the denominator.*

Example i.—*Find the value of* $\frac{5}{8}$ *of* £7, 16*s.* 3*d.*

After dividing the *pence* by 8, there remain 7 pence over, which is expressed as part of the quotient (see page 80) by the fraction $\frac{7}{8}$ of a penny.

N.B.—We do not in such cases reduce the pence which are over to *farthings.*

$$\begin{array}{r|rcrcl}
 & \text{£} & & s. & & d. \\
 & 7 & . & 16 & . & 3 \\
 & & & & & 5 \\
\hline
8 & 39 & . & 1 & . & 3 \\
\hline
 & 4 & . & 17 & . & 7\frac{7}{8} \ \textit{Ans.}
\end{array}$$

Example ii.—*Multiply* £4, 2*s.* $3\frac{5}{6}$*d. by* $7\frac{3}{11}$.

Here we may either

(i) find $\frac{3}{11}$ of £4, 2*s.* $3\frac{5}{6}$*d.* and then *add the result* to 7 *times* £4, 2*s.* $3\frac{5}{6}$*d.*; or

(ii) reducing $7\frac{3}{11}$ to the improper fraction $\frac{80}{11}$, find $\frac{80}{11}$ of £4, 2*s.* $3\frac{5}{6}$*d.*

Adopting the latter method, we first multiply $\frac{5}{6}$*d.* by 10, obtaining $8\frac{1}{3}$*d.*, so we set down $\frac{1}{3}$ and "carry" 8, &c.

Also, when we reach the division of the pence by 11, there remain over $2\frac{2}{3}$*d.*, and $2\frac{2}{3} \div 11 = \frac{8}{3} \times \frac{1}{11} = \frac{8}{33}$.

$$\begin{array}{r|rcrcl}
 & \text{£} & & s. & & d. \\
 & 4 & . & 2 & . & 3\frac{5}{6} \\
 & & & & & 10 \\
\hline
 & 41 & . & 3 & . & 2\frac{1}{3} \\
 & & & & & 8 \\
\hline
11 & 329 & . & 5 & . & 6\frac{2}{3} \\
\hline
 & 29 & . & 18 & . & 8\frac{8}{33} \ \textit{Ans.}
\end{array}$$

Note.—It must be left to the judgment of the student to decide in any particular case which of the two above-mentioned methods to adopt. In this example the 2nd is evidently the better, but in many cases (for instance, in multiplying by $39\frac{1}{7}$) much labour would be wasted if the multiplier were reduced to an improper fraction.

---

* We know, from page 80, that the result is not altered by reversing the order of these operations, since it was there shown that $\frac{1}{4}$ of $3 = \frac{3}{4}$ of 1, &c.

EXAMPLE iii.—*Divide £27, 17s. $6\frac{3}{11}$d. by $5\frac{3}{10}$.*

Here we *must* reduce the *divisor* to an improper fraction.

Now (£27, 17*s.* $6\frac{3}{11}$*d.*) $\div 5\frac{3}{10}$
= (£27, 17*s.* $6\frac{3}{11}$*d.*) $\div \frac{53}{10}$
= (£27, 17*s.* $6\frac{3}{11}$*d.*) $\times \frac{10}{53}$.

Hence we multiply by 10 and divide the result by 53, obtaining the quotient £5, 5*s.* 2*d.*, with 16 pence remaining over and $\frac{8}{11}$*d.* not yet brought down.

Now $16\frac{8}{11}d. \div 53$
$= \frac{184}{11} \times \frac{1}{53} = \frac{184}{583}d.$

∴ the complete quotient is £5, 5*s.* $2\frac{184}{583}$*d.*

$$
\begin{array}{rl}
 & \text{£} \quad\ \ s. \quad\ \ d. \\
 & 27 \;.\; 17 \;.\; 6\frac{3}{11} \\
 & \qquad\qquad\quad 10 \\ \hline
53\,) & 278 \;.\; 15 \;.\; 2\frac{8}{11}\,(\underline{5}\ \text{£} \\
 & \underline{265} \\
 & 13 \\
 & \underline{20}\ s. \\
53\,) & 275\,(\underline{5} \\
 & \underline{265} \\
 & 10 \\
 & \underline{12}\ d. \\
53\,) & 122\,(\underline{2} \\
 & \underline{106} \\
 & 16\frac{8}{11} \div 53 \\
 & = \frac{184}{11} \times \frac{1}{53} = \frac{184}{583}.
\end{array}
$$

Ans. £5, 5*s.* $2\frac{184}{583}$*d.*

In many cases it is best to avoid *compound* multiplication and division, by expressing the given compound quantity in *one single denomination.*

EXAMPLE iv.—*Find the value of* $\dfrac{142\frac{1}{7}}{83\frac{3}{14}}$ *of* 9*s.* $8\frac{1}{2}$*d.*

$$\frac{142\frac{1}{7}}{83\frac{3}{14}} \text{ of } 116\tfrac{1}{2}d. = \frac{\overset{199}{\cancel{995}}}{\underset{1}{\cancel{7}}} \times \frac{\overset{\overset{1}{\cancel{2}}}{\cancel{14}}}{\underset{\underset{1}{\cancel{233}}}{\cancel{1165}}} \times \frac{\overset{1}{\cancel{233}}}{\underset{1}{\cancel{2}}}d. = 199d. = \underline{16s.\ 7d.\ \textit{Ans.}}$$

EXAMPLE v.—*Find the value of* $£235\frac{1}{3} \times \dfrac{3\frac{4\frac{1}{2}}{12}}{20}$.

$$£235\tfrac{1}{3} \times \frac{3\frac{4\frac{1}{2}}{12}}{20} = £\frac{706}{3} \times \frac{3\frac{9}{24}}{20} = £\frac{706}{3} \times \frac{3\frac{3}{8}}{20} = £\frac{\overset{353}{\cancel{706}}}{\underset{1}{\cancel{3}}} \times \frac{\overset{9}{\cancel{27}}}{8 \times \underset{10}{\cancel{20}}}$$

$$= £\frac{317{,}7}{8{,}0} = £39\tfrac{57}{80} = \underline{£39,\ 14s.\ 3d.\ \textit{Ans.}}$$

Note.—The value of £$\frac{57}{80}$ may easily be found *mentally*, for £$\frac{57}{80} = \frac{57}{4}s. = 14\frac{1}{4}s.$

XXIII. FRACTIONS.

To Express one Concrete Quantity as the Fraction of another of the same kind.

The method follows at once from the definition of a fraction.

For instance, as 20 *shillings* make £1,

$$\therefore 1 \textit{ shilling} \text{ is } \tfrac{1}{20} \text{ of £1,}$$
$$\therefore 3 \textit{ shillings} \text{ is } \tfrac{3}{20} \text{ of £1,}$$
$$\text{and } 17 \textit{ shillings} \text{ is } \tfrac{17}{20} \text{ of £1; and so on.}$$

Similarly, as 112 *lbs.* make 1 cwt.,

$$\therefore 1 \textit{ lb.} \text{ is } \tfrac{1}{112} \text{ of } 1 \textit{ cwt.}$$
$$\therefore 5 \textit{ lbs.} \text{ is } \tfrac{5}{112} \text{ of } 1 \textit{ cwt.}\text{; and so on.}$$

Hence, in order to express the first of two quantities as the fraction of a second quantity of the same kind, we express both quantities in terms of the same unit, *i.e.* we reduce them to a *common denomination*; then *the number of these units in the first gives the numerator, and the number of these units in the second gives the denominator* of the fraction which, when expressed in its lowest terms, is the fraction required.

Example i.—*Reduce* 5*s.* 4*d.* *to the fraction of* £1.

As 5*s.* 4*d.* = 16 *fourpences*,
and £1 = 60 *fourpences*,

$$\therefore \text{the req}^{\text{d}} \text{ fraction is } \tfrac{16}{60} = \tfrac{4}{15}.$$

Ans. 5*s.* 4*d.* is $\tfrac{4}{15}$ of £1.

Example ii.—*What fraction of* 3 *cwts. is* 2 *qrs.* 7 *lbs.?*

As 2 qrs. 7 lbs. = $2\tfrac{1}{4}$ *qrs.*,
and 3 cwts. = 12 *qrs.*,

$$\therefore \text{the req}^{\text{d}} \text{ fraction is } \frac{2\frac{1}{4}}{12} = \frac{9}{12 \times 4} = \frac{3}{16}.$$

Ans. 2 qrs. 7 lbs. is $\tfrac{3}{16}$ of 3 cwts.

N.B.—In this question the *order* in which the quantities are mentioned is *reversed*. The question is identical with the following:—

Reduce 2 *qrs.* 7 *lbs. to the fraction of* 3 *cwts.*

EXAMPLE iii.—*Express $\frac{5}{9}$ of £2, 15s. 9d. as the fraction of $\frac{2}{3}$ of $\frac{7}{9}$ of 12s. 6d.*

$$\tfrac{5}{9} \text{ of } £2,\ 15s.\ 9d. = \tfrac{5}{9} \text{ of } 55\tfrac{3}{4}s.,$$
$$\tfrac{2}{3} \text{ of } \tfrac{7}{9} \text{ of } 12s.\ 6d. = \tfrac{2}{3} \text{ of } \tfrac{7}{9} \text{ of } 12\tfrac{1}{2}s.,$$

$$\therefore \text{ the req}^{\text{d}} \text{ fraction is } \frac{\frac{5}{9} \times 55\frac{3}{4}}{\frac{2}{3} \times \frac{7}{9} \times 12\frac{1}{2}} = \frac{\overset{1}{\cancel{5}}}{\underset{1}{\cancel{9}}} \times \frac{223}{4} \times \frac{3}{\underset{1}{\cancel{2}}} \times \frac{\overset{1}{\cancel{9}}}{7} \times \frac{\overset{1}{\cancel{2}}}{\underset{5}{\cancel{25}}} = \frac{669}{140} \ \underline{Ans.}$$

EXAMPLE iv.—*What fraction of 18 tons is $\frac{3}{5}$ of 5 cwts. 3 qrs. 12 lbs.*

$$\tfrac{3}{5} \text{ of } 5 \text{ cwts. } 3 \text{ qrs. } 12 \text{ lbs.} = \tfrac{3}{5} \text{ of } 23\tfrac{3}{7}\ qrs.,$$
$$18 \text{ tons} = 18 \times 20 \times 4\ qrs.,$$

$$\therefore \text{ the req}^{\text{d}} \text{ fraction is } \frac{\frac{3}{5} \times 23\frac{3}{7}}{18 \times 20 \times 4} = \frac{\overset{1}{\cancel{3}}}{5} \times \frac{\overset{41}{\cancel{164}}}{7} \times \frac{1}{\underset{6}{\cancel{18}} \times 20 \times \underset{1}{\cancel{4}}}$$
$$= \frac{41}{4200} \ \underline{Ans.}$$

EXAMPLE v.—*Express 2 tons 13 cwts. 3 qrs. 12 lbs. 4 ozs. in cwts. and the fraction of a cwt.*

$$2 \text{ tons } 13 \text{ cwts. } 3 \text{ qrs. } 12 \text{ lbs. } 4 \text{ ozs.} = 53\frac{3\frac{12\frac{1}{4}}{28}}{4} \text{ cwts.}$$

$$= 53\frac{3\frac{7}{16}}{4} \text{ cwts.} = \underline{53\tfrac{55}{64} \text{ cwts. } Ans}$$

Here the first step results from the following *mental* process:—

$$\text{As 1 lb. is } \tfrac{1}{28} \text{ of 1 qr.,} \quad \therefore 12\tfrac{1}{4} \text{ lbs. is } \frac{12\frac{1}{4}}{28} \text{ qrs.;}$$

$$\text{and as 1 qr. is } \tfrac{1}{4} \text{ of 1 cwt.,} \quad \therefore 3\frac{12\frac{1}{4}}{28} \text{ qrs. is } \frac{3\frac{12\frac{1}{4}}{28}}{4} \text{ cwts.}$$

XXIV. MISCELLANEOUS EXAMPLES.

In questions which involve both Multiplication and Division much labour may often be saved by *at first merely indicating the operations by the signs and then* cancelling *before completing the work.*

A (i). *Divide the continued product of* 54, 55 *and* 56 *by the continued product of* 14, 15 *and* 16.

$$\frac{\overset{\overset{9}{\cancel{18}}}{\cancel{54}} \times \overset{11}{\cancel{55}} \times \overset{\overset{1}{\cancel{4}}}{\cancel{56}}}{\underset{1}{\cancel{14}} \times \underset{\underset{1}{\cancel{3}}}{\cancel{15}} \times \underset{\underset{2}{\cancel{4}}}{\cancel{16}}} = \frac{99}{2} = 49\tfrac{1}{2} \ \underline{\textit{Ans.}}$$

A (ii). *How many miles an hour does a body travel which moves at the rate of* 16 *feet per second?*

As there are 60×60 *seconds* in 1 *hour*,
$\therefore$ the body travels $60 \times 60 \times 16$ *feet* per *hour*.

$$\text{And } 60 \times 60 \times 16 \textit{ feet} = \frac{60 \times \overset{\overset{1}{\cancel{2}}}{\cancel{60}} \times \overset{2}{\cancel{16}}}{\underset{\underset{11}{\cancel{88}}}{\cancel{1760}} \times \underset{1}{\cancel{3}}} \textit{ miles} = \frac{60 \times 2}{11} \textit{ miles}$$

$$= 10\tfrac{10}{11} \textit{ miles. } \underline{\textit{Ans.}}$$

B. *How many bits, each* $3\frac{2}{7}$ *inches long, could be cut from a yard of string, and what length would remain over?*

Expressing both lengths in the *same denomination*, namely, *sevenths* of an inch, we have

1 *yard*, or 36 *inches* = 252 *sevenths of an inch*.
and $3\frac{2}{7}$ *inches* = 23 *sevenths of an inch*.

Dividing 252 by 23 we find that 23 *sevenths* is contained 10 *times* in 252 *sevenths*, with 22 *sevenths* of an inch, *i.e.* $3\frac{1}{7}$ *inches* over.

```
23 ) 252 ( 10
     23
     ——
      22 sevenths.
```

Ans. 10 bits; $3\frac{1}{7}$ *in.* over.

C. *Multiply* $999\frac{20}{23}$ *by* 173, *shortly.*

$$\begin{aligned} 999\tfrac{20}{23} \times 173 &= 1000 \times 173 - \tfrac{3}{23} \times 173 \\ &= 173000 \quad - \tfrac{519}{23} \\ &= 173000 \quad - 22\tfrac{13}{23} = 172977\tfrac{10}{23} \ \underline{\textit{Ans.}} \end{aligned}$$

D. *Find the G.C.M. and L.C.M. of* $3\frac{3}{5}$, $2\frac{1}{4}$ *and* $3\frac{3}{8}$.

Reducing the given fractions to their *least common denominator* we have

$$3\tfrac{3}{5},\ 2\tfrac{1}{4},\ 3\tfrac{3}{8} = \frac{18}{5},\ \frac{9}{4},\ \frac{27}{8} = \frac{144}{40},\ \frac{90}{40},\ \frac{135}{40}.$$

Now the G.C.M. of 144, 90 and 135 is 9, and the L.C.M. is 2160.

∴ the G.C.M. of 144 *fortieths*, 90 *fortieths* and 135 *fortieths* is 9 *fortieths*, *i.e.* $\frac{9}{40}$ Ans., and the L.C.M. $\frac{2160}{40}$, *i.e.* 54 Ans.

E. *A boy spent* $\frac{3}{8}$ *of his money at one shop,* $\frac{4}{5}$ *of the remainder at another, and had* 1*s.* $0\frac{1}{2}$*d. left; how much had he at first?*

After he had spent $\frac{3}{8}$, he had $\frac{5}{8}$, of his money left.

He then spent $\frac{4}{5}$ of this remainder; *i.e.* $\frac{4}{\cancel{5}}$ of $\frac{\cancel{5}}{8} = \frac{4}{8}$ of the whole sum.

∴ he spent altogether $\frac{3}{8}+\frac{4}{8}$, *i.e.* $\frac{7}{8}$, of his money. After which he must have had $\frac{1}{8}$ of it left. Hence $\frac{1}{8}$ of his money $=$ 1*s.* $0\frac{1}{2}$*d.*

∴ he had at first 8 times 1*s.* $0\frac{1}{2}$*d.* $=$ 8*s.* 4*d.* Ans.

F. *Divide* £14, 15*s.* 2*d. between A and B so that A may have a third as much again as B.*

As A's share $= 1\frac{1}{3}$ times B's share,

∴ A's share $+$ B's share $= 2\frac{1}{3}$ times B's share $=$ £14, 15*s.* 2*d.*

Hence, dividing £14, 15*s.* 2*d.* by $2\frac{1}{3}$ we obtain B's share, £6, 6*s.* 6*d.*
And subtracting this from the whole we obtain A's share, £8, 8*s.* 8*d.* } Ans.

In order to *compare* fractions we may reduce them to a *common numerator* instead of to a common denominator, and thus sometimes save a considerable amount of labour.

G. *Arrange the fractions* $\frac{2}{19}$, $\frac{3}{29}$, $\frac{4}{37}$ *and* $\frac{5}{47}$ *in descending order of magnitude.*

$$\frac{2}{19},\ \frac{3}{29},\ \frac{4}{37},\ \frac{5}{47} = \frac{1}{\frac{19}{2}},\ \frac{1}{\frac{29}{3}},\ \frac{1}{\frac{37}{4}},\ \frac{1}{\frac{47}{5}}$$

$$= \frac{1}{9\frac{1}{2}},\ \frac{1}{9\frac{2}{3}},\ \frac{1}{9\frac{1}{4}},\ \frac{1}{9\frac{2}{5}}.$$

Now as the *numerators* of these fractions are all *equal*, the *greatest* fraction is that which has the *least denominator*, *i.e.* the third fraction; and so on.

Hence we have $\frac{4}{37}$, $\frac{5}{47}$, $\frac{2}{19}$, $\frac{3}{29}$ as the *descending* order.

XXV. SIMPLE PRACTICE.

An **Aliquot Part** of any quantity is *exactly* contained in that quantity.

For instance, 5*s*. is an *aliquot part* of £1, for 5*s*. is contained *exactly four times* in £1.

Also, 1*s*. 3*d*. is an *aliquot* part of 10*s*., for 1*s*. 3*d*. is contained *exactly eight times* in 10*s*.

Hence, if an Aliquot Part be expressed as a fraction in its lowest terms of the quantity, *the* **numerator** *is always* **1.**

For instance, 5*s*. is $\frac{1}{4}$ of £1; and 1*s*. 3*d*. is $\frac{1}{8}$ of 10*s*.

Practice is a method of finding, by aliquot parts, the value (or weight, &c.) of a *quantity*, when the value (or weight, &c.) of a *unit* of the same is given.

It is called **Simple,** or **Compound,** according as the *quantity* is *simple*, or *compound.*

For instance, the value of 712 tons at £1, 5*s*. 6*d*. per ton may be found by *Simple* Practice; the value of 5 tons 11 cwts. 1 qr. at £1, 5*s*. 6*d*. per ton by *Compound* Practice.

In **Simple Practice** the general method of procedure is as follows:—The *given value* (or weight, &c.) is separated into portions such that the first is some unit, or multiple of the unit, and each of the others is an aliquot part of this unit, or of some preceding portion.

For instance, to find the cost of 712 tons at £1, 7*s*. 6*d*. per ton, we separate the *given value* £1, 7*s*. 6*d*. into the portions £1; 5*s*.; and 2*s*. 6*d*., where £1 is a unit, 5*s*. is an aliquot part ($\frac{1}{4}$) of £1, and 2*s*. 6*d*. is an aliquot part ($\frac{1}{2}$) of 5*s*. We then reason thus:

The cost of 712 tons at **£1** per ton is evidently **£712.**

And, as 5*s*. is $\frac{1}{4}$ of £1, the cost of any number of things at 5*s*. each must be $\frac{1}{4}$ of their cost at £1 each.

∴ the cost of 712 tons at **5***s*. per ton is $\frac{1}{4}$ of £712, *i.e.* **£178.**

Also, as 2*s*. 6*d*. is $\frac{1}{2}$ of 5*s*., the cost of any number of things at 2*s*. 6*d*. each must be $\frac{1}{2}$ of their cost at 5*s*. each.

∴ the cost of 712 tons at **2***s*. **6***d*. per ton is $\frac{1}{2}$ of £178, *i.e.* **£89.**

Hence the cost of 712 tons at **£1, 7***s*. **6***d*. per ton is **£712 + £178 + £89**, *i.e.* £979.

Note.—The principle involved in the last step is (II.) on page 10, as employed in Compound Multiplication,

i.e. 712 times £1, 7*s*. 6*d*. = 712 times £1 + 712 times 5*s*. + 712 times 2*s*. 6*d*.

Hence it appears that by the method of Simple Practice the result of any question in Compound Multiplication may be obtained.

The following examples illustrate some of the many ways in which the method of Simple Practice may be employed:—

EXAMPLE i.—*Find the cost of* 233 *things at* 3*s.* 4*d. each.*

Here, as 3*s.* 4*d.* is an aliquot part of £1, namely $\frac{1}{6}$ of £1, it is only necessary to *divide* the cost of 233 things at £1 each, *by* 6, in order to obtain their cost at 3*s.* 4*d.* each.

Written Work.

	£	s.	d.	
3*s.* 4*d.* = $\frac{1}{6}$	233	0	0	= cost at £1 each.
	Ans. 38	16	8	= .... 3*s.* 4*d.*...

EXAMPLE ii.—*Find the cost of* 3281 *lemons at* $1\frac{1}{2}$*d. each.*

3281 things at 1*s.* each would cost 3281*s.*; and, as $1\frac{1}{2}$*d.* is $\frac{1}{8}$ of 1*s.*, ∴ 3281 things at $1\frac{1}{2}$*d.* each will cost $\frac{1}{8}$ of 3281*s.*

	s.	d.	
$1\frac{1}{2}$*d.* = $\frac{1}{8}$	3281	0	= cost at 1*s.* each.
2,0	41,0	$1\frac{1}{2}$	= .... $1\frac{1}{2}$*d.* .....
	Ans. £20, 10*s.* $1\frac{1}{2}$*d.*		

We therefore divide 3281*s.* by 8, and then express the result in £, *s.* *d.*

EXAMPLE iii.—*Find the cost of* 1526 *things at* 8*s.* 4*d. each.*

We separate 8*s.* 4*d.* into the aliquot parts 6*s.* 8*d.* ($\frac{1}{3}$ of £1), and 1*s.* 8*d.* ($\frac{1}{4}$ of 6*s.* 8*d.*).

	£	s.	d.	
6*s.* 8*d.* = $\frac{1}{3}$	1526	0	0	= cost at £1 each.
1*s.* 8*d.* = $\frac{1}{4}$	508	13	4	= .... 6*s.* 8*d.*...(i)
	127	3	4	= .... 1*s.* 8*d.*...(ii)
	Ans. 635	16	8	= .... 8*s.* 4*d.*

Dividing £1526 (the cost at £1) by 3, we obtain the cost at 6*s.* 8*d.* (i). Dividing the cost at 6*s.* 8*d.* by 4, we obtain the cost at 1*s.* 8*d.* (ii). Finally, adding (i) and (ii) together, we obtain the cost at 8*s.* 4*d.*

EXAMPLE iv.—*Find the cost of* 425 *articles at* £3, 7*s.* $9\frac{1}{4}$*d. each.*

We separate £3, 7*s.* $9\frac{1}{4}$*d.* into the portions— £3 (a multiple of the unit £1); 5*s.* ($\frac{1}{4}$ of £1); 2*s.* 6*d.* ($\frac{1}{2}$ of 5*s.*); 3*d.* ($\frac{1}{10}$ of 2*s.* 6*d.*), and $\frac{1}{4}$*d.* ($\frac{1}{12}$ of 3*d.*).

	£	s.	d.	
5*s.* = $\frac{1}{4}$	425	0	0	= cost at £1 each.
			3	
	1275	0	0	= .... £3 ....
2*s.* 6*d.* = $\frac{1}{2}$	106	5	0	= .... 5*s.* ....
3*d.* = $\frac{1}{10}$	53	2	6	= .... 2*s.* 6*d.*...
$\frac{1}{4}$*d.* = $\frac{1}{12}$	5	6	3	= .... 3*d.* ....
		8	$10\frac{1}{4}$	= .... $\frac{1}{4}$*d.* ....
	Ans. 1440	2	$7\frac{1}{4}$	= ......£3, 7*s.* $9\frac{1}{4}$*d.*

We then write down £425, the cost of 425 things at £1 each, which we multiply by 3, obtaining the cost at £3 each; &c.

EXAMPLE v.—*Find the cost of* 561 *tons at* £1, 14*s.* 10¼*d. per ton.*

Here two aliquot parts of £1 are used, namely, 10*s.* and 4*s.*, so we divide the top line first by 2, and then again by 5. Also, the top line (the cost at £1) is not cut off, but is included in the addition.

	£	s.	d.	
	561 .	0 .	0	= cost at £1 per ton.
10*s.* = $\frac{1}{2}$	280 .	10 .	0	= .... 10*s.* ....
4*s.* = $\frac{1}{5}$	112 .	4 .	0	= .... 4*s.* ....
8*d.* = $\frac{1}{6}$	18 .	14 .	0	= .... 8*d.* ....
2*d.* = $\frac{1}{4}$	4 .	13 .	6	= .... 2*d.* ....
¼*d.* = $\frac{1}{8}$		11 .	8¼	= .... ¼*d.* ....
Ans.	977 .	13 .	2¼	= ......£1, 14*s.* 10¼*d.*...

EXAMPLE vi.—*Find the cost of* 216 *lbs. at* 3*s.* 4½*d. per lb.*

	s.	
3*d.* = $\frac{1}{4}$	216	= cost at 1*s.*
	3	
	648	= .... 3*s.*
1½*d.* = $\frac{1}{2}$	54	= .... 3*d.*
	27	= .... 1½*d.*
2,0	72,9	= ...3*s.* 4½*d.*
	Ans. £36, 9*s.*	

or thus:—

	£	s.	
	216 .	0	= cost at £1.
3*s.* 4*d.* = $\frac{1}{6}$	36 .	0	= ......3*s.* 4*d.*
½*d.* = $\frac{1}{80}$		9	= .... ½*d.*
Ans.	£36,	9*s.*	= .... 3*s.* 4½*d.*

When the given price falls very little short of an exact number of pounds (or shillings), labour may be saved by first finding *the cost at this higher value,* and then **subtracting from it the excess cost,** as in the following examples:—

EXAMPLE vii.—*Find the value of* 4125 *sheep at* £2, 18*s. each.*

Here we multiply the cost at £1 by **3**, obtaining the cost at **£3** (i). We then find the cost at **2***s.*, and **subtract** it from (i).

	£	s.	d.	
2*s.* = $\frac{1}{10}$	4125 .	0 .	0	= value at £1 each
			3	
	12375 .	0 .	0	= .... £3 ......(i)
	412 .	10 .	0	= .... 2*s.* ......
Ans.	11962 .	10 .	0	= .... £2, 18*s.* ...

EXAMPLE viii.—*Find the cost of* 1206 *things at* 17*s.* 4½*d. each.*

Here, as 17*s.* 4½*d.* falls short of £1 by only 2*s.* 7½*d.*, we separate 2*s.* 7½*d.* into aliquot parts, and having obtained the corresponding costs, we *add these and subtract from the top line in one operation.* (See page 39.)

	£	s.	d.	
2*s.* = $\frac{1}{10}$	1206 .	0 .	0	= cost at £1 each.
6*d.* = $\frac{1}{4}$	120 .	12 .	0	= .... 2*s.* ......
1½*d.* = $\frac{1}{4}$	30 .	3 .	0	= .... 6*d.* ......
	7 .	10 .	9	= .... 1½*d.* ....
Ans.	1047 .	14 .	3	= ...... 17*s.* 4½*d.*...

It sometimes happens that one of the portions into which we separate the given value is so small an aliquot part of the preceding portion that its value cannot be obtained by ordinary short division in one step, in which case we insert *an extra step, which we cancel* before the final addition, or subtraction.

EXAMPLE ix.—*Find the cost of* 4763 *tons at* 19*s.* 7*d. per ton.*

To obtain the cost at 5*d.* from the cost at £1, we insert an extra step (the cost at 5*s.*), which we cancel before subtracting.

	£	s.	d.	
$5s. = \frac{1}{4}$	4763	0	0	= cost at £1 per ton.
$5d. = \frac{1}{12}$	~~1190~~	~~15~~	~~0~~	= .... ~~5s.~~ ....
	99	4	7	= .... 5*d.* ....
Ans.	4663	15	5	= .... 19*s.* 7*d.* .....

When the quantity, the cost of which is required, is expressed as a mixed number, and the fractional part is such that the corresponding fraction of £1 can be exactly expressed in current coin, we proceed thus:—

EXAMPLE x.—*Find the cost of* $87\frac{3}{4}$ *tons at* £1, 9*s.* 3*d. per ton.*

As $\frac{3}{4}$ of a ton, at £1 per ton, would cost 15*s.*

∴ $87\frac{3}{4}$ tons, at £1 per ton, will cost £87, 15*s.*

	£	s.	d.	
$4s. = \frac{1}{5}$	87	15	0	= cost at £1 per ton.
$5s. = \frac{1}{4}$	17	11	0	= .... 4*s.* ....
$3d. = \frac{1}{20}$	21	18	9	= .... 5*s.* ....
	1	1	$11\frac{1}{4}$	= .... 3*d.* ....
Ans.	128	6	$8\frac{1}{4}$	= .... £1, 9*s.* 3*d.*

If, however, the fractional part of the quantity is such that the corresponding fraction of £1 cannot be expressed exactly in current coin, we proceed thus:—

EXAMPLE xi.—*Find the cost of* $341\frac{2}{7}$ *tons at* £1, 3*s.* $1\frac{1}{4}d.$ *per ton.*

Here as $\frac{2}{7}$ of £1 would yield a fraction of a penny, the labour of the succeeding divisions would be greatly increased if we included this in the top line; we therefore find the cost of $\frac{2}{7}$ of a ton separately, and include the result in the final addition.

	£	s.	d.
	1	3	$1\frac{1}{4}$
			2
7)	2	6	$2\frac{1}{2}$
		6	$7\frac{3}{14}$

	£	s.	d.	
$2s. = \frac{1}{10}$	341	0	0	= cost of 341 tons at £1 per ton.
$1s. = \frac{1}{2}$	34	2	0	= .... 2*s.* ....
$1d. = \frac{1}{12}$	17	1	0	= .... 1*s.* ....
$\frac{1}{4}d. = \frac{1}{4}$	1	8	5	= .... 1*d.* ....
		7	$1\frac{1}{4}$	= .... $\frac{1}{4}d.$ ....
		6	$7\frac{3}{14}$	= cost of $\frac{2}{7}$ ton.
Ans.	394	5	$1\frac{13}{28}$	= cost of $341\frac{2}{7}$ tons at £1, 3*s.* $1\frac{1}{4}d.$

EXAMPLE xii.—*Find the quantity of coal in* $953\frac{1}{4}$ *truck-loads, averaging* 9 *tons* 13 *cwts.* 2 *qrs. per truck.*

	tons.	cwts.	qrs.	
10 cwts. = $\frac{1}{2}$	953	. 5	. 0	= quantity at 1 ton per truck.
		.	9	
	8579	. 5	. 0	= .... 9 tons ....
2 cwts. = $\frac{1}{5}$	476	. 12	. 2	= .... 10 cwts. ....
1 cwt. = $\frac{1}{2}$	95	. 6	. 2	= .... 2 cwts. ....
2 qrs. = $\frac{1}{2}$	47	. 13	. 1	= .... 1 cwt. ....
	23	. 16	. $2\frac{1}{2}$	= .... 2 qrs ....
	9222	. 13	. $3\frac{1}{2}$	= ...... 9 tons 13 cwts. 2 qrs...

Ans. 9222 tons 13 cwts. $3\frac{1}{2}$ qrs.

Many questions which have the form of a question on Compound Practice may with advantage be treated by the methods of Simple Practice.

EXAMPLE xiii.—*A bankrupt pays a dividend of* 17*s.* $10\frac{1}{2}$*d. in the pound; find the loss sustained by a creditor to whom he owes* £892, 13*s.* 4*d.*

The bankrupt pays a *dividend* of 17*s.* $10\frac{1}{2}$*d.* in the pound.
i.e. he pays 17*s.* $10\frac{1}{2}$*d.* for each £1 he owes.
∴ his creditors lose 2*s.* $1\frac{1}{2}$*d.* on each £1 due to them.

	£	*s.*	*d.*	
2*s.* = $\frac{1}{10}$	892	. 13	. 4	= loss at the rate of £1 in the pound.
$1\frac{1}{2}$*d.* = $\frac{1}{16}$	89	. 5	. 4	= .... 2*s.* ....
	5	. 11	. 7	= .... $1\frac{1}{2}$*d.* ....
Ans.	94	. 16	. 11	= .... 2*s.* $1\frac{1}{2}$*d.* ....

EXAMPLE xiv.—*Find the cost of* 17 *cwts.* 3 *qrs.* 21 *lbs. at* 4*s.* 2*d. per qr.*

Here, as 17 cwts. 3 qrs. 21 lbs. = $71\frac{3}{4}$ qrs., we proceed thus:—

	£	*s.*	*d.*	
3*s.* 4*d.* = $\frac{1}{6}$	71	. 15	. 0	= cost at £1 per qr.
10*d* = $\frac{1}{4}$	11	. 19	. 2	= .... 3*s.* 4*d.* ...
	2	. 19	. $9\frac{1}{2}$	= .... 10*d.* ......
Ans.	14	. 18	. $11\frac{1}{2}$	= .... 4*s.* 2*d.* ...

EXAMPLE xv.—*Find the rent of* 893 A. 2 R. 20 P. *at* £1, 7*s.* 6*d. per acre.*

893 A. 2 R. 20 P. = 893 A. $2\frac{1}{2}$ R. = $893\frac{5}{8}$ A.

	£	*s.*	*d.*	
5*s.* = $\frac{1}{4}$	893	. 12	. 6	= rent at £1 per acre.
2*s.* 6*d.* = $\frac{1}{2}$	223	. 8	. $1\frac{1}{2}$	= .... 5*s.* ....
	111	. 14	. $0\frac{3}{4}$	= .... 2*s.* 6*d.* ....
Ans.	1228	. 14	. $8\frac{1}{4}$	= .... £1, 7*s.* 6*d.*...

XXVI. COMPOUND PRACTICE.

In **Compound Practice** *the compound quantity* is treated in the same way as *the given value* is treated in Simple Practice.

For instance, to find the cost of 5 tons 11 cwts. 2 qrs. at £1, 7*s.* 6*d.* per ton, we separate the *compound quantity* 5 tons 11 cwts. 2 qrs. into the portions 5 tons; 10 cwts.; 1 cwt.; and 2 qrs., where 5 tons is a multiple of the unit 1 ton; 10 cwts. is an aliquot part ($\frac{1}{2}$) of 1 ton; 1 cwt. is an aliquot part ($\frac{1}{10}$) of 10 cwts.; and 2 qrs. is an aliquot part ($\frac{1}{2}$) of 1 cwt. We then reason thus:

The cost of **5 tons** is 5 times £1, 7*s.* 6*d.*, *i.e.* **£6, 17*s.* 6*d.***

And, as 10 cwts. is $\frac{1}{2}$ of 1 ton, the cost of 10 cwts. must be $\frac{1}{2}$ of the cost of 1 ton.

∴ the cost of **10 cwts.** is $\frac{1}{2}$ of £1, 7*s.* 6*d.*, *i.e.* **13*s.* 9*d.***

Similarly, the cost of **1 cwt.** is $\frac{1}{10}$ of 13*s.* 9*d.*, or **1*s.* 4½*d.***

And the cost of **2 qrs.** is $\frac{1}{2}$ of the cost of 1 cwt., *i.e.* **8¼*d.***

Hence the cost of **5 tons 11 cwts. 2 qrs.**

is **£6, 17*s.* 6*d.*** + **13*s.* 9*d.*** + **1*s.* 4½*d.*** + **8¼*d.***, *i.e.* £7, 13*s.* 3¾*d.*

EXAMPLE i.—*Find the cost of* 1 *ton* 2 *cwts.* 3 *qrs.* 5 *lbs.* 8 *ozs. at* £5, 12*s. per cwt.*

	£	*s.*	*d.*	
2 qrs. = $\frac{1}{2}$	5	12	0	= cost of 1 cwt.
			2	
	11	4	0	
			11	
	123	4	0	= .... 22 cwts.
1 qr. = $\frac{1}{2}$	2	16	0	= .... 2 qrs.
4 lbs. = $\frac{1}{7}$	1	8	0	= .... 1 qr.
1 lb. = $\frac{1}{4}$		4	0	= .... 4 lbs.
8 ozs. = $\frac{1}{2}$		1	0	= .... 1 lb.
			6	= .... 8 ozs.
Ans.	127	13	6	= ..... 22 cwt. 3 qr. 5 lb. 8 oz.

Here, as the cost per *cwt.* is given, we multiply £5, 12*s.* by 22, obtaining the cost of 22 cwts., *i.e.* of 1 ton 2 cwts.

We now separate 3 qrs. 5 lbs. 8 ozs. into the aliquot parts—2 qrs. ($\frac{1}{2}$ of 1 cwt.); 1 qr. ($\frac{1}{2}$ of 2 qrs.); 4 lbs. ($\frac{1}{7}$ of 1 qr.): 1 lb. ($\frac{1}{4}$ of 4 lbs.); and 8 ozs. ($\frac{1}{2}$ of 1 lb.), &c.

EXAMPLE ii.—*Find the rent of* 11 *ac.* 3 *ro.* 24 *po. at* £2, 12*s.* 6*d. per acre.*

	£	*s.*	*d.*	
16 po. = $\frac{1}{10}$	2	12	6	= rent of 1 ac.
		12		
	31	10	0	= .... 12 ac.
		5	3	= .... 16 po.
Ans.	31	4	9	= .... 11 ac. 3 ro. 24 po.

Here, as 11 ac. 3 ro. 24 po. differs from 12 ac. by only 16 po., we adopt the "excess method".

EXAMPLE iii.—*Find the value of* 1 *oz.* 13 *dwts.* 16 *grains of gold at* £3, 17*s.* $10\frac{1}{2}$*d. per oz.*

	£	s.	d.	
10 dwts. = $\frac{1}{2}$	3	17	$10\frac{1}{2}$	= value of 1 oz.
2 dwts. = $\frac{1}{5}$	1	18	$11\frac{1}{4}$	= ....10 dwts.
1 dwt. = $\frac{1}{2}$		7	$9\frac{9}{20}$	= .... 2 dwts.
12 grs. = $\frac{1}{2}$		3	$10\frac{29}{40}$	= .... 1 dwt.
4 grs. = $\frac{1}{3}$		1	$11\frac{29}{80}$	= ....12 grs.
			$7\frac{63}{80}$	= .... 4 grs.
Ans.	6	11	$1\frac{3}{40}$	= .... 1 oz. 13 dwt. 16 grs.

Here, in dividing £1, 18*s.* $11\frac{1}{4}$*d.* by 5, there remains $2\frac{1}{4}$*d.* over from the pence, and $2\frac{1}{4}d. \div 5 = \frac{9}{4} \times \frac{1}{5} = \frac{9}{20}d.$

Similarly in dividing 7*s.* $9\frac{9}{20}$*d.* by 2, there remains $1\frac{9}{20}$*d.* over from the pence, and $1\frac{9}{20}d. \div 2 = \frac{29}{20} \times \frac{1}{2} = \frac{29}{40}d.$; and so on.

Again, in the addition, $\frac{1}{2} + \frac{1}{4} + \frac{9}{20} + \frac{29}{40} + \frac{29}{80} + \frac{63}{80} = \frac{40+20+36+58+29+63}{80}$ $= \frac{246}{80} = 3\frac{3}{40}d.$, so we set down $\frac{3}{40}$*d.* and "carry" 3 to the pence column.

Note.—In actual business, results are often only required *correct to the nearest penny*: in such case the fraction $\frac{3}{40}$*d.* in this result would be neglected. It would, however, still be necessary to retain the fractions of a penny in the working, for the sum of these yields 3 *complete pence.*

In some cases we use *both* kinds of Practice; for instance:—

EXAMPLE iv.—*Find the value of* 451 *tons* 12 *cwts.* 1 *qr.* 11 *lbs. at* £3, 13*s.* 4*d. per ton.*

Here we find the value of 451 tons at £3, 13*s.* 4*d.* per ton, by *Simple* Practice; and then the value of 12 cwts. 1 qr. 11 lbs., at £3, 13*s.* 4*d.* per ton, by *Compound* Practice, adding in the first result.

	£	s.	d.	
6*s.* 8*d.* = $\frac{1}{3}$	451	0	0	= value of 451 tons at £1 per ton.
6*s.* 8*d.* = $\frac{1}{3}$			3	
	1353	0	0	= .... £3 ....
	150	6	8	= .... 6*s.* 8*d.*.......
	150	6	8	= .... 6*s.* 8*d.*.......
	1653	13	4	= .... £3, 13*s.* 4*d.*...*

	£	s.	d.	
10 cwts. = $\frac{1}{2}$	3	13	4	= value of 1 ton.
2 cwts. = $\frac{1}{5}$	1	16	8	= .... 10 cwts.
1 qr. = $\frac{1}{8}$		7	4	= .... 2 cwts.
7 lbs. = $\frac{1}{4}$			11	= .... 1 qr.
4 lbs. = $\frac{1}{7}$			$2\frac{3}{4}$	= .... 7 lbs.
			$1\frac{4}{7}$	= .... 4 lbs.
	1653	13	4	= ....451 tons.*
Ans.	1655	18	$7\frac{9}{28}$	= ......451 tons 12 cwts. 1 qr. 11 lbs.

XXVII. INVOICES.

An **Invoice**, or **Bill**, is a written statement made by the seller and delivered to the buyer, with the goods, at the time of purchase, showing the *quantity*, *kind*, and *cost* of each article supplied.

Specimen of an Invoice, or Bill.

REV. W. JONES. LONDON, Jan. 21st, 1892.

Bought of SMITH & Co.,
School Stationers,
ALDERSGATE, E.C.

			£	s.	d.
	2 gross Penholders	2/3		4	6
	2 ,, Pencils	4/6		9	0
	3 doz. Copy-books	1/6		4	6
	6 Slates	5*d.*		2	6
			1	0	6

Here the column on the left of the £, *s. d.* columns gives the price per gross, dozen, or unit as the case may be, while opposite, in the £, *s. d.* columns, is placed the cost of the quantity of each article purchased. The column on the extreme left, intended for dates, is blank in this case, as all the articles were supplied on the date given at the head of the bill.

The *quantity* and *cost* of any **one** article mentioned in the Invoice is called an **Item.**

When *credit* is given (*i.e.* when the buyer does not pay "ready" money), a further statement is sent at intervals (usually of a quarter, or half-year), by the seller to the buyer, showing the dates and amounts of the invoices which then remain unpaid, with the total sum of money owing: this is called an **Account.**

Specimen of an Account.

REV. W. JONES. LONDON, June, 1892.

Dr. to SMITH & Co.,
School Stationers,
ALDERSGATE, E.C.

		£	s.	d.
Jan. 21	To Goods as per Invoice	1	0	6
Feb. 2	,, ,, ,, ...	2	7	2
April 19	,, ,, ,, ...	1	16	8
May 3	,, ,, ,, ...	4	12	5
		9	16	9

A **Detailed** Account is one in which each Invoice, or Bill, is reproduced in full; *i.e.* in which all the *Items* are mentioned.

Specimen of a Detailed Account.

MRS. BROWN. STAFFORD, Mids., 1892.

Bought of ROBINSON BROTHERS,

Drapers &c.,

85 and 86 Greengate Street.

			£	s.	d.
April 4	19 yds. Dutch carpet	2/11	2	15	5
—	Making same			5	
—	2 doz. drugget pins				5
May 2	$\frac{1}{2}$ doz. table-napkins............	16/6		8	3
—	27 yds. calico	7*d.*		15	9
June 7	1 doz. buttons				$2\frac{1}{4}$
—	$\frac{5}{8}$ yd. oil-cloth	1/9		1	$1\frac{1}{4}$
			4	6	$1\frac{1}{2}$

Here the *exact* cost of the last item is 1*s.* $1\frac{1}{8}$*d.* In such cases it is customary to charge the *farthing next above* the exact value.

Note.—Tradesmen often make an allowance, called "Trade Discount", for ready money, or for prompt payment of an account. For instance, suppose the tradesmen mentioned in the above specimen of a Detailed Account make it a rule to allow a deduction of 6*d.* in the pound on accounts paid within a month, then 2*s.* $1\frac{1}{2}$*d.* would be the discount subtracted from this particular account, namely, 6*d.* on each of the 4 pounds and $1\frac{1}{2}$*d.* on *5s.* (*i.e.* $\frac{1}{4}$ of £1), any remainder less than 5*s.* being neglected for this purpose.

In working the exercises on this chapter the student should endeavour to calculate the items with as little *written* work as possible.

Towards this end he should bear in mind

(i) That when the price of one article is given in pence and farthings, the price per dozen is obtained at once by taking a shilling for each *penny*, and **three-pence** for each *farthing* of the given price.

For instance, 1 doz. at $7\frac{3}{4}$*d.* each cost 7*s.* 9*d.*,

and 7 doz. at $9\frac{1}{4}$*d.* each cost 7 times 9*s.* 3*d.*; *i.e.* £3, 4*s.* 9*d.*

Hence, when the number of articles, the cost of which is required, but little exceeds, or falls short of, an *exact* number of *dozens*, it is well to calculate the *dozens* separately.

For instance, 37 at $2\frac{3}{4}$*d.* each cost 3 times 2*s.* 9*d.* + $2\frac{3}{4}$*d.*; *i.e.* 8*s.* $5\frac{3}{4}$*d.*

(ii) The *aliquot parts* of £1 and 1*s.*

For instance, 30 at 3*s.* 4*d.* each cost 30 times £$\frac{1}{6}$, *i.e.* £5.

(iii) That when the *cost* but little exceeds, or falls short of, an *exact* number of pounds, or shillings, the "excess method" may be used with advantage.

For instance, 17 at $11\frac{3}{4}$*d.* each cost 17*s.* − 17 *farthings*, *i.e.* 16*s.* $7\frac{3}{4}$*d.*

XXVIII. DECIMALS.

NOTATION.

We have seen, in Chapter I., that our system of notation is called the *Decimal* system on account of the *ten*-fold change in local value; a tenfold *increase* in each successive place towards the *left*, and, consequently, a corresponding *decrease* in each successive place towards the *right*.

Thus the local value of a figure in any place is **one-tenth** of the value it would possess in *the preceding place on the left*.

For instance, in the number 333, representing 3 *hundreds* + 3 *tens* + 3 *units*, the value of the middle 3 is *one-tenth* of that of the left-hand 3, and the value of the right-hand 3 is *one-tenth* of that of the middle 3.

If, then, we regard the position of the units' place as fixed, and *extend the operation of this law to places to the right of the units*, the local value of a figure in *the place immediately to the right of the units' place* will also be *one-tenth* of the value it would possess *in* the units' place; *i.e.* a figure standing in this new place will represent tenth-parts of unity, or *tenths*.

Similarly, the next figure on the right will represent tenth-parts of one-tenth of unity, or *hundredths*; and so on.

Hence it appears that figures placed *to the right* of the units' place represent *parts* of unity, or fractions, on the same system that figures placed *to the left* of the units' place represent *multiples* of unity, or integers; and such fractions are, consequently, called *Decimal* Fractions, or shortly, "Decimals."*

Thus, a **Decimal** is a number of *tenths, hundredths, thousandths*, &c.

i.e. in the case of a *decimal* fraction the *number of equal parts* into which the unit is divided is always some **power† of ten.**

Hitherto, when *integers only* have been thus represented, the units' place has always been occupied by *the figure on the extreme right* of the number; but when figures, representing *parts* of unity, stand *to the right* of the units' place, *it becomes necessary to mark the position of the units' place so that it may not be lost sight of.*

This is done by placing a *dot*, called the **Decimal Point**, *immediately to the right of the units' place.*

For instance, 33·3 represents 3 *tens* + 3 **units** + 3 *tenths*, and 333·33 represents 3 *hundreds* + 3 *tens* + 3 **units** + 3 *tenths* + 3 *hundredths*.

Again, ·07 represents 0 *tenths* + 7 *hundredths, i.e.* 7 *hundredths*; any *vacant decimal place* being occupied by a *cipher*.

* This development of the Arabic system is due to English mathematicians at the beginning of the 17th century.

† See page 12.

A decimal may be "read" in the same way as an integer:

For instance, $\cdot 34$ is 3 *tenths* + 4 *hundredths*,
and as we know, by the *place-law*, that **ten** *hundredths* make **one** *tenth*,

$\therefore$ 3 *tenths* = 30 *hundredths*,
and 3 *tenths* + 4 *hundredths* = 34 *hundredths*.

i.e. $\cdot 34$ may be read as "Thirty-four *hundredths*."

Similarly, $\cdot 257$, *i.e.* 2 *tenths* + 5 *hundredths* + 7 *thousandths*, may be read "Two-hundred-and-fifty-seven *thousandths*,"

i.e. we may read the decimal as if it were an integer, and then mention *the local value of the last figure.*

In practice, however, decimals are very commonly read figure by figure, the word "decimal" (or "point") being used to indicate the decimal point.

For instance, $\cdot 402$ is commonly read thus: "Decimal, four, nought, two."
And $25\cdot 0087$, thus: "Twenty-five, decimal, nought, nought, eight, seven."

Ciphers on the extreme right of a decimal do not affect its value.

For instance, $\cdot 3$ is 3 *tenths*, and $\cdot 3\mathbf{0}$ is 3 *tenths* + **0** *hundredths*;
i.e. $\cdot 3\mathbf{0}$ is neither greater, nor less, than $\cdot 3$.

Hence, **we may append ciphers**, or **strike off ciphers, on the extreme right** of a decimal with impunity.

To multiply a Decimal by 10,

Move the decimal point **one** *place to the* **right.**

For, by so doing, the *units* become *tens*, the *tenths* become *units*, the *hundredths* become *tenths*, &c.

i.e. each figure has its value increased *tenfold*, and, consequently, the entire decimal is multiplied by 10.

For instance, $3\cdot 45 \times 10 = 34\cdot 5$; and $\cdot 0057 \times 10 = \cdot 057$.

Similarly, to multiply a decimal by 100, 1000, &c., *move the decimal point* **two, three** &c. *places to the* **right,** *respectively.*

For instance, $12\cdot 503 \times 100 = 1250\cdot 3$; and $\cdot 08537 \times 1000 = 85\cdot 37$.

Conversely, to divide a Decimal by 10,

Move the decimal point **one** *place to the* **left.**

For, by so doing, the *units* become *tenths*, the *tenths* become *hundredths*, &c., and thus the entire decimal is divided by 10.

For instance, $3\cdot 45 \div 10 = \cdot 345$; and $\cdot 0057 \div 10 = \cdot 00057$.

Similarly, to divide a decimal by 100, 1000, &c., *move the decimal point* **two, three** &c. *places to the* **left,** *respectively.*

For instance, $1250\cdot 3 \div 100 = 12\cdot 503$; and $\cdot 08537 \div 1000 = \cdot 00008537$.

XXIX. DECIMALS.

Addition and Subtraction.

As Decimals are expressed in the same way as integers, it follows that they can be added, or subtracted, in the same way.

For instance, just as 3 *tens* and 4 *tens* make 7 *tens*,
so 3 *tenths* and 4 *tenths* make 7 *tenths*; and so on.

For convenience then we arrange the decimals to be added, or subtracted, so that the *tenths* are all in one vertical column, the *hundredths* all in another, and so on; and this is ensured if *all the decimal points are in one vertical column*; we then proceed exactly as in Simple Addition, or Subtraction, remembering, however, that we may, if desirable, *append ciphers to the extreme right* of any of the given decimals, and may also *reject ciphers on the extreme right* of any *result*.

Example i.—*Add together* 4·532, 17·06, ·3574 *and* 1·8.

Here, arranging the given decimals so that all the decimal points are in one vertical column, we add each vertical column, beginning with that on the extreme right, and "carry" just as in Simple Addition; not forgetting to insert the decimal point in the result immediately beneath the column of decimal points.

```
  4·532
 17·06
   ·3574
  1·8
 23·7494 Ans.
```

Example ii.—*Find the sum of* 129·546, 1·254 *and* 26.

Here it should be noticed that 26, being an integer, is placed on the *left* of the points' column. Also, that in writing down the result, two superfluous ciphers on the extreme right of the result are discarded.

```
129·546
  1·254
 26·
156·800
```

Ans. 156·8.

Example iii.—*From* 4·573 *take* 2·08751.

Here, having arranged the decimals with the points in column, we append two ciphers to the right of the upper line before subtracting.*

```
4·57300
2·08751
2·48549 Ans.
```

Example iv.—*Simplify* 4·37451 + 190·28 − 72·003 − ·608751 + 241 − 330·0876.

Here we proceed as in the example on page 8; *i.e.* we first *add* 4·37451, 190·28 and 241 together; we then *add* 72·003, ·608751 and 330·0876 together; and finally *subtract* the second result from the first.

```
  4·37451        72·003
190·28             ·608751
241·            330·0876
435·65451       402·699351
402·699351
 32·955159 Ans.
```

* Such extra ciphers need not necessarily be *actually written* down: after a little practice the student will append them *mentally*.

XXX. MULTIPLICATION OF DECIMALS.

The process of multiplying a *decimal* by an **integer** exactly corresponds to that of multiplying one integer by another.

For instance, just as 7 *units* × 2 = 14 *units*, or 1 *ten* + 4 *units;*
so 7 *tenths* × 2 = 14 *tenths*, or 1 *unit* + 4 *tenths*,
i.e. ·7 × 2 = 1·4:

and 7 *hundredths* × 2 = 14 *hundredths*, or 1 *tenth* + 4 *hundredths*,
i.e. ·07 × 2 = ·14. And so on.

Again as, on page 11, it was shown that

243 *units* × 75 = 18225 *units*, *i.e.* 243 × 75 = 18225,
so 243 *tenths* × 75 = 18225 *tenths*, *i.e.* 24·3 × 75 = 1822·5,
243 *hundredths* × 75 = 18225 *hundredths*, *i.e.* 2·43 × 75 = 182·25;
243 *thousandths* × 75 = 18225 *thousandths*, *i.e.* ·243 × 75 = 18·225; &c.,

the *number of decimal places* in any *product* being the *same* as the number in the corresponding *multiplicand*.

Hence, to multiply a Decimal by an **Integer**,

We may multiply as in Simple Multiplication, disregarding the decimal point during the operation, and afterwards insert a decimal point in the product, so as to mark off *as many* **decimal places** *as there are in the multiplicand.*

EXAMPLE i.—*Multiply* 2·105 *by* 782.

Multiplying 2105 by 782 we obtain the product 1646110.

Now in 2·105 there are *three* decimal places.

We therefore insert a decimal point in the product so as to mark off *three* decimal places.

Thus 2·105 × 782 = 1646·110.

Finally, *after inserting the decimal point*, we discard the cipher on the extreme right.

$$\begin{array}{r} 2{\cdot}105 \\ \underline{782} \\ 4\ 210 \\ 168\ 40\ \ \\ \underline{1473\ 5\ \ \ \ } \\ \underline{1646{\cdot}110} \end{array}$$

Ans. 1646·11

Again, a method of multiplying a *decimal* by a **decimal** may be deduced from the foregoing case.

For instance, we have already seen that ·7 × 2 = 1·4,
and we know, by the *place-law*, that ·2 is *one-tenth* of 2.
Hence ·7 × ·2 = *one-tenth* of 1·4, *i.e.* ·14 (see page 106).

Similarly, as ·02 is *one-hundredth* of 2,
·7 × ·02 = *one-hundredth* of 1·4, *i.e.* ·014.

And, as ·002 is *one-thousandth* of 2,
·7 × ·002 = *one-thousandth* of 1·4, *i.e.* ·0014.

And so on; the *number of decimal places* in any product being the *number in the multiplicand* **increased by** *the number in the multiplier.*

Hence, to multiply **Decimals** together,

We may multiply as in Simple Multiplication, disregarding the decimal points during the operation, and afterwards insert a decimal point in the product, so as to mark off in it *as many* **decimal places** *as there are, altogether, in the factors.*

EXAMPLE ii.—*Multiply* 134·25 *by* ·032.

Multiplying 3425 by 32 we obtain the product 429600.

Now in 134·25 there are *two* decimal places, and in ·032 there are *three* decimal places.

We therefore insert a decimal point in the product so as to mark off $2+3$, or *five* decimal places.

Thus $134{\cdot}25 \times {\cdot}032 = 4{\cdot}29600$, or, discarding the ciphers on the right, 4·296.

$$\begin{array}{r} 134{\cdot}25 \\ {\cdot}032 \\ \hline 268\ 50 \\ 4027\ 5 \\ \hline 4{\cdot}296\ 00 \end{array}$$

Ans. 4·296

EXAMPLE iii.—*Find the product of* ·0673 *and* ·012.

Multiplying 673 by 12 we obtain the product 8076.

Now in ·0673 there are *four* decimal places, and in ·012 there are *three* decimal places.

∴ in the product there are $4+3$, or *seven* decimal places.

So we must *prefix* three ciphers to the *left* of 8076 before we can mark the position of the decimal point.

$$\begin{array}{r} {\cdot}0673 \\ {\cdot}012 \\ \hline {\cdot}0008076 \end{array}$$

Ans.

Note.—In long operations, it is convenient to omit decimal points and *left*-hand ciphers in the working, as in the following example:—

EXAMPLE iv.—*Find the continued product of* ·0023, ·00008, 7·1 *and* 600.

We find the continued product of 23, 71, 8 and 600 to be 7838400.

Now in ·0023 there are *four* dec. places; in ·00008 there are *five* dec. places; in 7·1 there is *one* dec. place; and in 600 there is *no* dec. place.

Hence in the req^{d} product there must be $4+5+1+0$, or *ten* decimal places.

We therefore *prefix* three ciphers to the *left* of 7838400 before we can mark the decimal point.

Finally, we discard the two ciphers on the *right*.

$$\begin{array}{r} 23 \\ 71 \\ \hline 23 \\ 161 \\ \hline 1633 \\ 8 \\ \hline 13064 \\ 600 \\ \hline 7838400 \end{array}$$

$$\therefore {\cdot}0023 \times {\cdot}00008 \times 7{\cdot}1 \times 600 = {\cdot}0007838400$$
$$= {\cdot}00078384 \ \textit{Ans.}$$

XXXI. DECIMALS.

Division by an Integer.

The method is that of Simple Division, applied to decimals.

For instance, *to divide* 11·48 *by* 8.

1st step.

11 *units* ÷ 8 yield quot^t 1 *unit*, with rem^r 3 *units*, so we set down **1** in the *units' place* in the quotient, *marking the decimal point* immediately on the right of it. We now change the rem^r 3 *units* into 30 *tenths.*

$$8\,|\,\underline{11 \cdot 48}$$
$$1\cdot$$

2nd step.

30 *tenths* + 4 *tenths* make 34 *tenths*, and 34 *tenths* ÷ 8 yield quot^t 4 *tenths*, with rem^r 2 *tenths.* So we set down **4** in the *tenths' place* in the quotient and change the rem^r 2 *tenths* into 20 *hundredths.*

$$8\,|\,\underline{11 \cdot 48}$$
$$1 \cdot 4$$

3rd step.

20 *hundredths* + 8 *hundredths* = 28 *hundredths*, and 28 *hundredths* ÷ 8 yield quot^t 3 *hundredths*, with rem^r 4 *hundredths.* So we set down **3** in the *hundredths' place* in the quotient.

$$8\,|\,\underline{11 \cdot 48}$$
$$1 \cdot 43$$

Here, as a rem^r occurs, we can still continue the division, although the last figure in the given dividend has been dealt with, for the 4 *hundredths* which form the remainder can be changed to 40 *thousandths.*

We, therefore, append a cipher to the right of the dividend and proceed thus:—

4th step.

40 *thousandths* ÷ 8 yield quot^t 5 *thousandths*, with *no* rem^r; so we write **5** in the *thousandths' place* in the quotient.

$$8\,|\,\underline{11 \cdot 480}$$
$$1 \cdot 435$$

Thus the complete quotient is 1·435.

i.e. if 11·48 be divided into 8 equal parts, each of these parts is 1·435; or, 1·435 is the decimal which, when multiplied by 8, gives the product 11·48.

Hence, we divide as in Simple Division, marking the *decimal point in the quotient as soon as we reach the figure in the* **tenths'** *place in the dividend.*

N.B.—When *short* division is used this amounts to simply *copying down the decimal point in a vertical line.*

Also, if, after the last of the given figures in the dividend has been dealt with, a remainder occurs, we may append ciphers, one by one as required, to the right of the dividend and continue the division.

Note.—Such appended ciphers need not always be *written*; they may be appended *mentally.*

EXAMPLE i.—*Divide* 2·8455 *by* 35.

Here we divide, as in Simple Division, by the factors of 35, copying down the decimal point in a vertical line, and writing a cipher in the vacant *tenths'* place of the result.

$$35\left\{\begin{array}{r|l} 5 & 2{\cdot}8455 \\ \hline 7 & {\cdot}5691 \\ \hline & {\cdot}0813 \ \textit{Ans.} \end{array}\right.$$

EXAMPLE ii.—*Express the quotient of* $5 \div 8$ *as a decimal.*

As there are no *units* in the quotient, we mark the decimal point on the right of the 5, change the 5 *units* into *tenths*, and so on.

$$\begin{array}{r|l} 8 & 5{\cdot}000 \\ \hline & {\cdot}625 \ \textit{Ans} \end{array}$$

EXAMPLE iii.—*Divide* ·03702 *by* 600.

Dividing ·03702 by 6, we obtain quotient ·00617.

We now divide this result by 100, *i.e.* we move the decimal point *two places* to the left. (See p. 106.)

$$\begin{array}{r|l} 6 & {\cdot}03702 \\ \hline & {\cdot}00617 \end{array}$$

$$\therefore {\cdot}03702 \div 600 = {\cdot}0000617 \ \textit{Ans.}$$

EXAMPLE iv.—*Divide* 69·3 *by* 45000.

Dividing 69·3 by factors of 45 we obtain the quotient 1·54.

We now divide this result by 1000, *i.e.* we move the dec. point *three places* to the *left*.

$$45\left\{\begin{array}{r|l} 5 & 69{\cdot}3 \\ \hline 9 & 13{\cdot}86 \\ \hline & 1{\cdot}54 \end{array}\right.$$

$$\therefore 69{\cdot}3 \div 45000 = {\cdot}00154 \ \textit{Ans.}$$

EXAMPLE v.—*Divide* 2·5696 *by* 73.

$2 \div 73$ yields *no units*, so we *at once mark the dec. point in quotient.*

25 *tenths* $\div$ 73 yields no *tenths*, so we place a cipher in the *tenths'* place in the quotient; &c.

$$\begin{array}{r} 73)2{\cdot}5696(\,{\cdot}0352 \ \textit{Ans.} \\ \underline{2\ 19}\phantom{96(\,{\cdot}0352 \ \textit{Ans.}} \\ 379\phantom{6(\,{\cdot}0352 \ \textit{Ans.}} \\ \underline{365}\phantom{6(\,{\cdot}0352 \ \textit{Ans.}} \\ 146\phantom{(\,{\cdot}0352 \ \textit{Ans.}} \\ \underline{146}\phantom{(\,{\cdot}0352 \ \textit{Ans.}} \end{array}$$

EXAMPLE vi.—*Divide* 540·8 *by* 325.

$540 \div 325$ yields 1 *unit*, so we write 1 in the quotient, and then, before dealing with the *tenths*, mark the decimal point *to the right* of the 1.

$2158 \div 325$ yields 6 *tenths* and 208 *tenths* rem^r.

We now append ciphers one by one as required to the dividend.

$$\begin{array}{r} 325)540{\cdot}800(\,1{\cdot}664 \ \textit{Ans.} \\ \underline{325}\phantom{{\cdot}800(\,1{\cdot}664 \ \textit{Ans.}} \\ 2158\phantom{00(\,1{\cdot}664 \ \textit{Ans.}} \\ \underline{1950}\phantom{00(\,1{\cdot}664 \ \textit{Ans.}} \\ 2080\phantom{0(\,1{\cdot}664 \ \textit{Ans.}} \\ \underline{1950}\phantom{0(\,1{\cdot}664 \ \textit{Ans.}} \\ 1300\phantom{(\,1{\cdot}664 \ \textit{Ans.}} \\ \underline{1300}\phantom{(\,1{\cdot}664 \ \textit{Ans.}} \end{array}$$

XXXII. DIVISION OF DECIMALS.

We know (see page 17) that *the* **quotient is not altered** *if we multiply both divisor and dividend by the* **same** *number.*

For instance, the quotient of $36 \div 12$ is the same as that of 10 *times* $36 \div 10$ *times* 12, *i.e.*, of $360 \div 120$.

Similarly, the quotient of $\cdot 36 \div 1\cdot 2$ is the same as that of 10 *times* $\cdot 36 \div 10$ *times* $1\cdot 2$, *i.e.*, of $\mathbf{3\cdot 6 \div 12}$.

Also, the quotient of $\cdot 475 \div \cdot 05$ is the same as that of 100 *times* $\cdot 475 \div 100$ *times* $\cdot 05$, *i.e.*, of $\mathbf{47\cdot 5 \div 5}$.

And the quotient of $5\cdot 3 \div \cdot 625$ is the same as that of 1000 *times* $5\cdot 3 \div 1000$ *times* $\cdot 625$, *i.e.*, of $\mathbf{5300 \div 625}$.

Thus, the case of a *decimal* divisor can always, by multiplying both divisor and dividend by some power of ten, be reduced to that of an *integral* divisor.

Hence, when the divisor is a *decimal*,

we multiply both divisor and dividend by that power of ten which makes the divisor an integer, and then proceed as in Chapter XXXI.

EXAMPLE i.—*Divide* ·35 *by* ·8.

Multiplying both divisor and dividend by 10 we have

$$\cdot 35 \div \cdot 8 = 3\cdot 5 \div \mathbf{8}.$$

$$\begin{array}{r|l} 8 & 3\cdot 5 \\ \hline & \cdot 4375 \ \textit{Ans.} \end{array}$$

EXAMPLE ii.—*Divide* ·7 *by* ·0064.

Multiplying both divisor and dividend by 10000, we have

$$\cdot 7 \div \cdot 0064 = 7000 \div \mathbf{64}.$$

$$64 \left\{ \begin{array}{r|l} 8 & 7000 \\ 8 & 875 \\ \hline & 109\cdot 375 \ \textit{Ans.} \end{array} \right.$$

EXAMPLE iii.—*Divide* 3 *by* 31·25.

Multiplying both divisor and dividend by 100, we have

$$3 \div 31\cdot 25 = 300 \div \mathbf{3125}.$$

$$\begin{array}{l} 3125)300\cdot 000(\cdot 096 \ \textit{Ans.} \\ \quad\quad\ \ 281\ 25 \\ \quad\quad\ \ \ 18\ 750 \\ \quad\quad\ \ \ 18\ 750 \end{array}$$

EXAMPLE iv.—*Divide* 7·31 *by* ·0017.

Multiplying both divisor and dividend by 10000, we have

$$7\cdot 31 \div \cdot 0017 = 73100 \div \mathbf{17}.$$

Here the division is that of an *integer* by an *integer*, and the resulting quotient is the *integer* 4300.

$$\begin{array}{l} 17)73100(4300 \ \textit{Ans.} \\ \quad\ \ 68 \\ \quad\ \ \ 51 \\ \quad\ \ \ 51 \\ \quad\ \ \ \ 00 \end{array}$$

EXAMPLE v.—*Divide* ·013 *by* 2·9, *to five places of decimals.*

Multiplying both divisor and dividend by 10, we have ·013 ÷ 2·9 = ·13 ÷ 29.

Here, as there is no integer in the dividend, there can be none in the quotient, so we begin by marking the decimal point in the quotient.

And as 29 is not contained in 1, nor in 13, we next place *ciphers* in the *tenths*' and *hundredths*' places in the quotient.

We now carry on the division for three steps, and no further, as there are then *five places of decimals* in the quotient.

```
29)·13000(·00448
   116
    140
    116
     240
     232
       8
```

Ans. ·00448 +

The sign + in the result indicates that the division has not terminated.

Note.—It is evident that in such a case the division can never terminate however many ciphers we append to the dividend, for there is no digit by which 9 can be multiplied so as to yield a product ending in 0.

EXAMPLE vi.—*Find the integral part of the quotient of* 38·7563 ÷ ·41, *and the decimal remainder.*

Multiplying both divisor and dividend by 100, we have 38·7563 ÷ ·41 = 3875·63 ÷ 41.

In this case we cease dividing after dealing with the *units*' figure in the dividend.

```
41)3875·63(94·
   369
    185
    164
     21·63
```

Hence, 3875·63 ÷ 41 yields quot^t 94, rem^r 21·63;

∴ 38·7563 ÷ ·41 yields quot^t 94, rem^r ·21 63,

for although the *quotient* is not altered by multiplying both divisor and dividend by 100, the *remainder* 21·63 *which is a part of the dividend*, is 100 times the required rem^r, and must consequently be now *divided* by 100.

Ans. 94; rem^r ·2163.

EXAMPLE vii.—*Divide* ·416 *by* 3·25; ·416 *by* ·0325; *and* 4·16 *by* 32500.

(i) ·416 ÷ 3·25 = 41·6 ÷ 325.

```
325)41·600(·128 Ans. (i).
    32 5
     9 10
     6 50
     2 600
     2 600
```

Now, as the *same figures* occur in all three cases the operation need only be performed *once*.

(ii) ·416 ÷ ·0325 = 4160 ÷ 325.

Here the *working* is the same as in (i), but the decimal point in the dividend is moved *two places* to the *right*, ∴ the decimal point in the quotient will also also be moved *two places* to the *right*.

Hence in this case the result is 12·8 *Ans.* (ii).

(iii) 4·16 ÷ 32500 = ·0416 ÷ 325 (*dividing* both dividend and divisor by 100).

Here also the *working* is the same as in (i), but the decimal point in the dividend is moved *three places* to the *left*, ∴ the decimal point in the quotient must also be moved *three places* to the *left*.

Hence in this case the result is ·000128 *Ans.* (iii).

XXXIII. DECIMALS.

To Express a Decimal as a Vulgar Fraction, and a Vulgar Fraction as a Decimal.

If we "read" a decimal, in the way indicated on page 106, the local value of the right-hand figure gives the *denominator* of an equivalent *vulgar fraction.*

For instance, ·37 is 37 *hundredths, i.e.,* $\frac{37}{100}$;

And ·0319 is 319 *ten-thousandths, i.e.,* $\frac{319}{10000}$;

Also ·000001 is 1 *millionth, i.e.,* $\frac{1}{1000000}$.

Hence we see that in the *denominator* of the vulgar fraction thus obtained there must be **as many ciphers** as there are **decimal places** in the decimal.

If this vulgar fraction is not in lowest terms we reduce it.

Example i.—*Convert* ·025 *to a vulgar fraction in its lowest terms.*

$$\cdot 025 = 25 \textit{ thousandths} = \frac{25}{1000} = \frac{1}{40} \quad \underline{\textit{Ans.}}$$

Example ii. —*Reduce* 2·108 *to a mixed number.*

$$2\cdot 108 = 2\frac{108}{1000} = 2\frac{27}{250} \quad \underline{\textit{Ans.}}$$

Example iii.—*Express* 70·04 *as an improper fraction.*

$$70\cdot 04 = \frac{7004}{100} = \frac{1751}{25} \quad \underline{\textit{Ans.}}$$

Conversely, a vulgar fraction whose denominator is some *power of ten* (*i.e.* 10, 100, 1000, &c.) can at once be written in the form of a decimal.

For instance, $\frac{37}{100}$ is 37 *hundredths, i.e.,* ·37;

And $\frac{9}{10000}$ is 9 *ten-thousandths, i.e.,* ·0009;

Also $\frac{7261}{1000}$ is 7261 *thousandths, i.e.,* 7·261.

Thus a fraction having some *power of ten* as denominator is expressed as a decimal by taking the figures which form the numerator and inserting a decimal point so as to mark off **as many decimal places** as there are **ciphers** in the *denominator.*

Example iv.—*Write* $\frac{217}{1000}$ *as a decimal.*

$$\frac{217}{1000} = \cdot 217 \quad \underline{\textit{Ans.}}$$

EXAMPLE v.—*Express* $31\frac{17}{10000}$ *as a decimal.*

$$31\frac{17}{10000} = 31{\cdot}0017 \ \underline{Ans.}$$

EXAMPLE vi.—*Express* $\frac{53467}{100}$ *as a decimal.*

$$\frac{53467}{100} = 534{\cdot}67 \ \underline{Ans.}$$

In some few cases a vulgar fraction whose denominator is **not a** *power of ten*, may also **at sight** be expressed as a decimal; for we know (see page 68) that the value of a fraction is not altered by multiplying both its numerator and denominator by the same number; and, consequently, *if the denominator of the given fraction is evidently a* **factor of 10**, or **100**, or **1000**, &c., the equivalent decimal can be at once written down.

For instance,
$$\frac{3}{5} = \frac{6}{10} = {\cdot}6;$$
$$\frac{7}{25} = \frac{28}{100} = {\cdot}28;$$
$$\frac{3}{20} = \frac{15}{100} = {\cdot}15;$$
and
$$\frac{1}{8} = \frac{125}{1000} = {\cdot}125.$$

Again, we know (see page 80) that a vulgar fraction represents the *quotient resulting from the division of its numerator by its denominator.*

In the case, then, of a vulgar fraction which cannot easily by inspection be expressed as a decimal, we **perform this division.**

EXAMPLE vii.—*Express* $\frac{13}{64}$ *as a decimal.*

Dividing 13 by factors of 64 we obtain the quotient ·203125 as the req[d] decimal.

$$64\left\{\begin{array}{l|l} 8 & 13\cdot \\ 8 & 1{\cdot}625 \\ \hline & {\cdot}203125 \ \textit{Ans.} \end{array}\right.$$

EXAMPLE viii.—*Convert* $\frac{11}{625}$ *into a decimal.*

Dividing 11 by 625 we obtain the quotient ·0176 as the req[d] decimal.

Note.—In all work in Decimals involving Long Division, the "abridged" method (see p. 15) is, of course, available, if preferred.

```
625)11·0000(·0176 Ans.
    6 25
    4 750
    4 375
      3750
      3750
```

XXXIV. RECURRING DECIMALS.

We have already observed (see note on page 113) that there are cases in Division of Decimals in which *a remainder must always occur*, no matter how far we proceed with the work, while in other cases the division, sooner or later, *terminates.*

Hitherto we have confined our attention almost entirely to divisions which terminate; we now proceed to consider some examples of non-terminating division.

For instance, if we divide ·71 by 11, we obtain the quotient ·06454545 &c. &c. where the pair of figures 45 appear over and over again in the quotient, as we continue the division.

```
11 | ·71
     ·06454545...
```

Also, if we divide 8 by 37, we obtain the quotient ·216216 &c. &c. where the set of figures 216 recur over and over again in the quotient.

```
37)8·0000(·216216...
   7 4
     60
     37
     230
     222
       8
```

Now in each of the above instances we observe that, after a certain stage is reached the remainders reappear, and consequently, in the quotients, the figures which correspond to these remainders also reappear.

And this must always happen when the division does not terminate.

For instance, if we divide 1 by 7 it is evident that the division can never terminate, for there is no digit by which we can multiply 7 and obtain a product ending in 0.

```
7 | 1·0000000000
     ·1428571428....
```

And there are but *six* possible remainders, namely, 1, 2, 3, 4, 5 and 6. Hence, after *six* steps at most (if not before) some one of these remainders, say 3, which has already appeared, must *re*appear; and then all succeeding stages of the division are repetitions in regular order of the earlier ones.

Hence, in every non-terminating division of decimals some figure, or set of figures must, sooner or later, reappear over and over again in the quotient as the division is continued.

A decimal in which some *figure*, or *set of figures arranged in the same order*, is repeated over and over again without limit, is called a **Recurring**, or **Circulating**, Decimal.

For instance, ·47777.... and ·216216216.... are *recurring* decimals.

Decimals which, as distinguished from recurring decimals, contain but a *limited* number of figures are called **terminating** decimals.

The figure, or set of figures, which is repeated in a recurring decimal is called the recurring **Period**.

A recurring period is marked by placing a **dot** above the *first* and *last* figures in the period.

For instance, $\cdot 47777\ldots$ is written shortly thus, $\cdot 4\dot{7}$;
$\cdot 06454545\ldots$, thus, $\cdot 06\dot{4}\dot{5}$;
and $\cdot 216216216\ldots$, thus, $\cdot\dot{2}1\dot{6}$.

A recurring decimal is called *pure*, or *mixed*, according as **all** the figures on the right of the decimal point *do*, or *do not*, recur.

For instance, $6\cdot\dot{4}\dot{5}$, $\cdot\dot{2}1\dot{6}$ and $\cdot\dot{1}4285\dot{7}$ are *pure* recurring decimals.
But $\cdot 4\dot{7}$, $\cdot 06\dot{4}\dot{5}$ and $\cdot 9\dot{1}4285\dot{7}$ are *mixed* recurring decimals.

EXAMPLE i.—*Divide* $8\cdot 9$ *by* 120.

$8\cdot 9 \div 120 = \cdot 89 \div 12$.

$$\begin{array}{r|l} 12 & \cdot 89 \\ \hline & \cdot 074166\ldots \end{array}$$

Ans. $\cdot 0741\dot{6}$.

EXAMPLE ii.—*Divide* 23 *by* $\cdot 011$.

$23 \div \cdot 011 = 23000 \div 11$.

$$\begin{array}{r|l} 11 & 23000\cdot \\ \hline & 2090\cdot 9090\ldots \end{array}$$

Ans. $2090\cdot\dot{9}\dot{0}$.

Note.—In such cases as this the beginner sometimes falls into the error of placing a recurring dot upon an *integer*!

EXAMPLE iii.—*Divide* $\cdot 5$ *by* 28.

Here, in dividing by the factors of 28, we *first* divide by 4, which we foresee will yield a *terminating* quotient, and *afterwards* by the factor 7, which we expect to yield a recurring quotient. It will be found that labour is generally saved by adopting this order.

$$28\left\{\begin{array}{r|l} 4 & \cdot 5 \\ \hline 7 & \cdot 125 \\ \hline & \cdot 01785714285\ldots \end{array}\right.$$

Ans. $\cdot 01\dot{7}85714\dot{2}$.

EXAMPLE iv.—*Divide* $6\cdot 76$ *by* $8\cdot 08$.

$6\cdot 76 \div 8\cdot 08 = 676 \div 808$.

$$\begin{array}{r} 808)676\cdot 0(\cdot 8\dot{3}66\dot{3}\ \textit{Ans.} \\ \underline{646\ 4} \\ 29\ 60 \\ \underline{24\ 24} \\ 5\ 360 \\ \underline{4\ 848} \\ 5120 \\ \underline{4848} \\ 2720 \\ \underline{2424} \\ 296 \end{array}$$

Here, after six steps, we obtain the same figures, 296, in the rem^{r} which had before appeared four steps earlier. Thus the recurring period is 3663.

Note.—The appearance of the same figure twice over in the *quotient* is no sure sign that the quotient has begun to recur, *unless the corresponding remainders are alike.*

XXXV. RECURRING DECIMALS.

Vulgar Fractions expressed as Decimals.

We have seen, on page 115, that a vulgar fraction is converted into a decimal by dividing its numerator by its denominator, but the examples there considered were such as produced *terminating* decimals only. It follows from the last chapter that, in many cases, the decimal which results from this division will be a *recurring* decimal.

Example i.—*Express $\frac{13}{88}$ in the form of a decimal.*

$$88\left\{\begin{array}{l|l} 8 & 13\cdot \\ 11 & 1\cdot625 \\ \hline & \cdot1477272\ldots \end{array}\right. \qquad \underline{Ans.\ \cdot147\dot{7}\dot{2}.}$$

Thus we have two classes of vulgar fractions, namely:—

(1) those which produce *terminating* decimals;
and (2) those which produce *recurring* decimals.

We shall now show how these may be distinguished.

We have seen on page 115 that a vulgar fraction may be converted to a decimal, *without dividing*, if we can find a number by which to multiply both numerator and denominator such that the resulting new denominator is a *power of ten.*

For instance, $\frac{7}{25} = \frac{7}{5\times5} = \frac{7\times4}{10\times10} = \frac{28}{100} = \cdot28.$

Here, as the given den^r was 25, *i.e.* 5 × 5, it was evident that by multiplying both num^r and den^r by 4, *i.e.* by 2 × 2, the new den^r obtained, *i.e.* 5 × 2 × 5 × 2, would be a power of ten.

Similarly, $\frac{11}{400} = \frac{11}{100\times2\times2} = \frac{11\times25}{100\times10\times10} = \frac{275}{10000} = \cdot0275.$

Now as the only prime factors of 10 are 2 and 5, it follows that if *the denominator of a fraction*, **in its lowest terms,** *contains* **no other prime factors than 2 and 5,** the fraction will produce a *terminating* decimal; but that if it contain *any other* prime factor it *cannot* produce a *terminating* decimal, but must produce a *recurring* decimal, for there are no multipliers of 3, 7, 11, 13 &c. which yield a *power of ten* as product.

For instance, $\frac{1}{600}$ will produce a *recurring* decimal, since the denominator contains the factor **3**.

Note.—The condition that the fraction shall be in *lowest terms* before applying this test is important.

Hence we can, by inspection, or by resolving the denr into factors, predict the *number of places of non-recurring decimals* which any given fraction, expressed in its lowest terms, will, if converted into a decimal, yield; for each 10 in the denr will yield *one* place, and each additional 2, or 5, will also yield *one* place.

For instance, $\frac{7}{16}$ will produce a *terminating* decimal of four places, for $\frac{7}{16} = \frac{7}{2\times2\times2\times2} = \frac{7\times5\times5\times5\times5}{10\times10\times10\times10} = \frac{4375}{10000} = \cdot4375$;

And $\frac{1}{12500}$ will produce a *terminating* decimal of five places, for $\frac{1}{12500} = \frac{1}{100\times5\times5\times5} = \frac{1\times2\times2\times2}{100\times10\times10\times10} = \cdot00008$.

Again, $\frac{3}{11}$ will produce a *recurring* decimal with no places of *non*-recurring figures, for the denr contains neither 10, nor 2, nor 5.

And $\frac{1}{14}$ will produce a *recurring* decimal, since its denr contains the factor 7; and there will be one *non*-recurring place, since the denr contains one factor 2,

for $\frac{1}{14} = \frac{1}{7\times2} = \frac{1\times5}{7\times10} = \frac{5}{7}\div10 = \cdot\dot{7}1428\dot{5}\div10 = \cdot0\dot{7}1428\dot{5}$.

Similarly $\frac{19}{8800}$ will produce a *mixed recurring* decimal with five *non*-recurring places.

EXAMPLE ii.—*Which of the fractions* $\frac{1}{32250}$, $\frac{93}{128}$ and $\frac{17}{2222}$ *will produce terminating decimals?*

As 32250 is divisible by 3, $\frac{1}{32250}$ will produce a *recurring* decimal.

As $128 = 8\times16$, and thus contains no other prime factor than 2;

$\therefore \frac{93}{128}$ will produce a *terminating* decimal.

As 2222 is divisible by 11, $\frac{17}{2222}$ will produce a *recurring* decimal.

EXAMPLE iii.—*Find the number of places of non-recurring decimal figures in the decimals equivalent to* $\frac{493}{62500}$, $\frac{13}{224}$ and $\frac{1437}{2750}$.

$$62500 = 5\times5\times5\times5\times10\times10,$$

$\therefore \frac{493}{62500}$ will produce a *terminating* decimal of six places.

$$224 = 2\times2\times2\times2\times2\times7,$$

$\therefore \frac{13}{224}$ will produce a *mixed recurring* decimal with five places of *non*-recurring decimal figures.

$2750 = 10\times5\times5\times11$, (and the numr 1437 is *not* divisible by 11),

$\therefore \frac{1437}{2750}$ will produce a *mixed recurring* decimal, with three places of *non*-recurring decimal figures.

XXXVI. RECURRING DECIMALS.

TO EXPRESS A RECURRING DECIMAL AS A VULGAR FRACTION.

A recurring decimal may be multiplied, or divided, by a power of ten by moving the decimal point to right, or left, in the same way as was shown in the case of a terminating decimal.

For instance, as $\cdot\dot{4}$ signifies $\cdot 44444$ &c. &c. without limit, it is evident that, if we remove to the left of the point a *limited number* of this *unlimited number* of figures there will still remain, on the *right* of the point, an *unlimited number* of figures.

$$\text{Hence } \cdot\dot{4} \times 10 = 4\cdot 44444 \text{ \&c. \&c. without limit;}$$
$$i.e.\ \cdot\dot{4} \times 10 = 4\cdot\dot{4}.$$

Similarly $\cdot\dot{6}2\dot{3} \times 1000 = 623\cdot 623623$ &c. &c. $= 623\cdot\dot{6}2\dot{3}$.
Also $\cdot 4\dot{7} \times 10 = 4\cdot\dot{7}$; $\cdot 4\dot{7} \times 100 = 47\cdot\dot{7}$; and so on.

We shall now show how the vulgar fraction equivalent to any *pure* recurring decimal may be obtained.

For instance, *to find the vulgar fraction equivalent to* $\cdot\dot{4}$.

$$\begin{aligned} \text{As } \mathbf{10} \textit{ times } \cdot\dot{4} &= 4\cdot 44444\ldots\ldots\ldots \\ \text{and } \cdot\dot{4} &= \cdot 44444\ldots\ldots\ldots \\ \therefore \text{(subtracting) } \mathbf{9} \textit{ times } \cdot\dot{4} &= 4. \\ \text{Hence } \cdot\dot{4} &= \frac{4}{9}. \end{aligned}$$

Again, *to find the vulgar fraction equivalent to* $\cdot\dot{6}2\dot{3}$.

$$\begin{aligned} \text{As } \mathbf{1000} \textit{ times } \cdot\dot{6}2\dot{3} &= 623\cdot 623623623\ldots\ldots\ldots \\ \text{and } \cdot\dot{6}2\dot{3} &= \cdot 623623623\ldots\ldots\ldots \\ \therefore \text{(subtracting) } \mathbf{999} \textit{ times } \cdot\dot{6}2\dot{3} &= 623. \\ \text{Hence } \cdot\dot{6}2\dot{3} &= \frac{623}{999}. \end{aligned}$$

N.B.—We multiply the given decimal by 10, or 100, or 1000, &c., according as there are *one*, or *two*, or *three*, &c., *figures in the period.*

From such *results* as those above we obtain the following rule:—

(I.) To express a **pure** recurring decimal as a vulgar fraction.

For the *numerator*, write **the figures of the recurring period;**
for the *denominator*, write **as many nines as there are figures in the period.**

EXAMPLE i.—*Convert* $\cdot\dot{7}\dot{2}$ *and* $\cdot\dot{0}19\dot{8}$ *into vulgar fractions.*

$\cdot\dot{7}\dot{2} = \frac{72}{99} = \frac{8}{11}$ Ans. $\cdot\dot{0}19\dot{8} = \frac{198}{9999} = \frac{22}{1111} = \frac{2}{101}$ Ans.

Next, in the case of any *mixed* recurring decimal.

For instance, *to find the vulgar fraction equivalent to* $\cdot 4\dot{7}$.

$$\text{As } \mathbf{100} \textit{ times } \cdot 4\dot{7} = 47\cdot 77777\ldots\ldots\ldots$$
$$\text{and } \mathbf{10} \textit{ times } \cdot 4\dot{7} = 4\cdot 77777\ldots\ldots\ldots$$
$$\therefore \text{, subtracting, } \mathbf{90} \textit{ times } \cdot 4\dot{7} = 47 - 4.$$
$$\text{Hence} \qquad \cdot 4\dot{7} = \frac{47-4}{90}.$$

Again, *to find the vulgar fraction equivalent to* $\cdot 63\dot{1}2\dot{5}$.

$$\text{As } \mathbf{100000} \textit{ times } \cdot 63\dot{1}2\dot{5} = 63125\cdot 125125125\ldots$$
$$\text{and } \mathbf{100} \textit{ times } \cdot 63\dot{1}2\dot{5} = 63\cdot 125125125\ldots$$
$$\therefore \text{, subtracting, } \mathbf{99900} \textit{ times } \cdot 63\dot{1}2\dot{5} = 63125 - 63.$$
$$\text{Hence} \qquad \cdot 63\dot{1}2\dot{5} = \frac{63125-63}{99900}.$$

N.B.—We multiply the given decimal first by that power of ten which moves the decimal point to the *end* of the period; and next by that power of ten which moves the point up to the *beginning* of the period.

From such *results* as those above we obtain the following rule:—

(II.) To express a **mixed** recurring decimal as a vulgar fraction.

For the *numerator*, **subtract the number formed by the non-recurring figures from the number formed by all the figures as far as the end of the period;**

for the *denominator*, write **as many nines as there are figures in the period,** followed by **as many ciphers as there are non-recurring decimal figures.**

EXAMPLE ii.—*Reduce* $\cdot 23\dot{4}\dot{5}$ *to a vulgar fraction.*

$$\cdot 23\dot{4}\dot{5} = \frac{2345-23}{9900} = \frac{2322}{9900} = \frac{258}{1100} = \frac{129}{550} \textit{ Ans.}$$

EXAMPLE iii.—*Express* $8\cdot 708\dot{3}$ *in the form of a vulgar fraction.*

$$8\cdot 708\dot{3} = 8\frac{7083-708}{9000} = 8\frac{6375}{9000} = 8\frac{2125}{3000} = 8\frac{17}{24} \textit{ Ans.}$$
$$\text{or thus, } 8\cdot 708\dot{3} = \frac{87083-8708}{9000} = \frac{78375}{9000} = \frac{209}{24}.$$

The value of a *mixed* recurring decimal may also be deduced from that of a *pure* one.

For instance, as $\cdot 4\dot{7} \times 10 = 4\cdot\dot{7} = 4\frac{7}{9}$, $\therefore \cdot 4\dot{7} = 4\frac{7}{9} \div 10 = \frac{43}{90}$.

N.B.—It is important to notice that $\cdot\dot{9} = \frac{9}{9} = 1$; and, consequently, $\cdot 0\dot{9}$, $\cdot 4\dot{9}$, $\cdot 23\dot{9}$ &c. are equivalent to the *terminating* decimals $\cdot 1$, $\cdot 5$, $\cdot 24$ &c. respectively.

We have seen that every *pure* recurring decimal can be expressed as a vulgar fraction which has a den^r consisting of as many *nines* as there are figures in the period. If, therefore, we can find a number, by which to multiply both num^r and den^r, such that the resulting new den^r is all *nines*, we may, *without dividing*, express the fraction as a recurring decimal.

For instance, $\frac{2}{3} = \frac{2\times3}{3\times3} = \frac{6}{9} = \cdot\dot{6}$; $\frac{5}{11} = \frac{5\times9}{11\times9} = \frac{45}{99} = \cdot\dot{4}\dot{5}$;
also $\frac{2}{37} = \frac{6}{111} = \frac{54}{999} = \cdot\dot{0}5\dot{4}$; $\frac{3}{37} = \frac{81}{999} = \cdot\dot{0}8\dot{1}$; $\frac{4}{37} = \frac{108}{999} = \cdot\dot{1}0\dot{8}$; &c.

Hence all fractions in lowest terms which have *the same den^r* yield decimals having the same *number of figures in the period.*

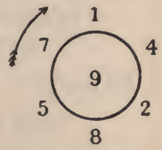
Also, by resolving 9, 99, 999 &c. into their prime factors, we can discover all the possible den^{rs} of fractions in lowest terms which yield periods of 1, 2, 3 &c. figures respectively:—

Thus the *only* den^{rs} of fractions yielding pure recurring decimals with periods of

one figure are **3** and 9;
two figures .... **11**, 33 and 99;
three .... **27**, **37**, 111, 333 and 999;
four .... **101**, 303, 909, 1111, 3333 and 9999;
five.............. **41**, 123, 369, **271**, 813, 2439, 11111, 33333 and 99999;
six.............. **7**, **13**, and either of these numbers, or their product, multiplied by any one or more of the nos. 3, 9, 27, 11 or 37.

Note.—The number 7, the only den^r less than 17 which yields a decimal with the *maximum* period, is noteworthy. For, as all the possible rem^{rs} are exhausted during the division, it is evident that, whatever be the *numerator* of the fraction, the *same figures* must appear in the quotient.

Thus $\frac{1}{7} = \cdot\dot{1}4285\dot{7}$, $\frac{2}{7} = \cdot\dot{2}8571\dot{4}$, $\frac{3}{7} = \cdot\dot{4}2857\dot{1}$,
$\frac{4}{7} = \cdot\dot{5}7142\dot{8}$, $\frac{5}{7} = \cdot\dot{7}1428\dot{5}$, $\frac{6}{7} = \cdot\dot{8}5714\dot{2}$.



Now, if we arrange the figures 1, 4, 2, 8, 5, 7 in a circle and read them in the direction of the arrow, beginning in turn with 1, 2, 4 &c., we obtain the periods of $\frac{1}{7}$, $\frac{2}{7}$, $\frac{3}{7}$ &c. respectively.

Moreover, as the sum of any opposite pair of figures thus arranged is 9, if we *commit to memory the first three*, namely, **"One, Four, Two,"** the other three figures are at once obtained by subtracting these in turn from 9; and thus the decimal value of $\frac{1}{7}$, $\frac{2}{7}$ &c., or the fractional value of any one of these periods, can be written down at once.

EXAMPLE iv.—*Reduce* $3\cdot43\dot{5}7142\dot{8}$ *to a vulgar fraction.*

$$3\cdot43\dot{5}7142\dot{8} = 3\cdot43\tfrac{4}{7}^{*} = 3\frac{43\frac{4}{7}}{100} = 3\tfrac{305}{700} = 3\tfrac{61}{140} \ \underline{Ans.}$$

EXAMPLE v.—*Reduce* $\frac{3}{35}$ *to a decimal.*

$$\frac{3}{35} = \frac{6}{70} = \frac{6}{7} \div 10 = \cdot\dot{8}5714\dot{2} \div 10 = \underline{\cdot0\dot{8}5714\dot{2} \ Ans.}$$

* See the above Note; also the last paragraph on page 121; also Appendix.

XXXVII. RECURRING DECIMALS.

ADDITION AND SUBTRACTION.

In order to add, or subtract, recurring decimals, we arrange them with the decimal points in a vertical line and repeat the recurring periods a sufficient number of times to enable us to see the recurring period in the sum, or difference; we then proceed as in the case of terminating decimals, *except that we omit to write down* the sum, or difference, of *the two columns on the extreme right*, from which we obtain the figure to be "carried."

We can determine beforehand to what extent to repeat the periods; for the *first column which contains none but recurring figures* must appear again after an interval not greater (though it may be less) than the L.C.M. of *the number of figures in each of the recurring periods*, and, consequently, the *sum*, or *difference*, of the figures in this column must also reappear.

EXAMPLE i.—*Add together* $\cdot 7$, $2\cdot 0\dot{3}\dot{1}$, $\cdot 1\dot{4}7\dot{5}$ *and* $26\cdot 0\dot{8}$.

Here we have recurring periods of 2, 3 and 1 figures respectively; and the L.C.M. of 2, 3 and 1 is 6. Also the first column containing none but recurring figures is the 2nd to the right of the decimal points. Hence this, and the five succeeding columns (as far as the vertical line) will yield the recurring period in the result. We, however, extend the periods *two* columns further, in order to obtain the figure to be "carried" to the column on the left of the vertical line.

·7	
2·0313131	31
·1475475	47
26·0888888	88
28·9677495	

Ans. $28\cdot 9\dot{6}7749\dot{5}$.

EXAMPLE ii.—*Subtract* $9\cdot 32\dot{1}5\dot{7}$ *from* $16\cdot\dot{4}80\dot{3}$.

Here we have periods of 3 and 4 figures respectively; and the L.C.M. of 3 and 4 is 12. Also the column containing 0 and 1 is the first recurring column; so we draw the vertical line to the left of this on its reappearance 12 columns further on.

16·48034803480348	03
9·32157157157157	15
7·15877646323190	

Ans. $7\cdot 15\dot{8}7764632319\dot{0}$

EXAMPLE iii.—*Find the sum of* $3\cdot 20\dot{7}\dot{4}$, $\cdot 0\dot{1}$ *and* $21\cdot 486\dot{4}\dot{1}$.

Here we expect a *two*-figure period in the result, but as both figures in this period are *alike*, we have really a period of *one* figure, namely $24\cdot 704\dot{9}$.

But we know (see page 121) that, as $\cdot\dot{9} = 1$, $\cdot 4\dot{9} = \cdot 5$. Hence in this case the result becomes the terminating decimal 24·705.

3·20747	47
·01111	11
21·48641	41
24·70499	

Ans. 24·705.

XXXVIII. RECURRING DECIMALS.

MULTIPLICATION AND DIVISION.

Similarly a recurring decimal may be multiplied, or divided, by either an *integer* or a *terminating* decimal, if we repeat the recurring period a sufficient number of times to enable us to see the recurring period in the product, or quotient.

EXAMPLE i.—*Multiply* $6\cdot248\dot{3}$ *by* $\cdot05$.

Here the two cancelled figures on the right are only multiplied for the sake of the figure to be "carried," and are not counted in marking the position of the decimal point.

$$\begin{array}{r} 6\cdot24833\cancel{3}\cancel{3} \\ \cdot05 \\ \hline \cdot31\ 24166\,.\,. \end{array}$$

Ans. $\cdot31241\dot{6}$.

EXAMPLE ii.—*Multiply* $1\cdot25\dot{7}$ *by* 64.

Here, multiplying $1\cdot2577\ldots$ by 4, and obtaining the carried figure as before, the product is $5\cdot0311\ldots$; also, multiplying $1\cdot2577\ldots$ by 60, we obtain the product $75\cdot466\ldots$.

We now add $5\cdot03\dot{1}$ and $75\cdot4\dot{6}$ by the method of the last chapter, obtaining the result $80\cdot49\dot{7}$.

$$\begin{array}{r} 1\cdot25777\cancel{7} \\ 64 \\ \hline 5\cdot03111\,. \\ 75\cdot4666\,.\,. \\ \hline 80\cdot4977\,.\,. \end{array}$$

Ans. $80\cdot49\dot{7}$.

Note.—It is only necessary to repeat the period in the multiplicand *once* in order to obtain the figure to be "carried;" for it follows, from page 122, that the *number of figures* in the *recurring period* of the various products will be the *same* as in the multiplicand.

EXAMPLE iii.—*Multiply* $4\cdot06\dot{8}\dot{5}$ *by* $1\cdot23$.

Multiplying by $\cdot03$ the product is $\cdot1220\dot{5}\dot{7}$.
Similarly, multiplying by $\cdot2$, the product is $\cdot813\dot{7}\dot{1}$; and multiplying by 1, the product is $4\cdot06\dot{8}\dot{5}$.

We now, *after the multiplication is finished*, repeat the periods of these products, in order to add them by the method of the previous chapter. *Note.*—The figures in thin type are those appended *after* the *multiplication* was finished in order to add the results.

$$\begin{array}{r|r} 4\cdot06\ 85\cancel{8}\cancel{5} & \\ 1\cdot23 & \\ \hline \cdot12\ 20\ 57 & 57 \\ \cdot81\ 37\ 17 & 17 \\ 4\cdot06\ 85\ 85 & 85 \\ \hline 5\cdot00\ 43\ 60 & \end{array}$$

Ans. $5\cdot0043\dot{6}\dot{0}$.

EXAMPLE iv.—*Divide* $37\cdot48\dot{3}$ *by* $\cdot05$.

$$\therefore\ 37\cdot48\dot{3} \div \cdot05$$
$$= 3748\cdot\dot{3} \div 5.$$

$$\begin{array}{r|l} 5 & 3748\cdot333\ldots \\ \hline & 749\cdot66\ldots. \end{array}$$

Ans. $749\cdot\dot{6}$.

When the multiplier, or divisor, is itself a *recurring* decimal, we reduce the decimals to vulgar fractions in order to perform the operation. This method also is to be preferred in certain cases of a *non*-recurring multiplier, or divisor. For instance:—

EXAMPLE v.—*Multiply* $1\cdot\dot{5}7142\dot{8}$ *by* $2\cdot17$.

$$1\cdot\dot{5}7142\dot{8} \times 2\cdot17 = 1\tfrac{4}{7}{}^{*} \times \frac{217}{100} = \frac{11}{\cancel{7}} \times \frac{\overset{31}{\cancel{217}}}{100} = \frac{341}{100} = \underline{3\cdot41} \textit{ Ans.}$$

EXAMPLE vi.—*Multiply* $3\cdot\dot{2}\dot{7}$ *by* $2\cdot8\dot{3}$.

$$3\cdot\dot{2}\dot{7} \times 2\cdot8\dot{3} = 3\tfrac{27}{99} \times 2\tfrac{83-8}{90} = 3\tfrac{3}{11} \times 2\tfrac{5}{6} = \frac{\overset{6}{\cancel{36}}}{11} \times \frac{17}{\cancel{6}}$$
$$= \frac{102}{11} = 9\tfrac{3}{11} = 9\tfrac{27}{99} = \underline{9\cdot\dot{2}\dot{7}} \textit{ Ans.}$$

EXAMPLE vii.—*Divide* $6\cdot\dot{7}$ *by* $2\cdot\dot{6}$.

$$6\cdot\dot{7} \div 2\cdot\dot{6} = 6\tfrac{7}{9} \div 2\tfrac{6}{9} = \frac{61}{\underset{1}{\cancel{9}}} \times \frac{\overset{1}{\cancel{9}}}{24}$$
$$= \frac{61}{24} = \underline{2\cdot541\dot{6}} \textit{ Ans.}$$

$$24\left\{\begin{array}{l|l} 8 & 61\cdot \\ \hline 3 & 7\cdot625 \\ \hline & 2\cdot54166.. \end{array}\right.$$

EXAMPLE viii.—*Simplify* $\dfrac{\cdot7\dot{0}\dot{9}}{\cdot\dot{8}\dot{1}} + \dfrac{2\cdot01\dot{6}}{1\cdot\dot{6}} \times \cdot\dot{0}\dot{1}$.

$$\frac{\cdot7\dot{0}\dot{9}}{\cdot\dot{8}\dot{1}} + \frac{2\cdot01\dot{6}}{1\cdot\dot{6}} \times \cdot\dot{0}\dot{1} = \frac{\;\frac{709-7}{990}\;}{\frac{81}{99}} + \frac{\;\frac{2016-201}{900}\;}{1\frac{6}{9}} \times \frac{1}{99}$$

$$= \frac{\overset{78}{\cancel{702}}}{\underset{10}{\cancel{990}}} \times \frac{\overset{1}{\cancel{99}}}{\underset{9}{\cancel{81}}} + \frac{\overset{11}{\cancel{\overset{121}{\cancel{1815}}}}}{900} \times \frac{\overset{1}{\cancel{9}}}{\underset{1}{\cancel{15}}} \times \frac{1}{\underset{1}{\cancel{\underset{11}{\cancel{99}}}}} = \frac{78}{90} + \frac{11}{900} = \frac{791}{900}$$
$$= \frac{7\cdot91}{9} = \underline{\cdot87\dot{8}} \textit{ Ans.}$$

---

* See note on page 122.

XXXIX. DECIMALS.

VALUE OF A DECIMAL OF A CONCRETE QUANTITY.

In the case of a *terminating* decimal the method corresponds to that of ordinary Reduction.

EXAMPLE i.—*Find the value of* ·6875 *of* £1.

As ·6875 of £1 = ·6875 of 20*s.*, we multiply ·6875 by 20 and mark off *four* places of decimals in the result, thus obtaining 13·7500*s.* We now *reserve the integral part*, 13*s.*, and reduce the remaining ·7500*s.* to pence. Multiplying ·7500 by 12, and again marking off *four* places of decimals, we obtain the integer 9*d.* Thus the complete result is 13*s.* 9*d.*

```
     ·6875
        20
s. 13·7500
        12
 d. 9·0000
```

Ans. 13*s.* 9*d.*

EXAMPLE ii.—*Find the value of* 2·55625 *of* 2 *tons.*

Multiplying 2·55625 by 2, we obtain 5·11250 as the number of tons, so we reserve the integer 5 as part of the answer, and reduce the remaining ·11250 of a ton to cwts. &c.

Note.—Ciphers on the right may of course be discarded at each step, if we then mark off at each step a number of decimal places correspondingly less.

```
      2·55625
            2
tons  5·11250
           20
cwts. 2·25000
```

Ans. 5 tons 2 cwts. 1 qr.

EXAMPLE iii.—*Find the value of* ·2896 *of* £15, 12*s.* 6*d.*

As ·2896 of £15, 12*s.* 6*d.* = ·2896 of $312\frac{1}{2}$*s.*
= ·2896 of 312·5*s.*,

we multiply ·2896 and 312·5 together, and mark off *five* places of decimals in the result, thus obtaining 90·5*s.*, *i.e.* £4, 10*s.* 6*d.*

Ans. £4, 10*s.* 6*d.*

```
      ·2896
      312·5
      14480
      5792
     2896
    8688
s. 90·50000
```

EXAMPLE iv.—*Find the value of* ·3425 *of* £1 − ·2375 *of a guinea* + ·875 *of* 7*s.* 6*d.*

Here we first express all the quantities in the same denomination, in this case, in *shillings*;

```
                  ·2375           ·875
·3425                21            7·5
   20              2375           4375
6·8500...(i)      4750           6125
                 4·9875...(ii)   6·5625...(iii)
```

We now add (i) to (iii) and subtract (ii) from the result, thus obtaining $8\cdot425\,s.$ Then reducing $\cdot425s.$ to pence, we obtain $5\cdot1d.$ Thus the complete result is $8s.\ 5\cdot1d.$

$$\begin{array}{r} 6\cdot5625 \\ 6\cdot8500 \\ \hline 13\cdot4125 \\ 4\cdot9875 \\ \hline s.\ \underline{8}\cdot425 \\ 12 \\ \hline d.\ \underline{5}\cdot100 \end{array}$$

Ans. $\underline{8s.\ 5\cdot1d.}$

Similarly we may proceed in the case of a recurring decimal, or we may convert the recurring decimal into a vulgar fraction.

EXAMPLE V.—*Find the value of* $\cdot3958\dot{3}$ *of* £1.

Here, in multiplying by 20 we multiply by 2, as in the last chapter, and then move the decimal point one place to the right in the product. Finally, we obtain the result, $7s.\ 10\cdot\dot{9}d.$, and, as $\cdot\dot{9} = 1$, this is equivalent to $7s.\ 11d.$

$$\begin{array}{r} \cdot395833\cancel{3}\cancel{3}\ldots \\ 20 \\ \hline s.\ \underline{7}\cdot9166\cancel{6}\cancel{6}\ldots \\ 12 \\ \hline d.\ \underline{10}\cdot9999\ldots\ldots \end{array}$$

Ans. $\underline{7s.\ 11d.}$

EXAMPLE VI.—*Find the value of* $\cdot\dot{3}8\dot{4}$ *of* £1, 7*s.* 6*d.* + $\cdot38\dot{4}$ *of* £1, 17*s.* 6*d.*

$$\cdot\dot{3}8\dot{4} \text{ of } £1,\ 7s.\ 6d. + \cdot38\dot{4} \text{ of } £1,\ 17s.\ 6d.$$

$$= \frac{384-3}{990} \times 27\tfrac{1}{2}s. \quad + \quad \frac{384-38}{900} \times 37\tfrac{1}{2}s.$$

$$= \frac{\overset{127}{\cancel{381}}}{\underset{\underset{\underset{6}{\cancel{18}}}{\cancel{90}}}{\cancel{990}}} \times \frac{\overset{\overset{1}{\cancel{5}}}{\cancel{55}}}{2}s. \quad + \quad \frac{\overset{173}{\cancel{346}}}{\underset{\underset{12}{\cancel{36}}}{\cancel{900}}} \times \frac{\overset{\overset{1}{\cancel{3}}}{\cancel{75}}}{\underset{1}{\cancel{2}}}s.$$

$$= \quad 10\tfrac{7}{12}\,s. \quad + \quad 14\tfrac{5}{12}\,s. = 25s. = \underline{£1,\ 5s.\ \textit{Ans.}}$$

EXAMPLE VII.—*Find the value of* $\cdot3\dot{4}2857\dot{1}$ *of 2 tons* 13 *cwts.* 3 *qrs.*

$\cdot3\dot{4}2857\dot{1}$ of 2 tons 13 cwts. 3 qrs.

$$= \cdot3\tfrac{3}{7}^{*} \text{ of } 53\tfrac{3}{4} \text{ cwts.} = \frac{\overset{\overset{3}{\cancel{6}}}{\cancel{24}}}{\underset{\underset{7}{\cancel{14}}}{\cancel{70}}} \times \frac{\overset{43}{\cancel{215}}}{\cancel{4}} \text{ cwts.} = \frac{129}{7} \text{ cwts.}$$

$$= 18\tfrac{3}{7} \text{ cwts.} \doteq \underline{18 \text{ cwts. } 1 \text{ qr. } 20 \text{ lbs. } \textit{Ans.}}$$

---

* See note on page 121.

XL. DECIMALS.

TO EXPRESS ONE CONCRETE QUANTITY AS THE DECIMAL OF ANOTHER OF THE SAME KIND.

This may always be done by expressing the given quantity as a *vulgar fraction* of the other (see Chap. XXIII.), and then reducing this vulgar fraction thus obtained to a decimal.

EXAMPLE i.—*Express* 2 *qrs.* 21 *lbs. as the decimal of* 2 *cwts.* 2 *qrs.*

2 qrs. 21 lbs. = $2\frac{3}{4}$ qrs.; 2 cwts. 2 qrs. = 10 qrs.;

$$\text{and } \frac{2\frac{3}{4}}{10} = \frac{11}{40} = \frac{1\cdot1}{4} = \underline{\cdot275 \ \textit{Ans.}}$$

EXAMPLE ii.—*What decimal of* 18 *yds.* 2 *ft.* 3 *in. is* 2 *yds.* 1 *ft.* 4 *in.?*

2 yds. 1 ft. 4 in. = $7\frac{1}{3}$ ft.; 18 yds. 2 ft. 3 in. = $56\frac{1}{4}$ ft.;

$$\text{and } \frac{7\frac{1}{3}}{56\frac{1}{4}} = \frac{22}{3} \times \frac{4}{225} = \frac{88}{675}$$

$$= \frac{88 \times 4^*}{675 \times 4} = \frac{352}{2700} = \frac{3\cdot52}{27} = \underline{\cdot1\dot{3}03\dot{7} \ \textit{Ans.}}$$

$$27 \begin{cases} 3 \,|\, \underline{3\cdot52} \\ 9 \,|\, \underline{1\cdot1733333...} \end{cases}$$
$$\cdot1303703...$$

If, however, the given quantity is to be expressed as the decimal of some *simple* quantity, we may, instead of the above method, reverse the "Reduction" method of the last chapter.

EXAMPLE iii.—*Reduce* 15*s.* $8\frac{1}{4}$*d. to the decimal of* £1.

$8\frac{1}{4}$*d.* = 8·25*d.*

Now to reduce 8·25*d.* to *shillings* we divide by 12, obtaining ·6875*s.*

Hence 15*s.* $8\frac{1}{4}$*d.* = 15·6875*s.*, so we set **15** to the left of ·6875.

$$\begin{array}{r|l} 12 & \underline{8\cdot25} \\ 2,0 & \underline{15\cdot6875} \\ & \cdot7\ 84375 \end{array}$$

Ans. ·784375.

And to express this in *pounds* we divide by 20 (*i.e.* we divide by **2**, and then move the decimal point *one* place to the left), obtaining £·784375 as the req[d] decimal.

---

* We multiply both numerator and denominator of the fraction by 4 at this stage, foreseeing that by so doing a more convenient denominator is obtained as the divisor in the final division.

EXAMPLE iv.—*Reduce £2, 17s. 10¾d. to the decimal of £5.*

10¾*d.* = 10·75*d.*

Now to reduce 10·75*d.* to *shillings* we divide by 12, obtaining ·895833...*s.*

∴ £2, 17*s.* 10¾*d.* = 57·895833...*s.*

And, to reduce 57·895833...*s.* to the decimal of £5, we divide by 100 (the number of shillings in £5), *i.e.* we merely move the decimal point *two* places to the left.

$$\begin{array}{r|l} 12 & 10\cdot 75 \\ \hline 1{,}00 & 57\cdot 895833\ldots \\ \hline & \cdot 578958\dot{3} \end{array}$$

Ans. $\cdot 578958\dot{3}$.

EXAMPLE v.—*Reduce* 1·53 *of* £2, 17*s.* 3½*d. to pence and the decimal of a penny.*

Reducing £2, 17*s.* 3½*d.* to pence, we obtain 687·5*d.*

We now multiply this by 1·53.

$$\begin{array}{lll} \pounds & s. & d. \\ 2\ . & 17\ . & 3\tfrac{1}{2} \\ 20 & & \\ \hline 57 & & \\ 12 & & \\ \hline \multicolumn{3}{l}{687\tfrac{1}{2} = 687\cdot 5d.} \end{array}
\qquad
\begin{array}{r} 687\cdot 5 \\ 1\cdot 53 \\ \hline 20625 \\ 34375 \\ 6875 \\ \hline 1051\cdot 875 \end{array}$$

Ans. 1051·875*d.*

EXAMPLE vi.—*Subtract* ·48 *of* 3 *weeks* 2 *days from* 2·47 *of* 5 *days* 1 *hour, and express the result in minutes.*

Here, in order to perform the subtraction, we first express both quantities in the common denomination *hours.*

We now subtract (i) from (ii), obtaining 33·91 *hours.*

Finally, to reduce 33·91 *hours* to *minutes,* we multiply it by 60.

$$\begin{array}{r} \cdot 48 \\ 23 \\ \hline 144 \\ 96 \\ \hline 1104 \\ 24 \\ \hline 264\cdot 96\ldots\text{(i)} \end{array}
\qquad
\begin{array}{r} 2\cdot 47 \\ 121 \\ \hline 247 \\ 494 \\ 247 \\ \hline 298\cdot 87\ldots\text{(ii)} \\ -\ 264\cdot 96 \\ \hline 33\cdot 91 \\ 60 \\ \hline 2034\cdot 60 \end{array}$$

Ans. 2034·6 mins.

XLI. MISCELLANEOUS EXAMPLES.

The devices illustrated in Chapter III. with integers may, of course, be employed in similar cases of multiplication and division of decimals. For instance:—

A. *Divide* $3\cdot12$ *by* $\cdot0125$.

$$3\cdot12 \div \cdot0125 = 31200 \div 125 = 31\cdot2 \times 8 = \underline{249\cdot6 \ \textit{Ans.}}$$

It is often convenient to express a division of decimals in the following form:—

B. *Divide* $9\cdot1 \times \cdot005 \times 3\cdot7$ *by* $\cdot074 \times 5\cdot2$.

$$\frac{9\cdot1 \times \cdot005 \times 3\cdot7}{\cdot074 \times 5\cdot2} = \frac{\overset{7}{\cancel{91}} \times 5 \times \overset{1}{\cancel{37}}}{\underset{2}{\cancel{74}} \times \underset{4}{\cancel{52}} \times 10} = \frac{35}{80} = \frac{3\cdot5}{8} = \underline{\cdot4375 \ \textit{Ans.}}$$

Here we multiply both num^r and den^r by 10000, thus replacing all the decimals by integers; *i.e.* we multiply $9\cdot1$ by 10, $\cdot005$ by 1000, and $3\cdot7$ by 10, and therefore the entire num^r by 10000. Similarly we deal with the den^r. We then "cancel" before performing the division.

C. *Show that* $\frac{2}{17} = \cdot117647\frac{1}{17}$; *and then, without further division, obtain the value of* $\frac{2}{17}$ *to twelve places of decimals.*

Dividing 2 by 17 to six places of decimals, we obtain the quotient $\cdot117647$ and rem^r $\cdot000001$.

$$\therefore \tfrac{2}{17} = \cdot117647\tfrac{1}{17} \ \textit{Ans.} \text{ (i).}$$

Now $\frac{1}{17} = \frac{1}{2}$ of $\frac{2}{17} = \frac{1}{2}$ of $\cdot117647\frac{1}{17} = \cdot058823\frac{9}{17}$,

$$\therefore \tfrac{1}{17} \text{ part of } \textit{one-millionth} = \cdot000000058823\tfrac{9}{17}.$$

Hence $\frac{2}{17} = \cdot117647058823\frac{9}{17}$ *Ans.* (ii).

D. *Find the least integer which contains both* $4\cdot15$ *and* $49\cdot8$ *each an integral number of times.*

$4\cdot15$ is 415 *hundredths*, and $49\cdot8$ is 4980 *hundredths*.

The L.C.M. of 415 and 4980 is found to be $5 \times 12 \times 83$.

$\therefore$ The L.C.M. of 415 *hundredths* and 4980 *hundredths* is $5 \times 12 \times 83$ *hundredths.*

i.e. $\frac{5 \times 12 \times 83}{100}$, or $\frac{3 \times 83}{5}$, contains both the given decimals an integral number of times, but is not itself an integer.

Hence the required least *integer* is evidently $\frac{3 \times 83}{5} \times 5 = \underline{249 \ \textit{Ans.}}$

XLII. THE UNITARY METHOD,

OR SINGLE RULE OF THREE.

We shall now consider a class of questions involving both multiplication and division.

The first eight examples illustrate the "Unitary" method of solution; the rest the "Fractional Factor" method.

EXAMPLE i.—*If* 8 *lbs. of tea cost* 14*s. what do* 3 *lbs. of the tea cost?*

Here the cost of 8 lbs. is given; from which we obtain the cost of the unit* (*i.e.* of **1** lb.) *by dividing the given cost by* **8**.

The cost of 3 lbs. is then found *by multiplying the cost of* 1 *lb. by* **3**.

Written work.

8 *lbs.* cost 14*s.*

$\therefore$ 1 *lb.* costs $\frac{1}{8}$ of 14*s.*

Hence 3 *lbs.* cost $\frac{3}{8}$ of 14*s.* $= \frac{21}{4}$*s.*

$=$ 5*s.* 3*d.* *Ans.*

Note.—We do not actually perform the operations at each stage, but merely indicate them until the final stage of the reasoning is reached.

EXAMPLE ii.—*If* 5 *lbs. of tea cost* 9*s.* 2*d.*, *how many lbs. of the same kind of tea cost* £10, 10*s.* 10*d.?*

Here we first reduce the two sums of money to the same denomination, viz. *twopences.* Then, observing that the answer required is *weight,* not *money,* we re-state the fact given in the question, arranging it so that "lbs." shall occupy the right-hand column.

Written work.

s.	*d.*		£	*s.*	*d.*
9 .	2		10 .	10 .	10
6			20		
55			210		
			6		
			1265		

For 55 *twopences* we buy 5 *lbs.*

$\therefore$ ... 1 *twopence* ....$\frac{1}{55}$ of 5 *lbs.*

Hence ... 1265 *twopences* .....$\frac{1265}{55}$ of 5 *lbs.* $= \frac{1265}{11}$ *lbs.*

$=$ 115 *lbs.* *Ans.*

---

* Hence the name "Unitary" method.

In the solution of the last example we expressed the compound quantities in *one* denomination, as is generally more convenient. Sometimes, however, it is better to use Compound Multiplication and Division; for instance:—

EXAMPLE iii.—*If* 7 *articles cost* £6, 10*s.* $2\frac{3}{4}$*d.*, *what is the cost of* 3 *of them?*

7 *articles* cost £6, 10*s.* $2\frac{3}{4}$*d.*

$\therefore$ 1 *article* costs $\dfrac{\text{£6, 10s. }2\frac{3}{4}\text{d.}}{7}$

Hence 3 *articles* cost $\dfrac{\text{£6, 10s. }2\frac{3}{4}\text{d.}}{7} \times 3 = \dfrac{\text{£19, 10s. }8\frac{1}{4}\text{d.}}{7}$

$= \underline{\text{£2, 15s. }9\frac{3}{4}\text{d.}}$ *Ans.*

EXAMPLE iv.—*If* 6 *men could hoe a field in* 10 *days, how long should* 15 *men take to do it?*

Written work.

6 *men* can do the work in 10 *days*

$\therefore$ 1 *man*.........................6 times 10 *days*

Hence 15 *men* .... $\frac{1}{15}$ of 60 *days* = $\underline{4 \textit{ days}}$ *Ans.*

Note.—Here we *multiply* in the second line; for 1 man would take *longer* than 6 men.

EXAMPLE v.—*How far should* 40 *lbs. be carried by rail for the money that pays the carriage of* 1 *cwt. for* 130 *miles?*

Written work.

	lbs.		miles.
	112	is carried	130
$\therefore$	1	....	130×112
Hence	40	....	$\frac{1}{40}$ of 130×112

$= 13 \times 28 = \underline{364 \text{ miles}}$ *Ans.*

Note.—Here, also, we *multiply* in the second line; for the money that pays the carriage of 112 lbs. for 130 miles will pay the carriage of 1 lb. for a *greater* distance.

In the foregoing examples the work is arranged with the whole of the reasoning written down. The reasoning may, however,

be easily performed *mentally* and the result alone be written. For instance:—

EXAMPLE vi.—*Find the cost of* 7 *articles at the rate of* 4*s.* 9*d. per dozen.*

1st step.

As "*cost*" is wanted, we first set down **57***d.*, the given cost of 12 things.

2nd step.

As the cost of 1 thing is found by dividing the cost of 12 things by 12, we now set **12** *under* the 57*d.*

3rd step.

As the cost of 7 things is 7 times the cost of 1 thing, we next append the factor **7**.

Finally, we simplify this result.

Written work.

$$\overset{d.}{\frac{57 \times 7}{12}} = \frac{133}{4}$$

$$= 33\tfrac{1}{4} = \underline{2s.\ 9\tfrac{1}{4}d.\ \textit{Ans.}}$$

EXAMPLE vii.—*How far will a train, travelling at the rate of* 40 *miles per hour, go in* 18 *minutes?*

1st step.

As "*distance*" is wanted, we first set down **40** *miles*, the given distance travelled in 60 minutes.

2nd step.

And, as the distance travelled in 1 minute is found by dividing the distance travelled in 1 hour by 60, we next set **60** *under* the 40 miles.

Written work.

$$\overset{miles.}{\frac{40 \times 18}{60}}$$

$$= \underline{12\ \textit{miles Ans.}}$$

3rd step.

Then, as the train goes 18 times as far in 18 minutes as it goes in 1 minute, we append the factor **18**, and simplify the result.

EXAMPLE viii.—*If* 14 *men can mow* 95 *acres of grass in* 5 *days, how many men would do as much in* 7 *days?*

Here it should be noticed that the *number of acres* in no way affects the reasoning, and may be ignored.

1st step.

As the *number of "men"* is wanted, we first write down **14** *men*, the given number who do the work in 5 days.

2nd step.

Now to do the work in 1 day we require 5 times as many men, so we *multiply* by **5**.

Written work.

$$\overset{men.}{\frac{14 \times 5}{7}}$$

$$= \underline{10\ \textit{men Ans.}}$$

3rd step.

And to do the work in 7 days we require *fewer* men than do it in 1 day, in fact $\frac{1}{7}$ of this number, so we now *divide* by **7**.

After some practice on the last plan, the student will, without difficulty, be able to dispense with the "unitary" process by *combining the two last steps*, thus:—

EXAMPLE ix.—*If a labourer earn £6, 6s. in 7 weeks, how much will he earn in 10 weeks?*

Written work.

	weeks.			s.	
In	7	he	earns	126	
∴ "	10	"	"	$126 \times \frac{10}{7}$	$\overset{s.}{=180} = $ £9 *Ans.*

Explanation.

The top line of the above written work is the *fact* given in the question, not, however, in this case, quoted just as it stands, but transposed so that *money* (the name of the required answer) heads the *right-hand* column.

The lower line may be supposed to be arrived at by the following mental process:—

In 1 week he earns $\frac{1}{7}$ of what he earns in seven,
∴ in 2 weeks " $\frac{2}{7}$ " " "
" 3 " " $\frac{3}{7}$ " " " ; and so on.
∴ in 10 weeks he earns $\frac{10}{7}$ of what he earns in seven.

Or we may reason thus:—

"10 weeks is *more* than 7 weeks, so *more* would be earned, and we therefore must multiply 126*s.* by the *improper* fraction $\frac{10}{7}$ (for to multiply by an *improper* fraction *increases*, but to multiply by a *proper* fraction *decreases*, the quantity so treated)".

The student is advised to adopt the above plan of arranging the written work in solving the Rule of Three questions which occur in most of the higher rules, *e.g.* Stocks, &c.

Sometimes, however (*e.g.* in the Chain Rule), it is better to omit all "wording", and to write down only the result, as is done in the following example:—

EXAMPLE x.—*If I lend a man £40 for 3 months, for how long ought he to lend me £60 in return?*

1st step.

"*Time*" is wanted, so we set down 3 months.

2nd step.

As £60 is greater than £40, the favour is requited by lending £60 for *less* time than £40 was lent, in fact for $\frac{40}{60}$ of that time, so we multiply 3 months by the *proper* fraction $\frac{40}{60}$.

Written work.

months.

$$3 \times \frac{40}{60}$$

= 2 *months Ans.*

XLIII. THE UNITARY METHOD (*continued*),

OR DOUBLE RULE OF THREE.

The questions we are now about to consider differ only from those of the preceding chapter in requiring a *repeated application* of the same process of reasoning.

EXAMPLE i.—*If* 14 *men in* 7 *days earn* £9, 16*s.*, *what would* 20 *men earn in* 4 *days at the same rate?*

14 *men* in 7 *days* earn .... £$9\frac{4}{5}$

∴ 1 *man* in 7 *days* earns .... $\frac{1}{14}$ of £$9\frac{4}{5}$

∴ 1 *man* in 1 *day* earns.... $\frac{1}{7}$ of $\frac{1}{14}$ of £$9\frac{4}{5}$

∴ 20 *men* in 1 *day* earn .... $\frac{1}{7}$ of $\frac{1}{14}$ of £$9\frac{4}{5} \times 20$

∴ 20 *men* in 4 *days* earn...... $\frac{1}{7}$ of $\frac{1}{14}$ of £$9\frac{4}{5} \times 20 \times 4$

= £8 *Ans.**

In questions which involve many quantities the above process is very cumbrous when the whole of the reasoning is *written down.* The reasoning, however, may easily be performed *mentally*, thus;

EXAMPLE ii.—*If* 6 *labourers, working* 8 *hours a day, mow* 21 *acres in* 9 *days, in how many days will* 5 *labourers, working* 9 *hours a day, mow* 35 *acres?*

1st step. As "time" is wanted, we first set down 9 *days.*

2nd step. As 1 man would take 6 times as long to do the same work as 6 men take, we next *multiply* by 6.

3rd step. As a man who works 1 hour a day would take 8 times as long to do the the same work as a man who works 8 hours a day, we next *multiply* by 8.

Written work.

days.

$$\frac{9 \times 6 \times 8 \times 35}{21 \times 5 \times 9}$$

= 16 *days Ans.**

4th step. As 1 acre takes but $\frac{1}{21}$ of the time that 21 acres take, we *divide* by 21.

[We have now obtained the time, namely, $\frac{9 \times 6 \times 8}{21}$ days, in which 1 man, working 1 hour a day, mows 1 acre.]

5th step. As 5 men would take but $\frac{1}{5}$ of the time of 1 man, we next *divide* by 5.

6th step. As 5 men, working 9 hours a day, would take but $\frac{1}{9}$ of the time they take when they work 1 hour a day, we next *divide* by 9.

7th step. As 35 acres will take them 35 times as long as 1 acre, we *multiply* by 35.

* In order to save space the "cancelling" is omitted.

"**Fractional Factor**" method. To the student familiar with Fractions this method is as easy to apply as the Unitary method, while it has the advantages of greater rapidity and conciseness.

EXAMPLE iii.—*If* 90 *men can dig a ditch* 50 *yards long in* $4\frac{1}{2}$ *days, how many men can dig a ditch* 360 *feet long in* 27 *days?*

Written work.

	yards.		days.		men.
A ditch	50	long is dug in	$4\frac{1}{2}$	by	90,
∴ „	120	„ „ „	27	„	$90 \times \frac{120}{50} \times \frac{4\frac{1}{2}}{27}$
					$= \underline{36 \textit{ men Ans.}}$

Explanation.

1st step.

Set down as the top line the fact given in the question, so arranging that "men" (the name of the answer) heads the *right-hand* column.

2nd step (change in *length of trench* alone considered).

Set 120 under the 50 yards, and consider what effect lengthening the trench will have on the number of men required. As a *longer trench* would require *more men*, we multiply 90 by the *improper* fraction $\frac{120}{50}$.

3rd step (change in *number of days* alone considered).

Set 27 under the $4\frac{1}{2}$ days, and consider what effect more time will have on the number of men required. As *more time* enables the work to be done by *fewer men*, we now append as a second multiplier the *proper* fraction $\frac{4\frac{1}{2}}{27}$.

In questions involving many changes the "wording" may be omitted and the result alone written, thus:—

EXAMPLE iv.—*If* 3 *compositors, in* 12 *days of* 10 *hours each, can set in type* 360 *pages averaging* 60 *lines per page and* 40 *letters per line, in how many days of* 8 *hours each would* 9 *compositors set up* 480 *pages averaging* 45 *lines per page and* 50 *letters per line?*

1st step. Set down **12** *days.*

2nd step. (Change in *number of men* alone considered).

9 men take *less* time than 3 men to do the same work, so we multiply by the *proper* fraction $\frac{3}{9}$.

Written work.

days.

$$12 \times \frac{3}{9} \times \frac{10}{8} \times \frac{480}{360} \times \frac{45}{60} \times \frac{50}{40}$$

$$= \frac{25}{4} = \underline{6\tfrac{1}{4} \textit{ days Ans.}}$$

3rd step. (Change in *length of day* alone considered).

Men who work fewer hours per day take *more* days to do the same work, so we multiply by the *improper* fraction $\frac{10}{8}$. And so on.

BOOKWORK.—PART II.

XLIV. VARIOUS APPLICATIONS OF THE FOREGOING METHODS.

A. AGENTS OF DIFFERENT POWERS, &c.

A (i). *If* 4 *men can do as much as* 7 *boys, how long would* 12 *men and* 5 *boys take to do the work which* 8 *men alone can do in* 39 *days?*

[Here we first express the question in terms of *one* agent, namely boys.]

As 4 men = 7 boys $\therefore$ **8 men = 14 boys.**

also 12 men = 21 boys $\therefore$ **12 men and 5 boys = 26 boys.**

[Now in place of the given question we solve the following:—
"*If* **14 boys** *can do the work in* 39 *days, how long will* **26 boys** *take?*"]

$$\text{Hence the req}^{\text{d}}\text{ time} = \overset{\text{days.}}{39} \times \tfrac{14}{26} = \underline{21 \textit{ days Ans.}}$$

Note.—In this, and some of the following examples, for the sake of clearness the cancelling is omitted.

A (ii). *If a man can do three times, and if a woman can do twice, as much work as a boy, how long would* 7 *men,* 5 *women and* 3 *boys together take to do the work which* 3 *men,* 4 *women and* 5 *boys can accomplish in* 51 *days?*

[We first substitute the equivalent number of boys for each set of men and women.]

Here 1 man = 3 boys; and 1 woman = 2 boys

$\therefore$ **3 men + 4 women + 5 boys** = 9 + 8 + 5 boys
= **22 boys.**

Also **7 men + 5 women + 3 boys** = 21 + 10 + 3 boys
= **34 boys.**

[Thus we obtain, in place of the given question, the following:—
"*If* 22 *boys do the work in* 51 *days, how long would* 34 *boys take?*"]

$$\text{Hence the req}^{\text{d}}\text{ time} = \overset{\text{days.}}{51} \times \tfrac{22}{34} = \underline{33 \textit{ days Ans.}}$$

A (iii). *If* 10 *sheep, or* 15 *lambs, eat* 40 *bushels of turnips in* 7 *days, how long will* 36 *bushels last* 6 *sheep and* 18 *lambs?*

It is evident from the question that

10 sheep eat as much as 15 lambs;

$\therefore$ 6 sheep .... $15 \times \frac{6}{10} = 9$ lambs

$\therefore$ 6 sheep + 18 lambs.................. $9 + 18 = 27$ lambs.

[Thus instead of the given question we have the following:—

"*If* 15 *lambs eat* 40 *bushels in* 7 *days, how long will* 27 *lambs take to eat* 36 *bushels?*"]

$$\text{Hence the req}^{\text{d}}\text{ time} = \overset{\text{days.}}{7} \times \frac{15}{27} \times \frac{36}{40} = \underline{3\tfrac{1}{2}\ \textit{days Ans.}}$$

A (iv). *Thirty-three masons could build a certain wall in* 47 *days, but, after working for* 11 *days, fifteen of them "strike"; in how many days after this will the wall be finished?*

In the 11 days that 33 men work they do $\frac{11}{47}$ of the work; we have now to find how long the 18 men left after the strike will take to do the remaining $\frac{36}{47}$ of the work.

$$\text{Hence the req}^{\text{d}}\text{ time} = \overset{\text{days.}}{11} \times \frac{33}{18} \times \frac{36}{11} = \underline{66\ \textit{days Ans.}}$$

B. Assets, Dividends, &c.

A person is *insolvent* when he owes more than the value of his possessions; *i.e.* when his *debts* are greater than his **assets.**

The persons to whom he owes money are called his **creditors.**

When the debtor is made a **bankrupt** his assets are fairly distributed among his creditors, *i.e.* in place of each £1 he owes, some fraction of a pound, called a **dividend**, is paid.

B (i). *A bankrupt pays a dividend of* 5*s.* 8*d. in the pound: what was due to a creditor who received* £61, 1*s.* 2*d.?*

5*s.* 8*d.* = $5\frac{2}{3}$*s.*; £61, 1*s.* 2*d.* = $1221\frac{1}{6}$*s.*

The creditor received $\overset{s.}{5\frac{2}{3}}$ in place of $\overset{£}{1}$

$\therefore$ .... $1221\frac{1}{6}$ .... $1 \times \dfrac{1221\frac{1}{6}}{5\frac{2}{3}}$

$$= \frac{431}{2} = \underline{£215,\ 10s.\ \textit{Ans.}}$$

B (ii). *A bankrupt's debts amount to £4563; his assets to £1872; the legal expenses connected with his bankruptcy amount to £105: what loss will a creditor sustain whose claim is for £507?*

The sum available for distribution among creditors = £1872 − £105
= £1767.

∴ the loss to creditors on debts of £4563 = £4563 − £1767 = £2796.

$$\therefore \text{ the loss to a creditor whose claim is } £507 = \overset{£}{2796} \times \frac{507}{4563} = \overset{£}{\frac{932}{3}}$$

= £310, 13*s.* 4*d.* *Ans.*

C. Rates, Taxes, &c.

Taxes are charges on property which are fixed by Parliament, and are payable to the Government of the country.

Income-tax is assessed (*i.e.* calculated) as a charge (subject to certain conditions and exceptions) on each *pound* of a person's *income.*

Rates differ from taxes in being fixed by, and payable to, the Council or Corporation (*i.e.* the local governing body) of a county, town, or parish. Rates are assessed as a charge on each *pound* of the *rental* value of the land, house, &c., occupied by a person.

The total sum of money (*e.g.* salary, profits on trade, &c.) which comes to a person *annually** is called his *gross** income.

After fixed "outgoings" (*e.g.* rates, taxes, &c.) have been deducted, the remainder is called his *net** income.

C (i). *The rates on a certain property, assessed at* 1*s.* 10*d. in the pound, amounted to* £26, 5*s.* 3*d.*: *what was the rateable value of the property?*

£26, 5*s.* 3*d.* = $525\frac{1}{4}$*s.*

Now $\overset{s.}{1\frac{5}{6}}$ was paid on $\overset{£}{1}$

$$\therefore \quad 525\tfrac{1}{4} \ldots\ldots\ldots\ldots\ 1 \times \frac{525\frac{1}{4}}{1\frac{5}{6}} = \frac{\overset{191}{\cancel{2101}}}{\underset{2}{\cancel{4}}} \times \frac{\overset{3}{\cancel{6}}}{\cancel{11}} = \overset{£}{\frac{573}{2}}$$

= £286, 10*s.* *Ans.*

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* *Annual*, from Latin *annus*, a year. *Gross*=thick, *i.e.* whole. *Net*=neat, *i.e.* trimmed.

C (ii). *A man's gross income was £540; find his net income after paying income-tax at the rate of 5d. in the pound.*

£1, or 240*d.*, is reduced by the tax to 235*d.*

∴ Each £1 *gross* becomes £$\frac{235}{240}$ *net*

∴ £540................ £$540 \times \frac{235^*}{240} = \frac{2115}{4} =$ £528, 15*s.* *Ans.*

N.B.—**Net* income is obtained from *gross* by multiplying by the *proper* fraction $\frac{240-tax}{240}$, the tax being expressed in *pence per* £1.

C (iii). *A man's net income after deducting income-tax at 6d. in the pound is £819; find his gross income.*

£1, or 240*d.*, is reduced by the tax to 234*d.*

∴ £$\frac{234}{240}$ *net* results from £1 *gross*

∴ 1 .... $1 \times \frac{240}{234}$

∴ 819 .... $819 \times \frac{240^*}{234} = 7 \times 120 =$ £840 *Ans.*

N.B.—* *Gross* income is obtained from *net* by multiplying by the *improper* fraction $\frac{240}{240-tax}$, the tax being expressed in *pence per* £1.

D. Fractions.

D (i). *Three-eighths of a certain number is 174; find the number.*

$\frac{3}{8}$ of the req^d number $= 174$

∴ $\frac{1}{8}$ .... $= \frac{174}{3}$

∴ the whole, or $\frac{8}{8}$, ... $= \frac{174}{3} \times 8 = 464$ *Ans.*

D (ii). *What number exceeds its seventh part by 78?*

By the question, 6 *sevenths* of the req^d number $= 78$

∴ the whole, or 7 *sevenths* .... $= 78 \times \frac{7}{6} = 91$ *Ans.*

D (iii). *What number is that the sum of whose fifth and eighth parts is 39?*

As $\frac{1}{5} + \frac{1}{8} = \frac{13}{40}$, ∴ $\frac{13}{40}$ of the req^d number $= 39$

∴ $\frac{40}{40}$ .... $= 39 \times \frac{40}{13} = 120$ *Ans.*

D (iv). *After paying $\frac{1}{3}$ of the money in my purse to one person, and then $\frac{1}{6}$ of what remained to another, I had* 10*d. left. How much money was there in my purse at first?*

To the first person I paid $\frac{1}{3}$ of my money, and then had left $\frac{2}{3}$ of it.

To the second person I then paid $\frac{1}{6}$ of $\frac{2}{3} = \frac{1}{9}$ of my money.

$\therefore$ altogether I paid away $\frac{1}{3} + \frac{1}{9} = \frac{4}{9}$ of my money,

$\therefore$ I had left at last $\frac{5}{9}$ of my money.

Hence 5 *ninths* of the req^{d} sum $= \overset{d.}{10}$

$\therefore$ the whole, or 9 *ninths*...................... $= 10 \times \frac{9}{5} = \underline{18d.\ Ans.}$

D (v). *The liquid in a cask filled $\frac{2}{3}$ of the cask; after* 24 *gallons had been drawn out the remaining liquid filled $\frac{2}{7}$ of the cask: how many gallons would the cask hold?*

As $\frac{2}{3} - \frac{2}{7} = \frac{8}{21}$, $\therefore \frac{8}{21}$ of the cask contained $\overset{\text{gallons.}}{24}$

$\therefore \frac{21}{21}$ .... $24 \times \frac{21}{8} = \underline{63\ gal.\ Ans.}$

E. Time and Work.

E (i). *A man, A, alone could mow a field in* 6 *days; another, B, alone could mow it in* 8 *days: how long would they take together?*

N.B.—In questions of this class we first find the *fractional part of the work* done by each agent in a unit of time.

Here A mows $\frac{1}{6}$, and B $\frac{1}{8}$, of the field in 1 day.

$\therefore$ together they mow $\frac{1}{6} + \frac{1}{8} = \frac{7}{24}$ of the field in $\overset{\text{day}}{1}$

$\therefore$ .... $\frac{1}{24}$ .... $\frac{1}{7}$

$\therefore$ .... $\frac{24}{24}$ .... $\frac{24}{7} = \underline{3\tfrac{3}{7}\ days\ \ Ans.}$

E (ii). *A cistern can be filled from one tap in* 24 *minutes or be emptied by a valve in* 44 *minutes. If the cistern is empty and both tap and valve are open, how long will it take to fill?*

The tap *fills* $\frac{1}{24}$, the valve *empties* $\frac{1}{44}$, of the cistern in 1 min.

$\therefore$ with both open, $\frac{1}{24} - \frac{1}{44} = \frac{5}{264}$ is filled in $\overset{\text{min.}}{1}$

$\therefore$ the whole, or $\frac{264}{264}$, ... $1 \times \frac{264}{5}$

$= \underline{52\tfrac{4}{5}\ min.\ \ Ans.}$

E (iii). *A and B working together can mow 7 acres in 3 days; B and C, 9 acres in 5 days; A and C, 8 acres in 4 days. How long would A, B and C together take to mow 23 acres?*

$$A + B \text{ mow } \overset{\text{acres.}}{\tfrac{7}{3}},\ B + C \text{ mow } \overset{\text{acres.}}{\tfrac{9}{5}},\ A + C \text{ mow } \overset{\text{acres.}}{2} \text{ in } 1 \text{ day.}$$

$$\therefore, \text{ by addition, } 2A + 2B + 2C \text{ mow } \tfrac{92}{15} \text{ acres in } \overset{\text{day.}}{1}$$

$$\therefore A + B + C \ldots\ldots\ldots \tfrac{46}{15} \ldots\ldots\ldots\ldots 1$$

$$\therefore A + B + C \ldots\ldots\ldots 23 \ldots\ldots\ldots\ldots 1 \times \frac{23}{\frac{46}{15}}$$

$$= 23 \times \tfrac{15}{46} = 7\tfrac{1}{2} \textit{ days Ans.}$$

E (iv). *A and B can do a piece of work alone in 12 and 16 days respectively; they work together for 3 days when A leaves, and 2 days later B is joined by C, after which the work is finished in 3 more days. How long would C take to do the work alone?*

A does $\frac{1}{12}$, and B $\frac{1}{16}$, of the work per day.

$\therefore$, in the 3 days A works he does $\frac{3}{12}$, or $\frac{1}{4}$, of the work;

and in the 8 days B works he does $\frac{8}{16}$, or $\frac{1}{2}$, of the work.

$\therefore$, without C's help, $\frac{1}{4} + \frac{1}{2} = \frac{3}{4}$ of the work is done.

Hence C does $\frac{1}{4}$ of the work in 3 days.

$\therefore$ C could do the whole in 12 *days. Ans.*

F. Time and Distance.

The *rate*, or *velocity*, of a body which moves uniformly, is expressed by the *distance* it travels in some *unit of time.*

For instance, when we say that a train is travelling at the rate of 30 miles an hour, we mean that if it continued to move on steadily for 1 hour it would go 30 miles; if for four hours, it would go 4 times 30 miles; if for $\frac{1}{4}$ hour, it would go $\frac{1}{4}$ of 30 miles; and so on.

F (i). *How many feet per second does a body travel which moves uniformly at the rate of 50 miles an hour?*

$$\text{In } 60 \times 60 \textit{ seconds} \text{ it goes } \overset{\text{miles.}}{50}$$

$$\therefore \ldots\ 1 \textit{ second} \ldots\ldots\ldots \frac{50}{60 \times 60} = \overset{\text{feet.}}{\frac{50 \times 1760 \times 3}{60 \times 60}} = 50 \times \overset{\text{feet.}}{\frac{22^*}{15}}$$

$$= 73\tfrac{1}{3} \textit{ ft. per sec. Ans.}$$

F (ii). *How many miles per hour correspond to* 44 *feet per second?*

In 1 sec. the distance is 44 feet.

$$\therefore \text{ in } 60 \times 60 \ldots\ldots\ldots\ldots\ldots\ 44 \times 60 \times 60 = \frac{44 \times 60 \times 60}{3 \times 1760} \text{ miles} = 44 \times \frac{15}{22}^{*} \text{ miles}$$

$\underline{30 \textit{ mi. per hour Ans.}}$

N.B.—From the results marked thus * in Exs. F (i) and (ii) we see that the multiplier $\frac{22}{15}$ converts *mi. per hr.* into *ft. per sec.*; and, conversely, the multiplier $\frac{15}{22}$ converts *ft. per sec.* into *mi. per hr.*

F (iii). *How long will a bicyclist whose rate is* 12 *miles an hour take to go* 60 *miles?*

12 miles is the distance in 1 hour.

$$\therefore 60 \ldots\ldots\ldots\ldots\ldots\ 1 \times \frac{60}{12}^{\dagger} = \underline{5 \textit{ hours Ans.}}$$

F (iv). *Two persons, A and B, start at the same time from places* 35 *miles apart, A at the rate of* 3 *miles an hour, and B at the rate of* 4 *miles an hour, and walk to meet each other: how long do they take?*

3 + 4, or 7 miles is the distance gained in 1 hour.

$$\therefore 35 \ldots\ldots\ldots\ldots\ldots\ 1 \times \frac{35}{7}^{\dagger} = \underline{5 \textit{ hours Ans.}}$$

F (v). *A and B start at the same time from places* 9 *miles apart, B at the rate of* 6 *miles an hour, and A following him at the rate of* 8 *miles an hour; at what time will A overtake B?*

8 − 6, or 2 miles is gained by A in 1 hour.

$$\therefore 9 \ldots\ldots\ldots\ldots\ldots\ 1 \times \frac{9}{2}^{\dagger} = \underline{4\tfrac{1}{2} \textit{ hours Ans.}}$$

N.B.—The results† in Exs. (iii), (iv) and (v) may be generalized:—

(I). In the case of a single body moving uniformly, $\textit{time} = \dfrac{\textit{distance}}{\textit{rate}}$.

i.e. the *number of units of length* in the distance to be travelled, when divided by the rate (*expressed in terms of the* same unit of length and *some unit of time*), gives the *number of such units of time* occupied.

(II). When two bodies move in *opposite* directions, the *time* taken to gain a given distance $= \dfrac{\textit{distance}}{\text{sum } \textit{of rates}}$.

(III). When two bodies move in the *same* direction, the *time* taken to gain a given distance $= \dfrac{\textit{distance}}{\text{difference } \textit{of rates}}$.

F (vi). *How long will a train* 90 *yards long, going at the rate of* 20 *miles per hour, take to pass completely over a bridge* 130 *yards long?*

The train begins to cross the bridge when the *engine* is at A; it has not *completely* crosed it until the *guard's van* is over and the *engine* has reached B.

i.e. the engine must travel a distance = length of bridge + length of train
= 130 + 90 = 220 yds.

$$\text{But } \overset{\text{yds.}}{20 \times 1760} \text{ is traversed by the engine in } \overset{\text{seconds.}}{60 \times 60}$$

$$\therefore \quad 220 \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad 60 \times 60 \times \frac{220}{20 \times 1760}$$

$$= 22\tfrac{1}{2} \textit{ secs. Ans.}$$

F (vii). *A train, travelling at the rate of* 40 *miles an hour, whilst inside a tunnel, meets another train of half its length, travelling at* 60 *miles an hour, and passes it completely in* $4\frac{1}{2}$ *seconds. Find the length of the tunnel, if the first train passes completely through it in* 4 *minutes* $37\frac{1}{2}$ *seconds.*

In order to pass each other completely the engines must separate by a distance = the sum of the lengths of the trains = $1\frac{1}{2}$ times the length of the 1st train;
and the rate at which they separate = the sum of their rates = 40 + 60 = 100 mi. per hour = $100 \times \frac{22}{15}$ *ft. per sec.* (See p. 143.)

$$\therefore \text{ in } 4\tfrac{1}{2} \text{ sec. they separate by } 100 \times \frac{22}{15} \times 4\tfrac{1}{2} \text{ ft.}$$

$$\therefore \textbf{ the length of the 1st train} = 100 \times \frac{22}{15} \times 4\tfrac{1}{2} \div 1\tfrac{1}{2} \text{ ft.} = 440 \text{ ft.}$$

Again, in order to pass completely through the tunnel, the 1st train must travel a distance = length of tunnel + its own length.

$$\text{But in 1 sec. it goes } 40 \times \frac{22}{15} \text{ feet}$$

$$\therefore \text{ in 4 min. } 37\tfrac{1}{2} \text{ secs., or } 277\tfrac{1}{2} \text{ secs} \ldots\ldots\ldots \; 40 \times \frac{22}{15} \times \frac{555}{2} = \frac{440 \times 111}{3} \text{ ft.}$$

$$\text{Hence, the length of the tunnel} = \frac{440 \times 111}{3} - 440 \text{ feet} = 440 \times \left(\frac{111}{3} - 1\right) \text{ ft.}$$

$$= 440 \times \frac{108}{3} \text{ feet} = \tfrac{1}{4} \times \frac{108}{3} \text{ miles} = \underline{3 \textit{ mi. Ans.}}$$

To this section belong questions relating to the motion of the hands of a clock. As the long hand makes a complete circuit of of the face in an hour it travels past 60 of the *minute-spaces*

marked on the dial in 1 hour, while the short hand travels past 5 of these *spaces* only.

∴ *the long hand gains* **55** *minute-spaces in* **1** *hour.*

i.e. the difference of their rates is 55 minute-spaces per hour.

F (viii). *At what time between* 4 *and* 5 *o'clock are the hands of a watch pointing in the same direction?*

At 4 o'clock the long hand points to XII., and the short hand to IIII., *i.e.* they are 20 minute-spaces apart.

So the long hand will overtake the short hand and they will point in the same direction when it has gained 20 minute-spaces.

But the long hand gains 55 *min.-spaces* in 1 hour.

$$\therefore \ldots\ldots\ldots\ldots\ldots\ldots\ldots 20 \textit{ min.-spaces} \ldots 1 \times \frac{20}{55} = \frac{20 \times 60}{55} = 21\frac{9}{11} \text{ min.}$$

Ans. $21\frac{9}{11}$ *minutes past* 4.

F (ix). *At what time between* 8 *and* 9 *o'clock are the hands pointing in opposite directions?*

They will point in opposite directions when the long hand is 30 *minutes-spaces* behind the short hand. But at 8 o'clock the long hand is 40 min.-spaces behind

∴ the long hand must gain 10 min.-spaces.

But it gains 55 min.-spaces in 1 hour.

$$\therefore \ldots\ldots\ldots 10 \ldots\ldots 1 \times \frac{10}{55} = \frac{10 \times 60}{55} = \frac{120}{11} \text{ min.}$$

Ans. $10\frac{10}{11}$ *min. past* 8.

F (x). *At what times between* 5 *and* 6 *o'clock are the hands at right angles?*

The hands are at right angles when they are 15 minute-spaces apart, and this happens twice in the hour.

1st when the long hand is 15 min.-spaces *behind* the short hand
2nd.. *in front* of..............

Now at 5 o'clock the hands are 25 min.-spaces apart.

Hence they will be at right angles, 1st when the long hand has gained 10 min.-spaces, and again when it has gained 40 min.-spaces;

i.e. in $\frac{10}{55}$, and $\frac{40}{55}$, of an hour; or at $10\frac{10}{11}$, and $43\frac{7}{11}$, *min. past* 8. *Ans.*

G. RACES.

G (i). *A can beat B by* 5 *yards in* 100 *yards, and B can beat C by* 5 *yards in* 100 *yards: by what distance would A beat C in* $\frac{1}{4}$ *mile?*

	yds.		yds.
A runs	**100**	while B runs	**95**
But B	100	.... C	95
∴ B	**95**	.... C	$95 \times \frac{95}{100} = \frac{361}{4}$
i.e. A	100	.... C	$\frac{361}{4}$
∴ A	440	.... C	$\frac{361}{4} \times \frac{440}{100} = \frac{361 \times 11}{10} = 397\frac{1}{10}$

∴ A can give C $440 - 397\frac{1}{10} = 42\frac{9}{10}$ yds. *Ans.*

G (ii). *A can give B* 10 *points in* 50 *at billiards, B can give C* 25 *in* 100, *and C can give D* 50 *in* 500*: how many points should A give D in* 1000*?*

	points.		points.
A scores	**50**	while B scores	**40**
But B	100	.... C	75
∴ B	**40**	.... C	$75 \times \frac{40}{100} = 30$
And C	500	.... D	450
∴ C	**30**	.... D	$450 \times \frac{30}{500} = 27$
i.e. A	50	.... D	27
∴ A	1000	.... D	$27 \times 20 = 540$

Hence A should give D $1000 - 540 = 460$ *Ans.*

H. CHAINS.

H (i). *If* 3 *oranges are worth* 7 *apples, and* 9 *apples are worth* 5 *pears, and* 2 *pears are worth* 15 *strawberries, and* 80 *strawberries cost* 6*d.*, *what is the value of* 4 *doz. oranges?*

1 *orange* is worth $\frac{7}{3}$ of an *apple*

$= \frac{7}{3}$ of $\frac{5}{9}$ of a *pear*

$= \frac{7}{3}$ of $\frac{5}{9}$ of $\frac{15}{2}$ of a *strawberry*

$= \frac{7}{3}$ of $\frac{5}{9}$ of $\frac{15}{2}$ of $\frac{6}{80}$ of a *penny*.

∴ 48 *oranges* are worth $\frac{7}{3} \times \frac{5}{9} \times \frac{15}{2} \times \frac{6}{80} \times 48$ *pence*,

$= 35d. =$ 2*s.* 11*d.* *Ans.*

XLV. RATIO AND PROPORTION.

RATIO.

The **ratio** of one number to another, or of one quantity to another of *like* kind, is the relation which the first bears to the second with respect to *magnitude*, expressed by the number of times the first contains the second, or by the fraction which the first is of the second.

For instance, the ratio of 8 to 2 is expressed by the number 4,
the ratio of 7 to 3 is expressed by the fraction $\frac{7}{3}$,
the ratio of 3 cwts. to 1 ton 2 cwts. is expressed by the fraction $\frac{3}{22}$.

The two numbers, or quantities, thus compared are called the **terms** of the ratio, the first being sometimes called the *antecedent*, and the second the *consequent*.

A ratio is indicated by two dots, one over the other, placed between the terms, or by placing the first term over the second with a line between them.

For instance, the ratio of 5*s.* to 6*s.* 6*d.*, thus, 5*s.* : 6*s.* 6*d.*; or thus, $\frac{5s.}{6s.\ 6d.}$.

Note.—The *continuous* notation 2 : 3 : 4 indicates *three* ratios, namely, the ratio of 2 to 3, of 3 to 4, and of 2 to 4.

The **terms** of a ratio must always be of *like* kind.

For instance, we *can* compare the magnitude of 3 *cwts.* with that of 4 *tons*; but we *cannot* compare the magnitude of 3 *cwts.* with that of 4 *miles*.

A ratio whose *first* term is *greater* than its *second* is sometimes called a *ratio of greater inequality*; and a ratio whose first term is *less* than its second, a *ratio of less inequality*.

EXAMPLE i.—*Express in simplest form the ratio £2, 5s. : £3, 7s. 6d.*

$$\text{The ratio} = \frac{2\frac{1}{4}}{3\frac{3}{8}} = \frac{18}{27} = \frac{2}{3}. \qquad \underline{\textit{Ans. } 2:3.}$$

EXAMPLE ii.—*What length bears to a mile the ratio 5 : 11?*

$$\frac{5}{11} \text{ of a mile} = \frac{5}{11} \times 1760 \text{ yds.} = \underline{800 \text{ yds. } \textit{Ans.}}$$

EXAMPLE iii.—*If 4 be added to the numerator of the fraction $\frac{16}{23}$, what number must be added to the denominator in order that the value of the fraction may be unchanged?*

The new den^r must be greater than 16 + 4, or 20, in the ratio of 23 to 16; *i.e.* the new den^r is $\frac{23}{16}$ of $20 = \frac{115}{4} = 28\frac{3}{4}$

$\therefore$ the req^d number is $28\frac{3}{4} - 23 = 5\frac{3}{4}$ *Ans.*

PROPORTION.

Four numbers, or quantities, are called **proportionals** when the *ratio* of the *first* to the *second* is **equal** to the *ratio* of the *third* to the *fourth.*

For instance, 8, 12, 14*s.* and 21*s.* are proportionals,

for the ratio of 8 to 12, or $\frac{8}{12} = \frac{2}{3}$;

and the ratio of 14*s.* to 21*s.* or $\frac{14s.}{21s.} = \frac{14}{21} = \frac{2}{3}$ also;

and thus the ratio of 8 (the first) to 12 (the second) is equal to the ratio of 14*s.* (the third) to 21*s.* (the fourth).

Also, 9*s.*, 7*s.* 6*d.*, 3 tons, and 2 tons 10 cwts. are proportionals,

for the ratio of 9*s.* to 7*s.* 6*d.*, or $\frac{9s.}{7\frac{1}{2}s.} = \frac{18}{15} = \frac{6}{5}$;

and the ratio of 3 tons to 2 tons 10 cwts., or $\frac{3 \text{ tons}}{2\frac{1}{2} \text{ tons}} = \frac{6}{5}$ also.

The statement of the fact that two ratios are equal is called a *proportion*, and is often written thus:—

8 : 12 : : 14*s.* : 21*s.* .... (I).

and read thus: "8 *is to* 12 *as* 14*s.* *is to* 21*s.*"

or written thus: 8 : 12 = 14*s.* : 21*s.* .... (II).

or thus: $\frac{8}{12} = \frac{14s.}{21s.}$ .... (III).

It follows from (III) that any two equal fractions form a proportion, which may be read in four ways.

For instance, as $\frac{2}{3} = \frac{4}{6}$, ∴ 2 is to 3 as 4 is to 6,

also as $\frac{4}{6} = \frac{2}{3}$, ∴ 4 is to 6 as 2 is to 3,

also as $\frac{3}{2} = \frac{6}{4}$, ∴ 3 is to 2 as 6 is to 4,

also as $\frac{6}{4} = \frac{3}{2}$, ∴ 6 is to 4 as 3 is to 2.

Since the two terms of a ratio must always be *alike*, it follows that the 1st and 2nd terms of a proportion must always be alike, and the 3rd and 4th terms also alike.

For instance, 2 tons : 3 tons : : 4 acres : 6 acres is a *correct* statement; but 2 tons : 3 acres : : 4 tons : 6 acres is *incorrect.*

EXAMPLE iv.—*The first, third and fourth terms of a proportion are* 7*s.*, 8 *and* 12 ; *find the second term.*

The req[d] quantity is *greater* than 7*s.* *in the ratio of* 12 *to* 8;

i.e. the req[d] quantity = $\frac{12}{8}$ of 7*s.* = $10\frac{1}{2}$*s.* = 10*s.* 6*d.* *Ans.*

EXAMPLE v.—*Find a fourth proportional to 2s., 2s. 6d., and 1 yd. 2 ft.*

The req^d quantity is *greater than* $1\frac{2}{3}$ yds. in the ratio of $2\frac{1}{2}s.$ to $2s.$

$$\text{i.e. the req}^{d}\text{ quantity} = \frac{2\frac{1}{2}}{2} \text{ of } 1\tfrac{2}{3} \text{ yds.} = \tfrac{5}{4} \times \tfrac{5}{3} \text{ yds.} = \underline{2 \text{ yds. } 3 \text{ ins. } Ans.}$$

Note.—All the Exercises on Chapter XLII., it should be noticed, consist in finding a *fourth proportional* to three given terms; and the "Fractional Factor" (see pages 134, 136) expresses *the ratio in which the 3rd term must be increased or diminished*, to produce the req^d 4th term.

The *first* and *fourth* terms of a proportion are called the **extremes**; and the *second* and *third* terms, the **means**.

When the terms of a proportion are *abstract* numbers the **product** *of the extremes is equal to the* **product** *of the means.*

For instance, in the proportion $3 : 5 :: 15 : 25$
the product of the extremes is $3 \times 25 = 75$
and the product of the means is $5 \times 15 = 75$ also.

Three numbers are said to be in **Continued Proportion** when the ratio of the *first* to the *second* is equal to the ratio of the *second* to the *third*;
and the *second* is called a **mean proportional** between the 1st and 3rd.

For instance, 4, 6, 9 are in continued proportion, for $\frac{4}{6} = \frac{6}{9}$, and 6 is a mean proportional between 4 and 9.

Also 9 is a *third* proportional to 4 and 6.

EXAMPLE vi.—*Find a mean proportional between* 11 *and* 99.

Here 11 : the req^d number :: the req^d number: 99

[But product of means = product of extremes.]

$$\begin{aligned}\text{Hence, the } \textit{square} \text{ of the req}^{d}\text{ number} &= 11 \times 99 \\ &= 11 \times 11 \times 3 \times 3 \\ &= 33 \times 33 \\ \therefore \text{ the req}^{d}\text{ number} &= \underline{33 \ \ Ans.}\end{aligned}$$

Note.—It is shown in Geometry that in similar figures (*i.e.* figures which are of *exactly the same shape* though of different *size*) corresponding lines are proportionals.

For instance, if two triangles are of the *same shape*, and the longest side of the first is 5 *times* the length of the longest side of the second, then the shortest side of the first will also be 5 *times* the length of the shortest side of the second, and so on.

XLVI. PROPORTIONAL PARTS AND PARTNERSHIP.

To divide a given number, or quantity, into parts proportional to given numbers; *i.e.* into parts which shall be in given ratios; we proceed thus:—

EXAMPLE i.—*Divide £21 into two parts proportional to 2 and 5.*

As $2+5=7$, and $\frac{1}{7}$ of £21 = £3.

∴ the req^d parts are £3 × 2 and £3 × 5, *i.e.* *£6 and £15 Ans.*

EXAMPLE ii.—*Divide 375 into parts proportional to 3, 8 and 14.*

$3+8+14=25$, and $\frac{1}{25}$ of $375=15$.

Hence the req^d parts are 15×3; 15×8 and 15×14.

i.e. 45; 120; *and* 210 *Ans.*

EXAMPLE iii.—*Divide 3 cwts. in the ratio* 42 : 53 : 17.

$42+53+17=112$. Hence the req^d parts are

$\frac{42}{112}$ of 3 cwts.; $\frac{53}{112}$ of 3 cwts.; and $\frac{17}{112}$ of 3 cwts.

or, 1 *cwt.* 14 *lbs.*; 1 *cwt.* 47 *lbs.*; *and* 51 *lbs. Ans.*

Note.—In this, and succeeding Examples, in order to save space, part of the "working" is omitted.

EXAMPLE iv.—*A and B are partners in a business; A's capital is* £1200 *and B's is* £950. *Divide profits amounting to* £344 *fairly between them.*

[Here we have to divide £344 into parts proportional to 1200 and 950.]

$1200+950=2150$.

$$\left.\begin{array}{l}\text{Hence A's share of the profits is } \frac{1200}{2150} \text{ of } £344 = £192 \\ \text{and B's } \ldots\ldots\ldots\ldots \frac{950}{2150} \text{ of } £344 = £152\end{array}\right\} \textit{Ans.}$$

From the above examples we obtain the following rule:—

To divide a given quantity into parts proportional to given numbers:

Add together the given numbers; divide the given quantity by the result; and multiply the quotient by each of the given numbers in succession.

Note.—It is generally best to *merely indicate* the above division and

multiplication by signs, and not actually to perform them until any possible cancelling has been performed. (See Ex. iii and Ex. iv.)

EXAMPLE v.—*Divide £65, 2s. into parts proportional to* $\frac{1}{3}$, $\frac{2}{5}$, $\frac{3}{10}$.

$$\frac{1}{3}+\frac{2}{5}+\frac{3}{10}=\frac{10}{30}+\frac{12}{30}+\frac{9}{30}=\frac{31}{30}.$$

Hence the req^{d} parts are

$$\frac{10}{31}^{*} \text{ of } £65\tfrac{1}{10};\quad \frac{12}{31} \text{ of } £65\tfrac{1}{10};\quad \text{and } \frac{9}{31} \text{ of } £65\tfrac{1}{10};$$

i.e. £21; £25, 4*s.*; and £18, 18*s.* *Ans.*

EXAMPLE vi.—*Divide* 2·18 *into parts proportional to* ·8, 1·7, ·95 *and* 2.

$$\cdot 8+1\cdot 7+\cdot 95+2=5\cdot 45.$$

Hence the req^{d} parts are

$$\frac{\cdot 8}{5\cdot 45} \text{ of } 2\cdot 18;\quad \frac{1\cdot 7}{5\cdot 45} \text{ of } 2\cdot 18;\quad \frac{\cdot 95}{5\cdot 45} \text{ of } 2\cdot 18;\quad \text{and } \frac{2}{5\cdot 45} \text{ of } 2\cdot 18$$

$$\text{or, } \frac{80}{545}\times 2\cdot 18;\quad \frac{170}{545}\times 2\cdot 18;\quad \frac{95}{545}\times 2\cdot 18;\quad \text{and } \frac{200}{545}\times 2\cdot 18,$$

i.e. ·32; ·68; ·38; and ·8 *Ans.*

We shall now give some examples of a rather more complex nature:—

EXAMPLE vii.—*Divide £252 between A, B and C, so that for every £2 A receives B receives £5, and for every £3 B receives C receives £7.*

[Here, in order to obtain the *continuous* ratio of the shares, we take the L.C.M. of 5 and 3, the two numbers which relate to B's share and replace the given ratios by equivalent ones having this common number, 15, instead of 5 and 3 respectively.]

$$\frac{\text{A's share}}{\text{B's share}}=\frac{2}{5}=\frac{6}{15}, \text{ and } \frac{\text{B's share}}{\text{C's share}}=\frac{3}{7}=\frac{15}{35};$$

∴ A's share : B's share : C's share : : 6 : 15 : 35.

[We now divide £252 into parts proportional to 6, 15 and 35.]

$$6+15+35=56.$$

Hence the shares are $\frac{6}{56}$ of £252; $\frac{15}{56}$ of £252; $\frac{35}{56}$ of £252.

i.e. £27; £67, 10*s.*; £157, 10*s.* *Ans.*

---

* For $\frac{10 \textit{ thirtieths}}{31 \textit{ thirtieths}}=\frac{10}{31}$, and so on.

EXAMPLE viii.—*Divide £125 between A, B and C so that A may have four times as much as B, and C one-fourth of the sum of A's and B's shares.*

B's share $= \frac{1}{4}$ of A's, and C's share $= \frac{1}{4}$ of A's $+ \frac{1}{4}$ of B's;

$\therefore$ C's share $= \frac{1}{4}$ of A's $+ \frac{1}{16}$ of A's $= \frac{5}{16}$ of A's.

Hence A's share : B's : C's : : A's : $\frac{1}{4}$ of A's : $\frac{5}{16}$ of A's

$$:: 1 : \tfrac{1}{4} : \tfrac{5}{16}.$$

$$\text{Now } 1 + \tfrac{1}{4} + \tfrac{5}{16} = \tfrac{16}{16} + \tfrac{4}{16} + \tfrac{5}{16} = \tfrac{25}{16}.$$

$\therefore$ the shares are $\frac{16}{25}$ of £125; $\frac{4}{25}$ of £125; $\frac{5}{25}$ of £125.

i.e. £80; £20; £25 *Ans.*

N.B.—In questions on Partnership in which the capitals of the partners are employed for unequal intervals of time, we *substitute for the given capitals* their equivalents *referred to* the same unit of time.

EXAMPLE ix.—*A begins to trade with a capital of £600. At the end of 4 months he takes a partner, B, with a capital of £500. How must they divide the profits, £224, at the end of the year?*

A has £600 employed for 12 months,
which is equivalent to £600 × 12 for 1 month.
B has £500 employed for 8 months,
which is equivalent to £500 × 8 for 1 month.
Hence the profits must be divided in the ratio

$$600 \times 12 : 500 \times 8$$

i.e. in the ratio 7200 : 4000; &c.

EXAMPLE x.—*Three men, A, B and C, together hire the grazing of a pasture for £50, 5s. A puts on 70 sheep for 6 months, B puts on 90 sheep for 5 months, and C puts on 45 sheep for 3 months. What share of the rent should each pay?*

A must pay for the food of	70	sheep for	6 months.
i.e. A	70 × 6	....	1 month.
B	90	....	5 months.
i.e. B	90 × 5	....	1 month.
C	45	....	3 months.
i.e. C	45 × 3	....	1 month.

Hence we must divide £50, 5*s.* into parts proportional to
70 × 6, 90 × 5, and 45 × 3; *i.e.* to 420, 450, and 135; &c.

EXAMPLE xi.—*A and B enter into partnership. A puts in £2000, but at the end of 3 months withdraws £500 of this, and again at the end of 8 months withdraws £300. At the end of the year B's share of profits : A's : : 4 : 5. Find B's capital.*

A's working capital is

£2000 for 3 mo. + £1500 for 5 mo. + £1200 for 4 mo., which is equivalent to (£6000 + £7500 + £4800) for 1 month.

= £18300 for 1 month = £1525 for a year.

Hence, as B's share of profits is $\frac{4}{5}$ of A's,

∴ B's capital is $\frac{4}{5}$ of £1525 = £1220 *Ans.*

EXAMPLE xii.—*A and B are partners, with capitals in the ratio 4 : 5. After 3 months they withdraw $\frac{1}{5}$ and $\frac{1}{3}$ of their capitals respectively, and 4 months later B again withdraws $\frac{1}{3}$ of his original capital. How must the profits, which at the end of the year amount to £581, be divided?*

A's capital is proportional to £4 for 3 mo. + $\frac{4}{5}$ of £4 for 9 mo.

= £12 + £$\frac{144}{5}$, for 1 month = £$\frac{204}{5}$ for 1 month.

B's capital is proportional to

£5 for 3 mo. + $\frac{2}{3}$ of £5 for 4 mo. + $\frac{1}{3}$ of £5 for 5 mo.

= £15 + £$\frac{40}{3}$ + $\frac{25}{3}$, for 1 month = £$\frac{110}{3}$ for 1 month.

Hence the profits must be divided in the ratio $\frac{204}{5} : \frac{110}{3}$; &c.

EXAMPLE xiii.—*Volumes proportional to the numbers 3, 4 and 7 of three different substances are mingled together. The weights of equal volumes of the substances are as 5 : 2 : 6 respectively. Find the weight of the amount of the 3rd substance contained in 52 lbs. of the mixture.*

The weight of 1 volume* of the 1st is proportional to 5
∴ .... 3 volumes **15**

.... 1 volume 2nd 2
∴ .... 4 volumes **8**

.... 1 volume 3rd 6
∴ .... 7 volumes **42**

Now 15 + 8 + 42 = 65.

∴ weight of 3rd substance in mixture = $\frac{42}{65}$ of 52 = $33\frac{3}{5}$ lbs. *Ans.*

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* It is immaterial to the reasoning what this particular "volume" is—whether pint, cubic foot, &c. – so long as by "1 volume" we always indicate the same bulk.

XLVII. AREAS.

Rectangular Surfaces.

A **rectangle** is a four-sided plane figure whose angles are *right* angles.

Its opposite sides are equal, and parallel.
The length and breadth of a rectangle are called its *dimensions*.

A **square** is a four-sided, right-angled plane figure whose sides are *all* equal.

Thus a square is one special kind of rectangle.
Rectangles, other than squares, are sometimes called *oblongs*.

The **perimeter** of a plane figure is the measure round it, *i.e.* the *sum* of the lengths of its sides.

For instance, the perimeter of a rectangle 5 inches long and 4 inches wide is $5 + 4 + 5 + 4$, or 18, inches.

The **area** of a plane figure is the amount of *surface* enclosed within its boundary.

The English units of measurement for areas are the square inch, square foot, &c. (See page 30.)

A *square inch of surface* is an amount of surface equal to that enclosed within the boundary of a *square* each side of which is *one inch* long.

N.B.—As figures of the same *size* are not necessarily of the same *shape*, it follows that a surface which has *an area of one square inch* (or foot, &c.) *is not necessarily in shape a square.*

For instance, the accompanying figure evidently has an area of *one square inch*, for by dividing it along the dotted line the two parts would together exactly cover the inch square represented above.

Thus a surface **an inch square** is *square in shape*; but a surface whose area is **a square inch** may be of *any shape.*

So also a surface **3 feet square** is *in shape a square*, whose sides are each 3 feet long; and its *area* is (as we know from page 30) 9 square feet; but a surface **3 square feet** in area may be of *any shape.*

We shall now show how the area of any **rectangular*** surface is calculated.

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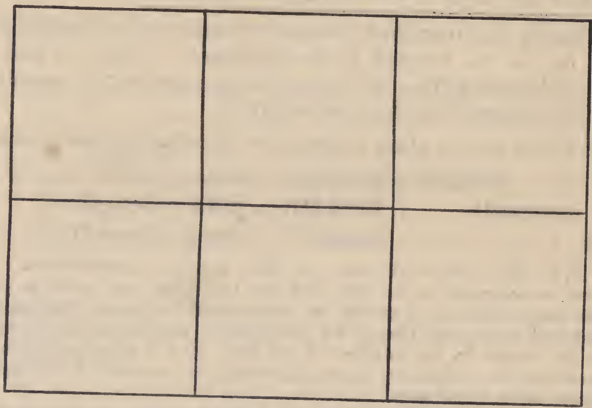
* The measurement of the surfaces of triangles, circles, &c., and of figures of irregular shape, belongs to Mensuration and Trigonometry.

CASE I. *When the dimensions are expressed by* **integers.**

If we take (for example) **six** squares, each having sides *one inch* long, and place them together thus—



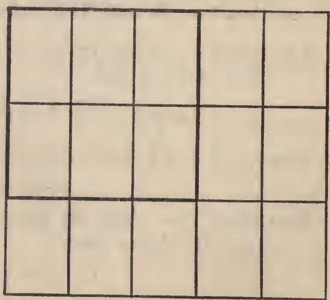
they form a rectangle 3 inches long and 2 inches wide.

∴ a rectangle 3 *inches* long and 2 *inches* wide has an area of $\underline{3 \times 2}$, or 6, *square inches.*

In the same way we might show that the area of a rectangle, say, **5** *feet* long and **4** *feet* wide, is $\underline{5 \times 4}$ *square feet.* And so on.

CASE II. *When the dimensions are expressed by* **fractions.**

If we take (for example) 15 little rectangles, each *half an inch* long and *one-third of an inch* wide, and arrange them thus—they form a larger rectangle 5 *thirds of an inch* long and 3 *halves of an inch* wide.



Now it is evident from the diagram that **one square inch** contains 6, while the larger rectangle contains 15, *of the small rectangles.*

Thus the area of the larger rectangle is $\frac{15}{6}$ of the area of the square inch; *i.e.* a rectangle $\frac{5}{3}$ of an *inch* long and $\frac{3}{2}$ of an *inch* wide has an area of $\underline{\frac{5}{3} \times \frac{3}{2}}$, or $\frac{15}{6}$, *square inches.*

In the same way we might show that the area of a rectangle, say,

$3\frac{1}{2}$ *feet* (*i.e.* 7 *half-feet*) long and $2\frac{3}{5}$ *feet* (*i.e.* 13 *fifths of a foot*) wide, is $3\frac{1}{2} \times 2\frac{3}{5}$ *square feet.* And so on.

Hence the following rule:—

To find the **area** of a **rectangular surface**:—

Multiply *the* **number** (whole or fractional) *of inches* (or feet, &c.) *in the* **length** *by the* **number** *of inches* (or feet, &c.) *in the* **breadth**, *and the result is the* **number** *of square inches* (or square feet, &c.) *in the* **area.**

The above rule is often represented shortly by the formula:—

$$\textbf{length} \times \textbf{breadth} = \textbf{area} \quad \text{(I).}$$

Or, conversely, and

$$\left.\begin{aligned} \textbf{breadth} &= \textbf{area} \div \textbf{length} \\ \textbf{length} &= \textbf{area} \div \textbf{breadth} \end{aligned}\right\} \quad \text{(II).}$$

N.B.—The shortened form (I) must not be misunderstood. Areas form no exception to the rule that the multiplier can never be *concrete* (see page 40). (I) is merely an abbreviated statement, and should be understood to mean that if we multiply the **number** (abstract) of units in the length by the **number** of the *same* kind of units in the breadth, we obtain (as was shown in the diagrams) the **number** of corresponding square units in the area.

We do *not* multiply "feet" by "feet" to obtain "square feet", but *number* (of feet) by *number* (of feet) to obtain *number* (of square feet).

This may strike the student at first as "a distinction without a difference"; but a little consideration will convince him that it is not so.

(In answer to the question "If a man earn a shilling an hour what will 3 men earn in 4 hours?" we do *not* multiply 3 *men* by 4 *hours* to obtain the result, 12 *shillings*!)

It is very important to notice that the length and breadth must both be expressed in terms of the **same unit** of length before we multiply their numbers together.

For instance, if a rectangle is 3 *feet* long and 2 *inches* wide, its area is neither 6 sq. *feet* nor 6 sq. *inches.*

EXAMPLE i.—*Find the area of a rectangle* 4 *ft.* 3 *in. long and* 2 *ft.* 4 *in. broad.*

Length $= 4\frac{1}{4}$ feet; and breadth $= 2\frac{1}{3}$ feet.

$\therefore$ area $= 4\frac{1}{4} \times 2\frac{1}{3}$, sq. feet $= 9\frac{11}{12}$ sq. ft. $=$ <u>1 *sq. yd.* 132 *sq. in. Ans.*</u>

EXAMPLE ii.—*Find the area of a rectangular field* 33 *yards wide and* 15 *chains long.*

15 *chains* $= 15 \times 22$ *yards.*

Hence, by (I), area $= 15 \times 22 \times 33$, sq. yds. $= \dfrac{15 \times 22 \times 33}{4840}$ acres

$= 2\frac{1}{4}$ ac. $=$ <u>2 *ac.* 1 *ro. Ans.*</u>

EXAMPLE iii.—*Find the length of a floor** 15 *ft. wide, the area of which is* 30 *sq. yds.* 5 *sq. ft.*

Area $=$ 30 sq. yds. 5 sq. ft. $=$ 275 *sq. feet*; width $=$ 15 *ft.*

Hence, by (II), length $= \frac{275}{15}$ *feet* $= \frac{55}{3}$ ft. $=$ 18 *ft.* 4 *in. Ans.*

EXAMPLE iv.—*If the price is* 3*d. per square foot, what length must be cut off a nine-inch board that the value of the piece cut off may be sixpence?*

For 6*d.* we obtain an *area* of 2 sq. ft. $= 2 \times 144$ *sq. inches*, and the *width* of the board is 9 *inches.*

$\therefore$, by (II), the *length* $= \dfrac{2 \times 144}{9}$ *inches* $=$ 32 in. $=$ 2 *ft.* 8 *in. Ans.*

EXAMPLE v.—*A roll of matting which is* 40 *yards long would just cover a floor* 20 *ft. long and* 16 *ft. wide. Find the width of the matting.*

[As the matting would just cover the floor, the area of the matting is the same as that of the floor.]

Area of matting $= 20 \times 16$ *sq. feet*; *length* of it $= 40 \times 3$ *feet*;

$\therefore$, by (II), *width* of matting $= \dfrac{20 \times 16}{40 \times 3}$ *feet* $=$ 2 *ft.* 8 *in. Ans.*

EXAMPLE vi.—*How many tickets, each* $2\frac{1}{4}$ *in. long and* $1\frac{1}{2}$ *in. wide, could be cut from a sheet of cardboard* $16\frac{1}{2}$ *in. long and* $13\frac{1}{2}$ *in. wide?*

[This is merely an example of Comp. Division, Class II. (see p. 46); *i.e.* we have to find *how many* of the smaller areas are contained in the larger.]

Area of card $= 16\frac{1}{2} \times 13\frac{1}{2}$ *sq. in.*; area of a ticket $= 2\frac{1}{4} \times 1\frac{1}{2}$ *sq. in.*;

$\therefore$ the req[d] number $= \dfrac{16\frac{1}{2} \times 13\frac{1}{2}}{2\frac{1}{4} \times 1\frac{1}{2}} = \dfrac{33 \times 27 \times 2}{9 \times 3} =$ 66 *Ans.*

EXAMPLE vii.—*How many bricks measuring* 9 *in. by* 4 *in. would pave a kitchen measuring* 18 *ft. by* 15 *ft.?*

Area of floor $= 18 \times 15$ *sq. ft.*; area of face of a brick $= \frac{3}{4} \times \frac{1}{3}$ *sq. ft.*

$\therefore$ the req[d] number $= \dfrac{18 \times 15}{\frac{3}{4} \times \frac{1}{3}} = 18 \times 60 =$ 1080 *Ans.*

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* Unless the contrary is stated, all *floors* are supposed to be *rectangular.*

EXAMPLE viii.—*How many yards of drugget 3 ft. 6 in. wide will cover a floor 21 ft. long by 15 ft. wide?*

1st Method (that of Example vi.).

[Here a "*yard*" of drugget is a *rectangle 3 ft. long by $3\frac{1}{2}$ ft. wide*, and we have to find *how many* such rectangles are contained in the area of the floor.]

Area of floor $= 21 \times 15$ sq. ft.; area of a "yard" $= 3 \times 3\frac{1}{2}$ sq. ft.

$$\therefore \text{ the } number \text{ of such "yards" req}^{d} = \frac{21 \times 15}{3 \times 3\frac{1}{2}} = \underline{30 \ Ans.}$$

2nd Method (that of Example v.).

Area of drugget $= 21 \times 15$ *sq. feet*; *width* of drugget $= 3\frac{1}{2}$ *feet*;

$$\therefore, \text{ by (II), } length \text{ of drugget} = \frac{21 \times 15}{3\frac{1}{2}} \, feet = 90 \text{ ft.} = \underline{30 \ yds. \ Ans.}$$

EXAMPLE ix.—*Find the cost of carpet, $\frac{3}{4}$ yard wide, at 3s. 6d. per yard for a floor 18 ft. 9 in. long and 11 ft. 3 in. wide.*

[$\frac{3}{4}$ yard $= 2\frac{1}{4}$ feet; and adopting, as preferable, the 1st method of Ex. viii., we regard a "yard" of carpet as a rectangle 3 ft. long by $2\frac{1}{4}$ ft. wide.]

Area of floor $= 18\frac{3}{4} \times 11\frac{1}{4}$ sq. ft.; area of a "yard" $= 3 \times 2\frac{1}{4}$ sq. ft.

$$\therefore \text{ the } number \text{ of "yards" req}^{d} = \frac{18\frac{3}{4} \times 11\frac{1}{4}}{3 \times 2\frac{1}{4}} = \frac{125}{4}$$

$$\text{Hence the } cost = \overset{s.}{3\frac{1}{2}} \times \frac{125}{4} = \frac{875}{8} s. = \underline{£5, \ 9s. \ 4\tfrac{1}{2}d. \ Ans.}$$

Note.—Such questions as this are not of a very *practical* nature, for allowance has to be made, in most cases, for waste in matching the pattern, &c.

EXAMPLE x.—*Find the cost of staining a margin 2 ft. wide all round a floor 21 ft. long and 16 ft. wide, at 6d. per sq. yard.*

1st Method.

Area of margin = Area of floor — area of unstained portion.

$= 21 \times 16$, sq. ft.-17×12, sq. ft.

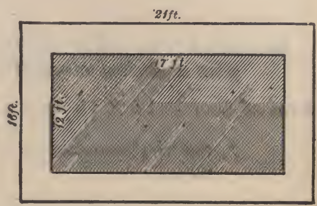
$= 336$ sq. ft. $- 204$ sq. ft.

$= 132$ sq. ft.

$= \frac{132}{9}$ sq. yds.

$\therefore$ cost $= \overset{d.}{6} \times \frac{132}{9} = 88d.$

$= \underline{7s. \ 4d. \ Ans.}$



2nd Method.

[Dividing the margin in the way shown in the diagram, we see that it consists of two equal rectangles each 19 ft. long by 2 ft. wide + two other equal rectangles each 14 ft. long by 2 ft. wide.]

Area of margin

$= \text{twice } (19 \times 2 + 14 \times 2)$ sq. ft.

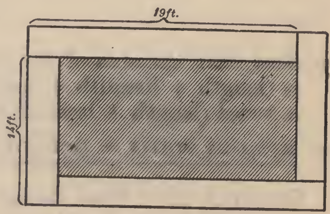
$= 4 \times (19 + 14)$ sq. ft.

$= 4 \times 33$, sq. ft.

$= \dfrac{4 \times 33}{9}$ sq. yds.

$\therefore$ cost $= \overset{d.}{6} \times \dfrac{4 \times 33}{9}$

$= \underline{7s.\ 4d.\ Ans.}$



EXAMPLE xi.—*If the value of matting* 2 *ft.* 3 *in. wide, at* 1*s.* 9*d. per yard, which covers a floor* 16 *ft. wide is* £3, 14*s.* 8*d.*, *how long is the floor?*

[This question is one of a class converse to that of Ex. ix. We consequently *reverse the order* of the operations.]

As £3, 14*s.* 8*d.* $= 74\frac{2}{3}s.$; and 1*s.* 9*d.* $= 1\frac{3}{4}s.$

$\therefore$ the *number of yards* of matting used $= \dfrac{74\frac{2}{3}}{1\frac{3}{4}} = \dfrac{128}{3}$.

And the area of a "yard" of it $= 3 \times 2\frac{1}{4}$, sq. ft.

[But the area of a "yard" multiplied by the *number* of yards = the area of the matting, *i.e.* of the floor.]

Hence, *area* of floor $= \overset{\text{sq. ft.}}{3} \times 2\frac{1}{4} \times \frac{128}{3} = 9 \times 32$ sq. ft.

$\therefore$, by (II), *length* of floor $= \dfrac{9 \times 32}{16}$ *feet* $= \underline{18 \textit{ feet } Ans.}$

Note.—In this and the preceding examples we have adopted the 1st method of Ex. viii., in preference to the 2nd method, as being more general in its application; *i.e.* the number of equal small rectangles (whether slabs of stone, sheets of paper, or "yards" of carpet) which will cover a larger rectangle is found *by dividing the area of the larger by the area of one of the smaller rectangles expressed in terms of the same unit.*

WALLS OF A ROOM.

In an ordinary room the walls consist of two equal rectangles which form the sides, and two other equal rectangles which form the ends.

Now if we suppose these four rectangles to be opened out so

as to form one long rectangle, as shown in the diagram, it is

evident that the area of the walls = area of this long rectangle
= (length + breadth + length + breadth) × height, of room
= twice (length + breadth) × height.

i.e. **area of walls = perimeter** of floor × **height** of room.

EXAMPLE xii.—*Find the area of the walls of a room* 18 *ft. long,* 15 *ft. broad, and* 12 *ft. high.*

$$\begin{aligned} \text{Area of walls} &= \text{perimeter} \times \text{height} \\ &= \text{twice } (18 + 15) \times 12, \text{ sq. ft.} \\ &= 2 \times 33 \times 12, \text{ sq. ft.} = \underline{792 \textit{ sq. ft. Ans.}} \end{aligned}$$

EXAMPLE xiii.—*How many yards of paper* 2 *ft. wide would just cover the walls of a room* 22 *ft. long,* 17 *ft. wide, and* 11 *ft. high?*

$$\begin{aligned} \text{Area of walls} = \text{perimeter} \times \text{height} &= 2 \times (22 + 17) \times 11, \text{ sq. ft.} \\ &= 2 \times 39 \times 11, \text{ sq. ft.} \end{aligned}$$

$$\text{Area of a "yard" of paper} = 3 \times 2, \text{ sq. ft.}$$

$$\therefore \text{ the } \textit{number} \text{ of yards req}^{\text{d}} = \frac{2 \times 39 \times 11}{3 \times 2} = \underline{143 \textit{ Ans.}}$$

EXAMPLE xiv.—*Find the cost of paper* 27 *inches wide, at* $1\frac{1}{2}$*d. per yard, sufficient to just cover the walls of a room* 17 *ft. long,* 14 *ft.* 6 *in. wide, and* 12 *ft.* 4 *in. high.*

$$\begin{aligned} \text{Area of walls} = \text{perimeter} \times \text{height} &= 2 \times (17 + 14\tfrac{1}{2}) \times 12\tfrac{1}{3}, \text{ sq. ft.} \\ &= 2 \times 31\tfrac{1}{2} \times 12\tfrac{1}{3}, \textit{ sq. feet.} \end{aligned}$$

$$\text{Area of a "yard" of paper} = 3 \times 2\tfrac{1}{4}, \textit{ sq. feet.}$$

$$\therefore \text{ the } \textit{number} \text{ of yards req}^{\text{d}} = \frac{2 \times 31\frac{1}{2} \times 12\frac{1}{3}}{3 \times 2\frac{1}{4}} = \frac{28 \times 37}{9}$$

$$\text{Hence the } \textit{cost} = \overset{d.}{1\tfrac{1}{2}} \times \frac{28 \times 37}{9} = 172\tfrac{2}{3}d. = \underline{14s.\ 4\tfrac{2}{3}d. \textit{ Ans.}}$$

Note.—If the dimensions of doors, windows, &c., are given, we subtract their areas from the total area of the walls in order to obtain the area to be covered with paper. It should, however, be observed that the results obtained are of little *practical* value, inasmuch as no allowance is made for waste in cutting, or in matching the pattern on, the paper. Moreover,

wall-paper is not sold by the yard but by the "piece", usually 12 yards long and 21 inches wide, so that an *exact* number of "pieces" must always be bought; *e.g.* if a fraction over 13 pieces was wanted, 14 complete pieces at least must be bought. The following is a more *practical* question.

EXAMPLE XV.—*Find the cost of paper* 21 *in. wide, at* 1*s.* 6*d. per piece of* 12 *yards, for the walls of a room* 18 *ft.* 6 *in. long,* 16 *ft.* 6 *in. wide, and* 11 *ft.* 3 *in. high, supposing that the amount of paper saved from windows, &c., together with two complete extra "pieces", is a sufficient allowance for waste.*

Area of walls $= 2 \times (18\frac{1}{2} + 16\frac{1}{2}) \times 11\frac{1}{4}$ sq. ft. $= 2 \times 35 \times \frac{45}{4}$, sq. ft.

Area of a "piece" of paper $= (12 \times 3) \times \frac{21}{12}$, sq. ft. $= 3 \times 21$, sq. ft.

$$\therefore \left.\begin{array}{l}\text{number of pieces which}\\ \text{would just cover the walls}\end{array}\right\} = \frac{2 \times 35 \times \frac{45}{4}}{3 \times 21} = 12\tfrac{1}{2}.$$

Hence number of pieces to be bought $= 13 + 2 = 15$.

$\therefore$ Cost $= 1\frac{1}{2}s. \times 15 =$ £1, 2*s.* 6*d.* *Ans.*

EXAMPLE XVI.—*How many square feet of sheet-lead would line an open tank measuring inside* 4 *ft. in length,* 3 *ft.* 6 *in. in width, and* 1 *ft.* 10 *in. in depth?*

Area of *sides and ends* $=$ perimeter $\times$ depth

$= 2 \times (4 + 3\frac{1}{2}) \times 1\frac{5}{6}$, sq. feet $= 2 \times 7\frac{1}{2} \times 1\frac{5}{6}$, sq. ft.

area of *bottom* $= 4 \times 3\frac{1}{2}$, sq. ft.

$\therefore$ total area of lead req^d $= 2 \times 7\frac{1}{2} \times 1\frac{5}{6} + 4 \times 3\frac{1}{2} = 41\frac{1}{2}$ sq. ft. *Ans.*

EXAMPLE XVII.—*The cost of lining an open rectangular reservoir* 25 *yards long and* 5 *ft.* 6 *in. deep, with tiles six inches square, at* 1*s.* 6*d. per dozen, was* £67, 18*s.* 6*d.* *Find the width of the reservoir.*

£67, 18*s.* 6*d.* $= 2717$ *sixpences*; 1*s.* 6*d.* $= 3$ *sixpences*:

$\therefore$ number of *dozens* of tiles used $= \frac{2717}{3}$.

$\therefore$ no. of tiles used $= \frac{2717}{3} \times 12 = 2717 \times 4$. But area of a tile is $\frac{1}{4}$ sq. ft.

$\therefore$ the total area covered with tiles $= \frac{1}{4} \times 2717 \times 4 = 2717$ sq. ft.

But area of the two *sides* of reservoir $= 75 \times 5\frac{1}{2} \times 2 = 825$ sq. ft.

$\therefore$ area of the *ends* + area of *bottom* $= 2717 - 825 = 1892$ sq. ft

i.e. the *width* $\times 5\frac{1}{2} \times 2 +$ the *width* $\times 75 = 1892$ sq. ft.

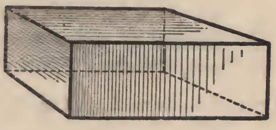
or, the *width* $\times (11 + 75) = 1892$ sq. ft.

$\therefore$ the *width* $= \frac{1892}{86}$ ft. $= 22$ ft. *Ans.*

XLVIII. VOLUMES.

RECTANGULAR SOLIDS.

A **rectangular solid** is a body bounded by *six rectangular* surfaces.



These are called its *faces*.

Its opposite pairs of faces are equal and parallel.

It has three *dimensions*, namely, *length*, *breadth* (or width), and *thickness* (or height).

A **cube** is a body bounded by *six equal square* surfaces.

Thus a cube is one special kind of rectangular solid. Rectangular solids, other than cubes, are sometimes called *rectangular parallelepipeds.*

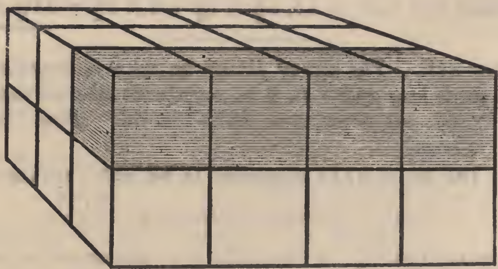
The **volume**, or capacity, or content, of a body is the amount of *space* enclosed within its bounding surfaces.

The English units of measurement for volumes are the cubic inch, cubic foot, &c. (See pages 30, 31.)

A *cubic inch of volume* is an amount of space equal to that enclosed by the faces of a *cube*, each edge of which is *one inch* long.

We shall now show how the volume of a **rectangular** solid is calculated.

If we take (for example) 24 cubes each having edges *one foot* long, and arrange them thus—



they form a rectangular solid 4 feet long, 3 feet wide and 2 feet thick: for there are evidently **three** *rows of* **four** *cubes* in the upper layer, and the same number in the lower layer. ∴ the volume of a rectangular solid, 4 *feet* long, 3 *feet* wide and 2 *feet* thick, is $\underline{4 \times 3 \times 2}$, or 24, *cubic feet.*

In the same way we might show that the volume of a rectangular solid, say, 7 *inches* long, 5 *inches* wide and 3 *inches* thick, is $7 \times 5 \times 3$ *cub. in.*

Also, by a method similar to that exemplified for areas, we might show that the volume of a rectangular solid, say, $\frac{7}{4}$ *in.* long, $\frac{5}{3}$ *in.* wide and $\frac{3}{2}$ *in.* thick, is $\frac{7}{4} \times \frac{5}{3} \times \frac{3}{2}$ *cub. in.* And so on.

Hence the following rule:—

To obtain the **number** of *cubic inches* (or feet, &c.) in the **volume of a rectangular solid.**

Find the **continued product** of the **numbers** (integral or fractional) which measure its **length, breadth** and **height** (or thickness), in *inches* (or feet, &c.); the result is the **number** of *cubic inches* (or cub. ft., &c.) in its **volume.**

This rule is often expressed shortly by the formula—

$$\left.\begin{aligned} \text{length} \times \text{breadth} \times \text{height} &= \text{volume} \\ \text{or, area of a face} \times \text{3rd dimension}^* &= \text{volume} \end{aligned}\right\} \ldots\ldots\text{(I).}$$

$$\left.\begin{aligned} \text{Hence, conversely, \&c.,}\quad \text{length} &= \frac{\text{volume}}{\text{breadth} \times \text{height}} \\ \text{or, 3rd dimension} &= \frac{\text{volume}}{\text{area of a face}} \end{aligned}\right\} \ldots\ldots\text{(II).}$$

Note.—For the sense in which the words volume, length, &c., are used in these formulæ, see note on page 156.

It is very important to observe that the length, breadth, and height must all be expressed in terms of the **same unit.**

EXAMPLE i.—*Find the volume of a rectangular solid* 5 *ft.* 3 *in. long,* 4 *ft.* 8 *in. wide and* 3 *ft.* 6 *in. thick.*

The volume $= 5\frac{1}{4} \times 4\frac{2}{3} \times 3\frac{1}{2}$, cub. ft. $= \frac{343}{4}$ cub. ft. $= 85\frac{3}{4}$ cub. ft.

$= $ 3 *cub. yd.* 4 *ft.* 1296 *in. Ans.*

EXAMPLE ii.—*A beam* 24 *feet long and* 9 *inches wide contains* $11\frac{1}{4}$ *cubic feet of timber; how thick is the beam?*

By (II), the *thickness* of beam $= \dfrac{11\frac{1}{4}}{24 \times \frac{3}{4}}$ *ft.* $= \frac{5}{8}$ ft. $= 7\frac{1}{2}$ *in. Ans.*

EXAMPLE iii.—*A uniform iron bar,* $\frac{3}{4}$ *of a sq. in.* in section,† *contains a cubic foot of metal: how long is it?*

By (II), *length* $= \dfrac{1728}{\frac{3}{4}}$ *inches* $= \dfrac{1728 \times 4}{3 \times 12}$ *feet* $=$ 192 *ft. Ans.*

---

* *i.e.* the dimension not involved in the area of the particular face considered.

† *i.e.* the *face*, to which the length of the bar is perpendicular, is $\frac{3}{4}$ sq. in. in *area*.

EXAMPLE iv.—*How many bricks each* 9 *in. long,* $4\frac{1}{2}$ *in. wide and* 3 *in. thick, would (making no allowance for mortar) be required for a wall* 17 *yards long,* 6 *ft.* 6 *in. high and* $13\frac{1}{2}$ *in. thick?*

$$\text{Volume of wall} = 51 \times 6\tfrac{1}{2} \times \frac{13\frac{1}{2}}{12}, \text{ cubic feet.}$$

$$\text{Volume of a brick} = \tfrac{3}{4} \times \tfrac{3}{8} \times \tfrac{1}{4}, \text{ cubic feet.}$$

$$\therefore \textit{number} \text{ of bricks req}^{\text{d}} = \frac{51 \times 6\frac{1}{2} \times \frac{13\frac{1}{2}}{12}}{\frac{3}{4} \times \frac{3}{8} \times \frac{1}{4}} = \underline{5304 \textit{ Ans.}}$$

EXAMPLE v.—*How many gallons of water will a tank* 5 *feet long,* 2 *ft.* 6 *in. wide and* 3 *ft. deep hold?*

[Remembering that "*a pint of water weighs a pound and a quarter*", *i.e.* that a gallon of water weighs 10 lbs.; and that a *cubic foot of water weighs* 1000 *ozs.*, we proceed thus:—]

$$\textit{Content} \text{ of tank} = 5 \times 2\tfrac{1}{2} \times 3, \textit{ cub. feet.}$$

$$\therefore \textit{weight} \text{ of water it will hold} = 5 \times \tfrac{5}{2} \times 3 \times 1000, \textit{ ozs.}$$

$$= 5 \times \tfrac{5}{2} \times 3 \times \tfrac{1000}{16}, \textit{ lbs.}$$

$$\therefore \text{ the quantity of water} = 5 \times \tfrac{5}{2} \times 3 \times \tfrac{1000}{16} \times \tfrac{1}{10}, \textit{ gallons}$$

$$= \underline{234\tfrac{3}{8} \textit{ gallons Ans.}}$$

EXAMPLE vi.—*To what depth would* 25 *gallons of water fill a tank* 10 *feet long and* 2 *ft.* 8 *in. wide?*

As a gallon of water weighs 10 lbs., *i.e.* 160 ozs.

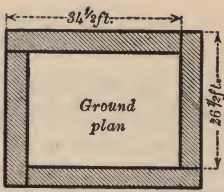
$\therefore$ the *volume* of a gallon of water is $\frac{160}{1000}$ of a cubic foot.

$\therefore$ the volume of 25 gallons is $\frac{16}{100} \times 25$, cub. ft. $= 4$ cub. ft.

$$\text{Hence, by (II), } \textit{depth} = \frac{4}{10 \times 2\frac{2}{3}} \text{ feet} = \tfrac{3}{20} \text{ ft.} = \underline{1\tfrac{4}{5} \textit{ in. Ans.}}$$

EXAMPLE vii.—*How many cubic feet of brickwork are there in a wall* 10 *feet high and* 18 *inches thick, surrounding a rectangular plot which measures outside the wall,* 36 *feet in length and* 28 *feet in breadth?*

[From the accompanying ground-plan it is evident that *two* lengths of wall each $34\frac{1}{2}$ ft. long + *two* lengths each $26\frac{1}{2}$ ft. long, would surround the plot. Hence—]



Number of cubic ft. of brickwork

$= \text{total length of wall} \times \text{thickness} \times \text{height}$

$= \textit{twice}\left(34\frac{1}{2} + 26\frac{1}{2}\right) \times 1\frac{1}{2} \times 10$, cub. ft.

$= 2 \times 61 \times \frac{3}{2} \times 10$, cub. ft.

$=$ 1830 *cub. ft. Ans.*

EXAMPLE viii.—*How many cubic feet of wood are there in a box, with lid, 4 feet long, 3 feet wide and 2 feet high, the board of which it is made being $1\frac{1}{2}$ in. thick?*

1st method.

[Here we find the *internal* dimensions of the box by subtracting *twice* the thickness of the wood from each of the given *external* dimensions; then]

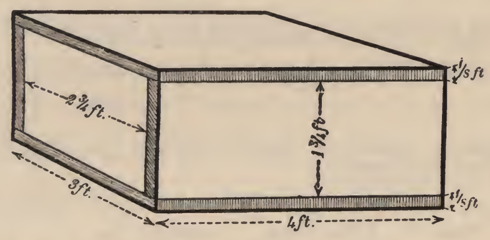
Volume of *wood* = vol. of box, *outside* measure — vol. *inside* measure,

$= 4 \times 3 \times 2$, cub. ft. $- 3\frac{3}{4} \times 2\frac{3}{4} \times 1\frac{3}{4}$, cub. ft.

$= 24$ cub. ft. $- 18\frac{3}{64}$ cub. ft. $= 5\frac{61}{64}$ *cub. ft. Ans.*

2nd method.

[Here we dissect the box in the way shown in the diagram, and find the volume of lid, side and end as separate pieces.]



Vol. of *lid* and *bottom* = twice (*area of lid*) × (*thickness of wood*)

$= \text{twice } (4 \times 3) \times \frac{1}{8}$, cub. ft.

Vol. of the two *sides* = twice (*area of side*) × (*thickness of wood*)

$= \text{twice } \left(4 \times 1\frac{3}{4}\right) \times \frac{1}{8}$, cub. ft.

Vol. of the two *ends* = twice (*area of end*) × (*thickness of wood*)

$= \text{twice } \left(2\frac{3}{4} \times 1\frac{3}{4}\right) \times \frac{1}{8}$, cub. ft.

$\therefore$ total vol. of wood $= \frac{2}{8} \times \left(4 \times 3 + 4 \times 1\frac{3}{4} + 2\frac{3}{4} \times 1\frac{3}{4}\right)$, cub. ft.

$= \frac{1}{4} \times \left(\;12 \;+\; 7 \;+\; 4\frac{13}{16}\;\right)$, cub. ft.

$= \frac{1}{4} \times 23\frac{13}{16}$, cub. ft. $= 5\frac{61}{64}$ *cub. ft. Ans.*

XLIX. DUODECIMALS.

In the calculation of rectangular areas and volumes, builders, glaziers, &c., use a *duodecimal** method.

A *foot*, whether of length, area, or volume, is divided into 12 equal parts called **primes**; a *prime* into 12 equal parts called **seconds**; a *second* into 12 equal parts called **thirds**; and so on.

A *linear* **prime** $= \frac{1}{12}$ of a *foot* (*i.e.* 1 **inch**);

a *linear* **second** $= \frac{1}{12}$ of a *prime* $= \frac{1}{(12)^2}$ of a *foot*, or $\frac{1}{144}$ *ft.*;

a *linear* **third** $= \frac{1}{12}$ of a *second* $= \frac{1}{(12)^3}$ of a *ft.*; or $\frac{1}{1728}$ *ft.*;

and so on.

A *superficial* **prime** $= \frac{1}{12}$ of a *square foot* (*i.e.* 12 *sq. in.*);

a *superficial* **second** $= \frac{1}{(12)^2}$ of a *sq. ft.*, or $\frac{1}{144}$ *sq. ft.* (*i.e.* 1 **sq. in.**);

a *superficial* **third** $= \frac{1}{(12)^3}$ of a *square foot*; or $\frac{1}{1728}$ *sq. foot*;

and so on.

A *cubic* **prime** $= \frac{1}{12}$ of a *cubic foot* (*i.e.* 144 *cub. ins.*);

a *cubic* **second** $= \frac{1}{(12)^2}$ of a *cu. ft.*, or $\frac{1}{144}$ *cu. ft.* (*i.e.* 12 *cu. in.*);

a *cubic* **third** $= \frac{1}{(12)^3}$, or $\frac{1}{1728}$, of a *cu. ft.* (*i.e.* 1 **cu. in.**);

and so on.

The notation commonly used,† whether for length, area, or volume, is the following:—

primes are indicated thus, $'$; *seconds*, thus, $''$; *thirds*, thus, $'''$; *fourths*, thus, $^{\text{iv}}$; *fifths*, thus, $^{\text{v}}$; and so on.

For instance, the length, 3 ft. $4'.\,6''$;
the area, 5 sq. ft. $7'.\,8''.\,9'''$;
the volume, 8 cub. ft. $2'.\,0''.\,4'''.\,5^{\text{iv}}$.

EXAMPLE i.—*Express the length* 3 *yds.* 2 *ft.* $7\frac{3}{8}$ *in. in duodecimals.*

$$\tfrac{3}{8} \text{ in.} = \tfrac{3}{8} \text{ of a } prime = \tfrac{3}{8} \times 12, \; seconds = 4\tfrac{1}{2}''$$

Hence 3 yds. 2 ft. $7\frac{3}{8}$ in. $=$ $\underline{11 \; ft. \; 7'.\,4''.\,6''' \; Ans.}$

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* Latin, *duodecim*, twelve.

† Sometimes, however, the words "*feet*", "*inches*" and "*parts*" are used: *e.g.* 3 ft. 5 in. 6 pts. for 3 ft. 5 primes 6 seconds. Sometimes also 1 *foot* is indicated thus, $1'$; and *then* 1 *inch is written thus*, $1''$.

EXAMPLE ii.—*Express in duodecimals* 5 *sq. ft.* $27\frac{3}{4}$ *sq. in.*

[As *sq. in.* are superficial (*i.e.* surface) seconds we reduce *sq. in.* to primes by *dividing* by 12. Hence]

5 sq. ft. $27\frac{3}{4}$ sq. in. = 5 sq. ft. $2'.\,3\frac{9}{12}''$ = 5 *sq. ft.* $2'.\,3''.\,9'''$ *Ans.*

EXAMPLE iii.—*Reduce* 7 *cub. ft.* $635\frac{1}{4}$ *cub. in. to duodecimals.*

[As *cub. in.* are cubic thirds, we reduce *cub. in.* to cub. seconds by *dividing* by 12, and then *cub. seconds* to *cub. primes* by again dividing by 12. Hence]

7 cub. ft. $635\frac{1}{4}$ cub. in. = 7 cub. ft. $52''.\,11\frac{1}{4}'''$
= 7 cub. ft. $4'.\,4''.\,11\frac{3}{12}'''$
= 7 *cub. ft.* $4'.\,4''.\,11'''.\,3^{iv}$ *Ans.*

EXAMPLE iv.—*Express* 8 *ft.* $5'.\,10''$ *in yards, feet and inches.*

8 ft. $5'.\,10''$ = 2 yds. 2 ft. $5\frac{10}{12}$ in. = 2 *yds.* 2 *ft.* $5\frac{5}{6}$ *in. Ans.*

EXAMPLE v.—*Reduce* 13 *sq. ft.* $7'.\,4''.\,8'''$ *to sq. ft. and sq. in.*

[As surface *seconds* are *sq. in.* we reduce surface *primes* to sq. in. by multiplying by 12. Hence]

13 sq. ft. $7'.\,4''.\,8'''$ = 13 sq. ft. $88\frac{8}{12}$ sq. in. = 13 *sq. ft.* $88\frac{2}{3}$ *sq. in. Ans.*

EXAMPLE vi.—*Express* 17 *cub. ft.* $6'.\,10''.\,5'''.\,3^{iv}.\,9^{v}$ *in cub. ft. and in.*

[As cubic *thirds* are *cub. in.* we reduce cub. *seconds* to *cub. in.* by *multiplying* by 12; cub. *primes* to *cub. in.* by *multiplying* by 144 ; cub. *fourths* to *cub. in.* by *dividing* by 12, &c. Hence]

17 cub. ft. $6'.\ 10''.\ 5'''.\ 3^{iv}.\ 9^{v}$

$120\frac{3}{12}$

$864\frac{9}{144}$

= 17 cub. ft. . $989\frac{45}{144}'''$ = 17 *cub. ft.* $989\frac{5}{16}$ *cub. in. Ans.*

N.B.—To *multiply*, or *divide*, by **12**, in Duodecimals, we have merely to *shift all the figures one place to the left*, or *right*, respectively (just as in *Decimals* we multiply or divide by *10*).

For instance, (3 ft. $7'.\,5''$) ÷ 12 = $3'.\,7''.\,5'''$;
and ($5'.\,8''.\,4'''$) × 60 = (5 ft. $8'.\,4''$) × 5 = 28 ft. $5'.\,8''$.

Areas.

We shall now show how, when the dimensions of a rectangle are expressed in duodecimals, its area may be conveniently found.

The method depends upon the following considerations:—

(I) As a *prime* is $\frac{1}{12}$ of a *foot*, a second is $\frac{1}{144}$ of a *foot*, &c., *whether the foot be linear or superficial*, it follows that—

Linear *primes* × **feet** give area in *superficial* **primes**;*

For instance, a rectangle whose dimensions are 5′, and 2 *ft.*, has area $\frac{5}{12} \times 2$, or $\frac{10}{12}$, *sq. ft.* = 10′ (superficial).

linear *seconds* × **feet** give area in superficial **seconds**;

For instance, a rectangle whose dimensions are 5″, and 2 *ft.*, has area $\frac{5}{144} \times 2$, or $\frac{10}{144}$, *sq. ft.* = 10″ (superficial).

linear *thirds* × **feet** give area in superficial **thirds**;

For instance, a rectangle whose dimensions are 5‴, and 2 *ft.*, has area $\frac{5}{1728} \times 2$, or $\frac{10}{1728}$, *sq. ft.* = 10‴ (superficial)

and so on.

Linear *primes* × **primes** give area in superficial **seconds**;

e.g. a rect. 5′ by 2′ has area $\frac{5}{12} \times \frac{2}{12}$, or $\frac{10}{144}$, sq. ft. = 10″.

linear *seconds* × **primes** give area in superficial **thirds**;

e.g. a rect. 5″ by 2′ has area $\frac{5}{144} \times \frac{2}{12}$, or $\frac{10}{1728}$, sq. ft. = 10‴.

linear *thirds* × **primes** give area in superficial **fourths**;

e.g. a rect. 5‴ by 2′ has area $\frac{5}{(12)^3} \times \frac{2}{12}$, or $\frac{10}{(12)^4}$, sq. ft. = 10^{iv}.

and so on.

Similarly, linear *primes, seconds, thirds,* &c., multiplied by **seconds**, give area in superficial **thirds, fourths, fifths,** &c., respectively. And so on.

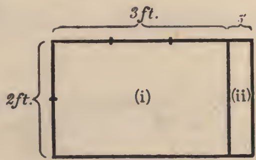
The above results are summarized in the statement that in Duodecimals—

The *order* of a product is the **sum** of the *orders* of the factors.

e.g. seconds × *thirds* give *fifths.*

(II) The area of any rectangle = the *sum* of the areas of all the rectangles into which it is divided.

For instance, the area of a rectangle 3 ft. 5′ long by 2 ft. wide = area of rect. 3 ft. by 2 ft.....(i) + area of rect. 5′ by 2 ft....(ii)



* *i.e.* a *number* of linear primes multiplied by a *number* of linear feet gives the *number* of superficial primes in the area of a rectangle of those dimensions. See note on p. 156.

EXAMPLE vii.—*Find the area of a rectangle* 3 *ft.* 5′. 4″ *long and* 2 *ft. wide.*

We set 2, the number of *feet* in the width, under 3, the number of *feet* in the length, and then, beginning on the right, we proceed as in compound multⁿ.

It follows, from (I) and (II) above, that the result thus obtained is the area of the rectangle in superficial feet, primes, and seconds.

$$\begin{array}{ccccc} \text{ft.} & & ' & & '' \\ 3 & . & 5 & . & 4 \\ 2 & & & & \\ \hline 6 & . & 10 & . & 8 \end{array}$$

Ans. 6 *sq. ft.* 10′. 8″.

EXAMPLE viii.—*Find the area of a rectangle* 7 *ft.* 3′. 4″ *by* 2′.

We set 2, the No. of *primes* in the width, under 3, the No. of *primes* in the length, and then, multiplying as before, we set *each partial result* **one place to the right** *of the column from which it is obtained*; for by (I), when we multiply a number of linear *seconds* by a number of linear *primes* we obtain area in superficial *thirds*; and so on.

$$\begin{array}{ccccccc} \text{ft.} & & ' & & '' & & ''' \\ 7 & . & 3 & . & 4 & . & \\ & & 2 & & & & \\ \hline & & 14 & . & 6 & . & 8 \end{array}$$

Ans. 1 *sq. ft.* 2′. 6″. 8‴.

EXAMPLE ix.—*Find the area of a rectangle* 5 *ft.* 2′ *long and* 4 *ft.* 3′ *wide.*

Setting 4 ft. 3′ under 5 ft. 2′ we first multiply by 4 in the manner shown in Ex. vii; then by 3 in the manner shown in Ex. viii. Finally we add these results.

$$\begin{array}{lccccccl} & \text{ft.} & & ' & & '' & & \\ & 5 & . & 2 & . & & & \\ & 4 & & 3 & & & & \\ \hline \text{(ii)}\ldots\ldots & 20 & . & 8 & & & & \ldots\ldots\text{(i)} \\ \text{(iv)}\ldots\ldots & & & 15 & . & 6 & & \ldots\ldots\text{(iii)} \\ \hline & 21 & . & 11 & . & 6 & & \end{array}$$

Ans. 21 *sq. ft.* 11′. 6″.

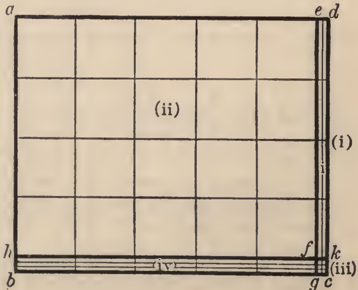
Note.—The accompanying diagram illustrates the above process.

Here each of the *larger squares* represents a *square foot*;

each very thin *rectangle*, a *superficial prime* (*i.e.* 12 sq. in., or a rect. 1 *ft.* long and 1′ wide);

each very *little square*, a *superficial second* (*i.e.* a sq. in.).

And rectangle $abcd$ = rectangles $efkd + ahfe + fgck + hbgf$
(i) (ii) (iii) (iv)
= 8′ + 20 sq. ft. + 6″ + 15′
= 21 sq. ft. 11′. 6″, as above.



EXAMPLE X.—*Find the area of a rectangle the dimensions of which are* 3 *yds.* 2 *ft.* 5′. 9″ *and* 5 *yds.* 1 *ft.* 7′. 6″.

1st step. (Multiply by 11.)
6 × 11 = 66; set down **6″**, carry *5*.
7 × 11 = 77; 77 + *5* = 82; set down **10′**, carry *6*.
16 × 11 = 176; 176 + *6* = **181**.

2nd step. (Multiply by 5.)
6 × 5 = 30; set down **6‴**, carry *2*.
7 × 5 = 35; 35 + *2* = 37; set down **1″**, carry *3*.
16 × 5 = 80; 80 + *3* = 83; set down **11′**, and carry *6*, *i.e.* set **6** in the column headed *ft.*

3rd step. (Multiply by 9.)
6 × 9 = 54; set down **6**^iv^, carry *4*.
7 × 9 = 63; 63 + *4* = 67; set down **7‴**, carry *5*.
16 × 9 = 144; 144 + *5* = 149; set down **5″**, and carry 12′, *i.e.* **1** *sq. ft.*

4th step. Add the results, as in Comp. Addⁿ.

ft.	′	″	‴	iv
16	7	6		
11	5	9		
181	10	6		
6	11	1	6	
1	0	5	7	6
189	10	1	1	6

Ans. 189 *sq. ft.* 10′. 1″. 1‴. 6^{iv}.

Note.—We may extend the duodecimal arrangement to the *left* of the column headed "*feet*", should the number of feet in the given dimensions be inconveniently large. Then the two next columns on the left of the "feet" column would contain *dozens* and *gross* of feet, respectively.

For instance, to find *by the duodecimal method*, the area measuring 17 yards 2 ft. 6 in. by 16 yards 1 ft. 4 in., we might proceed thus:—

17 yds. 2 ft. 6 in.
= 53 ft. 6 in. = 4 doz. 5 ft. . 6′

16 yds. 1 ft. 3 in.
= 49 ft. 3 in. = 4 doz. 1 ft. . 3′

Now, when we multiply by 4 *dozens* of feet, we set each partial result *one step to the left* of the column treated, &c.

gross.	doz.	ft.	′	″
	4	5	6	
	4	1	3	
17	10	0		
	4	5	6	
	1	1	4	6
17	15	6	10	6

Ans. 2634 *sq. ft.* 10′. 6″.

VOLUMES.

In the same way as for *areas* it may be shown that for volumes

(I) The *order* of a product is the sum of the *orders* of the factors.

For instance, a rectʳ solid one face of which is 5 *superficial* primes in area and its 3rd dimension 2 *linear* seconds, has volume $\frac{5}{12} \times \frac{2}{144}$ *cub. ft.* = $\frac{10}{1728}$ *cub. ft.* = 10 *cubic* thirds.

(II) The volume of a rectangular solid = the sum of the volumes of all the rectʳ solids into which it is divided.

EXAMPLE xi.—*Find the volume of a rectangular solid, the dimensions of which are* 3 *ft.* 4′. 6″, 2 *ft.* 7′ *and* 2 *ft.* 1′. 6″.

Here we first find the *area* of a *face* (i), and then obtain the *volume* (ii) by multiplying the area of the face by the 3rd dimension of the solid; the process, in both cases, being exactly similar to that shown in Ex. x.

```
ft.      '      "      '''     iv
 3   .   4   .  6
 2   .   1   .  6
 6   .   9   .  0
         3   .  4   .  6
         1   .  8   .  3   .  0
 7   .   2   .  0   .  9          (i)
 2   .   7
14   .   4   .  1   .  6
 4   .   2   .  2   .  5   .  3
18   .   6   .  3   . 11   .  3   (ii)
```

Ans. 18 *cub. ft.* 6′. 3″. 11‴. 3^iv.

The converse of the above operations, *i.e.* Division, may be performed in Duodecimals, but is not of much practical value.

EXAMPLE xii.—*Find, by duodecimals, the length of a rectangle* 8 *ft.* 3′. 6″ *wide and* 194 *sq. ft.* 3′. 4″. 1‴ *in area.*

When we consider how many times 8 is contained in 19 (the first two figures of 194) we must remember that 2, the quotient figure, is 2 *tens*, and so we multiply the *entire divisor* by 20, and not by 2.

```
ft.  '  "  sq. ft.   '     "     '''   iv    ft.    '   "
8 . 3 . 6)194  .  3  .  4  .  1  .  (20 + 3 . 5 . 2
          165  . 10  .  0
           28  .  5  .  4
           24  . 10  .  6
            3  .  6  . 10  .  1
            3  .  5  .  5  .  6
                  1  .  4  .  7  .  0
                  1  .  4  .  7  .  0
```

Ans. 23 *ft.* 5′ 2″.

Note.—If we adopt the method suggested in the Note on page 170, the work would stand thus:—

```
ft.   '    "  doz. sq. ft.  '    "    '''   iv doz.  ft.    '    "
8 . 3 . 6)16  .  2  .  3  .  4  .  1  .  (1  . 11  .  5  .  2
           8  .  3  .  6
           7  . 10  .  9  .  4
           7  .  7  .  2  .  6
                 3  .  6  . 10  .  1
                 3  .  5  .  5  .  6
                       1  .  4  .  7  .  0
                       1  .  4  .  7  .  0
```

Ans. 23 *ft.* 5′. 2″.

L. APPROXIMATION.

DECIMALS.

In calculations made for commercial, and other practical, purposes it would often happen that, if the *exact* result were obtained, some small part of it would be practically useless.

For instance, a very small fraction of a *penny* in a *money* result; for it could not be paid in our coinage.

Yet the labour involved in obtaining this small part of the exact result would often be considerable.

The methods we are now about to consider enable us, if we operate in Decimals, to **approximate** to an exact result with any degree of nearness that may be desired, without such waste of labour as would often occur in obtaining the *complete* result.

Note.—All results based upon measurement, no matter how completely the calculations themselves are carried out, must of necessity be but *approximate*; since in the measurements themselves, which depend upon our imperfect powers of observation, *absolute accuracy* is unattainable.

The degree of nearness to the exact result to which an approximation should be carried depends, of course, upon the particular purpose for which the calculation is made.

In the case of *money*, a result "correct to the nearest penny" is sufficiently accurate for most *practical* purposes.

Within the limit fixed upon, the *closest possible* approximation will sometimes be a little *greater*, and sometimes a little *less*, than the *exact* result; *i.e.* the **error** may be either one of defect or of excess.

For instance, taking *pence* as the limit in money,

$4d.$ would be a closer approximation to $3\frac{5}{8}d.$ than $3d.$ would be.

So, also, if we fix the limit at *two* places of decimals,

$5 \cdot 64$ is nearer to $5 \cdot 6375$ than $5 \cdot 63$ is;

for $5 \cdot 64$ exceeds $5 \cdot 6375$ by only $\cdot 0025$, but $5 \cdot 63$ falls short of it by $\cdot 0075$.

N.B.—Similarly, the closest approximation to $\cdot 5986173$

in *six* places of decimals is $\cdot 598617$

...*five*.......................... $\cdot 59862$

...*four* .... $\cdot 5986$

... *three* .... $\cdot 599$

... *two*........................... $\cdot 60$

Hence the following general rule:—

If the first of the decimal figures discarded be less than 5, *the last figure retained is unaltered; but if the first of the figures discarded be* 5, *or greater than* 5, *the last figure retained is* **increased by one.**

For, by so doing we ensure that the *error* can never be more

than *half* of what it otherwise might be without this correction; that is, can never be more than **half** *of the value of* **unity** *standing in the place of the last figure retained.*

For instance, the error in taking ·38 as a *two*-place approximation for ·3751 is ·0049, *i.e.* is *less* than ·005, or less than *half* of ·01; whereas if we took ·37 as a *two*-place approximation for ·3799, the error would be ·0099, or very nearly ·01.

EXAMPLE i.—*Find the nearest approximation in two places of decimals to the sum of* 4·563825, 120·468539, ·054938 *and* 30·5.

Here, and in all similar cases, we retain *two* columns of decimal places beyond the number required in the result, in order to obtain the figures to be "carried".

$$\begin{array}{r} 4·5638 \\ 120·4685 \\ ·0549 \\ 30·5 \\ \hline 155·587 \end{array} \qquad \textit{Ans. } 155·59.$$

EXAMPLE ii.—*Find, to the nearest thousandth of unity, the difference between* (i) $3·41\dot{6}$ *and* 1·253075; (ii) 12·75 *and* 3·216675.

Here we require the results to **three** places of decimals.

(i) $\begin{array}{r} 3·41666 \\ 1·25307 \\ \hline 2·1635 \end{array}$ *Ans.* 2·164.

(ii) $\begin{array}{r} 12·75 \\ 3·21667 \\ \hline 9·5333 \end{array}$ *Ans.* 9·533.

To obtain a result "*correct within one-thousandth of the* **whole**", we find the **first four** *significant** figures of the result.

For, if we divide any number by 1000, the first figure in the quotient has the same *local* value as the *fourth* figure in the original number.

For instance, the number 17·4625. One-thousandth of it is ·0174625 (i). The four-figure approx^n to it is 17·46, which differs from it by ·0025, *i.e.* by *less* than (i), or one-thousandth of the whole.

Similarly, to obtain a result correct within **one-millionth of the whole,** we approximate to the first **seven** figures. And so on.

EXAMPLE iii.—*Find, within one-thousandth of the whole, the sum of* 423·56285, $81·28\dot{3}$, 17·46025 *and* $120·\dot{8}\dot{2}$.

Here, as we require only a four-figure result, we shall need but *one* place of *decimals* in the *result*, so we retain *three* in the working.

$$\begin{array}{r} 423·562 \\ 81·283 \\ 17·460 \\ 120·828 \\ \hline 643·13 \end{array} \qquad \textit{Ans. } 643·1.$$

* In such a decimal as ·003057, 3 is the first *significant* figure, and 3, 0, 5, are the first *three* significant figures.

Approximate results in Multiplication and Division, *when the multipliers and divisors contain only* one *or* two *figures*,* may be easily obtained by the methods of Chap. XXXVIII. For instance:

EXAMPLE iv.—*Find, to three places of decimals*, $1\cdot73425876 \times 4\frac{1}{4}$.

We multiply (i) by 4, retaining five places and adding in the carried figure from four times 8.
We then divide (i) by 4 to five places, and add the two results.

$$\begin{array}{r|l} 4 & 1\cdot734258\ldots(\text{i}) \\ & \quad\ \ 4 \end{array}$$

$$\begin{array}{r} 6\cdot93703\ldots. \\ \cdot43356\ldots. \\ \hline 7\cdot37059\ldots. \end{array} \quad \textit{Ans.}\ 7\cdot371.$$

To *Decimalize* a sum of English money approximately (*i.e.* to express it as the decimal of £1)—

As a *florin* (or 2*s.*) = $\frac{1}{10}$ of £1 = £·1,
and a *shilling* = $\frac{1}{2}$ of a florin = $\frac{1}{2}$ of £·1 = £·05,

any number of shillings mentioned in the sum of money can be at once decimalized, by *writing the number of* florins *it contains in the* first *place, and then representing the* odd shilling (if there be one) *by a* 5 *in the* second *place, in decimals.*

For instance, 13*s.* (*i.e.* 6 florins + 1*s.*) = £·65;
18*s.* = £·9; and 17*s.* = £·85.

And then any number of pence and farthings mentioned in the sum of money may be treated by the method of *aliquot parts*, thus:

EXAMPLE v.—*Decimalize* £2, 13*s.* $3\frac{1}{2}$*d. to five places.*

Explanation.

1st step.
£2, 13*s.* = £2 + 6 fl. + 1*s.* = £2·65.
2nd step.
3*d.* = $\frac{1}{4}$ of 1*s.* = $\frac{1}{4}$ of £·05 = £·0125.
3rd step.
$\frac{1}{2}$*d.* = $\frac{1}{6}$ of 3*d.* = $\frac{1}{6}$ of £·0125 = £·002083.

Written Work.

$$\begin{array}{r} £\quad\quad\quad\ \ \\ 2\cdot65\quad\quad\ \ \\ \cdot0125\quad\ \ \\ \cdot002083\ldots \\ \hline 2\cdot664583\ldots \end{array}$$

Ans. £2·66458.

EXAMPLE vi.—*Decimalize* £17, 18*s.* $8\frac{3}{4}$*d. to five places.*

Explanation.

Remembering that a florin = £·1,
a shilling = £·05, and sixpence = £·025.
1st step. £17, 18*s.* = £17·9.
2nd step. 8*d.* = $\frac{1}{3}$ of a fl. = $\frac{1}{3}$ of £·1 = £·0333.
3rd step. $\frac{3}{4}$*d.* = $\frac{1}{8}$ of 6*d.* = $\frac{1}{8}$ of £·025 = £·003125.

$$\begin{array}{r} £\quad\quad\quad\ \ \\ 17\cdot9\quad\quad\quad\ \ \\ \cdot033333 \\ \cdot003125 \\ \hline 17\cdot936458 \end{array}$$

£17·93646 *Ans.*

---

* For methods of dealing with larger multiplications and divisions, see pages 177, &c.

Similarly we may treat other concrete quantities. For instance:

EXAMPLE vii.—*Express 3 tons 13 cwts. 2 qrs. 15 lbs. as a decimal of a ton to four places.*

Explanation.

Here cwts. are dealt with in exactly the same way as shillings in Ex. v.

1st step.
3 tons 13 cwts. = 3·65 ton.

2nd step.
2 qrs. = $\frac{1}{2}$ of 1 cwt. = $\frac{1}{2}$ of ·05 ton.

3rd step.
8 lbs. = $\frac{1}{7}$ of 2 qrs. = $\frac{1}{7}$ of ·025 ton.

4th step.
7 lbs. = $\frac{1}{8}$ of 2 qrs. = $\frac{1}{8}$ of ·025 ton.

Written Work.

Tons.
3·65
·025
·003571...
·003125...
3·681696...

Ans. 3·6817 tons.

Note.—To decimalize money to *three* places *mentally.*

As 6*d.* = 25 *thousandths* of £1, and the number of *farthings* in 6*d.* is 24,

(i) The *number of farthings* in any sum *less* than 6*d.* gives all the *thousandths* of £1 in that sum.

For instance, 3$\frac{3}{4}$*d.* = £·015; and 14*s.* 5$\frac{1}{4}$*d.* = £·721, to three places.

(ii) The *number of farthings* + 1 in any sum between 6*d.* and 9*d.* gives all the *thousandths* of £1 in that sum.

For instance, 7$\frac{1}{2}$*d.* = £·031; and 13*s.* 8*d.* = £·683, to three places.

(iii) The *number of farthings* + 2 in any sum between 9*d.* and 1*s.* gives the nearest approximation in *thousandths* of £1 to its value.

For instance, 16*s.* 10$\frac{3}{4}$*d.* = £·845; and 17*s.* 11$\frac{1}{4}$*d.* = £·897, to three places.

In the converse operation, that of reducing a decimal of £1 to £, *s. d.*; if the result be required *to the nearest penny*, we may ignore figures beyond the *fourth* place of decimals; if to the nearest farthing, beyond the *fifth* place.

EXAMPLE viii.—*Find, to the nearest penny, the value of* £13·578675.

Here, in multiplying by 20, we first multiply ·578 by 2, *carrying* from the fourth place, and then move the dec. point in the result one place to the right (*i.e.* multiply by 10).

$$
\begin{array}{r}
£13{\cdot}578675 \\
\underline{\quad 2} \\
s.\,11{\cdot}57 \\
\underline{12} \\
d.\,6{\cdot}8
\end{array}
$$

Ans. £13, 11*s.* 7*d.*

Note.—Such results may also be obtained *mentally* by the converse of the rules given in the last note.

For instance, £·684 = 13*s.* 8*d.* to the nearest penny.

[For the 6 *tenths* represents 6 *fl.* or 12*s.*; a 5, taken from the 8, gives 1*s.*; and the remaining 34 *thousandths* = (34 − 1) *farthings*, *i.e.* 8*d.* nearly.]

Decimalized Practice.

Approximate results may often be very conveniently obtained by this method.

Example ix.—*Find, to the nearest penny, the value of* 3 *cwts.* 1 *qr.* 22 *lbs. at* £2, 13*s.* 6*d. per cwt.*

Here we decimalize the money, and then carry each successive division to *five* places of decimals, so as to ensure the result being correct to *three* places.

	£		
1 qr. $= \frac{1}{4}$	2·675	= value of	1 cwt.
	3		
	8·025	=	3 cwts.
14 lbs. $= \frac{1}{2}$	·66875	=	1 qr.
7 lbs. $= \frac{1}{2}$	·33437	=	14 lbs.
1 lb. $= \frac{1}{7}$	·16718	=	7 lbs.
	·02388	=	1 lb.
	£9·21918	= .....3 cwts. 1 qr. 22 lbs.	

Ans. £9, 4*s.* 5*d.*

Example x.—*Find, within a penny, the rent of* 423 *ac.* 2 *ro.* $31\frac{1}{2}$ *po. at* £1, 14*s.* 6*d. per acre.*

	£	
	423·5	
	·125	
	·0625	
	·00625	
	·00312...	
10*s.* $= \frac{1}{2}$	423·69687...	= rent at £1 per ac.
4*s.* $= \frac{1}{5}$	211·84843...	= .... 10*s.* ....
6*d.* $= \frac{1}{8}$	84·73937...	= .... 4*s.* ....
	10·59242...	= .... 6*d.* ....
	730·87710...	= ..... £1, 14*s.* 6*d.*...

Ans. £730, 17*s.* 6*d.*

Explanation.

To *five* places of decimals,

423 ac. 2 ro. = 423·5 ac., of which the rent, at £1 an ac. = £423·5

20 po. = $\frac{1}{4}$ of 2 ro. = $\frac{1}{4}$ of ·5 ac. = ·125 ac.,............. = £ ·125

10 po. = $\frac{1}{2}$ of 20 po. = $\frac{1}{2}$ of ·125 ac. = ·0625 ac., ... = £ ·0625

1 po. = $\frac{1}{10}$ of 10 po. = $\frac{1}{10}$ of ·0625 ac. = ·00625 ac., ... = £ ·00625

$\frac{1}{2}$ po. = $\frac{1}{2}$ of 1 po. = $\frac{1}{2}$ of ·00625 ac. = ·00312 ac.,... = £ ·00312

Hence, adding these results, we obtain £423·69687 as the rent of the whole at £1 per acre, from which we obtain by Practice the rent at £1, 14*s.* 6*d.* per ac.

The approximate result, £730·8771, we then convert to £, *s. d.*

CONTRACTED MULTIPLICATION.

We shall now show how the product of *any* two decimals may be approximately obtained.*

(I) We noticed, on page 11, that, in any multiplication sum, we might equally well *begin* with the *left-hand* figure instead of with the right-hand figure. This plan we shall now adopt.

For instance, in finding the product of 423 and 651, instead of the ordinary arrangement (i), we may multiply by the 6 *first*, when the work stands as in (ii).

```
(i)    423        (ii)  423
       651               651
       423             2538
      2115              2115
     2538                423
     275373           275373
```

Again, if we multiply a decimal by an *integer* figure, the decimal point in the result will stand *vertically below* the decimal point in the multiplicand.

For instance, in the product of 7·423 by 6, the decimal point in the result is vertically below that in the multiplicand.

```
  7·423
      6
 44·538
```

(II) But *every* multiplication of decimals can be performed with a multiplier consisting of *one single* **integer** figure followed by decimal figures, if we make use of the following principle:—

The **product** *is unaltered if we* **multiply** *one factor, and* **divide** *the other, by the* **same** *number.*

For instance, $4 \times 6 =$ (*ten* times 4) $\times$ (6 divided by *ten*).
also, $4 \times 6 =$ (a *hundred* times 4) $\times$ (6 divided by a *hundred*).

Hence we may, *without affecting the result, move the decimal point in the multiplier any number of places either to right or left, if we also move the decimal point in the multiplicand the same number of places in the* **opposite** *direction.*

For instance, 23·47 × 21·38 gives the same product as 234·7 × **2**·138;
·1425 × ·4362 .... ·01425 × **4**·362;
8105·75 × ·05637 .... 81·0575 × **5**·637.

As an illustration of the arrangements (I) and (II), we will now find the product of 261·4326 and ·2354.

Multiplying the multiplier by 10, and *dividing* the multiplicand by 10, we see that 261·4326 × ·2354 is equivalent to 26·14326 × 2·354. Then, arranging the work in the way suggested above, we obtain the product 61·54123404, with the decimal point in the result vertically below that in the multiplicand.

```
26·14326
   2·354
52·28652
 7 842978
 1 3071630
   10457304
61·54123404
```

* The method which follows is derived from that given in De Morgan's Arithmetic.

But, if we only required to know this result approximately, say to *two* places of decimals, then all the figures which here stand to the right of the vertical line might be omitted altogether, the two columns next on the left of this line only being retained for the sake of "carrying" in the addition.

```
26·1432 6
     2·254
----------
52·2865|2
 7 8429|78
 1 3071|630
   1045|7304
----------
61·5412|3404
```

All the work necessary might then be performed as follows:—

To find 26·14326 × 2·354 to *two* places of decimals.

As the result is required correct to *two* places of decimals only, we set 2, the first figure of the multiplier, under the figure which stands in the *fourth* place in decimals in the multiplicand, and begin there, adding in, however, **mentally anything there would have been to carry from the figure on the right in the multiplicand** (*i.e.* twice 2, and 1 carried, **5**).

```
26·14326
   2·354
--------
52·2865|
 7 8429|
 1 3071|
   1045|
--------
61·541
```

In the second line, when we multiply by 3, we begin at the figure which stands in the *third* place in decimals in the multiplicand.

And so on, in each successive line beginning to multiply at a figure of the multiplicand one place further to the left than in the preceding line, adding in mentally anything there would have been to carry from its right-hand neighbour; setting the first figure obtained of each line of partial results close to the vertical line; and *marking the decimal point in the final result* **vertically below** *the decimal point in the multiplicand.*

Here it will be observed that a little difficulty is caused to the eye, by reason of the distance of some of the figures of the multiplier from the part of the multiplicand treated.

This difficulty is avoided by the device of (III) **reversing the multiplier, when each figure of the multiplier will stand immediately under that figure of the multiplicand at which its work begins.**

Finally, then, the work will stand thus:—

```
26·14326
   4532
--------
52·2865|
 7 8429|
 1 3071|
   1045|
--------
61·541          Ans. 61·54.
```

EXAMPLE xi.—*Find* 2·530625 × 47·306547 *to one place of decimals.*

Here we take 2·530625 as multiplier, since it already has one, and only one, integer figure. Then, as we require to have *one* place of decimals correct in the result, we retain *three* places in the working, and so set 2, the first figure of the multiplier, under the 6 in the multiplicand.

```
  47·30656
5260 352
---------
  94·613
  23 653
   1 419
      28
---------
 119·71         Ans. 119·7.
```

EXAMPLE xii.—*Find the nearest approximation, in four places of decimals, to* $\cdot 1320566 \times \cdot\dot{4}1\dot{2}$.

First $\cdot 1320566 \times \cdot 412412......$

$= \cdot 01320566 \times 4 \cdot 124124 ...$

Now, as the result is req[d] to *four* places, we set 4, the integer figure of our multiplier, under 5, the figure in the *sixth* place of the multiplicand.

·0132056
214214
·052822
1320
264
52
1
·05446

Ans. ·0545.

Note.—Here the sum of the figures in the right-hand "carrying" column is **9**, so we carry **1** to the next column (since 9 is nearer to 10 than it is to 0). Similarly, in other "carrying" *columns*, for *sums* from **5** to **15** we may carry **one**; from **15** to **25**, carry **two**; and so on.

EXAMPLE xiii.—*Find, within a millionth of the whole, the product of* 7834·25 *and* ·9362825.

Here we require the first *seven* figures of the result (see page 173).

Now $7834 \cdot 25 \times \cdot 9362825$

$= \mathbf{783} \cdot 425 \times \mathbf{9} \cdot 362825.$

And as 9 times 700 is more than 6000, we see that there will be *four integer* figures in the result.

Hence we require *three dec.* figures.

We therefore retain *five* places of decimals in the working.

783·42500
52 82639
7050·82500
235 02750
47 00550
1 56685
62673
1566
391
7335·0712

Ans. 7335·071.

EXAMPLE xiv.—*Find the integral part of the continued product of* 171·421534, ·842056, *and* 246·327057.

We rearrange the decimal points so that *both* multipliers have *one integer* figure, thus—

$246 \cdot 327057 \times 171 \cdot 421534 \times \cdot 842056$

$= 246 \cdot 327057 \times 17 \cdot 1421534 \times 8 \cdot 42056$

$= 2463 \cdot 27057 \times \mathbf{1} \cdot 71421534 \times \mathbf{8} \cdot 42056.$

Then, as we require the result correct so far as the integral part only is concerned, we retain *two* places of decimals in the *working*.

N.B.—Here, in adding the first column of the first result (i), we obtain **9**, which we count as **10**, thus making some allowance for a probable carried figure from the *unwritten* column on its right.

2463·270
6502 48
19706·16
985 30
49 26
1 23
14
20742·10......(i)
351241 71
20742·10
14519 46
207 42
82 96
4 14
20
10

Ans. 35556.

In finding any *power** of a decimal higher than the cube, much labour may be saved by making use of the following principle:—

The **index of the power** *of the product of two powers of any number is the* **sum** *of the indices of those two powers.*

For instance, $3^4 \times 3^2 = 3^6$;
for $3^4 \times 3^2 = (3 \times 3 \times 3 \times 3) \times (3 \times 3) = 3 \times 3 \times 3 \times 3 \times 3 \times 3 = 3^6$.

Hence, in raising a number to a higher power, by utilizing some of the lower powers first obtained, we can reduce the number of steps in the multiplication.

For instance, to find 2^{12} we proceed thus:—As $\mathbf{2^2 = 4}$
$\therefore\ \mathbf{2^4}$ (*i.e.* $2^2 \times 2^2$, or 4^2) $= \mathbf{16}$
$\therefore\ \mathbf{2^8}$ (*i.e.* $2^4 \times 2^4$, or 16×16) $= \mathbf{256}$
Hence $\mathbf{2^{12}} = 2^8 \times 2^4 = 256 \times 16 = \mathbf{4096}$.

Thus we obtain the 12th power of a number in *four* steps instead of *eleven*, if we multiplied successively by the number itself.

Example xv.—*Find* $(1 \cdot 04)^{10}$ *to three places of decimals.*

1st step.

Here we first obtain the square, or 2nd power, of 1·04, by adding 1·04 to $\frac{4}{100}$ of 1·04.

2nd step.

We now square 1·0816 (retaining *five* places in the working), and obtain thus the 4th power of 1·04.

3rd step.

We next square 1·16985, thus obtaining the 8th power of 1·04.

4th step.

Finally, we multiply 1·36852 (the 8th power of 1·04) by 1·0816 (the 2nd power of 1·04), thus obtaining the 10th power of 1·04.

* In both the 2nd and 3rd stages we increase the sum of the right-hand column by 1. (See Note on Ex. xiv.)

```
 1·04
  ·0416
 ────────
 1·08160...2nd power
   61801
 ────────
 1·08160
    8652
     108
      64
 ────────
 1·16985*...4th power
 5 89611
 ────────
 1·16985
   11698
    7018
    1052
      92
       5
 ────────
 1·36851*...8th power
   61801
 ────────
 1·36851
   10948
     136
      81
 ────────
Ans. 1·480.   ...10th power.
```

Example xvi.—*Find* $236 \cdot 75 \times (1 \cdot 05)^6$ *correct to three places.*

Here as $(1 \cdot 05)^6$, when obtained, has to be multiplied by *hundreds* (*i.e.* 236) we must, in calculating $(1 \cdot 05)^6$, retain *seven* dec. places in the *working.*

---

* See page 12.

CONTRACTED DIVISION.

We shall now show how in Long Division the labour involved in obtaining the last few figures in the quotient may be curtailed.

Suppose, for example, that we require the quotient of $436 \cdot 563875 \div 17 \cdot 6824$ correct to *three* places of decimals.

If we performed the division in the ordinary way (adopting the integral divisor method of Chapter XXXII.), the work would stand thus:—

Hence the required quotient is $24 \cdot 689$; for the figure in the *fourth* decimal place in the quotient, obtained by *inspecting* the remainder, would evidently be *less* than 5. (See Rule on p. 172.)

```
176824)436563|8·750(24·689
       353648|
        82915|8
        70729|6
        12186|27
        10609|44
         1576|835
         1414|592
          162|2430
          159|1416
            3|1014
```

Now if all the figures on the right of the vertical line are omitted, the same *quotient* can be obtained as follows:—

Having obtained the first quotient figure in the usual way, we then, instead of bringing down **8**, strike off the last figure, **4**, in the divisor, and obtain the second figure in the quotient by considering how many times the divisor 17682 is contained in 82915.

Then, when we multiply this divisor by the new figure, 4, in the quotient, we add in mentally anything there would have been to carry from the figure cancelled (*i.e.* 4 times 2 + 1 carried = **9***).

And so on, *cancelling one fresh figure in the divisor at each stage.*

The work then stands thus:—

```
17~~6~~~~8~~~~2~~~~4~~)4365638·75(24·689
         353648
          82915
          70729*
          12186
          10609
           1576
           1414
            162
            158
              4
```

Here the chief difficulty consists in deciding *when to begin the cancelling process.* Now it will have been observed that

(i) The number of quotient figures which can be so obtained is **two less than the number of figures in the divisor.**

Also, before beginning the work, *we can discover by inspection*

(ii) The **local value of the first figure in the quotient.**

And (i) and (ii) will together enable us to decide how many quotient figures (if any) must be obtained by the *ordinary* process.

For instance, $536 \cdot 214575 \div 132 \cdot 654$.

Here it is evident, without shifting the dec. points, that there will be *one integral* figure in the quotient.

If then we require the result to *six* places of decimals we must continue the work until the quotient contains *seven* figures.

But *four* of these only can be obtained by the cancelling process.

Hence the first *three* must be obtained by the ordinary process.

If, however, the result were required to *two* places of decimals only, then the quotient need only contain *three* figures, and so one figure in the divisor might be cancelled before starting, and we should only have to consider the division of 536·21 by 132·65.

N.B.—In order to avoid possible serious error, the student should *invariably*, *before any cancelling is done*, decide *by inspection* what the **local value of the first significant figure in the quotient** will be; *i.e.* whether it will represent *tens*, *units*, *tenths*, or *hundredths*, &c. The quotient may then be *planned out and the decimal point marked* before beginning the actual work.

EXAMPLE xvii.—*Divide* ·0632815 by 146·52065 *to six places.*

·0632815 ÷ 146·52065
= 6328·1500 ÷ 14652065

Now it is evident that the first *significant* figure in the quotient will represent *ten-thousandths*; *i.e.* there will be *three* ciphers after the decimal point, so we have to perform but *three* steps in division in order to obtain the quotient to *six* places of decimals. (The *seventh* figure we obtain by *inspection.*) We therefore at first retain but *five* figures of the divisor.

```
14652)63281(·0004319
      58608
      -----
       4673
       4395
       ----
        278
        146
        ---
        132      Ans. ·000432.
```

EXAMPLE xviii.—*Find within a millionth of the whole the quotient of* 6930·5 ÷ 91·5024.

Here we have to find the first *seven* figures of the quotient, and as *four* can, with this *six* figure divisor, be obtained by the cancelling process, we see that the first *three* must be obtained by the ordinary process.

Now in 6930·5 ÷ 91·5024 it is evident (comparing 6000 with 90) that the first **two** figures in the quotient will be *integers*.

Here the *eighth* figure in the quotient (obtained by *inspecting* the rem^r) is not written, as, being less than 5, it will not affect the req^d result. See Rule on p. 172.

```
915024)69305000(75·74118
       6405168
       -------
        5253320
        4575120
        -------
         6782000
         6405168
         -------
          376832
          366009
          ------
           10823
            9150
           -----
            1673
             915
            ----
             758
             732
             ---
              26
Ans. 75·74118.
```

Note.—We have not in Ex. xviii., as hitherto, adopted the beginners' method of moving the decimal points so as to convert the divisor into a whole number.

This it should be noticed is by no means *necessary*; and the student will often be able to dispense with it, and obtain *by inspection of the numbers as given* the local value of the first significant quotient figure.

For instance, $61\cdot5638 \div \cdot04257$ will evidently yield *four* integer figures in the quotient since 4 *hundredths* is contained some *thousands* of times in 60.

The student who has acquired the abridged or Italian method of division (see page 15), will be able still further to curtail the written work.

Thus, the above example worked by this method, and without moving the decimal points, would stand thus:—

$$\begin{array}{rrl}
91\cdot\cancel{5}\cancel{0}\cancel{2}\cancel{4}\,) & \underline{6930\cdot5000} & (75\cdot74118 \\
 & \underline{525\ 3320} & \\
 & \underline{67\ 82000} & \\
 & \underline{3\ 76832} & \\
 & \underline{10823} & \\
 & \underline{1673} & \\
 & \underline{758} & \\
 & 26 &
\end{array}$$

EXAMPLE xix.—*Find, to three places of decimals,* $1\cdot278\dot{3} \div \cdot\dot{5}1\dot{6}$.

Here it is evident, without moving the decimal points, that the quotient will contain one integer figure (for $1 \div \frac{5}{10} = 2$). Hence we have to obtain altogether *four* figures in the quotient, and these must all be obtained by the cancelling process (for we evidently cannot here obtain any by the *ordinary* process). We must, therefore, retain six figures in the divisor, in order to allow of *four* being successively cancelled.

$$\begin{array}{rrl}
51\cancel{6}\cancel{5}\cancel{1}\cancel{6}\,) & 127833 & (2\cdot4749 \\
 & \underline{103303} & \\
 & 24530 & \\
 & \underline{20660} & \\
 & 3870 & \\
 & \underline{3615} & \\
 & 255 & \\
 & \underline{206} & \\
\underline{\textit{Ans. } 2\cdot475.} & 49 &
\end{array}$$

EXAMPLE xx.—*Find the value of* $£294,\ 17s.\ 3d. \times \dfrac{87}{36500}$ *to the nearest penny.*

Here we require the quotient approximately correct to *three* places, and one figure may be obtained by cancelling.

We need not, therefore, retain more than *three* places of decimals in the working of the multiplication.

$$£294\cdot8625 \times \frac{8\cdot7}{3650}$$

$$= \frac{£29\cdot48625 \times 8\cdot7}{365}$$

$$= £\cdot703$$

$$= \underline{14s.\ 1d.\ \textit{Ans.}}$$

$$\begin{array}{rrl}
 & £ & \\
 & 29\cdot48625 & \\
 & \underline{78} & \\
 & 235\cdot889 & \\
 & \underline{20\ 640} & £ \\
36\cancel{5}\,) & 256\cdot53 & (\cdot7028 \\
 & \underline{255\ 5} & \\
 & 1\ 03 & \\
 & \underline{73} & \\
 & 30 &
\end{array}$$

EXAMPLE xxi.—*Simplify* $\dfrac{428 \cdot 75 \times \cdot 6425}{(3 \cdot 1416)^2}$, *to two places of decimals.*

In cases of this kind, in order to decide how many figures to retain in the working, we make a rough *mental* calculation of the result before beginning the work thus:—

The product in the dividend will evidently be rather less than $400 \times \frac{7}{10}$, *i.e.* rather less than 280; and the divisor will evidently be rather less than 10.

Hence we expect a result nearly equal to 28.

$$\begin{array}{r} 3 \cdot 141600 \\ 61413 \\ \hline 9 \cdot 424800 \\ 314160 \\ 125664 \\ 3141 \\ 1884 \\ \hline 9 \cdot 869649 \end{array} \qquad \begin{array}{r} 42 \cdot 8750 \\ 5246 \\ \hline 257 \cdot 2500 \\ 17\ 1500 \\ 8575 \\ 2143 \\ \hline 275 \cdot 4718 \end{array}$$

$$\begin{array}{r} 986965\,)\,275472\,(\,27 \cdot 91 \\ 197393 \\ \hline 78079 \\ 69087 \\ \hline 8992 \\ 8882 \\ \hline 110 \\ 98 \\ \hline 12 \end{array}$$

Ans. 27·91.

Now, as we require the result to *two* places of decs., we must, with the two integer figures expected, have *four* figures in the quotient. Also, as the complete divisor would, if worked out in full, have 8 decimal places in it, all the quotient figures *must* be obtained by the cancelling process. So we must retain *six* figures in the divisor and not *less* than *six* in the dividend.

We therefore calculate the product $428 \cdot 75 \times \cdot 6425$ to three *places of decimals*, and $(3 \cdot 1416)^2$ to five places.

Note.—In such cases *one* extra place only need be retained in the working, as any small error thus resulting in the dividend will be counterbalanced by a corresponding one in the divisor.

EXAMPLE xxii.—*Find, to four places of decimals, the sum of the unlimited series* $\frac{1}{4} + \frac{1}{4^2} + \frac{1}{4^3} + \frac{1}{4^4} + \cdots\cdots\cdots$

Here each successive fraction is $\frac{1}{4}$ of the preceding one; so, having written the first as a decimal, we find the value of succeeding ones by dividing again and again by 4, retaining *6* places, and continuing the work until we reach a stage when *no significant figure* occurs in the first *six* places. We then add the results obtained.

$$\begin{array}{l} \cdot 25 \\ \cdot 0625 \\ \cdot 015612 \ldots\ldots \\ \cdot 003903 \ldots\ldots \\ \cdot 000975 \ldots\ldots \\ \cdot 000243 \ldots\ldots \\ \cdot 000060 \ldots\ldots \\ \cdot 000015 \ldots\ldots \\ \cdot 000003 \ldots\ldots \\ \cdot 000000 \ldots\ldots \\ \hline \end{array}$$

Ans. ·3333.

LI. PERCENTAGES.

The words "*per cent*"* signify "*for each hundred*".

For instance, 7 per cent of 2000 is 7 for each hundred in 2000, *i.e.* 7×20, or 140. But $\frac{7}{100}$ of 2000 is also 140.

Hence, 7 *per cent* is the same as $\frac{7}{100}$.

And so 7 per cent of $50 = \frac{7}{100}$ of $50 = 3\frac{1}{2}$.

A percentage of a number, or quantity, is a number of *hundredths* of it.

The *number* of hundredths is called the **rate** per cent.

Now any fraction can be expressed with den^r 100; for if its den^r is not a factor of 100, we can put it in the form of a complex fraction having den^r 1, and then multiply both num^r and den^r of this complex fraction by 100.

For instance, $\frac{2}{3} = \frac{\frac{2}{3}}{1} = \frac{\frac{2}{3} \times 100}{1 \times 100} = \frac{66\frac{2}{3}}{100} = 66\frac{2}{3}$ *hundredths.*

Hence any ratio can be expressed as a percentage.

For instance, the ratio of 3*s.* 6*d.* to £1 is expressed by the fraction $\frac{3\frac{1}{2}}{20}$, and $\frac{3\frac{1}{2}}{20} = \frac{3\frac{1}{2} \times 5}{20 \times 5} = \frac{17\frac{1}{2}}{100} = 17\frac{1}{2}$ per cent; *i.e.* 3*s.* 6*d.* is $17\frac{1}{2}$ per cent of £1.

EXAMPLE i.—*Find the percentage equivalent to the fraction* $\frac{11}{24}$.

[Here we express $\frac{11}{24}$ with den^r 100, and then the *numerator* is the req^d number.]

$$\frac{11}{24} = \frac{\frac{11}{24}}{1} = \frac{\frac{11}{24} \times 100}{1 \times 100} = \frac{45\frac{5}{6}}{100} = \underline{45\frac{5}{6} \text{ per cent. } \textit{Ans.}}$$

EXAMPLE ii.—*Find, in lowest terms, the fraction equivalent to* $21\frac{7}{8}$ *per cent.*

$$21\tfrac{7}{8} \text{ per cent} = \frac{21\frac{7}{8}}{100} = \frac{175}{800} = \frac{7}{32} \quad \underline{\textit{Ans.}}$$

EXAMPLE iii.—*Find* $3{\cdot}75$ *per cent of* 3920.

$$3{\cdot}75 \text{ per cent of } 3920 = \frac{3{\cdot}75}{100} \text{ of } 3920 = {\cdot}375 \times 392$$
$$= \underline{147 \ \textit{Ans.}}$$

* Latin, *centum*, a hundred.

EXAMPLE iv.—*Find 3 per cent of £235, 8s. 4d.*

$$3 \text{ per cent of } £235,\ 8s.\ 4d. = \frac{3}{100} \text{ of } £235,\ 8s.\ 4d.$$

$$= \frac{£706,\ 5s.\ 0d.}{100}$$

$$= \underline{£7,\ 1s.\ 3d.\ \ Ans.}$$

£	s.	d.
7,06	5	0
	20	
	1,25	
	12	
	3,00	

EXAMPLE v.—*How much per cent is £3, 10s. of £140?*

1st Method.

[Here we reduce £3, 10s. to the fraction of £140, and then express this fraction with den^r 100, when the numerator is the req^d percentage.]

$$\frac{£3,\ 10s.}{£140} = \frac{3\frac{1}{2}}{140} = \frac{1}{40} = \frac{\frac{1}{40} \times 100}{1 \times 100} = \frac{2\frac{1}{2}}{100} = \underline{2\tfrac{1}{2} \text{ per cent } Ans.}$$

2nd Method.

[Or we may regard the question as one in Proportion, namely—*to find a number which bears to* 100 *the ratio of* £3, 10*s. to* £140.]

$$i.e.\ \frac{\text{the req}^d \text{ no.}}{100} = \frac{3\frac{1}{2}}{140}$$

$$\therefore \text{ the req}^d \text{ no.} = \frac{3\frac{1}{2}}{140} \times 100 = \underline{2\tfrac{1}{2}\ \ Ans.}$$

EXAMPLE vi.—$5\frac{1}{2}$ *per cent of a certain number is* $41\frac{1}{4}$; *find the number.*

$$\frac{5\frac{1}{2}}{100},\ i.e.\ \frac{11}{200}, \text{ of the req}^d \text{ no.} = 41\tfrac{1}{4}$$

$$\therefore \text{ the req}^d \text{ no.} = 41\tfrac{1}{4} \div \frac{11}{200} = \underline{750\ \ Ans.}$$

<u>To find, *mentally*, 5 per cent of any sum of money, neglecting fractions of a penny.</u>

As 5 per cent $= \frac{5}{100} = \frac{1}{20}$, and 1*s.* is $\frac{1}{20}$ of £1, we have but to take 1s. for each £1 in the sum, then 6*d.* for any 10*s.* that remains, and 1*d.* for each *complete* 20*d.* still remaining.

For instance, 5 per cent of £157, 14*s.* 3*d.* = 157*s.* + 6*d.* + 2*d.*
= £7, 17*s.* 8*d.*

Hence, to find $2\frac{1}{2}$ per cent of any sum, first find 5 per cent of it, and then halve the result.

Note.—"Trade Discount" (see page 104) is often calculated as a *percentage* in this way.

To increase, or decrease, a given quantity by a given percentage.

This may, of course, be done by *first* finding the percentage and *then* adding it to, or subtracting it from, the given quantity.

For instance, *to decrease £191, 9s. 2d. by 4 per cent.*

$\frac{4}{100}$ of £191, 9*s.* 2*d.* = $\frac{1}{25}$ of £191, 9*s.* 2*d.*

= £7, 13*s.* 2*d.*

and £191, 9*s.* 2*d.* − £7, 13*s.* 2*d.* = £183, 16*s.*

$$25\left\{\begin{array}{l|rrr} & £ & s. & d. \\ 5\,) & 191 & 9 & 2 \\ 5\,) & 38 & 5 & 10 \\ \hline & 7 & 13 & 2 \end{array}\right.$$

It is, however, often much more convenient to combine the two operations.

For instance, to *increase* (say) £8 by (say) 3 per cent, we must to £8 add $\frac{3}{100}$ of £8.

Now $£8 + \frac{3}{100}$ of $£8 = \left(1 + \frac{3}{100}\right)$ of $£8 = £8 \times \frac{100+3}{100}$. Hence

(I.) *To* increase *by a given percentage*—

$$\textit{Multiply by the factor } \frac{100 + rate}{100}.$$

Again, to *decrease* (say) £13 by (say) 7 per cent, we must from £13 take $\frac{7}{100}$ of £13.

Now $£13 - \frac{7}{100}$ of $£13 = \left(1 - \frac{7}{100}\right)$ of $£13 = £13 \times \frac{100-7}{100}$. Hence

(II.) *To* decrease *by a given percentage*—

$$\textit{Multiply by the factor } \frac{100 - rate}{100}.$$

EXAMPLE vii.—*The population of a town decreased* 4·3 *per cent during a certain period. At the end of the period the population was* 73520; *what was it at the beginning?*

The req^{d} pop. when decreased by 4·3 per cent = 73520

i.e., the req^{d} pop. $\times \frac{100 - 4\cdot3}{100} = 73520$

or, the req^{d} pop. $\times \frac{95\cdot7}{100} = 73520$

∴ the req^{d} pop. $= 73520 \div \frac{95\cdot7}{100} = \underline{76823\ \textit{Ans.}}$

Cent per cent, or 100 per cent, of any number, or quantity, is the *whole*, since 100 per cent is $\frac{100}{100}$, or 1.

Hence, for example, if from any number, or quantity, **37** per cent be taken away, 100 − 37, or **63**, per cent of it is left.

EXAMPLE viii.—*In an examination* 45 *per cent of the total number of candidates were under* 15 *years of age; of these,* 65 *per cent were boys, and* 441 *were girls. Find the total number of candidates.*

$$\frac{45}{100} \text{ of the total no. were under 15,}$$

$$\text{and } \frac{65}{100} \text{ of } \frac{45}{100} \text{ of the total no. were } \textit{boys} \text{ under 15,}$$

$$\therefore \frac{35}{100} \text{ of } \frac{45}{100} \text{ of the total no. were } \textit{girls} \text{ under 15,}$$

$$\textit{i.e. } \frac{35}{100} \text{ of } \frac{45}{100} \text{ of the total no.} = 441,$$

$$\therefore \text{the total no.} = 441 \div \frac{35 \times 45}{100 \times 100} = \underline{2800 \ \textit{Ans.}}$$

EXAMPLE ix.—*What change per cent is made in the area of a rectangle by decreasing its length, and increasing its breadth, each* 5 *per cent?*

The original area = original length × original breadth.

$$\text{The new area} = \frac{95}{100} \text{ of original length} \times \frac{105}{100} \text{ of original breadth}$$

$$= \frac{9975}{10000} \text{ of original area.}$$

$$\therefore \text{the area is } \textit{decreased} \text{ by } \frac{25}{10000} \text{ of its original size}$$

$$= \frac{\cdot 25}{100} \text{ of its original size} = \underline{\cdot 25 \text{ per cent } \textit{Ans.}}$$

EXAMPLE x.—*One year the revenue of a country was* £3480560; *the next year it was* £3513200. *Find, correct to two places of decimals, the increase per cent.*

The increase on £3480560 = £3513200 − £3480560 = £32640.

[We have now, as in Ex. i., to reduce £32640 to the fraction of £3480560, and express this fraction with den^{r} 100; then its num^{r} gives the req^{d} number.

Now $\frac{£32640}{£3480560} = \frac{3264}{348056}$, and this fraction, when multiplied by 100, gives the percentage num^{r}. As, however, we require but 2 places of decimals, instead of appending two ciphers to 3264, we drop two figures from 348056, and then use contracted division.]

$$\frac{£32640}{£3480560} = \frac{3264}{348056} = \frac{\cdot 94}{100} \qquad 34\cancel{8}\cancel{0})\,3264\,(\cdot 94 \text{ nearly.}$$

$$3132$$

$$132$$

$$= \underline{\cdot 94 \text{ per cent } \textit{Ans.}}$$

LII. PROFIT AND LOSS.

If anything is sold for more than it cost it is said to be sold *at a profit*; if for less, *at a loss.* Profit, or loss, is often expressed as **a percentage of the cost price.**

Money employed in any business or other undertaking (by means of which the profit, if any, is gained) is called **capital.**

EXAMPLE i.—*Find the gain per cent of the cost price when an article which cost* 12*s. is sold for* 12*s.* 9*d.*

Here the *actual* profit is 9*d.*, and the capital employed is 12*s.* We have, therefore, to express 9*d.* as a percentage of 12*s.*

$$\text{Now } \frac{9d.}{12s.} = \frac{\frac{3}{4}}{12} = \frac{1}{16} = \frac{\frac{1}{16} \times 100}{100} = \frac{6\frac{1}{4}}{100} = \underline{6\tfrac{1}{4} \text{ per cent } Ans.}$$

EXAMPLE ii.—*Find the loss per cent of the cost price when articles which cost* £10 *per dozen are sold at* 16*s.* 6*d. each.*

Here the cost price of one article is $£\frac{5}{6}$ = 16*s.* 8*d.* Hence on 16*s.* 8*d.* capital employed the actual loss is 2*d.*

$$\text{Now } \frac{2d.}{16s.\ 8d.} = \frac{2}{200} = \frac{1}{100} = \underline{1 \text{ per cent } Ans.}$$

EXAMPLE iii.—*If an article which costs* 5*s. is sold at a loss of* $7\frac{1}{2}$ *per cent, what does it sell for?*

[Here we must *decrease* the cost by $7\frac{1}{2}$ per cent.]

$$\therefore \text{ the article sells for } \overset{s.}{5} \times \frac{100 - 7\frac{1}{2}^{*}}{100} = \overset{s.}{5} \times \frac{92\frac{1}{2}}{100} = \underline{4s.\ 7\tfrac{1}{2}d.\ Ans.}$$

EXAMPLE iv.—*Find the cost price of an article which sold, at a profit of* 8 *per cent, for* £9.

[Here the cost price *increased* by 8 per cent = the selling price.]

$$\begin{aligned} \textit{i.e.} \text{ the cost price} \times \frac{100 + 8}{100}^{*} &= £9 \\ \text{or, the cost price} \times \frac{108}{100} &= £9 \\ \therefore \text{ the cost price} &= £9 \div \frac{108}{100} \\ &= \underline{£8,\ 6s.\ 8d.\ Ans.} \end{aligned}$$

* See (I.) and (II.) on page 187.

EXAMPLE v.—*Find the cost price of an article which sold for 7s. 6d. at a loss of 10 per cent.*

[Here the cost price *decreased* by 10 per cent = the selling price.]

$$i.e.\text{ the cost price} \times \frac{100-10}{100} = 7\tfrac{1}{2}\text{s.}$$

$$\therefore \text{ the cost price} = 7\tfrac{1}{2} \div \frac{90}{100} = \underline{8s.\ 4d.\ \textit{Ans.}}$$

EXAMPLE vi.—*Find the gain per cent in buying eggs at 10d. per dozen and selling them at ten for a shilling.*

One egg *costs* $\frac{5}{6}d.$, and sells for $\frac{6}{5}d.$

$\therefore$ the *actual* gain is $\frac{6}{5} - \frac{5}{6} = \frac{11}{30}d.$, when the capital employed is $\frac{5}{6}d.$

[We have, therefore, to express $\frac{11}{30}d.$ as a percentage of $\frac{5}{6}d.$]

$$\text{Now } \frac{\frac{11}{30}}{\frac{5}{6}} = \frac{11}{30} \times \frac{6}{5} = \frac{11}{25} = \frac{11 \times 4}{25 \times 4} = \frac{44}{100} = \underline{44 \text{ per cent } \textit{Ans.}}$$

EXAMPLE vii.—*If by selling goods for £14, 6s. 10 per cent of their cost is gained, at what price should they have been sold so as to gain 20 per cent?*

[Here the cost price *increased* by 10 per cent = the selling price.]

$$i.e.\text{ the cost price} \times \frac{110}{100} = £14\tfrac{3}{10},$$

$$\therefore \text{ the cost price} = £14\tfrac{3}{10} \times \frac{100}{110},$$

and this must be *increased* by 20 per cent.

$$\therefore \text{ the req}^{\text{d}} \text{ selling price} = £14\tfrac{3}{10} \times \frac{100}{110} \times \frac{120}{100} = \underline{£15,\ 12s.\ \textit{Ans.}}$$

EXAMPLE viii.—*If, by selling a horse for 19 guineas, 5 per cent of its cost is lost, what would have been gained or lost per cent by selling it for 21 guineas?*

$$\text{The cost } \textit{decreased} \text{ by 5 per cent} = £19 \times \frac{21}{20}$$

$$i.e.\text{ the cost} \times \frac{95}{100} = £19 \times \frac{21}{20}$$

$$\therefore \text{ the cost} = £19 \times \frac{21}{20} \times \frac{100}{95} = £21.$$

Now in selling for 21 guineas what *cost* £21, the *actual gain* is 21s., and this we have to express as a percentage of the *cost*.

$$\text{Hence } \frac{21s.}{£21} = \frac{1}{20} = \frac{5}{100} = \underline{5 \text{ per cent gain } \textit{Ans.}}$$

EXAMPLE ix.—*If 24 lbs. of tea costing 1s. 9d. per lb. be mixed with 30 lbs. costing 2s. 4d. per lb., at what price per lb. must the mixture be sold so as to gain $12\frac{1}{2}$ per cent?*

The cost price of the 54 lbs. of mixture is $\overset{s.}{1\frac{3}{4}} \times 24 + \overset{s.}{2\frac{1}{3}} \times 30 = 112s.$

$\therefore$ .... 1 lb. .... $= \frac{112}{54}s. = \frac{56}{27}s.$

Hence the req^d selling price $= \frac{56}{27}s.$ *increased* by $12\frac{1}{2}$ per cent.

$$= \frac{56}{27}s. \times \frac{112\frac{1}{2}}{100} = \frac{56}{27}s. \times \frac{225}{200} = \underline{2s.\ 4d.\ Ans.}$$

EXAMPLE x.—*If a merchant buys 360 gallons of spirit for £540, and makes a profit of 12 per cent by retailing it at 28s. per gallon, how much water does he add to it?*

The cost of 1 gal. of *mixture increased* by 12 per cent = 28s.

i.e. the cost of 1 gal. of *mixture* $\times \frac{112}{100} = 28s.$

$\therefore$ the cost of 1 gal. of *mixture* $= \overset{s.}{28} \times \frac{100}{112} = 25s.$

But the cost of 1 gal. of *pure* spirit $= \frac{540 \times 20}{360}s. = 30s.$

Hence $\frac{25}{30}$, or $\frac{5}{6}$, of each gal. of mixture is pure spirit, and $\frac{1}{6}$, consequently, is water; *i.e.* the ratio of spirit to water = 5 : 1.

$\therefore$ to 360 gals. of spirit he adds $\frac{1}{5}$ of 360 gals. of water = 72 gals. *Ans.*

EXAMPLE xi.—*A makes an article and sells it to B at a profit of 20 per cent of what it cost him to make it. B sells it to C at a loss of 12 per cent of what B gave for it. C sells it for £63, 16s., making a profit of 16 per cent on what it cost him. What did it cost A to make?*

The cost to C *increased* by 16 per cent $= £63\frac{4}{5}$;

i.e. the cost to C $\times \frac{116}{100} = £63\frac{4}{5}$, $\therefore$ the cost to C $= £63\frac{4}{5} \times \frac{100}{116}$.

The cost to B *decreased* by 12 p. c. = what he sold it to C for;

i.e. cost to B $\times \frac{88}{100} = £63\frac{4}{5} \times \frac{100}{116}$, $\therefore$ cost to B $= £63\frac{4}{5} \times \frac{100}{116} \times \frac{100}{88}$.

The cost to A *increased* by 20 p. c. = what he sold it to B for;

i.e. the cost to A $\times \frac{120}{100} = £63\frac{4}{5} \times \frac{100}{116} \times \frac{100}{88}$,

$\therefore$ the cost to A $= £63\frac{4}{5} \times \frac{100}{116} \times \frac{100}{88} \times \frac{100}{120} = \underline{£52,\ 1s.\ 8d.\ Ans.}$

LIII. PERCENTAGES.

COMMISSION, BROKERAGE, &c.

An *agent* is a person who is employed by another to buy or sell, or to collect rents, &c., for him. The agent usually receives for his trouble a *percentage* of the price of the goods he buys or sells, or of the amount of rents, &c., he collects. This percentage is called his **commission**. Architects, auctioneers, &c., are often paid for their services in the same way. Some agents are called *brokers*, and the percentage they receive is called **brokerage**.

If a person insures his goods against fire, he pays annually to the insurance company a certain *percentage* of their supposed value; this percentage is called a **premium**.

These, being *percentages*, are all calculated in exactly the same way.

For instance, 3 per cent *commission* on £750 is $\frac{3}{100}$ of £750;

and $\frac{1}{2}$ per cent *brokerage* on £3000 is $\frac{1}{200}$ of £3000.

Hence the rule—*Multiply the sum* (on which the commission is to be calculated) *by the* **rate** *per cent, and divide the result by* **100.**

Note.—In this connection such an expression as "2*s.* 6*d.* per cent" is sometimes used, and must be understood to mean 2*s.* 6*d.* per £100.
Thus, 2*s.* 6*d.* per cent = £$\frac{1}{8}$ per £100 = $\frac{1}{8}$ per cent.

EXAMPLE i.—*Find, to a penny, the commission on* £853, 16*s.* 9*d.* *at* 3 *per cent.*

$$\begin{array}{r} \text{£} \quad s. \quad d. \\ 853 \,.\, 16 \,.\, 9 \\ 3 \\ \hline \text{£}25{,}61 \,.\, 10 \,.\, 3 \\ 20 \\ \hline s.\ 12{,}30 \\ 12 \\ \hline d.\ 3{,}63 \end{array}$$

Ans. £25, 12*s.* 3*d.*

EXAMPLE ii.—*Find, to the nearest penny, the brokerage on* £4566, 10*s.* *at* 2*s.* 6*d.* *per cent.*

Brokerage = $\frac{1}{800}$ of £4566, 10*s.*

$$\begin{array}{r} \text{£} \quad s. \quad d. \\ 8\,\underline{|4566 \,.\, 10 \,.\, 0} \\ 5{,}70 \,.\, 16 \,.\, 3 \\ 20 \\ \hline s.\ 14{,}16 \\ 12 \\ \hline d.\ 1{,}95 \end{array}$$

Ans. £5, 14*s.* 2*d.*

EXAMPLE iii.—*After the deduction of commission at* $3\frac{1}{2}$ *per cent a landlord's net rental was* £2123; *find his agent's commission.*

The gross rental *decreased* by $3\frac{1}{2}$ per cent = £2123;

i.e. gross rent[1] $\times \frac{96\frac{1}{2}}{100}$* = £2123, $\therefore$ gross rent[1] = £2123 $\times \frac{100}{96\frac{1}{2}}$ = £2200.

Hence agent's commission = $3\frac{1}{2}$ per cent of £2200 = £77 *Ans.*

* See (II.) on page 187.

LIV. SIMPLE INTEREST.

Interest is the payment made for the use of money.

The money, for the use of which the interest is paid, is called the **Principal**.

The interest is always *a percentage* of the principal, and this percentage is generally charged for *each year* the principal is used.

For instance, if money be borrowed at 5 "per cent per annum", then for *each* £100 borrowed, £5 is due yearly as interest.

If then, £800 be borrowed, £40 is due yearly as interest at 5 per cent.

The percentage *number* is called the **Rate** per cent.

For instance, if 4 be the rate per cent per annum, then a year's interest on £100 is £4; on £50 is £2; on £700 is £28; and so on.

Note.—The words "per annum" are often omitted, but, unless the contrary is stated, must always be understood.

The symbol %. or the abbreviation p. c., is used for "*per cent*".

The *sum* of the principal and interest is called the **Amount.**

For instance, at 4 per cent, £104 is the *amount* of £100 in a year.

When the interest is paid year by year as it becomes due, or when, if unpaid, *no interest is charged on overdue interest*, the principal is said to be lent at **Simple** interest.

Thus, in the case of simple interest, the principal remains unchanged year by year, and consequently the interest for 2, or 3, &c. years is simply *double*, or *treble*, &c. of the interest for *one* year.

If, however, interest is charged on overdue interest as well as on the principal, the principal is said to be lent at *compound* interest. (See Chap. LV.)

As interest is *a percentage*, the process of finding the interest on any sum at any rate for **one** year exactly corresponds to that of finding any other percentage—such as Commission, for instance, *i.e. the principal multiplied by the rate per cent and divided by* 100 gives the interest for **one** year.

And the *simple* interest for 2, 3, &c. years is, as we have already seen, merely *double*, *treble*, &c., respectively, of the interest for *one* year. Similarly the interest for $\frac{1}{2}$, $\frac{2}{3}$, &c. of a year is *half*, *two-thirds*, &c., respectively, of the interest for *one* year.

Hence the following rule:—

To find the Simple Interest on a given sum of money at a given rate per cent for a given time.

Multiply *the principal by the rate per cent, and by the number* (whole, or fractional) *of years, and* **divide** *the result by* **100.**

The examples which follow illustrate various ways of performing the multiplication and division involved in the application of the foregoing Rule.

Note.—Before proceeding to apply the "rule" it should be noticed that, when once the definitions are known, the interest in any given case may be found independently by the "Unitary" method.

For instance, *to find the simple interest on* £650, *at* 3 *per cent per annum, for* 2 *years.*

$$\begin{array}{lllll} & \pounds & & \pounds & \\ \text{On} & 100 & \text{the interest is} & 3 & \text{for a year.} \\ \therefore \ldots & 1 & \ldots\ldots\ldots\ldots & \dfrac{3}{100} & \ldots\ldots\ldots \\ \therefore \ldots & 650 & \ldots\ldots\ldots\ldots & \dfrac{3}{100} \times 650 & \ldots\ldots\ldots \end{array}$$

Hence the interest for 2 years is $\pounds\dfrac{3 \times 650}{100} \times 2$; *i.e.* $\pounds\dfrac{650 \times 3 \times 2}{100}$ ......(i).

It would be well for the beginner to work a few exercises at full length thus, before using the rule enabling him to write down the required interest at once in the form (i).

EXAMPLE i.—*Find the simple interest on* £1757, 18*s.* 4*d.* *for* 2 *years at* 3 *per cent per annum.*

The req^d int. $= \dfrac{(\pounds 1757,\ 18s.\ 4d.) \times 3 \times 2}{100}$.

$= \underline{\pounds 105,\ 9s.\ 6d.\ \textit{Ans.}}$

$$\begin{array}{r} \pounds \quad\ \ s. \quad d. \\ 1757\ .\ 18\ .\ 4 \\ 6 \\ \hline \pounds 105{,}47\ .\ 10\ .\ 0 \\ 20 \\ \hline s.\ 9{,}50 \\ 12 \\ \hline d.\ 6{,}00 \end{array}$$

[In such a case as this the work is easily performed by *compound* mult^n and div^n.]

EXAMPLE ii.—*Find the simple interest, for* $2\frac{1}{2}$ *years at* $3\frac{3}{4}$ *per cent, on* £370.

$$\text{The req}^{\text{d}} \text{ int.} = \frac{\pounds 370 \times 3\frac{3}{4} \times 2\frac{1}{2}}{100}$$

$$= \pounds 37\cancel{0} \times \frac{15}{4} \times \frac{\overset{1}{\cancel{5}}}{2} \times \frac{1}{\underset{2\cancel{0}}{\cancel{10}\cancel{0}}}$$

$$= \pounds\frac{555}{16} = \pounds 34\tfrac{11}{16}$$

$$= \underline{\pounds 34,\ 13s.\ 9d.\ \textit{Ans.}}$$

[Here it is convenient to perform the mult^n and div^n in *vulgar fractions.*]

When the interest is required for a number of *months* they must be considered as *twelfths of a year*.

EXAMPLE iii.—*Find the amount of* £514 *in* 7 *months at* $2\frac{1}{2}$ *per cent per annum.*

$$\text{The int.} = \frac{£514 \times 2\frac{1}{2} \times \frac{7}{12}}{100}.$$

$$= £\overset{257}{\cancel{514}} \times \frac{\cancel{5}}{\underset{1}{\cancel{2}}} \times \frac{7}{12} \times \frac{1}{\underset{20}{\cancel{100}}}$$

$$= £\frac{1799}{240} = 1799d. \qquad \begin{array}{r} 12\,)\,1799 \\ 2,0\,)\,14,9 + 11d. \\ £7 + 9s. \end{array}$$

$$= £7,\ 9s.\ 11d.$$

Hence the req^d amount = £514 + £7, 9*s.* 11*d.* = £521, 9*s.* 11*d.* *Ans.*

When the interest is required for a number of *days* they must be considered as 365ths of a year.*

If the day of the month of the beginning, and that of the end, of the period be given, it must be remembered that, in adding the days in the calendar months and parts of months involved, *the first of the days mentioned must* **not** *be included.*

For instance, from June 3 to June 4 is but *one* day.
Hence, from April 6 to July 13 = 24 + 31 + 30 + 13 = 98, days.

Note.—It is well to remember that 73 is a factor of 365, and hence that the fractions $\frac{73}{365}$, $\frac{146}{365}$, $\frac{219}{365}$, $\frac{292}{365}$ reduce to $\frac{1}{5}$, $\frac{2}{5}$, $\frac{3}{5}$, $\frac{4}{5}$ of a year respectively.

EXAMPLE iv.—*Find the amount of* £207, 1*s.* 8*d. in* 219 *days at* 5 *per cent.*

$$\text{Int.} = (£207,\ 1s.\ 8d.) \times \frac{\overset{1}{\cancel{5}}}{100} \times \frac{\overset{3}{\cancel{219}}}{\underset{5}{\cancel{365}}}$$

$$= (£207,\ 1s.\ 8d.) \times \frac{3}{100}$$

$$= £6,\ 4s.\ 3d.$$

$$\begin{array}{rrr} £ & s. & d. \\ 207\ . & 1\ . & 8 \\ & & 3 \\ \hline £6,21\ . & 5\ . & 0 \\ 20 & & \\ \hline s.\ 4,25 & & \\ 12 & & \\ \hline d.\ 3,00 & & \end{array}$$

Hence, req^d amt. = £207, 1*s.* 8*d.* + £6, 4*s.* 3*d.* = £213, 5*s.* 11*d.* *Ans.*

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* Even if the year be a *leap* year, and the extra day be counted in the num^r, it is customary to retain 365 in the den^r.

Bankers, &c., have no dealings in any coin less than a penny; hence, in calculating interest, fractions of a penny in a result are either neglected altogether, or the result is given to the *nearest* penny.

Note.—Interest at 5 per cent, or $2\frac{1}{2}$ per cent, for **one** year may be found **mentally** in the same way as was shown for these percentages on p. 186.

(Notice, however, that a result so obtained, if multiplied by 2, 3, &c., cannot be relied upon to a penny as the interest for 2, 3, &c. years.)

As int. for *a year* at 5 p. c. is found by taking 1*s*. for each £1 of principal, and as a month is $\frac{1}{12}$ of a year, it follows that to find *mentally*

Int. at 5 p. c. for **one month**:—Take 1*d*. for each £1 of principal.

For instance, the int. on £413, 15*s*. 9*d*. for *one month* at 5 per cent is 413*d*. = £1, 14*s*. 7*d*.; or, to the *nearest* penny, £1, 14*s*. 8*d*.

EXAMPLE V.—*Find, neglecting fractions of a penny, the interest on* £521 *from Aug.* 17 *to Dec.* 31 *at* 2 *per cent.*

From Aug. 17 to Dec. 31 = 14 + 30 + 31 + 30 + 31 = 136 days.

$$\text{Hence req}^{\text{d}}\text{ int.} = £521 \times \frac{2}{100} \times \frac{136}{365}$$
$$= £\frac{521 \times 2 \times 136 \times 2}{100 \times 365 \times 2}{}^{*}$$
$$= £\frac{283424}{73000}$$
$$= £3\cdot882$$
$$= \underline{£3,\ 17s.\ 7d.\ \ Ans.}$$

521
544
2084
2084
2605
73) 283·424 (3·882 £
219
644
584
602
584
184

[* Here, instead of cancelling, we multiply both num^{r} and den^{r} by 2, thus obtaining the divisor 73000 instead of a less convenient one. We then perform the division in decimals as far as 3 places.]

EXAMPLE VI.—*Find, within a penny, the interest on* £1874, 17*s*. 2*d*. *from Jan.* 23 *to June* 30, 1896, *at* $4\frac{1}{2}$%.

8 + 29 + 31 + 30 + 31 + 30 = 159, days.

$$\therefore \text{req}^{\text{d}}\text{ int.} = £1874\cdot86 \times \frac{9}{2} \times \frac{159}{365} \times \frac{1}{100}$$
$$= \frac{£1874\cdot86 \times 1\cdot431}{73}$$
$$= \underline{£36,\ 15s.\ \ Ans.}$$

1874·86
13 41
1874·86
749 94
56 24
1 87
$7\cancel{3}$) 2682·92 (36·752 £
492
549
382
17

[Here we have applied Contracted methods in decimals (see Chap. L.), and abridged division (see page 15).]

N.B.—The division by 73000, generally involved in calculating interest for a number of days, may be avoided altogether by using what is called the "*Third, tenth* and *tenth*" rule, as follows:—

(i) For the num^r of the interest-fraction multiply the decimalized principal by the rate p. c. and *double* the number of days (thus making the den^r 73000).

(ii) To the num^r thus obtained add $\frac{1}{3}$ of itself; $\frac{1}{10}$ of this third, and again $\frac{1}{10}$ of this latter.

(iii) Divide the sum by 100,000, *i.e. move the dec. point* 5 *places to the left.*

The result, decreased by a farthing for each £10 in it, is the req^d interest.

For instance, *to find the interest on* £817, 13*s. for* 67 *days at* $2\frac{1}{2}$ *per cent.*

$$\text{The int.} = £\frac{817{\cdot}6 \times 2\frac{1}{2} \times 67 \times 2}{100 \times 365 \times 2}$$

$$= £\frac{273896{\cdot}}{73000} \text{ .... (i)}$$

$$= £3,\ 15s.$$

$$\begin{array}{r|r}
 3 & 273896{\cdot} \\
10 & 91298{\cdot} \\
10 & 9129{\cdot} \\
 & 913{\cdot} \\
\hline
 & £3{\cdot}752\cancel{36}
\end{array}
\begin{array}{l} \left.\begin{array}{l} \\ \\ \\ \\ \end{array}\right\} \text{...... (ii)*} \\ \text{......... (iii)} \end{array}$$

[* It is unnecessary to retain any decimals during this part of the work.]

The reason of the rule is due to the fact that

$$73000 + \tfrac{1}{3} \text{ of } 73000 + \tfrac{1}{10} \text{ of } \tfrac{1}{3} \text{ of } 73000 + \tfrac{1}{10} \text{ of } \tfrac{1}{10} \text{ of } \tfrac{1}{3} \text{ of } 73000$$
$$= 73000 \times (1 + \tfrac{1}{3} + \tfrac{1}{30} + \tfrac{1}{300}) = 73000 \times \tfrac{137}{100}$$
$$= 100010, \text{ which is very nearly } 100000.$$

Now if we, in this way, mult. both num^r and den^r of the interest-fraction by $\frac{137}{100}$ we do not alter its value, while we change the den^r from 73000 to 100010. Then, if we divide this new num^r by 100000 instead of by 100010, our divisor being too small by about $\frac{1}{10000}$ of itself, our quotient will be too great by about $\frac{1}{10000}$ of itself, *i.e.* by about $\frac{1}{4}d.$ in each £10.

We have seen, in a variety of examples, how when principal, rate, and time are given, the interest (or amount) can be found. We shall now consider the three *converse* cases.

CASE I.—To find the **Rate** when principal, time, and interest (or amount) are given.

EXAMPLE vii.—*At what rate per cent would* £47, 5*s. be the simple interest of* £350 *for* 3 *years?*

[We first find the interest at **one** per cent.]

$$\text{Int. at } \mathbf{1} \text{ per cent} = \frac{£350 \times 1 \times 3}{100} = £\frac{21}{2}.$$

[Now as the rate is a *factor* of the interest, it is evident that the interest at 2, 3, &c. per cent is *double, treble*, &c. the int. at *one* per cent. We therefore obtain the req^d rate by dividing *the given interest by the interest at* **one** *per cent.*]

$$\text{Hence req}^d \text{ rate per cent} = \frac{£47\frac{1}{4}}{£\frac{21}{2}} = \frac{189}{4} \times \frac{2}{21} = 4\tfrac{1}{2}\ \textit{Ans.}$$

EXAMPLE viii.—*At what rate per cent would £520 amount to £551, 4s. in 2 years at simple interest?*

[By subtracting the given principal from the given amount we obtain the *interest* at the req^d rate, and thus reduce this question to the form of the preceding one.]

The *int.* at req^d rate $= £551\frac{1}{5} - £520 = £31\frac{1}{5}$.

But int. at **1** per cent $= \frac{£520 \times 1 \times 2}{100} = £\frac{52}{5}$.

Hence req^d rate $= \frac{£31\frac{1}{5}}{£\frac{52}{5}} = \frac{156}{52} = 3$ *Ans.*

Note.—The beginner must be careful not to mistake the *amount* for the *principal* when he finds the interest at 1 per cent.

CASE II.—To find the **Time** when principal, rate, and interest (or amount) are given.

The method of procedure in this case is exactly similar to that of Case I.

EXAMPLE ix.—*In what time would the simple interest on £92, 10s. be £12, 19s. at 4 per cent?*

The int. for req^d no. of years $= £12\frac{19}{20}$.

But int. for one year $= \frac{£92\frac{1}{2} \times 4 \times 1}{100} = £\frac{37}{10}$.

Hence the req^d no. of years $= \frac{£12\frac{19}{20}}{£\frac{37}{10}} = \frac{259}{37 \times 2} = \frac{7}{2} = 3\frac{1}{2}$ *Ans.*

EXAMPLE x.—*In what time would £650 amount to £661, 7s. 6d. at 3 per cent?*

The *int.* for the req^d time $= £661\frac{3}{8} - £650 = £11\frac{3}{8}$;

And int. for *one* year $= \frac{£650 \times 3 \times 1}{100} = £\frac{39}{2}$.

Hence req^d no. of years $= \frac{£11\frac{3}{8}}{£\frac{39}{2}} = \frac{91}{8} \times \frac{2}{39} = \frac{7}{12}$.

Ans. 7 months.

Note.—All the preceding examples might have been regarded as questions in Double Rule of Three, and have been treated by the "ratio" method of page 136.

For instance, *Find the simple interest on £370 for $2\frac{1}{2}$ yrs. at $3\frac{3}{4}$ per cent per annum.*

This is equivalent to the following question:—

"*If £$3\frac{3}{4}$ be the interest on £100 for* 1 *year*, what is the interest on £370 for $2\frac{1}{2}$ years?"

$$\text{Hence req}^{\text{d}}\text{ int.} = £3\tfrac{3}{4} \times \frac{370}{100} \times \frac{2\frac{1}{2}}{1} = £34,\ 13s.\ 9d.$$

Again, *At what rate per cent per annum would £47, 5s. be the simple interest on £350 for 3 years?*

This is equivalent to the following question:—

"If £$47\frac{1}{4}$ be the interest on £350 for 3 years, *what is the interest on* £100 *for* 1 *year?*"

$$\text{Hence req}^{\text{d}}\text{ rate} = 47\tfrac{1}{4} \times \frac{100}{350} \times \frac{1}{3} = 4\tfrac{1}{2}.$$

CASE III.—To find the Principal when rate, time, and interest (or amount) are given.

EXAMPLE xi.—*On what principal is £17 the simple interest at 3 per cent for 4 years?*

[We first find the *interest* on £100.]

Int. on £100 at 3% for 4 yrs. = £12.

$$\begin{array}{lll} & £ & & £ \\ \text{Hence, as} & 12 & \text{is the interest on} & 100 \\ \therefore & 1 & \dots\dots\dots\dots & \frac{100}{12} \\ \text{and} & 17 & \dots\dots\dots\dots & \frac{100 \times 17}{12} = \underline{£141,\ 13s.\ 4d.\ \textit{Ans.}} \end{array}$$

Note.—We have inserted here the full reasoning of the "Unitary" method. The student who has acquired the "ratio" method would omit the 2nd line of reasoning.

EXAMPLE xii.—*What sum would amount to £466, 11s. at $2\frac{1}{2}$ per cent, simple interest, in 3 years?*

[We first find the *amount* of £100.]

Interest on £100 at $2\frac{1}{2}$% for 3 yrs. = £3 × $2\frac{1}{2}$ = £$7\frac{1}{2}$.

∴ *amount* of £100 .... = £$107\frac{1}{2}$.

$$\begin{array}{llll} & £ & & £ \\ \text{Hence, as} & 107\frac{1}{2} & \text{is the } \textit{amount} \text{ of} & 100 \\ \therefore & 466\frac{11}{20} & \dots\dots\dots\dots & 100 \times \dfrac{466\frac{11}{20}}{107\frac{1}{2}} = \underline{£434\ \textit{Ans.}} \end{array}$$

The following are examples of rather more difficult questions:—

EXAMPLE xiii.—*A man lent £1600 at a certain rate per cent, and £850 at $\frac{1}{2}$ per cent less. His annual income from both sources was £118, 5s. Find the rates.*

If he had lent both sums at the higher rate his income would have been £$118\frac{1}{4}$ + the extra int. at $\frac{1}{2}$ per cent on £850

$$= £118\tfrac{1}{4} + £850 \times \tfrac{1}{2} \times \tfrac{1}{100} = £118\tfrac{1}{4} + £4\tfrac{1}{4} = £122\tfrac{1}{2}.$$

But income at one per cent on both sums $= £\dfrac{2450 \times 1}{100} = £\dfrac{49}{2}$.

Hence the higher of the req^d rates $= 122\frac{1}{2} \div \frac{49}{2} = 5$
and ∴ the lower .... $= 4\frac{1}{2}$ } *Ans.*

EXAMPLE xiv.—*Divide £2400 into two parts such that the simple interest on one part, at 3 per cent for $2\frac{1}{3}$ years, shall be equal to that on the other at $4\frac{1}{2}$ per cent for 2 years.*

$$\text{The 1st part} \times \frac{3 \times 2\frac{1}{3}}{100} = \text{the 2nd part} \times \frac{4\frac{1}{2} \times 2}{100}.$$

$$\therefore \frac{\text{the 1st part}}{\text{the 2nd part}} = \frac{4\frac{1}{2} \times 2}{3 \times 2\frac{1}{3}} = \frac{9}{7}.$$

[We must, therefore, divide £2400 into parts proportional to 9 and 7.]

Hence the req^d parts are £$2400 \times \frac{9}{16}$ = £1350
and £$2400 \times \frac{7}{16}$ = £1050 } *Ans.*

EXAMPLE xv.—*A man opens a banking account on Jan. 31, 1895, by depositing £120; on Feb. 14 he pays in £28; on Mar. 23 he draws £41, 10s., and on May 7 he draws £32. Find the interest at 2 per cent due to him from the bank on June 30.*

There is due to him int. from

Jan. 31 to Feb. 14 (*i.e.* for 14 days) on £120.........	3360	(i)
Feb. 14 ... Mar. 23 (*i.e.* for 37 days) ... £148.........	10952	(i)
Mar. 23 ... May 7 (*i.e.* for 45 days) ... £106·5......	9585	(i)
May 7 ... June 30 (*i.e.* for 54 days) ... £74·5......	8046	(i)
	31943	......(ii)
	2	....(iii)
	63886	....(iii)
	21295·	...(iv)
	2129·	...(iv)
	213·	...(iv)
	·87523	...(iv)

[We multiply each of the four sums by *double* the no. of days for which int. is due on it (i).
Add these products (ii).
Multiply the sum by the rate p.c. (iii).
And then apply the "third, tenth and tenth" rule (iv).]

Ans. 17*s.* 6*d.*

LV. COMPOUND INTEREST.

When the *interest* as it becomes due *is added to the principal*, the money is said to be put out at **Compound Interest.**

For instance, if £200 be put out at *compound* interest at 5 per cent per annum for 3 years, at the end of the 1st year there is due
the interest on £**200** at 5 p. c. for a year, *i.e.* £**10.**

At the end of the 2nd year there is due
the interest on £**210** at 5 p. c. for a year, *i.e.* £**10**, **10***s.*

At the end of the 3rd year there is due
the interest on £**220½** at 5 p. c. for a year, *i.e.* £**11**, **0***s.* **6***d.*

Thus the *compound* interest for the 3 years is £**31**, **10***s.* **6***d.*
whereas the *simple* interest for the 3 years is but £10 × 3 = £30.

Hence in the case of Compound Interest, the interest for any one year (or other period) is the interest on the **amount** at the end of the previous year (or period).

Thus the original principal + a year's interest on it
= the 2nd principal, *i.e.* the *amount* at the end of *one* year.
The 2nd principal + a year's interest on it
= the 3rd principal, *i.e.* the *amount* at the end of *two* years. And so on.

And the difference between the final *amount* and the original principal is the compound interest for the whole time.

Note.—Unless it is otherwise expressly stated, the interest is always understood to be due *annually*. Also, fractions of a penny in final results may be neglected, unless the result is required to the *nearest* penny.

EXAMPLE i.—*Find the amount at compound interest of* £843, 15*s.* *in* 3 *years at* 4 *per cent per annum.*

1st year.	*2nd year.*	*3rd year.*
£ *s.* *d.*	£ *s.* *d.*	£ *s.* *d.*
843 . 15 . 0	877 . 10 . 0	912 . 12 . 0
4	4	4
33,75 . 0 . 0	35,10 . 0 . 0	36,50 . 8 . 0
20	20	20
15,00	2,00	10,08
		12
		,96

£ *s.* *d.*	£ *s.* *d.*	£ *s.* *d.*
843 . 15 . 0 1st P.	877 . 10 . 0 2nd P.	912 . 12 . 0 3rd P.
33 . 15 . 0 1st I.	35 . 2 . 0 2nd I.	36 . 10 . 0 3rd I.
877 . 10 . 0 2nd P.	912 . 12 . 0 3rd P.	949 . 2 . 0 4th P.

Ans. £949, 2*s.*

Here we have used comp. mult^{n} and div^{n} in the working in order to show how one ignorant of decimals might obtain the result.

But, as we shall now show, there is great gain in conciseness when decimals are properly used.

In the following examples, worked in decimals, we *multiply by the rate per cent and divide by* 100 *simultaneously*; and we do *not write down the multiplier.*

For instance, in multiplying 123·64 by 3 and dividing the result by 100, instead of setting down the multiplier 3 and, after the multiplication is done, moving the decimal point *two* places to the *left*, thus:—

123·64
3
3·7092

we multiply by an *unwritten* 3, setting *each figure of the result* as soon as it is obtained *two* places to the *right*, and mark the decimal point *vertically below* that in the multiplicand, thus:—

123·64
3·7092

In this way the interest for any year is obtained in a position suitable for adding it to the principal.

We need not retain more than 5 places of decimals in the working.

Example ii.—*Find the amount at compound interest of* £520 *in* 4 *years at* 3 *per cent.*

£
520· ....1st Prin.
15·60...........1st yr.'s Int.
535·6.............2nd Prin.
16·068 ....2nd yr.'s Int.
551·668 ....3rd Prin.
16·55004 ......3rd yr.'s Int.
568·21804 ......4th Prin.
17·04654~~12~~...4th yr.'s Int.
585·26458 ......5th Prin. = amt in 4 yrs.
20
5,29160
12
3,49920

Ans. £585, 5*s.* 3*d.*

Note.—The final multiplication by 20 and 12 will be omitted in future, as the shillings and pence are easily obtained at sight. (See p. 175.)

Example iii.—*Find the compound interest for* 3 *years at* 4 *per cent on* £724, 6*s.* 8*d.*

Here, as we require but 5 places of decs. in the working at each stage, we *begin to multiply at the figure in thick type*, not forgetting to *carry* from its right-hand neighbour. Having obtained the *amount* for 3 years we subtract the 1st principal, thus obtaining the *interest* for 3 years. (See p. 201.)

£
724·33**3**33...1st Prin.
28·97333...1st yr.'s Int.
753·30**6**66...2nd Prin.
30·13226...2nd yr.'s Int.
783·43**8**92...3rd Prin.
31·33755...3rd yr.'s Int.
814·77647...4th Prin. = amt in 3 yrs.
724·33333...1st Prin.
90·44314...Int. for 3 yrs.

Ans. £90, 8*s.* 10*d.*

The method of Aliquot Parts may often be used with advantage, especially when the rate per cent is a fractional number.

For instance, for the rate per cent 5.

As $\frac{5}{100} = \frac{1}{20}$, instead of multiplying the principal by 5 and setting the figures *two* places to the right, we may divide the principal by 20, *i.e.* divide by 2 and set all the figures one place to the right.

Again, for the rate per cent $2\frac{1}{2}$.

As $\frac{2\frac{1}{2}}{100} = \frac{5}{200} = \frac{1}{40}$, we divide the principal by 4 and set the figure one place to the right.

Also, for the rate per cent $4\frac{3}{4}$.

As $\frac{4\frac{3}{4}}{100} = \frac{4}{100} + \frac{3}{400} = \frac{4}{100} + \frac{2+1}{400} = \frac{4}{100} + \frac{1}{200} + \frac{1}{400}$, we first *multiply* the principal by 4, setting the figures *two* places to the right (i); then *divide* the principal by 2, setting the figures *two* places to the right (ii); and then divide the latter result simply by 2 (iii).

(i), (ii), and (iii) together make up the year's interest at $4\frac{3}{4}$ per cent.

And so on.

EXAMPLE iv.—*Find, to the nearest penny, the amount at compound interest of £1273, 13s. 9d. in 2 years at $2\frac{1}{2}$ per cent.*

		£	
*£1273, 13s. =		1273·65	
6d. =		·025	
3d. =		·0125	
	$\frac{1}{40}$	1273·6875	...1st Prin.
		31·84218	...1st yr.'s Int.
	$\frac{1}{40}$	1305·52968	...2nd Prin.
		32·63824	...2nd yr.'s Int.
Ans. £1338, 3s. 4d.		1338·16792	...3rd Prin. = amt in 2 yrs.

EXAMPLE v.—*Find the compound interest on £261, 10s. for 2 years at $2\frac{3}{4}$ per cent.*

As $\frac{2\frac{3}{4}}{100} = \frac{2}{100} + \frac{1}{200} + \frac{1}{400}$, we first multiply by an unwritten 2 and set the figures *two* places to the right (i), then divide the principal by 200 (ii), and then divide the last result by 2 (iii). (i), (ii), and (iii) together make up the 1st year's interest.

	£	
$\frac{1}{200}$	261·5	.............1st Prin.
	5·230 ...(i)	...1st yr.'s Int.
$\frac{1}{2}$	1·3075 ...(ii)	
	·65375...(iii)	
$\frac{1}{200}$	268·69125	............2nd Prin.
	5·37382	...........2nd yr.'s Int.
$\frac{1}{2}$	1·34345	
	·67172	
	276·08024	............amt in 2 yrs.
	261·5	............1st Prin.
	14·58024	............Int. for 2 yrs.

Ans. £14, 11s. 7d.

* See page 174.

The next example illustrates the case in which the interest, due yearly, is required for a *part of a year.*

EXAMPLE vi.—*Find the amount at compound interest of* £1025 *in* $2\frac{1}{2}$ *years at* 3%.

Having obtained the 3rd Principal as before, we now require the interest on it at 3 per cent per ann. for *half* a year.

Now 3 p. c. per year = $1\frac{1}{2}$ p. c. per *half* year.

We ∴ (i) divide the 3rd prin. by 100; and then (ii) divide this result by 2.

(i) and (ii) together make the half-year's int.

£	
1025·	..............1st Prin.
30·75	..........1st yr.'s Int.
1055·75	..........2nd Prin.
31·6725	.......2nd yr.'s Int.
1087·4225	.......3rd Prin.
10·87422 5·43711	} ...$\frac{1}{2}$ yr.'s Int.
1103·73383	......reqd amt.

Ans. £1103, 14*s.* 8*d.*

The next two examples illustrate cases in which *the interest is added to the principal at intervals of less than a year.*

EXAMPLE vii.—*Find to the nearest penny the amount, at compound interest payable half-yearly, of* £433, 13*s.* 4*d.* *in* $1\frac{1}{2}$ *years at 5 per cent.*

As 5 per cent means 5 per cent per *year*, *i.e.* $2\frac{1}{2}$ per cent per *half*-year, and as $1\frac{1}{2}$ years = 3 half-years, we must calculate interest for three periods at $2\frac{1}{2}$ per cent per period.

Ans. £467, 0*s.* 3*d.*

	£	
$\frac{1}{40}$	433·66666	......1st P.
	10·84166	......1st I.
$\frac{1}{40}$	444·50832	......2nd P.
	11·11270	......2nd I.
$\frac{1}{40}$	455·62102	......3rd P.
	11·39052	......3rd I.
	467·01154	......reqd amt.

EXAMPLE viii.—*Find the compound interest on* £825 *for a year at* $2\frac{1}{2}$% *payable quarterly.*

$2\frac{1}{2}$% per ann. = $\frac{2\frac{1}{2}}{4}$, or $\frac{5}{8}$, per cent per quarter,

and 1 year = 4 quarters,

so we calculate interest for 4 periods at $\frac{5}{8}$ per cent per period.

Now $\frac{5}{8} \div 100 = \frac{4+1}{800} = \frac{1}{200} + \frac{1}{800}$,

so *we first divide the princ. by* **200** (i), *and then this result by* **4** (ii).

(i) and (ii) together make up the quarter's int.

Ans. £20, 16*s.* 4*d.*

£	
825·	..............1st P.
4·125 1·03125	} ...1st I.
830·15625	......2nd P.
4·15078 1·03769	} ...2nd I.
835·34472	......3rd P.
4·17672 1·04418	} ...3rd I.
840·56562	......4th P.
4·20282 1·05070	} ...4th I.
845·81914	......amt in 1 yr.
825·	......1st P.
20·819	......Int. reqd.

EXAMPLE ix.—*Find the difference between the simple and compound interest on £231, 12s. 6d. for 3 years at 4 per cent.*

£	
231·625	....1st P.
9·26500	....1st I.
240·89	....2nd P.
9·6356	....2nd I.
250·5256	....3rd P.
10·02102	....3rd I.
260·54662	....amt in 3 yrs.
231·625	....1st P.
28·92162	....comp. int. for 3 yrs.
27·795	= 1st int. × 3 = simp. int. for 3 yrs.
1·1266	= reqd diff.

Ans. £1, 2*s.* 6*d.*

In the preceding examples we have seen how the compound interest, or amount, may easily be obtained for a small number of years (or periods). If, however, the number of years (or periods) exceeds *seven*, a different method of treatment is preferable.

Compound interest for a **large number** of periods.

As £100, at (say) **4**% amounts in a year to £104

$\therefore$ 1, ... $\frac{104}{100} = £1{\cdot}04$

$\therefore$ any prin. (say) 630, ... $630 \times 1{\cdot}04$

Thus at **4** per cent
the **2nd Prin.** = **1st Prin.** × 1·04
... **3rd Prin.** = 2nd Prin. × 1·04 = **1st Prin.** × 1·04 × 1·04
... **4th Prin.** = 3rd Prin. × 1·04 = **1st Prin.** × 1·04 × 1·04 × 1·04
and so on.

But the 3rd, 4th, &c., Prins. are the *amounts* in **2**, **3**, &c., years respectively.

$\therefore$ at **4** per cent, amount in **1** year = 1st Prin. × **1·04**
.... **2** years = 1st Prin. × $(\mathbf{1{\cdot}04})^2$
.... **3** ...... = 1st Prin. × $(\mathbf{1{\cdot}04})^3$,
.... = ...
.... **10** ...... = 1st Prin. × $(\mathbf{1{\cdot}04})^{10}$. And so on.

Similarly, at **3**%, amount in **2** years = 1st Prin. × $(\mathbf{1{\cdot}03})^2$,
.... **7** ...... = 1st Prin. × $(\mathbf{1{\cdot}03})^7$. And so on.

Hence, if (i) *we decimalize the fraction* $\frac{100 + \text{rate}}{100}$; (ii) *raise this decimal to the power whose index is the given number of years* (or periods); and (iii) *multiply the principal by this power*; we obtain an expression for the final **amount**.

For instance, the amount of £253 in 8 years at $2\frac{1}{2}$% is £253 × $(1{\cdot}025)^8$.

EXAMPLE x.—*Find the amount of £2500 in 10 years at 3%.*

$$\text{Req}^{d}\ \text{am}^{t} = £2500 \times (1\cdot03)^{10}$$
$$= £2500 \times 1\cdot3439164^{*}$$
$$= £\frac{10000}{4} \times 1\cdot3439164$$
$$= £\frac{13439\cdot164}{4}$$
$$= £3359\cdot791$$
$$= £3359,\ 15s.\ 10d.\ \textit{Ans.}$$

* Here, in order to save space, we have omitted the work of calculating $(1\cdot03)^{10}$. For this part of the work compare Exs. xv, xvi on page 180. We must calculate $(1\cdot03)^{10}$ to 7 places of decimals here in order to have 3 places of decs. in the product $2500 \times (1\cdot03)^{10}$.

Note.—The amount of labour in calculating Comp. Int. for a large no. of periods may be greatly reduced by the use of Logarithms.

The preceding examples are all varieties of the same general question—to find the compound interest (or amount) when principal, rate, and time are known.

There remain the three converse cases to be considered.

CASE I.—To find the Rate when principal, compound interest (or amount), and time are given.

This case is considered in the chapter on Square Root.

CASE II.—To find the Time when principal, rate, and interest (or amount) are given.

EXAMPLE xi.—*In what time would £600 amount to £694, 11s. 6d. at 5 per cent compound interest?*

Here the given am^{t} is £694·575.

[The req^{d} time may be found by trial; *i.e.* we find the amount in 1, 2, 3, &c. years until we obtain either the given amount, or the first amount which exceeds it.]

Ans. 3 years.

£	
600	............1st P.
30	............1st I.
630	............2nd P.
31·5	.........2nd I.
661·5	.........3rd P.
33·075	......3rd I.
694·575	= am^{t} in 3 years.

EXAMPLE xii.—*In what time would £423, 13s. 9d. be the compound interest, to the nearest penny, on £3715 at 4 per cent?*

Here £3715 + £423·6875 = **£4138·6875**, the *amount* in req^{d} time.

After obtaining the 4th P. we observe that the given amount in the req^{d} time lies between the amounts in 2 and 3 years.

Hence the req^{d} time is 2 yrs. + some fraction of a year.

3715	..............1st P.
148·60	.........1st I.
3863·6	.........2nd P.
154·544	......2nd I.
4018·144	......3rd P. or am^{t} in 2 yrs.
160·72576	...3rd I.
4178·86976	...4th P. or am^{t} in 3 yrs.

In order to find this fraction we proceed thus:—

$$\text{Am}^{\text{t}} \text{ in req}^{\text{d}} \text{ time} - \text{am}^{\text{t}} \text{ in 2 yrs.} = £4138{\cdot}6875 - £4018{\cdot}144$$
$$= £120{\cdot}5435$$
$$\text{Am}^{\text{t}} \text{ in 3 yrs.} - \text{am}^{\text{t}} \text{ in 2 yrs.} = £160{\cdot}72576$$

$$\therefore \frac{\text{req}^{\text{d}} \text{ fraction}}{1 \text{ year}} = \frac{£120{\cdot}5435}{£160{\cdot}72576} = \frac{3}{4}.$$

```
160·72576)120·5435(·75
          112 5
          -----
             80
```

Note.—As the given int. was approximate, we obtain the req$^{\text{d}}$ fraction of a year by Approximation.

$\therefore$ req$^{\text{d}}$ time $= 2\frac{3}{4}$ yrs. *Ans.*

CASE III.—To find the Principal when the rate, time, and compound interest (or amount) are given.

EXAMPLE xiii.—*What sum would amount at compound interest to £813, 16s. 2d. (nearly) in 3 years at 5 per cent?*

[In this case the method of page 205 is convenient.]

£1 in 3 yrs. at 5 per cent amounts to $£1 \times (1{\cdot}05)^3$

Hence as $\overset{£}{1} \times (1{\cdot}05)^3$ is the amount of $\overset{£}{1}$

$\therefore$ 1 .... $\dfrac{1}{(1{\cdot}05)^3}$

and $\therefore$ 813·8083 .... $\dfrac{813{\cdot}8083}{(1{\cdot}05)^3} = £703$ *Ans.*

Note.—To save space the approximate calculation of $\dfrac{£813{\cdot}8083}{(1{\cdot}05)^3}$ is omitted. For the method see next Ex.

EXAMPLE xiv.—*What sum put out at compound interest for 3 years at 4 per cent would gain £64, 12s. 4d.?*

£1 in 3 yrs. at 4% comp. int. amounts to $£(1{\cdot}04)^3$

$\therefore$ 1 .... gains *int.* $£(1{\cdot}04)^3 - £1$
$= £{\cdot}124864.$*

Hence as $\overset{£}{{\cdot}124864}$ is the comp. int. on $\overset{£}{1}$

$\therefore$ 64·6166 .... $\dfrac{64{\cdot}6166}{{\cdot}124864} = \overset{£}{£517, \text{ 10s.}}$ *Ans.*

```
*1·04              ·124864)64·6166(517·5 nearly.
   416                     62 4320
------                     -------
1·0816                      2 1846
  43264                     1 2486
--------                    ------
1·124864                      9360
                              8740
                              ----
                               620
```

* Here we square 1·04 by adding to 1·04, $\frac{4}{100}$ of 1·04, *i.e.* we mult. by 4, setting the figures two places to the right; and so on. See p. 202.

The following are examples of other problems involving the methods of Compound Interest.

EXAMPLE XV.—*The population of a city is* 765240, *and its annual increase is at the rate of* 2·7 *per cent; what will be the population at the end of* 3 *years?*

$$\left.\begin{array}{r}\frac{1}{40}\\ \frac{1}{500}\end{array}\right\}\begin{array}{|l}765240 \text{ ....1st pop.}\\ \left.\begin{array}{r}19131\\ 1530{\cdot}48\end{array}\right\}\text{...1st yr.'s inc.}\end{array}$$

$$\begin{array}{l}\hline 785901{\cdot}48\text{......2nd pop.}\\ \left.\begin{array}{r}19647{\cdot}53\\ 1571{\cdot}80\end{array}\right\}\text{...2nd yr.'s inc.}\\ \hline 807120{\cdot}81\text{......3rd pop.}\\ \left.\begin{array}{r}20178{\cdot}02\\ 1614{\cdot}24\end{array}\right\}\text{...3rd yr.'s inc.}\\ \hline 828913{\cdot}07\text{......pop. at end of 3 yrs.}\end{array}$$

Ans. 828913.

Here as we only require the result correct to the nearest *integer*, we retain but 2 places of decimals in the working.

Now $\frac{2{\cdot}7}{100} = \frac{27}{1000} = \frac{25+2}{1000} = \frac{1}{40} + \frac{1}{500}$. We, therefore, (i) divide the original pop. by 40; then (ii) divide the original pop. by 500; (i) and (ii) together make up the 1st year's increase. And so on.

Note.—If for the word "increase" we substitute "decrease", then, instead of adding, we *subtract* at each stage, to obtain the final pop.

EXAMPLE XVI.—*The difference between the simple and compound interest on a certain sum for* 2 *years at* 3 *per cent is* £1, 10*s. Find the sum.*

[We first find the difference between the simple and comp. int. on £100 for the time.]

Simple int. on £100 for 2 yrs. at 3% is £6

Comp.... $£3 + £\frac{103 \times 3}{100} = £\frac{609}{100}$

Hence the difference is $\overset{£}{\frac{9}{100}}$ on a principal of $\overset{£}{100}$

$\therefore$ .... $1\frac{1}{2}$ .... $100 \times \dfrac{1\frac{1}{2}}{\frac{9}{100}}$

$$= £\frac{5000}{3} = \underline{£1666,\ 13s.\ 4d.\ Ans.}$$

EXAMPLE xvii.—*£1030 is borrowed for 2 years, at 6 per cent compound interest, to be repaid, principal and interest, in two equal annual instalments. Find the amount of an instalment.*

It is evident that the two instalments, together with the int. on the *first* instal. for *one* year, must be equal to the *amount* of £1030 in 2 years.

$$i.e.\ \text{req}^{\text{d}}\text{ instal.} + \text{req}^{\text{d}}\text{ instal.} \times 1\cdot 06 = £1030 \times (1\cdot 06)^{2}*$$

$$\text{or, req}^{\text{d}}\text{ instal.} \times (1 + 1\cdot 06) = £1030 \times (1\cdot 06)^{2}$$

$$\therefore \text{ the req}^{\text{d}}\text{ instal.} = £\frac{1030 \times 1\cdot 06 \times 1\cdot 06}{2\cdot 06}$$

[* See p. 205.]

$$= £561,\ 16s.\ \textit{Ans.}$$

EXAMPLE xviii.—*If £400 amounts to £532, 8s. in 3 years at compound interest, what would £5000 amount to in 6 years at the same rate per cent?*

[We know, from page 205, that

$$£1 \text{ amounts in 3 yrs. to } £1 \times \left(\frac{100 + \text{rate}}{100}\right)^{3}$$

$$\text{and } £1 \ldots\ldots\ldots\ldots 6 \ldots\ldots\ldots £1 \times \left(\frac{100 + \text{rate}}{100}\right)^{6}$$

i.e. when the number of years is doubled the Amount of £1 is squared. Similarly, ... *trebled* .... *cubed.* And so on.]

$$\text{Hence, as } \overset{£}{400} \text{ amounts in 3 yrs. to } \overset{£}{532\cdot 4}$$

$$\text{and, consequently, } 1 \ldots\ldots\ldots\ldots 3 \ldots\ldots \frac{532\cdot 4}{400}$$

$$\therefore 1 \ldots\ldots\ldots\ldots 6 \ldots\ldots \left(\frac{532\cdot 4}{400}\right)^{2}$$

$$\text{and } \therefore 5000 \ldots\ldots\ldots\ldots 6 \ldots\ldots \left(\frac{532\cdot 4}{400}\right)^{2} \times 5000$$

$$= \frac{5324 \times 5324 \times 50}{400 \times 400}$$

$$= £8857,\ 16s.\ 1d.\ \textit{Ans.}$$

Note.—If a quantity be subject to a periodical *decrease* at a given rate per cent, the remainder at the end of a given number of periods may be found by multiplying the original quantity by $\frac{100 - \text{rate}}{100}$ *raised to the power whose index is the given number of periods.* (Cf. p. 205.)

LVI. DISCOUNT AND PRESENT WORTH.

PRACTICAL, OR BANKERS', DISCOUNT.

When one merchant purchases goods of another he does not usually pay for the goods in ready money, but by giving a *Bill of Exchange.*

A Bill of Exchange is a document authorizing the person to whom it is given to receive a specified sum of money *at the end of a stated time.*

The person to whom the "Bill" is given need not, however, wait until the end of the stated time for his money; he can *sell the Bill for cash** to a Banker, or Bill-broker.

But the amount of *cash* he receives for it will be *rather less* than the *sum named in the Bill*; (for, as interest can always be obtained for money, it is evident that a sum of *ready* money is more valuable than the *same sum at some future time.*)

When a Bill of Exchange is sold for cash it is said to be **Discounted.**

The sum of money named in the Bill is called its **Face Value.**

The amount of *cash* obtained for the Bill at the time it is discounted is called its (Commercial) **Present Worth** (or Value), and the difference between the *Face Value* and the *cash* value of the Bill is called the **Bankers'** (or Commercial) **Discount.**

The Bankers' **Discount** is *always* **the Interest** (at some rate per cent agreed upon) **on the Face Value** of the Bill, reckoned *for the time the Bill has still to run* before it is *due.*

Hence, the **Commercial Present Worth** of a Bill
= **Face Value − Bankers' Discount**
= **Face Value − Int. on Face Value.**

Note.—In this, as in any other commercial, Rule fractions of a penny need not be given in results.

EXAMPLE i.—*Find the commercial present worth of a bill for* £400, *discounted, at* 4 *per cent,* 58 *days before it was due.*

Banker's Disc. = *int.* on £400 for 58 days at 4%
= £2, 10*s.* 10*d.*†

Hence Commercial Present Worth = £400 − £2, 10*s.* 10*d.*
= £397, 9*s.* 2*d.* *Ans.*

* This sale is *conditional* on the bill not being "dishonoured"; *i.e.* the banker, or bill-broker, can claim the return of the money he advanced should the bill not be paid at maturity.

† The work involved in calculating this *interest* is omitted here, and in succeeding examples, as methods of calculation have been fully exemplified in Chap. LIV.

A Bill of Exchange which is *nominally* due on a certain day is not *legally* due until *three days later*; these 3 days are called **Days of Grace.**

For instance, a three-months' bill dated June 3, would be *nominally* due on Sep. 3, but not *legally* due until Sep. 6.

The Banker, or Bill-broker, always includes the days of grace in reckoning the time for which he charges discount.

Note.—In questions in which the *actual dates are mentioned* the days of grace should always be taken account of; but in questions in which the actual dates are *not* mentioned the days of grace may be supposed to be already included in the interval named.

SPECIMEN OF A BILL OF EXCHANGE.

[*Embossed Stamp* 1*s.* *per* £100.]

£100. *London, Sep. 7, 1896.*

Three months after date pay to my order one hundred pounds for value received.

William Jones.

To Mr. John Smith,
Birmingham.

Here we may suppose that Mr. Jones has supplied Mr. Smith with goods to the value of £100, and that Mr. Jones then writes a Bill of Exchange of the above form.

Mr. Jones is called the "*drawer*", and Mr. Smith the "*drawee*".

Mr. Smith next writes the word "accepted", and signs his name, across the face of the bill, returning it to Mr. Jones.

Mr. Smith having now "*accepted*" the three-months' bill, "*drawn*" (*i.e.* dated) Sep. 7, has made himself liable for the £100 on Dec. 10th.

Mr. Jones has now choice of the following three alternatives:—

(i) He may keep the bill until Dec. 10 and then obtain the whole sum of £100; or

(ii) He may at any time before Dec. 10 (say on Sep. 23), having signed his own name at the back of the bill (which constitutes his "order"), discount it for cash, obtaining from a banker, or bill-discounter, £100 *minus* the int. on £100 at, say 5, % for 78 days (*i.e.* the no. of days from Sep. 23 to Dec. 10); or

(iii) He may, having signed his name on the back, pass the bill on to a third person, Mr. Brown, to whom he owes money—in which case Mr. Brown has choice of the same three alternatives.

The person to whose order the bill is made payable is called the "*payee*". In the above specimen Mr. Jones is both drawer and payee; he might, however, have inserted the name of a third person as payee.

The above is but one of many forms of a bill of exchange.

A *Promissory Note* differs from a bill of exchange mainly in being *drawn by the debtor* instead of by the creditor. It can be *used* in the same way as a Bill. (See the Exercises, No. 29, for a specimen.)

EXAMPLE ii.—*Find the banker's charge for discounting on June 5, at 5%, a bill for £450, drawn May 21 at 3 months.*

[Here the days of grace must be taken account of.
As the bill was dated May 21, the money was *nominally* due on Aug. 21, and therefore *legally* due on Aug. 24.]

On June 5 the bill had $25 + 31 + 24 = 80$, days still to run.

Hence Bankers' disc. $= £450 \times \frac{5}{100} \times \frac{80}{365} = \underline{£4,\ 18s.\ 8d.\ \textit{Ans.}}$

Note.—Here, and in similar cases, we calculate the *discount* to the *nearest* penny.

EXAMPLE iii.—*Find the present worth of a bill for £1386, 15s. at 90 days' sight, presented for acceptance on Sep. 17, and discounted at 4% on Oct. 3.*

The bill was legally due on Dec. 19 (*i.e.* 93 days from Sep. 17),
∴ on Oct. 3 it had still to run $28 + 30 + 19 = 77$, days.
Hence disc., *i.e. int.* on £1386, 15s. for 77 days at 4%,
= £11, 14s.
∴ req^{d} P. W. = £1386, 15s. − £11, 14s. = $\underline{£1375,\ 1s.\ \textit{Ans.}}$

EXAMPLE iv.—*A three-months' bill drawn April 15, and discounted May 6 at 4%, realized £217 : find its face value.*

No. of days for which disc. was charged $= 25 + 30 + 18 = 73$
Now Face val. − Int. on Face val. = P. W.

$$\therefore \text{Face val.} - \text{Face val.} \times \frac{4}{100} \times \frac{73}{365} = \overset{£}{217}$$

$$\textit{i.e.}\ \text{Face val.} \times \left(1 - \frac{1}{25} \times \frac{1}{5}\right) = 217$$

$$\therefore \text{Face val.} = \overset{£}{217} \div \frac{124}{125} = \underline{£218,\ 15s.\ \textit{Ans.}}$$

EXAMPLE v.—*Show that a banker who discounts a bill at 5% when it has still 73 days to run, obtains interest at the rate of $5\frac{5}{99}$ per cent for his money.*

Here Banker's disc. on a bill for £100 would be $£5 \times \frac{1}{5} = £1$,
∴ the cash he advances on it = £100 − £1 = £99.

Hence on $\overset{£}{99}$ lent for $\frac{1}{5}$ yr. he gets $\overset{£}{1}$ int.

∴ ... 100 .... 1 .... $1 \times \frac{100}{99} \times 5 = £5\frac{5}{99}$;

i.e. he obtains *int. at the rate of* $5\frac{5}{99}$ *per cent per annum on his capital.*

Note.—Hence when a banker discounts a bill at a certain rate per cent he obtains rather more than that rate per cent on the money he advances.

We shall now consider *what sum, if charged as discount, would give the discounter interest at the discounting rate per cent, and no more, on the money he advances.*

It should, however, be clearly understood that this is purely a matter of theory, and of *no practical* importance, as Bills of Exchange, &c., which only run for *short* periods, are *always* discounted in the manner already exemplified.

Theoretical (often called "True") Discount.

As £100 at, say, 4 per cent amounts in a year to £104, it follows that, at 4 per cent,

£100 is the *theoretical pres. worth* of a debt of **£104** due at end of a year,
and £4 is the *theoretical discount* on .... **£104** ....

i.e. the Theoretical *Present Worth* of a debt due at some future time is that sum which, with interest, amounts in the given time to the debt;

and the Theoretical *Discount* is the *interest on the theoretical present worth of the debt* (not on the debt itself).

Hence questions on Theoretical Discount and Present Worth, for *short* periods, are but varieties of Case III in Simple Interest.

Ex. vi.—*Find the Theoretical Present Worth of a debt of £567, due in 3 months, supposing money to be worth 5%.*

£100 amounts in 3 mo. at 5% to £$101\frac{1}{4}$,

$\therefore$ Theo. Pres. Worth of a debt of £$101\frac{1}{4}$ is £100

$\therefore$ .... 567 ... $100 \times \dfrac{567}{101\frac{1}{4}} = \underline{£560\ \textit{Ans.}}$

Ex. vii.—*Find the "true" discount at 3 per cent on £282, 16s. due at the end of 4 months.*

£100 amounts in 4 mo. at 3% to £101,

$\therefore$ on £101 the "true" discount is £1

$\therefore$ ... $282\frac{4}{5}$ .... $1 \times \dfrac{282\frac{4}{5}}{101} = \underline{£2,\ 16s.\ \textit{Ans.}}$

Ex. viii.—*If the "true" discount on £756 due in 73 days is £6, what is the rate per cent?*

As £6 is the "true" disc. on £756 for $\frac{1}{5}$ year,

$\therefore$ £6 .... interest ... **£750** ....

Hence (see Case I, Simple Int.) the rate per cent is $\underline{4\ \textit{Ans.}}$

Ex. ix.—*On what sum, due in 6 months, is £8 the "true" discount at 5%?*

£100 in 6 mo. at 5% amounts to £$102\frac{1}{2}$,

$\therefore$ £$2\frac{1}{2}$ is the theoretical discount on £$102\frac{1}{2}$

$\therefore$ 8 .... $102\frac{1}{2} \times \dfrac{8}{2\frac{1}{2}} = \underline{£328\ \textit{Ans.}}$

Note.—We have already noticed (see pages 104, 186) that an allowance often made by a tradesman to a customer who pays ready money, is called **Trade Discount.** This is merely *a percentage* on the price of the goods; *i.e. time* is not considered in the calculation.

Long Period Present Value.

Here practice agrees with theory.

The **Present Value** (or Worth) of a given sum of money *due at the end* of a *number of years* is *that sum which, at compound interest,* at a given rate per cent, *amounts* in the time to the given sum.

Example x.—*Find, to the nearest penny, the Present Worth of* £800 *due in* 3 *years, allowing compound interest at* 4 *per cent.*

£1 amounts in 3 years at 4%, comp. int., to £$(1\cdot04)^3$*

$$\therefore\ \pounds(1\cdot04)^3 \text{ has Pres. Val. } \pounds1$$

$$\therefore\ 1 \ \ldots\ldots\ldots\ldots \frac{1}{(1\cdot04)^3}$$

$$\text{and } 800 \ \ldots\ldots\ldots\ldots \frac{800}{(1\cdot04)^3} = \pounds\frac{800}{1\cdot124864}$$

$$= 711\cdot188 = \pounds711,\ 3s.\ 9d.\ \textit{Ans.}$$

```
1·04
  416
1·0816    ... 2nd power.
  43264
1·124864 ... 3rd power.
```

```
1·124864)800·0000(711·188
         787 4048
          12 5952
          11 2486
           1 3466
           1 1249
             2217
             1225
              992
              899
               93
```

Note.—We square 1·04 by *adding* to 1·04 $\frac{4}{100}$ of 1·04. (See the method on p. 202.)

Example xi.—*How many pounds must a man invest, at* 3 *per cent compound interest, on his daughter's* 16*th birthday in order that she may come into* £1000 *on her* 21*st birthday?*

£1 amounts in 5 yrs. at 3%, comp. int., to £$(1\cdot03)^5$,

$$\therefore\ \pounds(1\cdot03)^5 \text{ has Pres. Val. } \pounds1$$

$$\therefore\ 1000 \ \ldots\ldots\ldots\ldots \frac{1000}{(1\cdot03)^5} = \pounds863\ \textit{Ans.}$$

* See page 187.

If by the payment of a sum of money down a person acquires the right to receive a fixed annual income, he is said to purchase an **Annuity.**

An annuity is called (i) *terminable*, or (ii) *perpetual*, according as it is agreed that it shall continue for (i) a certain number of years and then cease, or (ii) for an unlimited number of years.

The following examples show how simple cases of terminable annuities may be treated by the ordinary methods of Arithmetic.

EXAMPLE xii.—*Find, to the nearest pound, the present value of an annuity of* £80, *to continue for* 5 *years, the first payment to be made a year hence, allowing compound interest at* 3 *per cent.*

If an annuity of £1 for 5 years were *left unpaid*, there would be due at the *end* of the time (allowing comp. int. at 3%)

£1 (*i.e.* the last instalment)
$+$ £1·03 (*i.e.* amt. in 1 yr. at 3% of the last instal. but one)
$+ £(1\cdot03)^2$ (............ 2 yrs.................................... two)
$+ £(1\cdot03)^3$ (............ 3 .... three)
$+ £(1\cdot03)^4$ (............ 4 .... first instalment)
$= £1 + 1\cdot03 + 1\cdot0609 + 1\cdot0927 + 1\cdot1254 = £5\cdot309.$

Hence as **an annuity of £1** amounts to £5·309
$\therefore$ .... £80 .... $£5\cdot309 \times 80$

But **£1** amounts in 5 yrs. at 3% to $£(1\cdot03)^5 = £1\cdot1591$

Hence, as $\overset{£}{1\cdot1591}$ has Pres. Val. $\overset{£}{1}$

$\therefore\ 5\cdot309 \times 80$ .... $1 \times \dfrac{5\cdot309 \times 80}{1\cdot1591} = \underline{£366\ \textit{Ans.}}$

EXAMPLE xiii.—*What annuity, to continue for* 6 *years, the first instalment being due a year hence, could be purchased for* £1000, *when money is worth* 4 *per cent?*

Amount of an **annuity of £1**, left unpaid, for 6 yrs. at 4% comp. int.
$= £1 + 1\cdot04 + (1\cdot04)^2 + (1\cdot04)^3 + (1\cdot04)^4 + (1\cdot04)^5 = £6\cdot632974.$

But *amount* of **£1** in 6 yrs. at 4% is $£(1\cdot04)^6 = £1\cdot265318.$

Hence $\dfrac{£6\cdot632974}{1\cdot265318}$ is Pres. Val. of £6·632974 due in 6 yrs.

i.e. $\overset{£}{\dfrac{6\cdot632974}{1\cdot265318}}$ will purchase *an annuity of* $\overset{£}{1}$ *for* 6 *yrs.*

$\therefore$ 1000 .... $\dfrac{1\cdot265318}{6\cdot632974} \times 1000 = \underline{£190,\ 15s.\ 3d.}$

LVII. STOCKS AND SHARES.

Stocks.

When one person borrows money from another he borrows on the understanding (i) that he must pay interest at some rate per cent agreed upon; (ii) that he must, eventually, *repay the principal.*

The lender's *security* for his money is, usually, a **mortgage** on some property possessed by the borrower.

When, however, the Government of a country, or the Corporation of a town, borrows money (to meet some extraordinary expenditure not provided for by the ordinary revenue from taxes, or rates), it borrows on the understanding (i) that a fixed rate of interest will be paid continuously, but (ii) that it *can never be called upon to repay the principal.*

A lender, in this case, who wishes to recover his principal **sells his right to the fixed interest.**

The *security*, in this case, is the obligation of the Government, or Corporation, to pay the interest out of the taxes, or rates.

A person who has acquired *the right to the interest* of, say, £500 from a Government is said to "hold £500 *stock*"; and a person who buys, or sells, *this right to the interest* of £500 is said to buy, or sell, "**£500 stock**".

A person who buys, or sells, £100 *stock* does not, as a rule, pay, or receive, exactly £100 *cash* for it. Stock, like other things bought and sold, has a fluctuating market price—the price *rising*, or *falling*, according as the number of persons who wish to *buy* tends to exceed or fall short of the number who wish to *sell*.

The **market price** of stock is always expressed by the *number of pounds*, cash, which will purchase £100 *stock*.

For instance, when some particular stock is said to be "at 108", we must understand that £108 *cash* is then the price of **£100 stock** of that kind.

Any quantity of **Stock** may be bought, or sold.

For instance, a person might buy £321, 16*s.* 4*d.* *stock*; and, if the rate of interest on this stock were 3 per cent, he would then have *bought the right* to £9, 13*s.* 1*d.* a year; *i.e.* he would have *bought an* **annuity** of £9, 13*s.* 1*d.*

On some stocks the interest is paid *half-yearly*; on others *quarterly*; these payments are called **Dividends.**

Dividends are always calculated on the amount of **Stock** *held.*

Note.—The beginner must be very careful not to mistake *pounds stock* for *pounds cash*, or vice versa. For instance, in the statement "a man *invests* £1000 in the 3 per cents", the £1000 is *cash*; but in the statement "a man *holds* £1000 in the 3 per cents", the £1000 is *stock*.

The following examples* illustrate various cases which occur.

EXAMPLE i.—*Find the cost of* £825 *stock, at* 123.

[In other words:—"If £100 *stock* cost £123 *cash*, what will £825 *stock* cost?"]

$$\begin{array}{llll} & \text{Stock.} & & \pounds \\ \text{As} & 100 & \text{costs} & 123 \\ \therefore & 825 & \ldots\ldots & 123 \times \frac{825}{100} = \underline{\pounds 1014,\ 15s.\ \textit{Ans.}} \end{array}$$

EXAMPLE ii.—*How much stock, at* 93, *can be bought for* £217?

[In other words:—"If £93 cash will pay for £100 *stock*, how much stock will £217 cash pay for?"]

$$\begin{array}{llll} & \pounds & & \text{Stock.} \\ \text{As} & 93 & \text{pays for} & 100 \\ \therefore & 217 & \ldots\ldots\ldots & 100 \times \frac{217}{93} = \underline{\pounds 233,\ 6s.\ 8d.\ \textit{stock Ans.}} \end{array}$$

EXAMPLE iii.—*What annual income is obtained from* £8750 *stock paying* 3 *per cent?*

[In other words:—"If £100 *stock* yields an income of £3, what income will £8750 *stock* yield?"]

$$\begin{array}{lllll} & \text{Stock.} & & \pounds & \\ \text{As} & 100 & \text{yields} & 3 & \text{income} \\ \therefore & 8750 & \ldots\ldots & \multicolumn{2}{l}{3 \times \frac{8750}{100} = \underline{\pounds 262,\ 10s.\ \textit{Ans.}}} \end{array}$$

Note.—As the income is the interest, calculated on the *nominal* principal, *i.e.* on the amount of *stock*, $\text{Income} = \text{Stock} \times \frac{\text{rate p. c.}}{100}$ (I).

Hence, conversely, the amount of *stock* which yields a *given income* is found from the formula: $\text{Stock} = \text{Income} \times \frac{100}{\text{rate p. c.}}$ .... (II).

EXAMPLE iv.—*Find the annual income obtained by investing* £5800 *in a* 5 *per cent stock at* 116.

[In other words:—"If £116 *cash* will buy the right to £5 a year, how much a year will £5800 *cash* buy the right to?"]

$$\begin{array}{llll} & \pounds & & \pounds \\ \text{As} & 116 & \text{cash purchases an income of} & 5 \\ \therefore & 5800 & \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots & \frac{5 \times 5800}{116} = \underline{\pounds 250\ \textit{Ans.}} \end{array}$$

N.B.—From this result we see that the income may be found by multiplying the *sum invested* by the rate per cent, and dividing the result by the *market price of the stock*.

i.e. $\text{Income} = \frac{\text{cash invested} \times \text{rate p. c.}}{\text{market price of stock}}$ .... (III).

* In order to save space, the merely mechanical part of the working is omitted, various methods of performing this having already been fully exemplified. The student, however, is reminded that *the whole* of *his* work must be shown in an examination.

There is a special market, called the Stock Exchange, where stocks and shares are bought and sold.

The agents employed by the public to buy, or sell, for them are called *Stock-brokers.* The stockbroker's charge, called **Brokerage**, is *a percentage on the quantity of* **stock** bought, or sold.

Hence when stock is *bought*, the brokerage is **added** to the *market price* of the stock; when *sold*, it is **subtracted** from the *market price.*

Note.—The brokerage on Government stock is 2*s.* 6*d.* per £100 stock; *i.e.* $\frac{1}{8}$ per cent on the amount of stock dealt with. On certain other stocks the brokerage is $\frac{1}{4}$ per cent.

For instance, when a certain Government stock is at 112, the *cost* (through a broker) *to a buyer* would be £112$\frac{1}{8}$ cash per £100 stock bought; the *sum realized by a seller* would be £111$\frac{7}{8}$ cash per £100 stock sold.

[When brokerage is not specially mentioned in a question it need *not* be considered, but may be supposed to be already included in the stated price.]

EXAMPLE v.—*Find the cost of* £733, 6*s.* 8*d.* *stock at* 108, *allowing for brokerage at* $\frac{1}{8}$ *per cent.*

$$\begin{array}{llll} & \text{Stock.} & & £ \\ \text{As} & 100 & \text{costs} & 108\tfrac{1}{8} \end{array}$$

$$\therefore\ 733\tfrac{1}{3}\ \ldots\ldots\ 108\tfrac{1}{8} \times \frac{733\frac{1}{3}}{100} = \underline{£792,\ 18s.\ 4d.\ Ans.}$$

EXAMPLE vi.—*How much would be realized by the sale of* £2450 *stock at* 134? (*Brokerage* $\frac{1}{4}$ *per cent.*)

$$\begin{array}{llll} & \text{Stock.} & & £ \\ \text{As} & 100 & \text{realizes} & 133\tfrac{3}{4} \end{array}$$

$$\therefore\ 2450\ \ldots\ldots\ldots\ 133\tfrac{3}{4} \times \frac{2450}{100} = \underline{£3276,\ 17s.\ 6d.\ Ans.}$$

EXAMPLE vii.—*How much stock at* 95$\frac{1}{2}$ *could be bought for* £1020, *allowing* 2*s.* 6*d.* *per cent for brokerage?*

$$\begin{array}{llll} & £ & & \text{Stock.} \\ \text{As} & 95\tfrac{5}{8} & \text{is the cost of} & 100 \end{array}$$

$$\therefore\ 1020\ \ldots\ldots\ldots\ldots\ldots\ 100 \times \frac{1020}{95\frac{5}{8}} = \underline{£1066,\ 13s.\ 4d.\ \text{stock}\ Ans.}$$

EXAMPLE viii.—*How much stock at* 95$\frac{1}{2}$ *would be sold for* £1526? (*Brokerage* $\frac{1}{8}$ *p. c.*)

$$\begin{array}{llll} & £ & & \text{Stock.} \\ \text{As} & 95\tfrac{3}{8} & \text{is realized by the sale of} & 100 \end{array}$$

$$\therefore\ 1526\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ 100 \times \frac{1526}{95\frac{3}{8}} = \underline{£1600\ \text{stock}\ Ans.}$$

EXAMPLE ix.—*Find the net quarterly dividend, after deducting income-tax at 8d. in the pound, obtained by investing £1152 in $4\frac{1}{2}$ per cents at $107\frac{7}{8}$, the brokerage being $\frac{1}{8}$ per cent.*

The *gross* quarterly dividend on each £100 stock is $\frac{1}{4}$ of $£4\frac{1}{2} = £\frac{9}{8}$,

$\therefore$ the *net* quarterly dividend, less income-tax, $= £\frac{9}{8} \times \frac{232^*}{240} = £\frac{87}{80}$.

Hence, as £108, invested, yields div[d] $£\frac{87}{80}$,

$\therefore$ 1152 .... $\frac{87}{80} \times \frac{1152}{108} =$ £11, 12*s*. *Ans.*

EXAMPLE x.—*What rate per cent of interest on capital is obtained by investing in a 4 per cent stock at 85?*

As £4 is the interest obtained by investing £85, we have but to express the ratio of £4 to £85 *as a percentage.*

$$\text{Now } \frac{4}{85} = \frac{\frac{4}{85} \times 100}{100} = \frac{4\frac{12}{17}}{100} = 4\tfrac{12}{17} \text{ per cent } Ans.$$

EXAMPLE xi.—*Which investment pays better—3 per cent stock at 97, or 5 per cent stock at 161?*

[Here we compare the cost of equal incomes obtained from the two stocks.]

In the 1st stock an income of £3 is bought for £97,

$\therefore$ .... £1 .... $£\frac{97}{3} = £32\frac{1}{3}$.

In the 2nd stock an income of £5 .... £161,

$\therefore$ .... £1 .... $£\frac{161}{5} = £32\frac{1}{5}$.

Now $32\frac{1}{5}$ is *less* than $32\frac{1}{3}$, $\therefore$ in the 2nd stock *the same income* is obtained *by investing* less *capital*, *Ans.* 5 per cents at 161.

EXAMPLE xii.—*Find the change in income caused by selling out £2400 4 per cent stock at 135, and investing the proceeds in $2\frac{1}{2}$ per cent stock at 108.*

The income from £2400 stock at 4% is $£24 \times 4 = £96$.

Again, £2400 stock sells for $£135 \times 24$ cash.

And as £108 invested in the 2nd stock yields $£2\frac{1}{2}$ income,

$\therefore$ 135×24 .... $2\frac{1}{2} \times \frac{135 \times 24}{108} = £75$.

Hence the loss in income $= £96 - £75 =$ £21 *Ans.*

* See page 140.

Note.—The following terms are used in connection with stocks:—

THE FUNDS.—The bulk of the National Debt of England (incurred in time of war) called the Funded Debt, or the *Funds*, now consists of two parts: on one of which interest at the rate of $2\frac{1}{2}$ per cent per annum is paid; on the other, $2\frac{3}{4}$ per cent per ann. is paid, and will continue to be paid until 1903, when the int. on this part also will be reduced to $2\frac{1}{2}$ p. c.

CONSOLS.—The above stocks are called *Consols* (*i.e.* consolidated annuities). The dividends on Consols are paid through the Bank of England, *quarterly*, on April 5, July 5, Oct. 5, and Jan. 5. (See page 27.)

The $2\frac{3}{4}$ per cent Consols are sometimes spoken of as "Goschens", for it was when Mr. Goschen was Chancellor of the Exchequer in 1888 that the rate of interest, formerly 3%, was reduced to $2\frac{3}{4}$%. It was then further enacted that, in order to reduce the National Debt, the Government may in 1923 *redeem* (*i.e.* pay off) any amount of Consols at the rate of £100 cash for each £100 stock.

BONDS.—Foreign Government Stocks are so called from the documents given to the holders of such stocks. From these Bonds, certificates, called *coupons*, are cut which entitle the holder to his dividend each half-year until the Bond is redeemed.

PAR.—When the market price of £100 stock is £100 cash, the stock is said to be "*at par*" (Latin, *par*, equal). And the stock is said to be *above*, or *below*, *par*, according as the market price of £100 stock is greater, or less, than £100 cash.

PREMIUM.—Also, when the market price of £100 stock is *greater* than £100 cash, the stock is said to be "*at a premium*". Thus, if £100 stock is worth £104 cash, that stock is "at 4 p. c. *premium*".

DISCOUNT.—And, when the market price of £100 stock is *less* than £100 cash, the stock is said to be "*at a discount*". Thus, if £100 stock is worth but £95 cash, that stock is "at 5 p. c. *discount*".

A *stock-jobber* is a person who deals in stocks and shares. A stock-broker, the agent of the public, buys from, or sells to, a stock-jobber.

When the market price of stock on any day is quoted in the newspapers *two* prices are given; the higher is the price the broker pays for stock he buys from, and the lower the price he receives for stock he sells to, the stock-jobber. Thus, when a stock is quoted at "$108\frac{3}{4}$–$108\frac{7}{8}$", the broker *buys* at £$108\frac{7}{8}$, but *sells* at £$108\frac{3}{4}$, per £100 stock.

The market price of a stock is not necessarily high when the rate of interest paid is high; a more important consideration being the degree of *safety* the investment offers. Thus *English* Government stocks pay a low rate of interest, yet their market price is high compared with that of other stocks.

SHARES.

When it is proposed to carry on some business on a large scale, and, consequently, a large capital is required, a few persons specially interested in the undertaking (called Directors) draw up a statement (called a Prospectus) of what they propose and the amount of capital needed for the purpose. This capital they divide into a large number of equal parts, called **Shares**,* and invite the public to become partners in the business.

* In some undertakings (*e.g.* Railways) the capital is *not* divided up into *shares*, but is treated as *stock*.

The whole body of partners thus formed is called a **Company,** and the partners are called **Share-holders.**

The net profits are divided periodically among the shareholders, the **Dividend** being declared as a *percentage* of the capital.

A shareholder cannot require a company to refund the money he has paid for shares; but he may *sell his shares.*

The market price of shares rises, or falls, according as the company is more or less prosperous.

The market price of shares is generally expressed by the *number of pounds* cash (whole or fractional) which *one share* costs.

For instance, when £10 shares in a company are "at $12\frac{1}{2}$", then £$12\frac{1}{2}$ *cash* will purchase *the right to dividends on £10 of the company's capital.*

Dividends are calculated on the *nominal* value of shares.

The chief distinction between stocks and shares is that, while *any quantity of stock* may be bought, or sold, only **whole** *numbers of shares* are, as a rule, dealt with.

Shares are said to be "at a premium", "at par", or "at a discount", according as their *cash* value is greater than, equal to, or less than, their *nominal* value.

For instance, if a £5 share in a company costs £6 cash, the shares of that company are **at 1 premium**; if a £5 share costs £5 cash, the shares are **at par**; if a £5 share costs £$4\frac{1}{2}$ cash, the shares are **at $\frac{1}{2}$ discount.**

N.B.—2 premium is here very different from 2 *per cent* premium; for if a £10 share is at 2 *premium*, its cash value is **£12**; whereas if it is at 2 *per cent premium*, its cash value is **£$10\frac{1}{5}$**.

Brokerage is calculated usually either at the rate of $\frac{1}{2}$ per cent on the *nominal* value of shares, or else at so much *per share.*

For instance, the brokerage, at $\frac{1}{2}$ p. c., on 7 £10 shares is £$70 \div 200 = 7s.$

Note.—The word "**Limited**" now forms part of the title of most companies; it signifies that the liability of the shareholders is *limited to the number of their shares.* Formerly shareholders were fully responsible (like partners in a *firm*) for all the debts of a bankrupt company.

Some companies (*e.g.* Banks and Insurance Companies) do not require their whole capital for immediate use; in such cases only part of the nominal value of each share is *paid up*, and the company then may, in case of need, *call* upon its shareholders for all, or part, of what is not paid up. *Dividends* are then calculated on the *paid up capital.*

Companies often divide their shares, or stock, into different classes— **Debenture, Preference,** and **Ordinary.** Debenture holders have the security of a mortgage on the company's property, *e.g.* buildings, land, machinery, &c., and receive interest at a *fixed* rate. Holders of Preference shares, or stock, have the right to interest at a *fixed* rate to be paid out of profits *before* the claims of *ordinary* shareholders are considered; and then Ordinary shareholders are entitled to the whole of any remaining profits.

Hence Debentures offer *good security* with a *moderate* rate of interest; Ordinary shares, *no security* with the *possibility* of a *high* rate of interest.

EXAMPLE xiii.—*Find the cost of* 27 £10 *shares in a company, at* $12\frac{1}{2}$.

$$\text{As } 1 \text{ share costs } \overset{\pounds}{12\tfrac{1}{2}},$$

$$\therefore\ 27 \text{ shares cost } 12\tfrac{1}{2} \times 27 = \underline{\pounds 337,\ 10s.\ Ans.}$$

EXAMPLE xiv.—*Find the income from* 175 £2 *shares in a company paying a dividend of* 6 *per cent.*

$$\text{Income} = \pounds 2 \times 175 \times \frac{6}{100} = \underline{\pounds 21\ Ans.}$$

EXAMPLE xv.—*How many* £5 *shares at* $6\frac{1}{4}$ *could be bought for* £300?

$$\text{As } \overset{\pounds}{6\tfrac{1}{4}} \text{ will pay for } 1 \text{ share,}$$

$$\therefore\ 300 \ \ldots\ldots\ldots\ldots\ 1 \times \frac{300}{6\frac{1}{4}} = \underline{48 \text{ shares } Ans.}$$

EXAMPLE xvi.—*Find the cost of* 108 £1 *shares at* $\frac{1}{4}$ *premium.*

$$\text{As } 1 \text{ share costs } \overset{\pounds}{1\tfrac{1}{4}},$$

$$\therefore\ 108 \text{ shares cost } 1\tfrac{1}{4} \times 108 = \underline{\pounds 135\ Ans.}$$

EXAMPLE xvii.—*Find the amount of cash realized by the sale of* 75 £20 *shares, at* $2\frac{1}{2}$ *discount.* (*Brokerage* $\frac{1}{2}$ *p. c.*)

The broker sells the shares for $\pounds 17\frac{1}{2} \times 75 = \pounds 1312\frac{1}{2}$.

The brokerage ($\frac{1}{2}$ p. c. on the nominal value of the shares)

$$= \pounds 20 \times 75 \times \frac{1}{200} = \pounds 7\tfrac{1}{2}.$$

$$\therefore \text{ amount of cash realized} = \pounds 1312\tfrac{1}{2} - \pounds 7\tfrac{1}{2} = \underline{\pounds 1305\ Ans.}$$

EXAMPLE xviii.—*A man buys fifty* £50 *shares in a company,* £20 *paid up, at* $22\frac{1}{2}$, *and receives a dividend at the rate of* 4 *per cent. After paying a call of* £5 *per share, he receives a dividend at the rate of* $3\frac{1}{2}$ *per cent, and then sells his shares at* $26\frac{1}{4}$. *What amounts did he receive in dividends, and what capital did he sacrifice by the sale?*

$$\text{1st dividend} = \pounds 20 \times 50 \times \frac{4}{100} = \underline{\pounds 40\ Ans.\ \text{(i)}.}$$

$$\text{2nd dividend} = \pounds 25 \times 50 \times \frac{3\frac{1}{2}}{100} = \underline{\pounds 43,\ 15s.\ Ans.\ \text{(ii)}.}$$

Total cost of shares $= £22\frac{1}{2} \times 50 + £5 \times 50 = £27\frac{1}{2} \times 50.$

Result of sale $= £26\frac{1}{4} \times 50,$

$\therefore$ loss of capital $= £27\frac{1}{2} \times 50 - £26\frac{1}{4} \times 50$

$= £1\frac{1}{4} \times 50 =$ £62, 10*s.* *Ans.* (iii).

The following examples in stocks are more complex than those previously given:—

EXAMPLE xix.—*If when* $2\frac{3}{4}$ *per cent Consols were quoted at* $111 - 111\frac{1}{2}$ *a person invested* £2000, *and sold out when they were quoted at* $109\frac{3}{8} - 109\frac{5}{8}$, *what capital would he sacrifice?* (*Brokerage* $\frac{1}{8}$ *p.c.*).

He buys at $111\frac{1}{2}* + \frac{1}{8} = 111\frac{5}{8}$, and sells at $109\frac{3}{8} - \frac{1}{8} = 109\frac{2}{8}$;

$\therefore$ he reduces his capital in the ratio $\dfrac{109\frac{2}{8}}{111\frac{5}{8}} = \dfrac{874}{893}.$

$$\text{Hence his loss} = £2000 - £2000 \times \frac{874}{893} = £2000 \times \left(1 - \frac{874}{893}\right)$$

$$= £2000 \times \frac{19}{893} = £2000 \times \frac{1}{47} = £42,\ 11s. \text{ nearly } Ans.$$

EXAMPLE xx.—*What is the market price of* 4 *per cent Railway Debenture stock, if by buying this stock with the proceeds of the sale of* £3200 $2\frac{1}{2}$ *per cent Consols at* 102, *an increase of* £1, 10*s. in annual income results?* (*Brokerage on Consols* $\frac{1}{8}$; *on Railway stock,* $\frac{1}{4}$ *per cent.*)

Original income $= £32 \times 2\frac{1}{2} = £80.$

New income $= £80 + £1\frac{1}{2} = £81\frac{1}{2},$

and the amount of Ry. stock which yields this inc. $= £81\frac{1}{2} \times \frac{100}{4}$ stock.†

But the Consols sold for $£101\frac{7}{8} \times 32$ cash $= £3260$ cash.

Hence, as $\overset{\text{Stock.}}{81\frac{1}{2}} \times \frac{100}{4}$ is bought for $\overset{£}{3260}$,

$\therefore$ 100 .... $3260 \times \dfrac{100}{81\frac{1}{2} \times \frac{100}{4}} = £160.$

But this cost includes the brokerage,

$\therefore$ the market price of the Ry. stock $= 160 - \frac{1}{4} = 159\frac{3}{4}$ *Ans.*

* See Note, p. 220. † See Note, p. 217 (II).

Example xxi.—*A person invests £5500 partly in a 4 per cent stock at 130, and partly in a 3 per cent stock at 117, and receives the same income from each. Find the amount he invests in each stock, and his total income.*

An income of £4 results from investing £130 in the 1st stock,

∴ .... 1 .... $\frac{130}{4} = £\frac{65}{2}$ ....

Also .... 3 .... 117 ....2nd ......

∴ .... 1 .... $\frac{117}{3} = £39$ ....

Hence the sums invested must be in the ratio $\frac{65}{2} : 39$

i.e. 5 : 6.

[We therefore divide £5500 into parts proportional to 5 and 6.]

∴ the reqd parts are $\frac{5}{11}$ of £5500, $\frac{6}{11}$ of £5500,

i.e. £2500 must be invested in the 4 p. c.'s and £3000 .... 3 ...... } *Ans.* (i).

Again, total income = twice $£2500 \times \frac{4}{130}$ * $= £\frac{2000}{13}$

= £153, 16*s.* 11*d.* *Ans.* (ii).

Example xxii.—*If a person invests £1000 in $2\frac{3}{4}$ per cents at 99, how much must he also invest in 4 per cents at $112\frac{1}{2}$ so as to obtain interest at the rate of 3 per cent on his whole investment?*

On £100 invested in the 1st stock he obtains $£100 \times \frac{2\frac{3}{4}}{99} = £2\frac{7}{9}$ inc.

... £100 .... 2nd .... $100 \times \frac{4}{112\frac{1}{2}} = £3\frac{5}{9}$ ...

But the *average* income on £100 invested is £3.

Thus, on the 1st investment he obtains $\frac{2}{9}$ per cent *below* the average.

and .... 2nd .... $\frac{5}{9}$ .... *above* ....

Hence the sum invested in the 1st stock must bear to that invested in the 2nd the ratio of 5 to 2.

∴ the reqd sum = $\frac{2}{5}$ of £1000 = £400 *Ans.*

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* See note on page 217 (III).

LVIII. FOREIGN MONEY.

DECIMAL COINAGES.

With the exception of England and India, all important countries have *decimal* money systems.

In all such systems there are two *principal** coins, *the smaller of which is always some* **decimal part** (generally $\frac{1}{100}$) *of the larger.*

For instance, in France these coins are the *franc*, and the *centime* $= \frac{1}{100}$ of a franc.

In Germany they are the *mark*, and the *pfennig* $= \frac{1}{100}$ of a mark.

Compared with a non-decimal system a decimal system of money has the advantage of greatly simplifying calculations.

Reduction from one denomination to another involves *no labour*, being performed *at sight.*

For instance, as centimes are *hundredths* of francs, 6 francs 15 centimes is at once written either as 6·15 *francs*, or as 615 *centimes*; and 17 marks 8 pfennigs either as 17·08 *marks*, or as 1708 *pfennigs.*

Consequently, the complex processes of our Compound Rules are replaced by the Simple Rules in decimals.

For instance, 17 francs 5 centimes × 30 = 17·05 francs × 30 = 511·50 francs
= 511 francs 50 centimes.

Note.—On the other hand, our system is not altogether without its own advantages. The use of *three* principal coins enables us to express large sums with fewer figures. Also, the number of exact divisors in 240 is much greater than in 100.

The following table gives the names of the principal coins of the chief countries using a decimal system of money:—

Country.	Principal Coins.		Approximate Value in English money.†
France, Belgium, Switzerland	100 **centimes** (*c.* or *cts.*)	= 1 **franc** (*fr.* or *F.*)	$9\frac{1}{2}d.$
Italy	100 **centessimi**	= 1 **lira** (*l.* or £)	$9\frac{1}{2}d.$
Greece	100 lepta	= 1 drachma	$9\frac{1}{2}d.$
Spain	100 **centimos**	= 1 **peseta** (*pes.*)	$9\frac{1}{2}d.$
Germany	100 **pfennige** (*pf.*)	= 1 mark (*M.*)	$11\frac{3}{4}d.$
Denmark, Sweden, Norway	100 öre	= 1 kröna	13*d.*

* *i e.* those coins in terms of which a sum of money is expressed, or in which accounts are kept. In our non-decimal system we use three principal coins, namely, £, *s.* *d.*

† The values in English money will give the student an idea of the purchasing power of the coins (regarded as *tokens*) in their own country. But, as will be seen in Chap. LX., their *exchange* values fluctuate.

Country.	Principal Coins.		Approximate Value in English money.
Holland	**100 cents** (*c.*)	= **1 florin** (*fl.*) or **guilder**	20*d.*
Austria	**100 kreuzer** (*kr.*)	= **1 florin** (*fl.*) or **gulden** (*g.*)	20*d.*
Russia	**100 kopeks** (*kop.*)	= **1 rouble** (*R.*)	37½*d.*
Turkey	100 piastres	= 1 Turkish pound (£*T*)	18*s.* 0¾*d.*
United States	**100 cents**(*c.* or *cts.*)	= **1 dollar** ($)	50*d.*
Mexico and South America*	100 centavos	= 1 peso, or Mexican dollar	48*d.*
Japan	**100 sen**	= **1 yen**	48*d.*
Portugal	**1000 reis**	= **1 milreis** (*mlr.*)	53*d.*
Brazil	1000 reis	= 1 milreis	27*d.*
China	**1000 cash**	= **1 tael**	6*s.* 6*d.*

Notes.—France, Belgium, Switzerland, Italy, and Greece form what is known as the Latin Union; their coins are all alike in weight and fineness, differing only in name, and they circulate in any country of the Union.

The *cent* of the United States is very different from the *centime* of the Latin Union; 1 *cent* = about 5 *centimes.*

For most practical purposes, results *correct to the nearest* 5 *centimes* of the Latin Union are sufficiently accurate.

Besides the English, the Indian is the only important *non*-decimal coinage.

INDIAN MONEY TABLE.

4 pice = 1 anna; 16 **annas** = 1 **rupee.**

Notes.—The *nominal* value of a rupee is 1*s.* 10½*d.*; but its *exchange* value is now much less.

The sum of 100000 rupees (*Rs.*) is called a lac.

In all British colonies English coins are current.

Also in the Straits Settlements, Hong-kong, &c., a British *dollar*, equal in value to the Japanese *yen*, circulates.

All the above countries (except the United States) adopt in their currency the principle of **monometallism,** *i.e. one* metal only (usually gold) has a fixed standard value.

The United States still retains *bimetallism* (formerly in vogue in England and France), *i.e.* both silver and gold are *standards.*

Note.—In a country (*e.g.* India) which has no gold coinage, the standard of value is *silver.*

* Except Brazil.

EXAMPLE i.—*Add* 89 *fr.* 78 *c.*; 17 *fr.* 5 *c.*; 113 *fr.* 50 *c.* and 8 *fr.* 24 *c.**

$$\begin{array}{r} \text{fr.} \\ 89{\cdot}78 \\ 17{\cdot}05 \\ 113{\cdot}50 \\ 8{\cdot}24 \\ \hline 228{\cdot}57 \end{array}$$

Ans. 228 *fr.* 57 *c.*,
or approximately to the nearest 5 cents,
228 *fr.* 55 *c.*

EXAMPLE ii.—*Multiply* 7 *M.* 20 *pf. by* 24.

$$\begin{array}{r} \text{M.} \\ 7{\cdot}20 \\ 24 \\ \hline 28\ 80 \\ 144\ 0 \\ \hline 172{\cdot}80 \end{array}$$

Ans. 172 *M.* 80 *pf.*

EXAMPLE iii.—*Divide* 637 *fr.* 56 *c. by* 45.

$$\begin{array}{r|l} 5 & 637{\cdot}56 \\ \hline 9 & 127{\cdot}512 \\ \hline & 14{\cdot}168 \end{array}$$

Ans. 14 *fr.* 16·8 *c.*
or, to the nearest centime, 14 *fr.* 17 *c.*
or, to the nearest 5 centimes, 14 *fr.* 15 *c.*

EXAMPLE iv.—*Taking a mark as equivalent to* $11\frac{3}{4}d.$, *express* 427 *M.* 75 *pf. in* £, *s. d. to the nearest penny.*

This may conveniently be done by Practice.

From the value at 1*s.* we obtain the value at 1*d.*, and from that the value at $\frac{1}{4}d.$ We then *cancel* the 1*d.* line and subtract.

$$\begin{array}{r|rl} & s. & \\ 1d. = \frac{1}{12} & 427{\cdot}75 & = \text{value at } 1s. \text{ per mark.} \\ \frac{1}{4}d. = \frac{1}{4} & \cancel{35{\cdot}64}\,.. & = \ldots\ldots\ldots\ \cancel{1d.}\ \ldots\ldots\ldots \\ & 8{\cdot}91\,.. & = \ldots\ldots\ldots\ \tfrac{1}{4}d.\ \ldots\ldots\ldots \\ \hline & 418{\cdot}84 & = \ldots\ldots\ldots\ 11\tfrac{3}{4}d.\ \ldots\ldots\ldots \end{array}$$

Ans. £20, 18*s.* 10*d.*

EXAMPLE v.—*Express* £326, 13*s.* 7*d. in dollars and cents, supposing a dollar to be equivalent to* 4*s.* 2*d.*

As 1 dollar = 50*d.*, ∴ 1 cent = $\frac{1}{2}d.$

If, then, we reduce the English money to *half-pence* we obtain the equivalent number of *cents.*

$$\begin{array}{r} \pounds\quad s.\quad d. \\ 326\ .\ 13\ .\ 7 \\ 20 \\ \hline 6533 \\ 12 \\ \hline 78403 \\ 2 \\ \hline 156806 = \$1568{\cdot}06 \end{array}$$

Ans. \$1568, 6 *cts.*

---

* It is more usual abroad to *name the higher denomination only*, and instead of, *e.g.*, 8 *fr.* 24 *c.*, to write 8$^{\text{fr.}}$, 24; or *F* 8, 24; or 8·24 *fr.*

LIX. THE METRIC SYSTEM.

The **Metric System** is the *decimal* system of Weights and Measures devised by the French at the time of the great Revolution. It is now in general use in all the chief countries of Europe except Russia* and England, and there it has been adopted for *scientific* purposes.

The *standard unit of length* in this system is called the **metre** (Greek, *measure*). The standard units of area, volume, capacity, and weight are all derived from the metre, which is thus the fundamental unit of the system to which it gives its name.† All other units of the system are either **decimal** multiples of, or **decimal** parts of, their respective standard units. Those which are decimal **multiples** are denoted by the *Greek* prefixes

deca- (10), **hecto-** (100), **kilo-** (1000)‡;

those which are decimal **parts**, by the *Latin* prefixes

deci- $\left(\frac{1}{10}\right)$, **centi-** $\left(\frac{1}{100}\right)$, **milli-** $\left(\frac{1}{1000}\right)$.

For instance, a **deca**-metre is 10 metres; a **deci**-metre is $\frac{1}{10}$ of a metre; and a **kilo**-metre is 1000 metres; a **milli**-metre is $\frac{1}{1000}$ of a metre.

N.B.—The *Latin* prefixes denoting **parts** of a standard unit, *all end in the letter* **i**.

The metric, being a *decimal* system, possesses the following advantages over a non-decimal system which have already been noticed in the case of decimal money.

"Reduction" involves *no labour*, for it consists simply in either multiplying or dividing by a *power of ten.*

For instance, 5 *kilometres* is 5000 *metres*; 45 *metres* is ·045 of a *kilometre*; 6·5 *decametres* is 650 *decimetres*; 1·5 *decimetres* is ·15 of a *metre.*

Also, 8 *kilometres* 50 *metres* is either 8·050 *kilometres,* or 8050 *metres.*

Consequently, it is unnecessary to write the name of more than one denomination in the measure of any quantity. Thus the necessity for anything corresponding to the English "Compound" Rules is avoided, and all operations are performed by simple addition, subtraction, &c., in decimals.

Note.—The *decimal point* is not always used in foreign countries, a comma, or a gap, denoting its *position*; thus, $6^{m}05$, or 6,05 m., stands for 6·05 metres, *i.e.* for 6 metres 5 centimetres.

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* Russia and the United States of America, though possessing a *decimal coinage,* have not adopted a decimal system of Weights and Measures. In the United States the English Weights and Measures are still retained.

† The inventors of the system chose as the fundamental unit a ten-millionth part of a quarter-meridian of the globe; the length they thus obtained is preserved in a metal rod —the standard metre—kept at Paris.

‡ The Greek prefix **myria-** (10,000) is also sometimes used.

Moreover, when once the meanings of the *names* of the various units are known, the "tables" of the metric system (unlike our complex English tables) impose *no* tax on the memory.

Note.—Those parts of the following tables which are printed in thick type are specially useful.

LENGTH.

Standard unit, the **Metre.**

Multiples.

1 **kilo**-metre (*Km.*) = **1000** metres.
1 *hecto*-metre (*Hm.*) = 100 metres.
1 *deca*-metre (*Dm.*) = 10 metres.

Hence 10 Dm. = 1 Hm., 10 Hm. = 1 Km.
Also, **1 metre** = ·1 Dm. = ·01 Hm. = **·001 Km.**

Parts.

1 *deci*-metre (*dm.*) = $\frac{1}{10}$, or ·1, of a metre.
1 **centi**-metre (*cm.*) = $\frac{1}{100}$, or **·01**, of a metre.
1 *milli*-metre (*mm.*) = $\frac{1}{1000}$, or ·001, of a metre.

Hence 10 mm. = 1 cm., 10 cm. = 1 dm.
Also, **1 metre** = 10 dm. = **100 cm.** = 1000 mm.

Note.—The kilometre, metre, and centimetre are the units of length commonly employed; the others are seldom used.

For instance, the length **3675 metres** *might* be written and read as "3 *kilo*metres, 6 *hecto*metres, 7 *deca*metres, 5 metres"; but it is much more usual and convenient to write it thus, 3·675 *Km.*, and to read it thus, "3 *kilometres*, 675 *metres*".

EXAMPLE i.—*Express* 47·6 *metres* (i) *in kilometres*; (ii) *in centimetres.*

(i) [As 1000 metres = 1 kilometre, in order to express metres in kilometres, we have but to **divide** by 1000; i.e. *to move the decimal point* **three** *places to the* **left**. Hence—]

47·6 metres = ·0476 kilometres *Ans.* (i).

(ii) [As 1 metre = 100 centimetres, in order to express metres in centimetres, we have but to **multiply** by 100; i.e. *to move the decimal point* **two** *places to the* **right**. Hence—]

47·6 metres = 4760 centimetres *Ans.* (ii).

EXAMPLE ii.—*Find, in metres, the sum of* ·04 *Km.*, 86·5 *Dm.*, 3·4 *m.*, 3·4765 *Km.*, 213 *mm.*

[We first express all the quantities in the same denomination, *metres*, and then add the resulting decimals.]

Ans. 4385·113 *m.*

```
      m.
    40·
   865·
     3·4
  3476·5
      ·213
  --------
  4385·113
```

EXAMPLE iii.—*Multiply* 2 *Km.* 85 *m. by* 34.

[We first express the given quantity in the *single* denomination *kilometres*, and then multiply the resulting decimal by 34.]

Ans. 70·89 *Km.*; *or* 70 *Km.* 890 *m.*

```
   Km.
  2·085
     34
  -----
  8 340
 62 55
 ------
 70·890
```

EXAMPLE iv.—*Divide* 217 *m.* 50 *cm. by* 23.

[We first express the given quantity in the single denomination *metres*, and then divide the resulting decimal by 23, until the quotient obtained is sufficiently accurate for our purpose.

(i) If, in this case, we carry the division to *two* places of decimals, the result is 9 *metres* 45 *centimetres*, with the remainder 15 *centimetres*.

(ii) If the next figure in the quotient be obtained we have the more accurate result, 9 *metres* 45·6 *centimetres*, with the remainder 12 *millimetres*, *i.e.* 1·2 *centimetre*.]

```
       m.              m.
23)217·50         (9·45
   207
   ---                  m.
    10 5           0(9·456
     9 2
    ----
     1 30
     1 15
    -----
(i) ...... 15    150
                 138
                 ---
(ii) ............ 12
```

Correct to the nearest centimetre, the result is 9 *metres* 46 *centimetres*.

EXAMPLE v.—*Find the cost of* 16 *metres* 35 *centimetres of silk at* 5 *francs* 60 *centimes per metre.*

[16 metres 25 centimetres = 16·25 *metres*, and 5 francs 60 centimes = 5·6 *francs*. Hence]

```
                  francs.
Required cost =  5·6 × 16·25
              = 91·56 francs
              = 91 francs 56 centimes Ans.
```

```
 16·35
   5·6
 -----
 98 10
817 5
------
 91·560
```

EXAMPLE vi.—*If* 36 *metres* 75 *centimetres of cloth cost* 154 *francs* 35 *centimes, find, to the nearest* 5 *centimes, the value of* 21 *metres* 80 *centimetres of the cloth.*

metres. francs.
36·75 cost 154·35

$\therefore$ 21·8 ...... $154{\cdot}35 \times \dfrac{21{\cdot}8^*}{36{\cdot}75} = \overset{\text{fr.}}{91{\cdot}56}\ldots =$ 91 *fr.* 55 *c. Ans.*

* The working, having already been exemplified under Decimals, is omitted here.

AREA.

Standard unit, the **Square Metre**;

(*i.e.* an amount of surface equal to the area of a square each of whose sides is one metre long).

We have seen (page 30) that a table of *Square* measure is obtained by *squaring Long* measure.

Hence, as 1 metre = 100 centimetres,

∴ 1 **square metre** (*sq. m.*) = 10000 **sq. centimetres** (*sq. cm.*).

And, as 1 decametre = 10 metres.

∴ 1 **square decametre** (*sq. Dm.*) = 100 **square metres.**

And so on.

In measuring land* the *square decametre* (*i.e.* 100 square metres) is taken as the principal unit; this is called an **Are**, and the Greek and Latin prefixes are applied to this unit.

Thus 1 **hect**-are (*Ha.*) = 100 ares (*a.*).

1 dec-are (*Da.*) = 10 ares.

1 deci-are (*da.*) = $\frac{1}{10}$ of an are.

1 **centi**-are (*ca.*) = $\frac{1}{100}$ of an are (= 1 *sq. metre*).

Note.—The student must be careful here to avoid confusion:—

e.g. the decametre is a measure of *length*,

∴ a square *decametre* is not *ten* (but 100) square *metres.*

But the are is a measure of *surface*, ∴ a *decare* is ten *ares.*

VOLUME.

Cubic Measure.

Standard unit, the **Cubic Metre**;

(*i.e.* the volume of a cube whose edge is one metre in length,

A table of *Cubic* measure is obtained by *cubing Long* measure.

Hence, as 1 metre = 100 centimetres,

∴ 1 **cubic metre** (*c.m.*) = 1000000 **cubic centimetres** (*c.cm.*).

And, as 1 decimetre = $\frac{1}{10}$ of a metre,

∴ 1 **cubic decimetre** (*c.dm.*) = $\frac{1}{1000}$ of a **cubic metre.**

And so on.

The cubic metre is sometimes called a **stere**, to which, of course, the Latin and Greek prefixes may be applied.

Thus 1 deca-stere = 10 steres; 1 deci-stere = $\frac{1}{10}$ of a stere.

---

* In the case of a *very large* portion, the area would be expressed in square kilometres.

*Capacity.**

The principal unit of capacity is the *cubic decimetre*, which is called a **Litre**.

1 **hecto**litre (*Hl.*) = 100 litres.

1 decalitre (*Dl.*) = 10 litres.

1 decilitre (*dl.*) = $\frac{1}{10}$ of a litre.

1 centilitre (*cl.*) = $\frac{1}{100}$ of a litre.

N.B.—As a **litre** is a *cubic decimetre, i.e.* $\frac{1}{1000}$ of a cubic metre,

$\therefore$ **1000 litres = 1 cubic metre** .... (I).

WEIGHT.

Standard unit, the **Gramme**.

The gramme, like all the other standards of the Metric System, is derived from the fundamental unit of length, the *metre*.

A **gramme** is the weight of a *cubic centimetre* of pure water when at its greatest density.†

1 **kilo**gramme (*Kg.*) = 1000 grammes.

1 hectogramme (*Hg.*) = 100 grammes.

&c.

As the gramme is a very small unit, the *kilogramme* is the principal unit employed for general purposes (such as the English pound Av. is used for). For large bulks the following units are employed:— the **quintal** = 100 *kilogrammes*.

and the **tonneau** (*T.*) = 1000 *kilogrammes*.

N.B.—As 1 decimetre = 10 centimetres,

$\therefore$ 1 *litre, i.e.* 1 *cubic* decimetre, = 1000 *cubic* centimetres.

Hence, a **litre of water** weighs **1000 grammes**......(II).

Note.—It is unnecessary to give many easy examples in Area, Volume, and Weight, as they are worked in *exactly* the same way as those given under Length.

EXAMPLE vii.—*Find the cost of a plot of building land containing 2 hectares* 17 *ares, at* 3 *francs* 50 *centimes per square metre.*

3 hectares 17 ares = 217 ares
= 21700 sq. metres.

$\therefore$ req^{d} cost = $\overset{\text{fr.}}{3{\cdot}5} \times 21700$ = 75950 francs *Ans.*

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* Used for *Liquids*, &c., as stated on page 31.

† *i.e.* weighed under certain conditions as regards both atmospheric pressure and temperature.

EXAMPLE viii.—*Find the greatest* (i) *quantity,* (ii) *weight, of water which could be contained in a rectangular reservoir 23 metres 20 centimetres long, 16 metres 5 centimetres wide, and 2 metres 25 centimetres deep.*

(i) Volume of water $= 23{\cdot}2 \times 16{\cdot}05 \times 2{\cdot}25$, *cubic metres*
$= 23{\cdot}2 \times 16{\cdot}05 \times 2{\cdot}25 \times 1000$ *litres**
$= 232 \times 1605 \times 2{\cdot}25$ *litres*
$= 837810$ *litres*
$= 8378{\cdot}1$ *hectolitres Ans.* (i).

(ii) Hence weight of water $= 837810 \times 1000$ *grammes*†
$= 837810$ *kilogrammes*
$= 837{\cdot}81$ *tonneaux Ans.* (ii).

CONVERSION OF METRIC INTO ENGLISH MEASURES, AND VICE VERSÂ.

Correct to five places of decimals

1 metre = 39·37079 inches.

And 1 English gallon contains 277·274 cubic inches.

By help of these two equivalents we can, approximately, by the methods of Chapter L, convert any given Metric, into the corresponding English, measure, or *vice versâ.*

N.B.—Since 1 *metre* $= 39{\cdot}37079$ *inches,*
$\therefore$ 1 square *metre* $= (39{\cdot}37079)^2$ *square inches*;
and 1 cubic *metre* $= (39{\cdot}37079)^3$ *cubic inches.*

Note.—The following *rough* approximations should be borne in mind. Though not accurate enough for use in actual calculations, they afford the means of making a rough rapid, or even mental, test of results calculated from more accurate data. In this way any great error in the result of a long calculation (such as would be caused by the misplacing of a decimal point) may at once be detected.

A metre	is about 39 inches,	a litre	is about $1\frac{3}{4}$ pints‡
a kilometre	.... 5 furlongs,	a *hectolitre*	.... 22 *gallons.*
a centimetre	.... $\frac{3}{8}$ of an inch.	a kilogramme	.... $2\frac{1}{5}$ lbs. Av.,
a *hectare*	.... $2\frac{1}{2}$ *acres,*	a *gramme*	.... 15 *grains,*
an *are*	.... 4 *poles.*	a *quintal*	.... 2 *cwts.*

* See (I), on page 232. † See (II), on page 232. ‡ *i.e.* a "reputed quart".

EXAMPLE ix.—*Express 5 metres 24 centimetres in feet and inches correct to the nearest eighth of an inch.*

$$\begin{aligned} \text{5 metres 24 centimetres} &= 5{\cdot}24 \textit{ metres} \\ &= 5{\cdot}24 \times 39{\cdot}37079, \textit{ inches} \\ &= 206{\cdot}3 \textit{ inches} \\ &= \underline{17 \text{ ft. } 2\tfrac{1}{4} \text{ in. } \textit{Ans.}} \end{aligned}$$

$$\begin{array}{r} 39{\cdot}37\dot{0}\dot{7}\dot{9} \\ \underline{4\;25} \\ 196{\cdot}86 \\ 7\;87 \\ \underline{1\;57} \\ 206{\cdot}30 \end{array}$$

EXAMPLE x.—*Express 17 miles 250 yards in kilometres correct to the nearest metre.*

$$\begin{aligned} \text{17 mi. 250 yds.} &= 30170 \textit{ yds.} \\ &= 30170 \times 36 \textit{ in.} \\ &= \frac{30170 \times 36}{39{\cdot}37079} \textit{ metres} \\ &= 27587 \textit{ metres} \\ &= \underline{27{\cdot}587 \text{ Km. } \textit{Ans.}} \end{aligned}$$

$$\begin{array}{rrl} & 30170 & \\ & \underline{36} & \\ & 181020 & \\ & \underline{90510} & \text{metres.} \\ 3\dot{9}{\cdot}\dot{3}\dot{7}\dot{0}\dot{7}\dot{9}) & 1086120 & (27586{\cdot}9 \\ & \underline{787416} & \\ & 298704 & \\ & \underline{275595} & \\ & 23109 & \\ & \underline{19685} & \\ & 3424 & \\ & \underline{3149} & \\ & 275 & \\ & \underline{236} & \\ & 39 & \end{array}$$

Note.—Remembering that a *Km.* is about 5 fur., and applying this as a rough test, we see that our result is a reasonable one.

EXAMPLE xi.—*Express $2\frac{1}{2}$ hectolitres in gallons and pints correct to the nearest pint.*

$$\begin{aligned} 2{\cdot}5 \textit{ Hl.} = 250 \textit{ litres} &= \frac{250}{1000} \textit{ cubic metre.} \quad \text{[See (I), page 232.]} \\ &= \tfrac{1}{4} \textit{ cubic metre} \\ &= \tfrac{1}{4} \times (39{\cdot}37079)^3 \textit{ cubic inches} \\ &= \tfrac{1}{4} \times \frac{(39{\cdot}37079)^3}{277{\cdot}274} \textit{ gallons.*} \\ &= \underline{55 \text{ gallons } \textit{Ans.}} \end{aligned}$$

*We calculate the value of this expression to *one* place of decimals in the final quotient. See Chap. L for the method.

EXAMPLE xii.—*Express 1 acre 3 ro. 15 po. in ares, correct to the nearest square metre.*

$$\begin{aligned} \text{1 ac. 3 ro. 15 po.} &= 8923{\cdot}75 \textit{ sq. yds.} \\ &= 8923{\cdot}75 \times 9 \times 144 \textit{ sq. inches} \\ &= \frac{8923{\cdot}75 \times 9 \times 144}{(39{\cdot}37079)^2} \textit{ sq. metres.}\dagger \\ &= 7461 \textit{ sq. metres} \\ &= \underline{74{\cdot}61 \textit{ ares Ans.}} \end{aligned}$$

† We calculate this, to the nearest integer, by the methods of Chap. L. See Ex. xxi, p. 184.

Note.—For further examples of this class see Chapter LXI.

LX. FOREIGN EXCHANGES.

A debt due to a person in a foreign country might be paid in any one of the following three ways—

(1) by transmitting gold coins (**specie**);
(2) by transmitting gold bars (**bullion**);
(3) by means of a **Bill of Exchange** (see Chap. LVI.).

If either the first or second method be chosen it is necessary (in order to determine the amount of gold which must be sent to discharge any particular debt) to know the comparative value of a gold* coin of each of the two countries, *i.e.* the ratio of the amounts of pure gold* in them. This is called the **Par** of *Exchange.*

For instance, there is as much gold in 1000 *sovereigns* as there is in 2522 *ten-franc* pieces; and so the *par of exchange* between England and France is £1 = 25·22 *francs.*

The trouble and expense, however, of transmitting specie or bullion is considerable, and so these methods are very seldom adopted; the third is the usual method.

Now the *cost*, in the coin of one country, of a Bill of Exchange, payable in the coin of another country, is *not* determined from the *Par* of Exchange. Foreign Bills, like other marketable things, fluctuate in price, their market rate rising or falling with any increase or decrease in the demand for them. This market rate is called the **Course** of *Exchange*, or the **Rate** of *Exchange*, and the cost of a Foreign Bill depends upon the *course of exchange at the time* it is bought.

EXAMPLE i.—*Exchange £472, 10s. into francs at 25·16 francs for £1.*

$$£472{\cdot}5 = \overset{\text{francs.}}{25{\cdot}16} \times 472{\cdot}5 = \underline{11888{\cdot}10 \text{ francs.}}\ \textit{Ans.}$$

EXAMPLE ii.—*Exchange £320 into rupees when the rate of exchange is 1s. $2\frac{3}{8}$d. for 1 rupee.*

As $\overset{d.}{14\frac{3}{8}}$ is equivalent to $\overset{\text{rupee.}}{1}$

$\therefore 320 \times 240$ .... $\dfrac{320 \times 240}{14\frac{3}{8}}$ rupees

$= 5342\frac{14}{23}$ rupees

$=$ *Rs.* 5342, 10 *annas*, nearly. *Ans.*

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* *Silver*, in the case of two countries having a *silver standard.*

EXAMPLE iii.—*Exchange* 5380 *marks into sterling at* $20\cdot34\frac{1}{2}$ *m. for* £1.

$$\begin{array}{lll} \text{marks.} & & \pounds \\ 20\cdot345 & \text{are equivalent to} & 1 \\ \therefore 5380 & \ldots\ldots\ldots\ldots & 1\times\dfrac{5380}{20\cdot345}=\pounds264\cdot438\ldots \\ & & =\pounds264,\ 8s.\ 9d.\ \textit{Ans.} \end{array}$$

EXAMPLE iv.—*Exchange* 3560 *tael into sterling at* 2*s.* 11*d. for* 1 *tael.*

$$\begin{array}{lll} \text{tael.} & & d. \\ 1 & \text{is equivalent to} & 35 \\ \therefore 3560 & \ldots\ldots\ldots\ldots & 3560\times35=\pounds519,\ 3s.\ 4d.\ \textit{Ans.} \end{array}$$

Note.—The student will notice a remarkable difference between the *exchange* value and the *nominal* value, as given on page 225, of the coin of a country having a *silver* standard. This is mainly due to the depreciation of late years in the value of silver. For instance, the *exchange* value of the rupee is, owing to this cause, far below its *nominal* value.

Courses of exchange are published twice a week in the London daily newspapers, namely the rates then ruling "on 'Change" *in London* with respect to the money of various foreign countries, and also the rates *telegraphed from various foreign centres* as then ruling in them with respect to English money. These "quotations" are usually given in one of two ways; in the case of some countries the value of £1 *in the foreign coinage* is given; in the case of others, the value of *the principal foreign coin* is given in **pence**, or in shillings and pence.

For instance, "Paris, $25\cdot19\frac{1}{2}$" means 25 *francs* $19\frac{1}{2}$ *centimes* for **£1.**
"Vienna, $11\cdot97$" ...... 11 *gulden* 97 *kreuzer* ... **£1.**
"Lisbon, $40\frac{1}{2}$" ...... $40\frac{1}{2}$ *pence* ... **1 milreis.**

Note.—In the cases of St. Petersburg, Amsterdam, and New York, two ways are used. Quotations *telegraphed from* St. Petersburg appear in the form "$93\cdot70$", which means 93 *roubles* 70 *kopeks* for **£10**; but the *London* rate appears in the form "$25\frac{1}{4}$", which means $25\frac{1}{4}d.$ for 1 rouble.

The rate *telegraphed from* Amsterdam appears in the form "$12\cdot09$", which means 12 *florins* 9 **cents** for £1; but the *London* rate appears in the form "12 $2\frac{3}{4}$", which means 12 *florins* $2\frac{3}{4}$ **stivers*** for £1.

The rate *telegraphed from* New York appears in the form "$4\cdot86$", which means 4 *dollars* 86 *cents* for £1; whereas the *London* rate takes the form "$48\frac{5}{8}$", *i.e.* $48\frac{5}{8}d.$ for $1.

In such "quotations" the coin represented by **unity** is called the "fixed price"; and its value in other coins the "variable price".

Note.—The extent to which the course of exchange between two countries fluctuates above or below their par of exchange is limited by the cost of transmitting *gold*; for no one would care to buy *bills* with which to defray a debt, if he could *at less cost transmit gold*; these limits are called **specie points.**

* Here $2\frac{3}{4}$ is *not* a decimal of a florin; 1 *florin*=20 stivers.

EXAMPLE v.—*Exchange £218, 12s. 6d. into marks at* 20·38.

$$\begin{array}{llll} & \pounds & & \text{marks.} \\ \text{As} & 1 & \text{is equivalent to} & 20{\cdot}38 \\ \therefore & 218{\cdot}625 & \ldots\ldots\ldots\ldots & 20{\cdot}38 \times 218{\cdot}625 \\ & & & = \underline{4455{\cdot}58\,M.\ Ans.} \end{array}$$

EXAMPLE vi.—*Exchange £87, 13s. 5d. into Dutch money at* 12 $1\frac{3}{4}$.

$$\begin{array}{ll} \pounds & \text{florins.} \\ \text{As } 1 = 12\,\textit{florins}\ 1\tfrac{3}{4}\ \textit{stivers} = 12\tfrac{7}{80}\,\textit{florins} & = 12{\cdot}0875 \\ \therefore 87{\cdot}652\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots & = 12{\cdot}0875 \times 87{\cdot}652 \\ & = \underline{1059{\cdot}49\ \text{florins.}\ Ans.} \end{array}$$

When the Course of Exchange quotations are for cheques, or bills payable *at sight* (*i.e.* for bills payable without delay) they are called **short** rates; when the quotations are for bills the payment of which is deferred, they are called **long** rates.

For instance, "Paris, cheques, 25·16"; "Vienna, sight, 12·97"; "Hong-Kong, T.T.*, 2s. $1\frac{3}{8}$d.", are specimen quotations of *short* rates.

"St. Petersburg, 3 mo., 93·70"; "New York, 60 days, $48\frac{3}{4}$", are specimen quotations of *long* rates.

The difference at any particular date between a long, and the corresponding short, rate is the *discount* (at the current rate per cent of discount) for the time named in the long rate.

N.B.—In calculating this discount in order to find the short rate corresponding to a given long rate, or vice versâ, *results of sufficient accuracy are obtained by simply taking, to 3 places of decimals.* **the interest on the variable price** *mentioned, and either* **adding** *this interest to, or* **subtracting** *it from, the variable price, according as the case requires a greater or less variable price.*

The fluctuations in the Course of Exchange between two countries are due to several causes, the chief of which is called the *Balance of Trade.*

Thus, if the exports from England to France were equal in value to those from France to England, accounts between the two countries could be settled *without transmitting any gold*, and the **short** Rate of Exchange would coincide with the Par of Exchange. But when the exports from France to England exceed in value those from England to France, and the *balance of trade* is in favour of France, the demand in London for bills, by which to pay for French goods, increases and their price rises.

Now if this were the sole cause the rate in London on Paris, and that in Paris on London would be the same on any particular date. But this is seldom the case as there are persons who speculate in Bills of Exchange, thus increasing the demand for them sometimes in the one place more than the other. Also news affecting the demand for Bills may reach the one place some hours before the other.

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* Telegraphic transfers.

EXAMPLE vii.—*Find the cost in London of a bill for* 6550 *francs payable in Paris when the Course of Exchange, London on Paris, is* 25·21.

Here the expression "London on Paris, 25·21" gives the rate which rules on 'Change in London, and means that £1 cash paid in London will buy a bill for 25·21 francs payable at Paris.

Hence, as a bill for 25·21 francs payable in Paris costs £1 cash in London

∴ .... 6550 .... $\frac{6550}{25 \cdot 21}$ ....

$= £259 \cdot 817$

$= £259, 16s. 4d.$ *Ans.*

EXAMPLE viii.—*Find the "sight" quotations corresponding to the following "long" rates,* (i) *London on Paris,* 3 *mo.,* 25·31; (ii) *Berlin on London,* 8 *days,* 25·35; *the rate of discount being* 4%.

(i) "London on Paris, 3 mo., 25·31", means that

£1 cash paid in London buys a bill for 25·31 *francs payable in* 3 *mo. at Paris.*

Now the interest on 25·31 francs for 3 mo. at 4% is

$$\overset{\text{fr.}}{25 \cdot 31} \times \frac{1}{4} \times \frac{4}{100} = \overset{\text{fr.}}{\cdot 2531}.$$

And as £1 will buy fewer francs *payable at sight than in* 3 *mo.* we subtract ·2531 fr. from 25·31 fr., obtaining as the short rate required 25·06 fr. nearly.

Ans. (i) 25·06.

(ii) "Berlin on London, 8 days, 25·35", means that

25·35 *marks cash at Berlin buys a bill for £1 payable in* 8 *days at London.*

Now the interest on 25·35 marks for 8 days at 4% is ·02 marks nearly.

And as more marks must be paid for a bill for £1 payable *at sight* than for a bill for £1 payable *in* 8 *days*, we add ·02 marks to 25·35 marks, obtaining 25·37 marks as the required "short" rate.

Ans. (ii) 25·37.

In actual transactions in bills the debtor has to pay *brokerage* and the *cost of the government stamp.*

In England the former is usually $\frac{1}{10}$ % (*i.e.* 1 per *mille**), and the latter 1*s.* % (*i.e.* $\frac{1}{2}$ per *mille*).

*1 per *mille*, or 1 *p.m.*, means 1 per *thousand.*

EXAMPLE ix.—*Find the cost (including the usual brokerage and stamp) in London of a bill for* 845 *roubles payable at sight at St. Petersburg, when the Course of Exchange is—London on St. Petersburg,* 3 *mo.,* 93·70, *and the Bank rate of discount is* 4 *per cent.*

[As £10 cash in London will buy a bill for *fewer* roubles payable *at sight* than in 3 *months,* we must *subtract* the discount.]

Now the discount (*i.e.* the int. on 93·70 *R.* for 3 mo. at 4%) = ·937 *R.* nearly; and 93·70 − ·937 = 92·76 *R.* nearly.

Hence

a *bill* for 92·76 roubles payable *at sight* at St. P. costs £10 *cash* in London.

$\therefore$ .... 845 .... $\dfrac{10 \times 845}{92{\cdot}76}$ ....

$= 91{\cdot}095\ldots$

But the brokerage at 1 *p.m.* = £91·095 ÷ 1000 = ·091...

And the stamp at $\frac{1}{2}$ *p.m.* = $\frac{1}{2}$ of £·091 = ·045...

$\therefore$, by addition, the total cost = 91·231...

= £91, 4*s.* 7*d.* *Ans.*

There are various ways in which the payment of a debt by means of a foreign bill of exchange may be effected, and owing to the fact that the Course of Exchange between two commercial centres is not generally the same at both at the same date* (*e.g.* London on Paris might be 25·21 when Paris on London is 25·22), the question arises how the debtor may most cheaply pay his foreign creditor; whether

(i) by *direct remittance.*

For instance, a debtor A in London owing 1000 francs to a creditor B in Paris, may buy for cash *in London,* at the London rate, a bill for 1000 francs payable at Paris, and post it to B.

Or (ii) by instructing his creditor to *draw* on him.

For instance, the debtor A in London may instruct his creditor B in Paris to *draw* a bill payable at London for A's acceptance. This bill B would "negotiate" (*i.e.* sell) in Paris.

Or (iii) by *indirect* exchange.

For instance, A in London, instead of treating directly with B in Paris, may discharge his debt through an agent C in some other place, say Hamburg, when the courses of exchange between London and Hamburg, and Hamburg and Paris, are such that the cost to A would be less this way than if he effected a direct exchange.

The rate of exchange which results in this third case (when one or more places intervene) between the two principal places is called an **Arbitrated** *Course of Exchange.*

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* See page 237.

EXAMPLE x.—*A London merchant wishes to pay for* 1000 *florins' worth of goods, bought in Vienna, when the rates of Exchange are—London on Vienna,* 3 *mo.,* $12\cdot12\frac{1}{2}$; *Vienna on London, sight,* $11\cdot97\frac{1}{2}$; *and the rate of discount in London is* 3 *per cent; will it be better for him to remit to Vienna, or for his creditor there to draw upon him in London?*

A bill for $12\cdot12\frac{1}{2}$ *fl.* payable *in* 3 *mo.* at Vienna costs £1 cash in London.

Now the int. on $12\cdot12\frac{1}{2}$ *fl.* for 3 mo. at 3% is $\cdot09$ *fl.* nearly,

$$\text{and } 12\cdot12\tfrac{1}{2} - \cdot09 = 12\cdot03\tfrac{1}{2},^{*}$$

∴ a bill for $\overset{\text{fl.}}{12\cdot03\frac{1}{2}}$ payable *at sight* at Vienna costs $\overset{£}{1}$ cash in London,

∴ .... 1000 .... $\dfrac{1000}{12\cdot03\frac{1}{2}}$ (i) ....

Again, $11\cdot97\frac{1}{2}$ *fl. cash* is obtainable *in Vienna* in exchange for a *bill* for £1 payable *at sight in London,*

∴ 1000 *fl.* .... £$\dfrac{1000}{11\cdot97\frac{1}{2}}$ (ii) ....

Now (i) is less than (ii), for its denominator is greater than that of (ii), while the numerators are equal.

Hence, in this case, the merchant will pay his debt more cheaply by *remitting.*

EXAMPLE xi.—*Find the arbitrated rate of exchange at London on New York, viâ Paris and Berlin, when the direct rates are—*

London on Paris, $25\cdot26$ (francs for £1);
Paris on Berlin, $123\cdot54$ (francs for 100 marks);
Berlin on New York, $4\cdot22$ (marks for \$1).

[The course of exchange, London on New York, is expressed in *pence per dollar*; we therefore convert \$1 into *pence*, by the *Chain Rule* exemplified on p. 146.]

$$\begin{aligned}
\text{Hence } \$1 &= 4\cdot22 \textit{ marks} \\
&= 4\cdot22 \times \frac{123\cdot54}{100}, \textit{francs} \\
&= 4\cdot22 \times 1\cdot2354, \textit{francs} \\
&= \frac{4\cdot22 \times 1\cdot2354}{25\cdot26}, \textit{pounds} \\
&= \frac{4\cdot22 \times 1\cdot2354}{25\cdot26} \times 240, \textit{pence} \\
&= \underline{49\tfrac{1}{2}d. \text{ nearly } \textit{Ans.}}
\end{aligned}$$

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* Subtracting, because £1 will buy fewer "*sight*" than "3 *mo.*" florins.

In coins the precious metal is mixed with alloy (see page 25). The *degree of purity* of the mixture is called its **fineness**.

For instance, British *standard gold* is "$\frac{11}{12}$ fine",* for it contains 22 parts by weight of *pure* gold mixed with 2 parts of *alloy*, and the ratio of 22 to 24 is expressed by $\frac{11}{12}$.

EXAMPLE xii.—*Find the par of exchange between the United States and England, having given that the gold eagle (of* 10 *dollars),* $\frac{9}{10}$ *fine, weighs* 258 *grains, and that* 1869 *sovereigns are coined from* 480 *Troy ounces of standard gold* $\frac{11}{12}$ *fine.*

[This is another example of the "Chain Rule".]

$$\begin{aligned}
£1 &= \frac{480}{1869} \text{ ozs. of } \textit{English standard} \text{ gold} \\
&= \frac{480}{1869} \times \frac{11}{12} \text{ ozs. of } \textit{pure} \text{ gold} \\
&= \frac{480}{1869} \times \frac{11}{12} \times \frac{10}{9} \text{ ozs. of } \textit{U.S. standard} \text{ gold} \\
&= \frac{480}{1869} \times \frac{11}{12} \times \frac{10}{9} \times \frac{480}{258} \textit{ dollars} \\
&= \underline{\$4 \cdot 866 \ldots \textit{ Ans.}}
\end{aligned}$$

EXAMPLE xiii.—*English standard* (22 *carat*) *gold is worth* £3, 17*s.* 10½*d. per Troy ounce;* 1395 *German reichsmarks,* 900 (*per mille*) *fine, are coined from* 1 *pfund* (500 *grammes*) *of pure gold;* 1 *Troy ounce is equivalent to* 31·1035 *grammes; hence obtain the mint par of exchange between England and Germany.*

£3, 17*s.* 10½*d.* = £3·89375; and 22 *carat* gold is $\frac{11}{12}$ fine; hence, by the "Chain Rule",

$$\begin{aligned}
£1 &= \frac{1}{3 \cdot 89375} \text{ ozs. of } \textit{English standard} \text{ gold} \\
&= \frac{1}{3 \cdot 89375} \times \frac{11}{12}, \text{ ozs. of } \textit{pure} \text{ gold} \\
&= \frac{1}{3 \cdot 89375} \times \frac{11}{12} \times 31 \cdot 1035, \textit{ grammes} \text{ of } \textit{pure} \text{ gold} \\
&= \frac{1}{3 \cdot 89375} \times \frac{11}{12} \times 31 \cdot 1035 \times \frac{1395}{500}, \textit{ marks} \\
&= \underline{20 \cdot 43 \ldots \textit{ marks, Ans.}}
\end{aligned}$$

* Or "22 *carats* fine". See note to page 25.

LXI. THE METHOD OF NINE MULTIPLES.

The method consists in the formation of a table of the products of some constant number, or decimalized quantity, by each of the digits 1, 2, ...9; and then using the table so as to obtain, *by addition alone*, the products of this constant by various other numbers.

It is a labour-saving process capable of being applied to many practical purposes,—*e.g.* calculating the cost, or weight, &c., of various quantities at a fixed price, or weight, &c., per unit; changing metric into English weights and measures, and *vice versâ*; exchanges.

For instance, suppose we frequently had occasion to obtain the product of the decimal ·24375 by various numbers. Then we should make the following table consisting of 1, 2, ... 9 times this decimal, and preserve the table for future use.

Table of multiples of ·24375.

1	·24375
2	·48750
3	·73125
4	·97500
5	1·21875
6	1·46250
7	1·70625
8	1·95000
9	2·19375

Now suppose we happen to require the product of this decimal by 385·7.

From line 3 of the table (*by moving the decimal point* two *places to the* right) we obtain	300 times the decimal	= 73·125
From line 8 of the table (*by moving the decimal point* one *place to the* right) we obtain	80	= 19·500
From line 5 of the table we obtain	5	= 1·21875
From line 7 of the table (*by moving the decimal point* one *place to the* left) we obtain	·7	= ·170625
∴, by addition,	385·7	= 94·014375

Note.—The table is easily formed, for line 4 is obtained by *doubling* line 2; line 7 by *adding* lines 3 and 4, &c.

EXAMPLE i.—*Make a table of multiples for use in calculating the cost of coal (tons and cwts.) at 11s. 7d. per ton, and use it in finding the cost of* (i) 453 *tons;* (ii) 72 *tons* 13 *cwts.*

11*s.* 7*d.* = £·579166..

ton.	£
1	·579166
2	1·158333
3	1·737500
4	2·316666
5	2·895833
6	3·475000
7	4·054166
8	4·633333
9	5·212500

[It is unnecessary to carry the table beyond 6 places of decs. unless we require the cost of numbers of tons exceeding 1000.]

Now, from the table,

		£
(i)	400 tons cost	231·6666
	50	28·9583
	3	1·7375
∴, by addition,	453	262·362

= £262, 7*s.* 3*d.* *Ans.* (i).

[Since cwts. are *twentieths* of a ton, cwts. are decimalized by multiplying their number by 5.]

(ii) 72 tons 13 cwts. = 72·65 tons, and from the table—

		£
	70 tons cost	40·5417
	2	1·1583
	·6	·3475
	·05	·0289
∴, by addition,	72·65	42·076

= £42, 1*s.* 6*d.* *Ans.* (ii).

EXAMPLE ii.—*Given that* 1 *metre* = 39·37079 *inches, make a table for use in converting feet into metres; and use it in the case of* (i) 1756 *ft.;* (ii) 309 *ft.* $7\frac{1}{2}$ *in.*

ft.	metres.
1 =	·3047945
2 =	·6095890
3 =	·9143835
4 =	1·2191780
5 =	1·5239725
6 =	1·8287670
7 =	2·1335615
8 =	2·4383560
9 =	2·7431405

As 39·37079 in. = 1 metre,

$$\therefore 1 \text{ foot, or } 12 \text{ in.} = \frac{12}{39{\cdot}37079} \text{ metre}$$

$$= {\cdot}\mathbf{3047945} \text{ metre.}$$

Now, from the table,

$$\text{(i)}\quad 1756 \text{ ft.} = \left\{\begin{array}{r} \text{metres.} \\ 304{\cdot}7945 \\ 213{\cdot}3562 \\ 15{\cdot}2397 \\ 1{\cdot}8288 \\ \hline 535{\cdot}219 \end{array}\right.$$

535·219 = 535 *m.* 22 *cm.* nearly *Ans.* (i).

$$\text{(ii)}\quad 309 \text{ ft. } 7\tfrac{1}{2} \text{ in.} = 309{\cdot}625 \text{ ft.} = \left\{\begin{array}{r} \text{metres.} \\ 91{\cdot}4383 \\ 2{\cdot}7431 \\ {\cdot}1828 \\ {\cdot}0061 \\ {\cdot}0015 \\ \hline 94{\cdot}372 \end{array}\right.$$

94·372 = 94 *m.* 37 *cm.* nearly *Ans.* (ii).

Square Root

LXII. SQUARE ROOT.

The **square root** of a given number is the number whose *square** is equal to the given number.

For instance, 3 is the square *root* of 9, for 3^2, *i.e.* $3 \times 3, = 9$;
and 7 is the square *root* of 49, for 7^2, *i.e.* $7 \times 7, = 49$.

The square root of a number is indicated by the symbol $\surd$.

For instance, $\sqrt{9}$ and $\sqrt{49}$ are read "*square root of* 9" and "*square root of* 49"; or, shortly, "*root* 9", "*root* 49".†

The **squares** of the first twelve numbers are known from the multiplication table, namely,

$$1^2 = 1,\ 2^2 = 4,\ 3^2 = 9,\ 4^2 = 16,\ 5^2 = 25,\ 6^2 = 36,\ 7^2 = 49,$$
$$8^2 = 64,\ 9^2 = 81,\ 10^2 = 100,\ 11^2 = 121,\ 12^2 = 144.$$

Hence, conversely, the following *square roots* are also known,

$$\sqrt{1} = 1,\ \sqrt{4} = 2,\ \sqrt{9} = 3,\ \sqrt{16} = 4,\ \sqrt{25} = 5,\ \sqrt{36} = 6,\ \sqrt{49} = 7,$$
$$\sqrt{64} = 8,\ \sqrt{81} = 9,\ \sqrt{100} = 10,\ \sqrt{121} = 11,\ \sqrt{144} = 12.$$

Hence also $\sqrt{400} = 20$, $\sqrt{1600} = 40$, $\sqrt{2500} = 50$ &c.,

and $\sqrt{10000} = 100$, $\sqrt{40000} = 200$, $\sqrt{160000} = 400$, $\sqrt{250000} = 500$, &c.

The following squares, with the corresponding square roots, should also be known:— $13^2 = 169$, $14^2 = 196$,

$$15^2 = 225,\quad 16^2 = 256,\quad 17^2 = 289,\quad 18^2 = 324,\quad 19^2 = 361.$$

From these tables it is evident that comparatively few numbers have *integral* square roots.

For instance, as the square root of 16 is 4, and the square root of 25 is 5, the square root of any number, say 19, between 16 and 25 must be *greater than* 4, *and less than* 5, and so cannot be an *integer*.

In fact, the square root of such a number (*e.g.* 19) cannot be *exactly* found, though, as we shall see later, we may find an approximation as close as we please.

A number which has an *exact* square root is called a **perfect square**.

For instance, 81 is a *perfect square*, for its square root is 9;
and $\frac{16}{25}$ is a *perfect square*, for, as $\frac{4}{5} \times \frac{4}{5} = \frac{16}{25}$, its square root is $\frac{4}{5}$.

When the factors of a **perfect square** are known, its square root is also known.

For instance, the square root of $3 \times 3 \times 7 \times 7 \times 2 \times 2$, *i.e.* of $(3 \times 7 \times 2)^2$, is evidently $3 \times 7 \times 2$, or 42.

---

* See p. 12.

† The sign $\surd$ was originally the letter *r*, the initial of the word *radix*, Latin for "root".

EXAMPLE i.—*Find, by factors, the square root of the perfect square* 156816.

We see that the given number is exactly divisible by 4, and that the quotient is also divisible by 4. Also that 9801 is divisible by 9, and the quotient by 9. (See Tests of Divisibility, p. 52.)

$$\begin{array}{r|l} 4 & 156816 \\ 4 & 39204 \\ 9 & 9801 \\ 9 & 1089 \\ 11 & 121 \\ & 11 \end{array}$$

Hence $156816 = 4 \times 4 \times 9 \times 9 \times 11 \times 11$.

$\therefore \sqrt{156816} = 4 \times 9 \times 11 = \underline{396}$ *Ans.*

EXAMPLE ii.—*Find, by factors, the square root of* $\frac{5625}{5929}$.

$$\sqrt{\frac{5625}{5929}} = \sqrt{\frac{5 \times 5 \times 15 \times 15}{7 \times 7 \times 11 \times 11}} = \frac{5 \times 15}{7 \times 11} = \frac{75}{77} \text{ Ans.}$$

Note.—This method of finding square root is not of much practical use, for we cannot always say from inspection whether a number is a perfect square or not, or whether it has factors small enough to be easily found.

The following theorems are important:—

(I) *No perfect square can end in* 2, 3, 7, or 8.

For the square root of a perfect square must end in one of the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, or 9.

Hence any *perfect square* must either end in 0, or in *the units' figure of the* square *of one of the other digits, i.e.* in 0, or in the units' figure of 1, 4, 9, 16, 25, 36, 49, 64, or 81.

Hence the right-hand figure of a perfect square *must* be either 0, 1, 4, 9, 6, or 5, and $\therefore$ *cannot* be either 2, 3, 7, or 8.

Note.—Hence, although we cannot, without trial, say whether such a number as 496325 is, or is not, a perfect square, we *can* at a glance tell that 47638 is *not* a perfect square.

(II) *The* square *of any given number has either double,* or *one less than double, the number of figures in the given number.*

For, since the square of 10 is 100,

$\therefore$ the square of any single digit has either *one* or *two* figures in it.

And since the square of 100 is 10000,

$\therefore$ the square of any number between 10 and 100 has either *three* or *four* figures in it. And so on.

Hence the *number of figures* in the integral part of the square root of a number is found by counting the number of *couples of figures* in the number, beginning on the *right*, and including part of a couple, if such there be.

For instance, the *number of figures* in the square root of 64'00'00 is *three*, and the number of figures in the square root of 9'00'00 is also *three*.

(III) *The square of the sum of two numbers is equal to the sum of their squares plus twice their product.*

Take, for instance, the number 47 and regard it as the sum of 40 and 7.

$$\begin{aligned}\text{Then } (40+7)^2 &= 47 \times (40+7)\\ &= 47 \times 40 \quad + 47 \times 7^{*}\\ &= (40+7) \times 40 \quad + (40+7) \times 7\\ &= 40 \times 40 + 7 \times 40 + 40 \times 7 + 7 \times 7^{*}\\ &= 40^2 \quad + \textit{twice } 40 \times 7 \quad + 7^2.\end{aligned}$$

N.B.—Hence $47^2 - 40^2 = \textit{twice } 40 \times 7 + 7^2$
$= (\textit{twice } 40 + 7) \times 7.$

Similarly, in the case of any other numbers—

e.g. $365^2 - 360^2 = (\textit{twice } 360 + 5) \times 5.$

∴ *if from a perfect square* (*e.g.* 47^2) *we subtract the square of part of its root* (*e.g.* 40^2), *the remainder* (R)

= (*twice this part of the root* + *the remaining part*) × *the rem*^{g} *part.*

Hence the following

General method of finding the Square Root of a number.

For instance, *to find the square root of* 2209.

Marking the number off into periods of two figures, beginning on the right-hand side, we see, by (II), that the required root is a number of *two* digits.

$$\begin{array}{r|l} & 22'09\,(40+7\\ & 1600\\ \hline 80+7) & 609\\ & 609\\ \hline\end{array}$$

Also, we know (see page 244) that $\sqrt{1600} = 40$; and that $\sqrt{2500} = 50$,
∴ $\sqrt{2209}$ must lie between 40 and 50,
∴ the *tens'* figure of the root is **4**.

Now, from 2209 we subtract 40^2, *i.e.* 1600, and then we know [see (R) above] that the rem^r, 609, = (*twice* 40 + units' fig. of root) × units' fig.

We ∴ double 40 and use this double, 80, as a *trial divisor* of 609.

We thus obtain the units' figure of the root, 7, which we add to the 80, and then we multiply the completed divisor 87 by 7, obtaining the product 609.

Thus the square root of 2209 has been found to be 47.

---

* See (II), page 10.

Again, *to find the square root of* 133225.

Marking off the given number into "periods", of which in this case there are *three*, we know, from (II), that the req[d] root is a number of *three* digits.

1st stage.

As 133225 lies between 160000 and 90000,
i.e. between 400^2 and 300^2,
the hundreds', figure of the root is **3**.
Subtracting 300^2, *i.e.* 90000, from 133225, we obtain the first remainder 43225.

$$
\begin{array}{r|r|r}
 & 13'22'25 & 300 \\
 & 90000 & 60 \\
\cline{2-2}
600 & 43225 & 5 \\
60 & & \\
\cline{1-1}
660 & 39600 & \\
\cline{2-2}
720 & 3625 & \\
5 & & \\
\cline{1-1}
725 & 3625 & \\
\cline{2-2}
\end{array}
$$

2nd stage.

We now *double* 300, and use the double, *i.e.* 600, as a *trial divisor* of 43225.

Now, as 600 is contained more than 70 and less than 80 times in 43225, *we try* 7 as the tens' figure of the root. But on multiplying 670 by 70 we obtain a product greater than 43225.

Hence as 7 is too great for the second figure of the root, we are led to 6, the proper figure.

So we place 60 in the quotient; also add 60 to 600; multiply the completed divisor 660 by 60, and then subtract the product from 43225, obtaining the second remainder 3625.

3rd stage.

We now repeat the operations of the second stage.

Doubling 360, we obtain 720, which we use as a trial divisor of 3625, whence we arrive at the units' figure of the root, 5; for 725×5 yields a product $= 3625$.

Thus the square root of 133225 has been found to be 365.

N.B.—It follows, from (III) that the remainder at any stage, *i.e.* the difference between the given square and the square of the part of the root already obtained, is of the form (R); we therefore at each fresh stage form a new **trial divisor** by **doubling** *the part of the root already obtained;* and, having thence found the next figure of the root, *complete the divisor by the addition of this newly obtained part of the root.*

Note.—In practice it is usual to omit the *place* ciphers (such as are omitted in ordinary Long division; see page 15), and to bring down the periods in succession, when the above examples appear thus:—

$$
\begin{array}{r@{\,)\,}l}
 & 22'09\,(\,47 \\
 & 16 \\
\cline{2-2}
87 & 609 \\
 & 609 \\
\cline{2-2}
\end{array}
\qquad\qquad
\begin{array}{r@{\,)\,}l}
 & 13'32'25\,(\,365 \\
 & 9 \\
\cline{2-2}
66 & 432 \\
 & 396 \\
\cline{2-2}
725 & 3625 \\
 & 3625 \\
\cline{2-2}
\end{array}
$$

EXAMPLE iii.—*Find the square root of* 2220100.

Mark off the number into periods, beginning on the right.

1st stage.

Place in the quotient the square root of the greatest perfect square which is less than the left-hand period, in this case 1.

Subtract 1^2 from the left-hand period, and then bring down the next period, 22.

2nd stage.

Form a trial divisor by *doubling* the quotient, 1.

```
       2'22'01'0'0 ( 1490
       1
 24 ) 1 22
        96
289 ) 26 01
      26 01
         00
```

Ans. 1490.

Now, remembering that the new figure of the root will be placed to the right of the 2 in the trial divisor, we say, or think, "Twenty-*something* into 122?" And if we try 6 or 5 as an answer to this question we find that 6 times 26, and 5 times 25, both yield too great a product; we therefore conclude that 4 is the second figure of the root.
So we place 4 in both divisor and quotient;
multiply 24 by 4 and subtract the product 96 from 122.
We then bring down the next period, 01.

3rd stage.

Form a new trial divisor by doubling the quotient, 14.
Now, remembering that the new figure of the root will be placed to the right of 28 in the trial divisor, we say, or think, "Two hundred and eighty-*something* into 2601?" which leads us to 9 as the new figure of the root. So we place 9 in both divisor and quotient and multiply 289 by 9, obtaining the product 2601.

Thus 149 is the square root of 22201;
$\therefore$ 1490 .... 2220100.

EXAMPLE iv.—*Extract the square root of* 39790864.

Here, when we reach the 3rd stage, we have as trial divisor one thousand two hundred and sixty-*something*, which is *not contained* in the remainder 1008; we therefore place **0** in *both divisor and quotient*, bring down the next period, and proceed as usual.

```
          39'79'08'64 ( 6308
          36
   123 ) 3 79
         3 69
 12608 ) 1 00 8 64
         1 00 8 64
```

Hitherto we have applied the method to finding the square root of a *perfect square*, in which case the operation terminates, *i.e.* no final remainder occurs.

The process, however, can be applied to numbers *not* perfect squares; *i.e.* we can obtain either the integral part of the square root of *any* number; or its root correct to any number of places of decimals required.

EXAMPLE v.—*Find the perfect square nearest to* 65748.

Obtaining the integral part of the root of 65748, the remainder is 212.

∴ the square of 256 *falls short* of 65748 by 212.

Similarly, the square of 257 *exceeds* 65748 by 301.

∴ 65748 − 212, or

65536 is the reqd number.

```
       6'57'48(256
       4
  45)  257
       225
 506)  3248
       3036
        212
```

```
       6'57'48(257
       4
  45)  257
       225
 507)  3248
       3549
```

Square Root of a Decimal.

EXAMPLE vi.—*Find, correct to three places of decimals, the square root of* 401·7.

We mark off the *whole number* into periods as usual. Then, having obtained the *integral* part of the reqd root, 20, we insert the decimal point in the quotient, and proceed as before, appending as many periods of ciphers as we have occasion for.

```
          4'01·'70'00'00'00(20·0424
          4
 4004)      1 7000
            1 6016
   40082)     98400
              80164
     40084)  1823600
```

Ans. 20·042.

Note.—We continue the process until the figure in the *fourth* place in decimals is known, in order to see whether the required approximation is 20·042, or 20·043.

N.B.—In *marking the periods* in the case of a **decimal** we should always **begin** at the **decimal point**, proceeding to the *left* in the case of any *integral* part, and to the *right* for the *decimal.*

For instance, in $\sqrt{\cdot 15625}$ we *must* mark the periods thus:—·15'62'5

Square Root of a Vulgar Fraction.

CASE I.—When the *denominator* is a *perfect square*, we find the square roots of numerator and denominator *separately.*

N.B.—A *mixed number* must *first* be reduced to an *improper fraction,* in this case.

EXAMPLE vii.—*Find* (i) $\sqrt{\frac{1369}{5329}}$, *and* (ii) $\sqrt{1\frac{25}{144}}$.

(i)
```
      13'69(37           53'29(73
       9                 49
  67)  469         143)  429
       469               429
```

Ans. (i) $\frac{37}{73}$.

(ii) $\sqrt{1\frac{25}{144}} = \sqrt{\frac{169}{144}}$

$= \frac{13}{12}$

$= 1\frac{1}{12}$ *Ans.* (ii).

EXAMPLE viii.—*Find* $\sqrt{\frac{726}{2166}}$.

[Here 2166 is *not* a perfect square, but as the fraction is not in its lowest terms, we try the experiment of reducing it, and thus obtain a denominator which *is* a perfect square.]

$$\sqrt{\frac{726}{2166}} = \sqrt{\frac{121}{361}} = \frac{11}{19}\ \textit{Ans.}$$

CASE II.—When the *denominator* of the fraction is *not* a *perfect square*, we first reduce the fraction to a *decimal*, and then extract the square root of the result.

EXAMPLE ix.—*Find* $\sqrt{\frac{4}{3}}$.

$$\sqrt{\frac{4}{3}} = \sqrt{1\cdot\dot{3}}$$

$$= 1\cdot1547\ldots\ \textit{Ans.}$$

```
          1·'33'33'33'33 ( 1·1547
          1
   21 )   33
          21
  225 ) 1233
        1125
 2304 ) 10833
         9216
23087 ) 161733
```

Note.—Much labour would be wasted in such cases if we found the roots of num^r and den^r *separately*, as when this was done it would afterwards be necessary to divide the first result by the long decimal of the second result.

Method of Contracting the work of finding an Approximate Square Root.

(IV) *When any number of figures of a square root have been obtained by the ordinary process, one less than as many more figures may be obtained by* (contracted) division.*

For instance, suppose we require the sq. root of 2 to *six* places of decimals. We obtain the first *four* figures of the root by the ordinary process. *We then cancel the right-hand figure of the last trial divisor*, and obtain the next *three* figures of the root by *contracted division* (see Chap. L), thus—

[Here the last figure is 3, or nearly 4.

Thus the first seven figures of the root are 1·414213, but the *nearest* approximation in six places of decimals is 1·414214.]

```
        2·'00'00'00 ( 1·414213
        1
  24 )  1 00
          96
 281 )    400
          281
2824 )   11900
         11296
           604
           565
            39
            28
            11
```

* For a proof of the truth of this theorem, see Appendix.

Note.—The method of "Abridged Division" (see page 15), may be applied to square root.

We give the above example worked in three other forms:—

(i) At full length, using ordinary division.

(ii) .... the method of *abridged* division.

(iii) Using the method of *abridged* division, and *contracting* the latter part.

(i).
```
                (1·414213
            2·|00|00|00|00|00|00
              1
      24 ) 1 00
             96
       281 ) 4 00
             2 81
      2824 ) 1 19 00
             1 12 96
     28282 ) 6 04 00
               5 65 64
    282841 ) 38 36 00
               28 28 41
   2828423 ) 10 07 59 00
                8 48 52 69
                1 59 06 31
```

(ii).
```
                       (1·414213
            2·|00|00|00|00|00|00
      24 ) 1 00
        281 ) 4 00
        2824 ) 1 19 00
          28282 ) 6 04 00
           282841 ) 38 36 00
           2828423 ) 10 07 59 00
                       1 59 06 31
```

(iii).
```
                 (1·414213
          2·|00|00|00
    24 ) 1 00
      281 ) 4 00
     2824 ) 1 19 00
                6 04
                  39
                  11
```

Note.—In the last example we obtained the square root of 2 to *six* places of decimals, and we see from (i) that the remainder is then only ·000001590631.

Now we know from (III) that this rem^r $= 2 - (1\cdot414213)^2$. Thus by proceeding far enough we can find a number whose *square* differs from 2 by as small an amount as we please. But *there must always be a rem^r*, for *there is no significant digit whose square ends in* **0**.

Such expressions as $\sqrt{2}$, $\sqrt{3\cdot6}$, &c., where the roots cannot be *exactly* obtained, are called **surds**.

In contradistinction to surds, numbers which are *not* surds are called *rational* numbers.

It follows from the definition of the sign $\sqrt{\ }$, that $\sqrt{2} \times \sqrt{2} = 2$.

Note.—$2\sqrt{3}$ means $2 \times \sqrt{3}$, or $\sqrt{3} \times 2$; and so on.

EXAMPLE x.—*Find the value of* $\dfrac{3}{2\sqrt{2}}$ *correct to* 3 *places of decimals.*

$$\frac{3}{2\sqrt{2}} = \frac{3 \times \sqrt{2}}{2\sqrt{2} \times \sqrt{2}}$$ [For the value of a fraction is not altered by multiplying both num^r and den^r by the same quantity. See page 68.]

$$= \frac{3\sqrt{2}}{2 \times 2} = \frac{3\sqrt{2}}{4} = \frac{3 \times 1\cdot4142\ldots}{4}$$ [From the example above.]

$$= \frac{4\cdot2426\ldots}{4} = 1\cdot0606 = \underline{1\cdot061\ \textit{Ans.}}$$

N.B.—Here the first step, which is called **rationalizing the denominator**, saves much labour, namely, the division of 3 by double the long decimal root of 2.

Such an expression as $(5 - \sqrt{3}) \times (5 + \sqrt{3})$ is *rational*; for $(5 - \sqrt{3}) \times (5 + \sqrt{3}) = (5 - \sqrt{3}) \times 5 + (5 - \sqrt{3}) \times \sqrt{3}$*

$$= 25 - 5\sqrt{3} \quad + 5\sqrt{3} - 3$$
$$= 25 - 3 = \mathbf{22}.$$

Hence, if such an expression as $5 - \sqrt{3}$† occurs in the denominator of a fraction whose value is required, we *rationalize the denominator* by *multiplying* both numʳ and denʳ by the *complementary factor* $5 + \sqrt{3}$.†

Example xi.—*Find, the value of* $\dfrac{7}{3 + 2\sqrt{2}}$ *correct to three places of decimals.*

$$\frac{7}{3 + 2\sqrt{2}} = \frac{7 \times (3 - 2\sqrt{2})}{(3 + 2\sqrt{2}) \times (3 - 2\sqrt{2})} = \frac{7 \times (3 - 2\sqrt{2})}{9 - 8}$$
$$= \frac{7 \times (3 - 2 \times 1{\cdot}4142)}{1} = 7 \times {\cdot}1715 = \underline{1{\cdot}201 \textit{ Ans.}}$$

The *fourth* root of a given number is the number whose *fourth power* (see page 12) is equal to the given number.

(The sign $\sqrt[4]{}$ is used to indicate the fourth root.)

For instance, $\sqrt[4]{16} = 2$, for 2^4, *i.e.* $2 \times 2 \times 2 \times 2 = \mathbf{4} \times \mathbf{4} = 16$.

Hence the fourth root of a number may be found by *extracting the square root of its square root.*

The following are examples of the application of Square Root:—

Example xii.—*A square field contains* 10 *acres; find the length of its side.*

$$10 \text{ acres} = 10 \times 4840 \text{ square yards}$$

Hence, length of side $= \sqrt{10 \times 4840}$ yards $= \sqrt{100 \times 121 \times 4}$ yards
$$= 10 \times 11 \times 2 \text{ yards} = \underline{220 \text{ yards. } \textit{Ans.}}$$

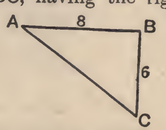
It is proved in Euclid, Book I., Proposition 47, that *in a right-angled triangle, the square on the side opposite to the right angle is equal to the sum of the squares on the other two sides.*

Hence, if the lengths of any two sides of a right-angled triangle are known, the length of the remaining side can be calculated.

For instance, if in the right-angled triangle ABC, having the right angle at C, AC is 8, and BC is 6, inches long, then the *square* of the length of AB $= 8^2 + 6^2$

$$= 64 + 36$$
$$= 100$$

$\therefore$ the length of AB $= \sqrt{100} = \underline{10 \text{ in.}}$



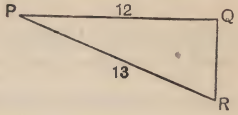
* By (II), page 10; see also Appendix F (vii).

† Or *vice versâ*.

Again, if in the right-angled triangle PQR having the right angle at Q, PR is 13, and PQ is 12 inches long, then the *square* of the length of QR $= 13^2 - 12^2$

$$= 169 - 144$$
$$= 25$$

$\therefore$ the length of QR $= \sqrt{25} = \underline{5 \text{ in.}}$



EXAMPLE xiii.—*A rectangular floor is* 11 *feet long and* 8 *feet* 3 *in. wide; find the distance between its opposite corners.*

$$\text{Req}^{\text{d}} \text{ distance} = \sqrt{11^2 + (8\tfrac{1}{4})^2} \text{ ft.} = \sqrt{\frac{121 \times 16 + 33 \times 33}{16}} \text{ ft.}$$
$$= \sqrt{\frac{121\,(16 + 9)}{16}} = \frac{55}{4} \text{ ft.} = \underline{13 \text{ ft. } 9 \text{ in. } \textit{Ans.}}$$

EXAMPLE xiv.—*The cost of turfing a rectangular cricket-ground twice as long as wide, at 3d. per square yard, is* £765, 12*s.* 6*d.; find the length of fencing required to surround it.*

Number of sq. yds. in area of ground

$$= \frac{£765,\ 12s.\ 6d.}{3d.} = 61250.$$

As the ground is twice as long as wide it can be divided into two equal squares.

$\therefore$ area of one of these squares $= 30625$ sq. yds.

$\therefore$ breadth of ground $= \sqrt{30625} = 175$ yds.

Hence perimeter of ground $= 175 \times 6 = \underline{1050 \text{ yds. } \textit{Ans.}}$

We will now consider the postponed case in Compound Interest (see page 206).

EXAMPLE xv.—*At what rate per cent compound interest will* £750 *amount to* £811, 4*s. in* 2 *years?*

$$£750 \times \left(\frac{100 + \text{req}^{\text{d}} \text{ rate}}{100}\right)^2 = \text{amount of } £750 \text{ in 2 yrs.*}$$

$$\therefore \left(\frac{100 + \text{req}^{\text{d}} \text{ rate}}{100}\right)^2 = \frac{£811\frac{1}{5}}{£750}$$

$$\therefore \frac{100 + \text{req}^{\text{d}} \text{ rate}}{100} = \sqrt{\frac{811\frac{1}{5}}{750}} = \frac{26}{25}$$

$$\textit{i.e. } 1 + \frac{\text{req}^{\text{d}} \text{ rate}}{100} = \frac{26}{25} \quad \therefore \frac{\text{req}^{\text{d}} \text{ rate}}{100} = \frac{1}{25}$$

$$\therefore \underline{\text{req}^{\text{d}} \text{ rate} = 4 \ \textit{Ans.}}$$

[* By the formula on p. 205.]

LXIII. CUBE ROOT.

The **cube root** of a given number is the number whose *cube** is equal to the given number.

For instance, 4 is the *cube root* of 64; for 4^3, *i.e.* $4 \times 4 \times 4 = 64$.

The symbol $\sqrt[3]{\ }$ indicates the *cube* root.

For instance, $\sqrt[3]{8}$ is read "the cube root of 8".

The following *cubes* should be known—

$1^3 = 1,\ 2^3 = 8,\ 3^3 = 27,\ 4^3 = 64,\ 5^3 = 125,\ 6^3 = 216,$
$7^3 = 343,\ 8^3 = 512,\ 9^3 = 729,\ 10^3 = 1000,\ 11^3 = 1331,\ 12^3 = 1728.$

Hence the following *cube roots* will be known—

$\sqrt[3]{1} = 1,\ \sqrt[3]{8} = 2,\ \sqrt[3]{27} = 3,\ \sqrt[3]{64} = 4,\ \sqrt[3]{125} = 5,\ \sqrt[3]{216} = 6.$

$\sqrt[3]{343} = 7,\ \sqrt[3]{512} = 8,\ \sqrt[3]{729} = 9,\ \sqrt[3]{1000} = 10,\ \sqrt[3]{1331} = 11,\ \sqrt[3]{1728} = 12.$

Hence also $\sqrt[3]{8000} = 20,\ \sqrt[3]{27000} = 30,\ \sqrt[3]{64000} = 40,$ &c.
and $\sqrt[3]{1000000} = 100,\ \sqrt[3]{8000000} = 200,\ \sqrt[3]{27000000} = 300,$ &c.

Note.—From the above tables it is evident that comparatively few numbers have an *exact* cube root; for, since $\sqrt[3]{64} = 4$ and $\sqrt[3]{125} = 5$, the cube root of any number (*e.g.* 96) between 64 and 125 must be greater than 4 and less than 5, and is *inexact*. (See Square Root, p. 244.)

A number which has an *exact* cube root is called a *perfect cube.*

For instance, 343 is a *perfect cube*, for $\sqrt[3]{343} = 7$
and $\frac{27}{125}$ is a *perfect cube*, for $\sqrt[3]{\frac{27}{125}} = \frac{3}{5}$.

When the factors of a perfect cube are known its cube root can at once be seen.

For instance, $\sqrt[3]{11 \times 11 \times 11 \times 5 \times 5 \times 5} = 11 \times 5$, or 55.

Hence in the case of *a perfect cube which can easily be factorized,* its cube root may be found thus—

EXAMPLE i.†—*Find, by factors, the cube root of the perfect cube* 583200.

$$5832000 = 8 \times 729 \times 1000$$
$$\therefore \sqrt[3]{5832000} = 2 \times 9 \times 10 = \underline{180\ \textit{Ans.}}$$

Note.—This method is of no great *practical* use; since we cannot always, without trial, tell whether the number whose cube root we require is a perfect cube, and whether it can easily be factorized, or not.

By reasoning similar to that of Theorem (I) on Square Root it may easily be shown that *a perfect cube can end in any digit.*

* See page 10. † See also Example K, page 63.

And, as in Square Root, so in Cube Root, *the number of figures in the integral part of the cube root of any number can be foretold.*

For, as $\sqrt[3]{1000} = 10$, the *cube root* of any number *less* than 1000 consists of **one** figure only;

And, as $\sqrt[3]{1000000} = 100$, the *cube root* of any number *between* 1000 and 1000000 consists of **two** figures. And so on.

Hence, if we mark off the figures of the number whose cube root is required into *sets of three*, beginning on the right, the *number of sets* (including part of a set if such there be) will be the *number of figures* in its cube root.

For instance, if we thus mark off the number 12'167, there are *two* such "periods", ∴ its cube root is a number of *two* digits.

And as there are three such "periods" in 8'365'427, its cube root is a number of *three* digits.

Note.—If the table of cubes on page 254 is known, the cube root of any number *less than a million which is known to be a* **perfect cube**, can be obtained *by inspection.*

For instance, take the perfect cube 157'464.

We see that its cube root consists of *two* figures.

And since 157464 lies between 50^3 and 60^3

∴ the *tens'* figure of the cube root must be **5**.

Also since 157464 ends in **4**, we know from the table of cubes that the *units'* figure of its cube root must be **4**. Thus $\sqrt[3]{157464} = 54$.

The following important Theorem corresponds to (III) in Square Root:—

The cube of the sum of any two numbers is equal to the cube of the first, plus thrice the square of the first multiplied by the second, plus thrice the first multiplied by the square of the second, plus the cube of the second, number.

Take, for instance, the number 47 and regard it as the sum of the two numbers 40 and 7.

$$\begin{aligned}
\text{Then } (40+7)^3 &= (40+7)^2 \times 47 \\
&= (40^2 + \textit{twice } 40 \times 7 + 7^2) \times 47^* \\
&= (40^2 + \textit{twice } 40 \times 7 + 7^2) \times (40+7) \\
&= (40^2 + \textit{twice } 40 \times 7 + 7^2) \times 40 \\
&\qquad + (40^2 + \textit{twice } 40 \times 7 + 7^2) \times 7^\dagger \\
&= 40^3 + \textit{twice } 40^2 \times 7 + 40 \times 7^2 \\
&\qquad + 40^2 \times 7 + \textit{twice } 40 \times 7^2 + 7^3 \\
&= 40^3 + \textbf{thrice } 40^2 \times 7 + \textbf{thrice } 40 \times 7^2 + 7^3
\end{aligned}$$

* By theorem (III) of Square Root. † By (II), page 10.

N.B.—Hence $47^3 - 40^3 = \textit{thrice } 40^2 \times 7 + \textit{thrice } 40 \times 7^2 + 7^3$
$= (\textit{thrice } 40^2 + \textit{thrice } 40 \times 7 + 7^2) \times 7.$

Similarly in the case of any other numbers—

e.g. $135^3 - 130^3 = (\textit{thrice } 130^2 + \textit{thrice } 130 \times 5 + 5^2) \times 5.$

∴ *If from a perfect cube* (*e.g.* 47^3) *we subtract the cube of part of its root* (*e.g.* 40^3), *the remainder* (R)

= (*thrice the square of this part of the root* + *thrice this part* × *the rem^g part* + *the square of the rem^g part*) × *the rem^g part.*

Hence the following

Method of finding the Cube Root of a number.

For instance, *to find the cube root of* 103823.

Marking the number off into "periods", beginning at the units' place, we see that the cube root is a number of *two* figures.

Now 103823 lies between 64000 and 125000

∴ $\sqrt[3]{103823}$ .... 40 and 50

∴ the *tens*' figure of the root must be **4**.

$$\begin{array}{r|l} & 103'823(40+7 \\ & \underline{64000} \\ 4800 & 39823 \\ 840 & \\ \underline{49} & \\ 5689) & \underline{39823} \end{array}$$

Subtracting 40^3 from the number we obtain the remainder 39823.

We now [see (R)] take *thrice* 40^2, *i.e.* 4800, as a *trial divisor* of 39823.

[Now 4800 is contained more than 8 times in 39823, but if we try 8 as the units' figure of the required root and thus *complete* the divisor according to the rule obtained from (R), the result when multiplied by 8 is *greater* than 39823.]

So we try 7 for the units' figure of the root, and complete the divisor by adding to 4800, *thrice* 40×7, and 7^2, when the complete divisor 5689 results. This we now multiply by 7 obtaining 39823.

Thus the cube root of 103823 is 47.

N.B.—In the case of a *three*-figure root a second trial divisor would be formed, and afterwards completed, in exactly the same way: for it follows from the above Theorem that the remainder at any stage (*i.e.* the difference between the given number and the cube of the part of its root already obtained) is of the form (R).

We therefore at each stage take as a **trial divisor**

thrice the square of the part of the root already obtained;

and having thence found the next figure of the root, *we complete the divisor* by adding on

thrice the part of the root previously obtained × *the new figure;*

and *the square of the new figure.*

Note.—In practice the *place* ciphers may be omitted (as in Square Root), and the periods brought down in succession.

We will illustrate the process in this form, in finding the cube root of 16974593.

We first mark off the number into periods thus,— 16ˈ974ˈ593.

1st stage.

We then place in the "quotient" **2**; *i.e.* the greatest number whose cube is less than the left-hand period, 16.

We subtract 2^3, *i.e.* **8**, from 16, and bring down the next period, 974.

```
16ˈ974ˈ593 ( 2
 8
 8974
```

Now as we bring down one period only at a time, we do not, in forming the first *trial divisor*, give 2 its *real* local value (*i.e.* 200), but merely its local value *relative* to the next figure of the root (*i.e.* **20**).

2nd stage.

So we take as first trial divisor *thrice* $\mathbf{20}^2$, *i.e.* 1200.

```
        16ˈ974ˈ593 ( 2
         8
1200 ) 8974
```

Now 1200 is contained 7 times in 8974, but if we *complete* the divisor by the help of 7, the result when multiplied by 7 is greater than 8974. The same also, we find on trial, is the case with 6. Thus we arrive at 5 as the new figure of the root.

We place **5** in the quotient, and now complete the divisor with

thrice **20** × 5, *i.e.* 300,
and 5^2, *i.e.* 25.

Adding 1200, 300, and 25 we obtain 1525 as the complete divisor.

We now multiply 1525 by 5, subtract the result from 8974, and then bring down the last period, 593.

```
        16ˈ974ˈ593 ( 25
         8
1200 |  8974
 300 |
  25 |
1525 )  7625
        1349539
```

3rd stage.

We now take *thrice* $25\mathbf{0}^2$, *i.e.* 187500, as a new trial divisor.

We thence obtain **7** as the third figure of the root.

To complete the divisor we add to 187500

thrice 250 × 7, *i.e.* 5250
and 7^2, *i.e.* 49.

obtaining 192799 as the complete divisor.

We now multiply 192799 by 7 obtaining 1349593 as the product.

```
          16ˈ974ˈ593 ( 257
           8
  1200 |  8974
   300 |
    25 |
  1525 )  7625
187500 | 1349593
  5250 |
    49 |
192799 ) 1349593
```

Thus the required cube root is found to be 257.

All that was said in the chapter on Square Root as regards the treatment of Fractions and Decimals applies also to Cube Root, and need not therefore be repeated here.

For instance, $\sqrt[3]{5\frac{13}{343}} = \sqrt[3]{\frac{1728}{343}} = \frac{12}{7} = 1\frac{5}{7}$.

EXAMPLE ii.—*Find the cube root of* 8869743000.

Here our first trial divisor, 1200, is *not* contained in the first remainder 869, so we place a **0** in the quotient, bring down the next period and form a *new trial divisor*, *i.e.* thrice 200^2.

Hence $\sqrt[3]{8869743} = 207$

$\therefore \sqrt[3]{8869743000} = 2070$.

```
                8'869'743'000 ( 2070
                8
120000  |        869743
  4200  |
    49  |
124249 )         869743
                 ------
                    000
```

Ans. 2070.

EXAMPLE iii.—*Find three significant figures of* $\sqrt[3]{\cdot 00064}$.

We mark off periods, beginning at the decimal point.

As the left-hand period consists of three ciphers, the first figure of the decimal cube root (a *non-significant* one) is 0.

Now 640 lies between 512 and 729

i.e. .... 8^3 and 9^3

so the first *significant* figure of the root is **8**.

We $\therefore$ subtract 8^3 from 640 and proceed with the work just as if the case were that of an *integer*, until two more figures of the root are obtained.

```
        ·000'640'000'000 ( ·0861
              512
  19200 |  128000
   1440 |
     36 |
  20676 )  124056
2218800 )  3944000
```

Ans. ·0861.

The work of finding an *approximate* cube root may be somewhat shortened; for *when* 3, 4... *figures of a cube root have been obtained by the ordinary process*, 1, 2... *more figures, respectively, may be obtained by* (contracted) division

Suppose, for instance, we require the cube root of $4\frac{1}{6}$ correct to 5 places of decimals.

We obtain the cube root to *three* places of decimals by the ordinary process. We have then *four* figures of the root and our last divisor is 772381, and the remainder 1157137. We now cancel the right-hand figure of the divisor, and obtain the next *two* figures of the root by Contracted Division, thus—

```
            4·'166'666'666 ( 1·60915
            1
    300 |   3 166
    180 |
     36 |
    516 )   3 096
7680000 |  70666666
  43200 |
     81 |
7723281 )  69509529
           --------
            1157137
             772328
            -------
             384809
```

Note.—There are other ways of approximating to the cube root of a number. The cube (or *any other*) root may be found approximately by the help of *Logarithms*, or by "*Horner's method*". Horner's method is exemplified in the Appendix.

The *sixth* root of a number may be found by taking *the cube root of its square root.*

For instance, $2^6 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^3 \times 2^3 = (2^3)^2$.

Thus the *square root* of 2^6 is 2^3, and the *cube root of this* is 2; which is evidently the *sixth* root of 2^6.

Similarly the *ninth* root of a number may be found by taking *the cube root of its cube root.*

EXAMPLE iv.—*Find the least number which must be added to* 148776, *that the sum may be a perfect cube.**

We begin the operation of finding the cube root, and thence discover that the given number falls short of the cube of 53 by 101.

$$
\begin{array}{r|l}
 & 148'776\,(53 \\
 & 125 \\
\hline
7500 & 23776 \\
450 & \\
9 & \\
\hline
7959 & 23877
\end{array}
$$

Ans. 101.

EXAMPLE v.—*Find the area of the entire surface of a cubical block the volume of which is* 17 *cub. ft.* 415 *cub. in.*

$$
\begin{aligned}
17 \text{ cub. ft. } 415 \text{ cub. in.} &= 29791 \text{ cub. in.} \\
\therefore \text{ length of side of cube} &= \sqrt[3]{29791} \text{ in.} \\
&= 31 \text{ in.} \\
\therefore \text{ area of one face} &= 31 \times 31 \text{ sq. in.} \\
&= 961 \text{ sq. in.} \\
\therefore \text{ area of entire surface} &= 961 \times 6 \text{ sq. in.} \\
&= 5766 \text{ sq. in.} \\
&= 40 \text{ sq. ft. } 6 \text{ sq. in. } \textit{Ans.}
\end{aligned}
$$

EXAMPLE vi.—*Three numbers are as* 3 : 4 : 5; *the sum of their cubes is* 74088; *find the numbers.*

$$
\begin{aligned}
\text{Since the numbers are as } & 3 : 4 : 5 \\
\therefore \text{ their } \textit{cubes} \text{ are as } & 3^3 : 4^3 : 5^3 \\
\textit{i.e.} \text{ as } & 27 : 64 : 125.
\end{aligned}
$$

We therefore divide 74088 into parts proportional to 27, 64, and 125. These parts will be found to be 343×27, 343×64, 343×125.

Hence the req[d] numbers are $\sqrt[3]{343 \times 27}$, $\sqrt[3]{343 \times 64}$, $\sqrt[3]{343 \times 125}$

i.e. 7×3, 7×4, 7×5

or 21, 28, 35 *Ans.*

---

* Compare with Ex. K, page 63.

LXIV. SCALES OF NOTATION.

A *Scale of Notation* is a systematic method of representing numbers by means of *figures.*

We know (see pages 2, 3) that the **radix**, or base, in the Common, or Decimal, Scale of Notation is the number *ten*; and that, consequently, *ten* different figures, and ten only, are used in this scale.

Also, that any other number may be made the *radix* of a Scale of Notation similar to the Common Scale.

For instance, just as, in the *common* scale, 473 means
4 *hundreds* + 7 *tens* + 3 *units*,

$$i.e.\ \ 4 \times \mathbf{10}^2 + 7 \times \mathbf{10} + 3,$$

so, in the scale of radix *eight*, 473 means

$$4 \times \mathbf{8}^2 + 7 \times \mathbf{8} + 3.$$

i.e. 4 *sixty-fours* + 7 *eights* + 3 units.

And, that the number of different figures required for any scale is the same as the number chosen as radix.

Thus in Scale Eleven, *ten* must be represented by a single figure, so the letter **t** is used.

And in Scale Twelve *ten* and *eleven* must be represented by single figures, so **t** and **e** are used.

For instance, in the scale of radix *twelve*, 3*t*7*e* stands for
three times (*twelve*)^3 + *ten* times (*twelve*)^2 + *seven* times *twelve* + *eleven.*

EXAMPLE i.—*Add* 8454, 7608, 726, *and* 1853 *in the Scale of Nine.*

3 + 6 + 8 + 4 = twenty-one* = 2 nines + 3 *units*;
set down **3**, and carry 2;
2 + 5 + 2 + 0 + 5 = fourteen = 1 nine + 5 units;
set down **5**, and carry 1;
1 + 8 + 7 + 6 + 4 = twenty-six = 2 nines + 8 *units*;
set down **8**, and carry 2;
2 + 1 + 7 + 8 = eighteen = 2 nines + 0 units.

(*Nine.*)
```
 8454
 7608
  726
 1853
20853 Ans.
```

EXAMPLE ii.—*Subtract* 7*t*58 *from* *e*90*t* *in Scale* 12.

8 from **t** leaves 2; set down **2**;
5 from *twelve* leaves 7; set down **7**, and carry 1;
1 + **t** from 9 + *twelve* leaves **t**; set down **t**, and carry 1;
1 + 7 from **e** leaves **3**.

(*Twelve.*)
```
e90t
7t58
3t72 Ans.
```

*The *mental* work, it will be noticed, is really performed in the *common* scale; though it *might*, of course be performed in Scale Nine by constructing a system of *Numeration* (see Numeration on page 2), and learning the corresponding Addition Table for the scale —*e.g.* "3 and 6 make *nine*, *nine* and 8 make 'eight*neen*', eight*neen* and 4 make two-*ny*-three; set down 3 and carry 2."

EXAMPLE iii.—*Multiply* 4256 *by* 54 *in Scale Seven.*

6×4 = twenty-four = 3 sevens + 3 *units*;
set down **3**, and carry 3;
$5 \times 4 + 3$ = twenty-three = 3 sevens + 2 *units*;
set down **2**, and carry 3;
$2 \times 4 + 3$ = eleven = 1 seven + 4 *units*;
set down **4**, and carry 1.
$4 \times 4 + 1$ = seventeen = 2 sevens + 3 units, &c.

(*Seven.*)

```
  4256
    54
 23423
31012
333543  Ans.
```

EXAMPLE iv.—*Divide* 344*t*7 *by* 8 *in Scale Eleven.*

8 into (3 elevens + 4), *i.e.* 8 into *thirty-seven*, **4**, and 5 over;
8 into (5 elevens + 4), *i.e.* 8 into *fifty-nine*, **7**, and 3 over;
8 into (3 elevens + t), *i.e.* 8 into *forty-three*, **5**, and 3 over;
8 into (3 elevens + 7), *i.e.* 8 into *forty*, **5**.

(*Eleven.*)

```
8)344t7
   4755  Ans.
```

To transfer a given number from one scale to another.

If we divide any number in the common scale, say 366, again and again by any one number, say 5, we know (see page 16) that

the first remainder is a number of *units*
the second .... *fives*
the third .... *twenty-fives*
And so on

```
5)366
 5)73 + 1
 5)14 + 3
     2 + 4
```

i.e. 366 = 2 *hundred-and-twenty-fives* + 4 *twenty-fives* + 3 *fives* + 1 *unit*

$= 2 \times 5^3 + 4 \times 5^2 + 3 \times 5 + 1$

$\therefore$ 366 in scale **10** = 2431 in scale **5**.

And if we divide a number, say 1435, in any scale, say Scale 8, again and again by any one number, say 6, *performing the division in Scale* 8, we obtain a similar result.

Thus 1435 in scale 8

$= 3 \times 6^3 + 4 \times 6^2 + 0 \times 6 + 5$

= 3405 in scale 6.

(Radix Eight.)

```
6)1435
 6)204 + 5 units
  6)26 + 0 sixes
      3 + 4 thirty-sixes
```

Hence the following Rule—

(I) *Divide the given number successively* (until the final quotient is less than the divisor) *by* **the radix of the new scale** *in which it is to be expressed*, **performing the division in the scale in which the number is given**; then the *remainders*, in order, are the figures of the number in its new scale, the *first* remainder being the *units*' figure.

EXAMPLE v.—*Change* 2738 *from the Common Scale to Scale* 4.

[By the above Rule we have but to divide 2738 successively by 4, performing the division in the ordinary way.]

$$\begin{array}{r l}
4\,)\underline{2738} & \\
4\,)\underline{684} & +\,2 \\
4\,)\underline{171} & +\,0 \\
4\,)\underline{42} & +\,3 \\
4\,)\underline{10} & +\,2 \\
2 & +\,2
\end{array}$$

Hence 2738 (scale 10) = 222302 (*scale* 4) *Ans.*

EXAMPLE vi.—*Change* 2653 *from Scale* 7 *to the Common Scale.*

[By the above Rule we divide 2653 successively by ten, *performing the division in Scale Seven.*]

(Radix Seven.)

$$\begin{array}{r l}
\text{ten}\,)\underline{2653} & \\
\text{ten}\,)\underline{203} & +\,8 \\
\text{ten}\,)\underline{13} & +\,1 \\
1 & +\,0
\end{array}$$

Hence 2653 (scale 7) = 1018 (*scale* 10) *Ans.*

Note.—In changing to the *common* scale, instead of the above process we may proceed thus—

$$\begin{aligned}
2653 \text{ in scale } 7 &= 2 \times 7^3 + 6 \times 7^2 + 5 \times 7 + 3 \\
&= 2 \times 243 + 6 \times 49 + 5 \times 7 + 3, \text{ in scale } 10 \\
&= 1018 \textit{ in scale } 10. \quad \textit{Ans.}
\end{aligned}$$

The various scales are often described by words derived from the names of the Latin numerals. Thus Scales 2, 3, 4, 5, 6, 7, 8, 9, 11 are the *Binary*, *Ternary*, *Quaternary*, *Quinary*, *Senary*, *Septenary*, *Octenary*, *Nonary*, *Undenary* Scales respectively; Scale 10 (the Common Scale) is the *Denary*, or the *Decimal*, and Scale 12, the *Duodenary*, or the *Duodecimal*, Scale.

EXAMPLE vii.—*Transfer* 201221 *from the Ternary to the Undenary Scale.*

[By the above Rule we divide 201221 successively by *eleven*, *performing the division in Scale* 3.]

(Radix 3.)

$$\begin{array}{r l}
\text{eleven}\,)\underline{201221} & \\
\text{eleven}\,)\underline{1210} & +\,\text{t} \\
11 & +\,4
\end{array}$$

Hence 201221 in scale 3
= 44 t *in scale* 11 *Ans.*

Note.—In such cases as this we may also, if preferred, proceed otherwise, first changing the given number into the *common* scale, and thence into the required scale. Thus—

$$\begin{aligned}
201221 \text{ in scale } 3 &= 2 \times 3^5 + 0 \times 3^4 + 1 \times 3^3 + 2 \times 3^2 + 2 \times 3 + 1 \\
&= 2 \times 243 + 0 + 27 + 18 + 6 + 1, \text{ in scale } 10 \\
&= 538 \text{ in scale } 10.
\end{aligned}$$

We now change 538 (scale 10) to scale 11, thus—

$$\begin{array}{r l}
11\,)\underline{538} & \\
11\,)\underline{48} & +\,t \\
4 & +\,4
\end{array}$$

Hence 201221 (scale 3) = 538 (scale 10)
= 44*t* (*scale* 11). *Ans.*

In order to transfer a *Vulgar* Fraction from one scale to another, we change both numerator and denominator *separately* into the new scale.

RADIX FRACTIONS.

A **Radix Fraction** in any scale corresponds to a *Decimal* fraction in the Common Scale.

For instance, just as $\cdot 413$ in scale 10 means $\frac{4}{10} + \frac{1}{10^2} + \frac{3}{10^3}$,

so $\cdot 413$ in scale 7 means $\frac{4}{7} + \frac{1}{7^2} + \frac{3}{7^3}$.

A *vulgar* fraction is changed into a *radix* fraction *in the same scale* by dividing the numerator by the denominator (as in Decimals) performing the division in the given scale.

To transfer a **Radix** fraction from one scale to another.

If we multiply a *decimal*, say $\cdot 875$, by any number less than *ten*, say 4; then $\cdot 875 \times 4 = 3 \cdot 5$

$$\therefore \cdot 875 = \frac{3\cdot 5}{4} = \frac{3}{4} + \frac{\cdot 5}{4}$$

But $\cdot 5 \times 4 = 2$

$$\therefore \cdot 5 = \frac{2}{4}, \quad \therefore \quad \frac{\cdot 5}{4} = \frac{2}{4^2}$$

$$\text{Hence } \cdot 875 = \frac{3}{4} + \frac{2}{4^2}$$

i.e. $\cdot 875$ *in scale* 10 = $\cdot 32$ *in scale* 4.

Again, if we multiply a *radix fraction* of any scale, say $\cdot 43$ in scale 9, by any number less than *nine*, say 6, performing the work in scale 9;

then, in scale 9, $\cdot 43 \times 6 = 2 \cdot 8$

$$\therefore \cdot 43 = \frac{2\cdot 8}{6} = \frac{2}{6} + \frac{\cdot 8}{6}$$

But, in scale 9, $\cdot 8 \times 6 = 5 \cdot 3$

$$\therefore \cdot 8 = \frac{5\cdot 3}{6}, \qquad \therefore \frac{\cdot 8}{6} = \frac{5\cdot 3}{6^2} = \frac{5}{6^2} + \frac{\cdot 3}{6^2}$$

and, in scale 9, $\cdot 3 \times 6 = 2$, $\quad \therefore \cdot 3 = \frac{2}{6}$, and $\therefore \frac{\cdot 3}{6^2} = \frac{2}{6^3}$

$$\text{Hence } \cdot 43 = \frac{2}{6} + \frac{5}{6^2} + \frac{2}{6^3}$$

i.e. $\cdot 43$ *in scale* 9 = $\cdot 252$ *in scale* 6.

Hence the following Rule—

(II) **Multiply** the **fractional** *part of the given radix fraction by the* **radix of the new scale**, and the fractional part of the result by the same radix, and so on; then the *integral* parts of the successive products are the figures in order of the transformed radix fraction.

EXAMPLE viii.—*Express the decimal* 286·78125 *as a radix fraction in Scale* 8.

N.B.—The integral and fractional parts must be dealt with separately.

Integral part.

By Rule (I)

$$\begin{array}{r} 8\,)\,286 \\ 8\,)\,35 + 6 \\ 4 + 3 \end{array}$$

286 = 436 in scale 8

Fractional part

By Rule (II)

$$\begin{array}{r} \cdot 78125 \\ 8 \\ \hline \mathbf{6}\cdot 25000 \\ 8 \\ \hline \mathbf{2}\cdot 00000 \end{array}$$

·78125 = ·62 in scale 8

Ans. 436·62 in scale 8.

EXAMPLE ix.—*Express the Senary radix fraction* ·45 *as a decimal.*

[By Rule (II) we multiply successively by *ten*, *performing the work in scale* 6.]

$$\begin{array}{r} (Six.) \\ \cdot 45 \\ \text{ten} \\ \hline \mathbf{8}\cdot 02 \\ \text{ten} \\ \hline \mathbf{0}\cdot 32 \\ \text{ten} \\ \hline \mathbf{5}\cdot 32 \\ \text{ten} \\ \hline \mathbf{5}\cdot 32 \end{array}$$

Hence ·45 in scale 6 = $\cdot 80\dot{5}$ in scale 10 *Ans.*

EXAMPLE x.—*Change* $e0t\cdot t4$ *from the scale of* 12 *to the scale of* 8.

[By Rule (I) we divide $e0t$ by **8**, *performing the work in scale* 12.]

$$\begin{array}{r} (Twelve.) \\ 8\,)\,e0t \\ 8\,)\,147 + 2 \\ 8\,)\,20 + 7 \\ 3 + 0 \end{array}$$

[By Rule (II) we multiply the *fractional* part successively by **8**, *performing the work in scale* 12.]

$$\begin{array}{r} (Twelve.) \\ \cdot t4 \\ 8 \\ \hline \mathbf{6}\cdot t8 \\ 8 \\ \hline \mathbf{7}\cdot 14 \\ 8 \\ \hline \mathbf{0}\cdot t8 \end{array}$$

Hence $e0t\cdot t4$ in scale twelve = $3072\cdot\dot{6}7\dot{0}$ in scale 8. *Ans.*

EXAMPLE xi.—*Express the decimal* $\cdot 08\dot{3}$ *as a radix fraction in Scale* 6.

Multiply the fractional part successively by 6, and, performing the work in the common scale, we find that

$\cdot 08\dot{3}$ in scale 10 = $\cdot 02\dot{5}$ in scale 6.

Now just as $\cdot\dot{9}$ in scale **10** = 1 (see page 121), so $\cdot\dot{\mathbf{5}}$ in scale **6** = 1.

$$\begin{array}{r} \cdot 08333\ldots\ldots \\ 6 \\ \hline \mathbf{0}\cdot 4999\ldots \\ 6 \\ \hline \mathbf{2}\cdot 9999\ldots \\ 6 \\ \hline \mathbf{5}\cdot 9999\ldots \end{array}$$

Hence $\cdot 08\dot{3}$ in scale 10 = ·03 in scale 6. *Ans.*

To express a recurring radix fraction as a vulgar fraction in the same scale.

The method follows from that given on page 121.

For instance, consider the mixed recurring radix fraction $\cdot 5\dot{4}\dot{3}$ in scale 8.

$$\text{Since } \cdot 5\dot{4}\dot{3} = 5\cdot 434343\ldots\ldots$$
$$\therefore (\text{eight})^3 \text{ times } \cdot 5\dot{4}\dot{3} = 543\cdot 4343\ldots\ldots\ldots$$
$$\text{and eight times } \cdot 5\dot{4}\dot{3} = 5\cdot 434343\ldots\ldots$$
$$\text{Hence, by subtraction, } [(\text{eight})^3 - \text{eight}] \text{ times } \cdot 5\dot{4}\dot{3} = 543 - 5$$
$$\therefore \cdot 5\dot{4}\dot{3} = \frac{543 - 5}{(\text{eight})^3 - \text{eight}}$$
$$= \frac{543 - 5}{770} \text{ in scale 8.}$$

Note.—This result corresponds exactly with that on page 121.

i.e. as in scale 10 there are in the den^r as many *nines* as there are recurring figs.
so in scale 8 .... *sevens* ....

And so on.

Hence we can write down at once the vulgar fraction equivalent to a given recurring radix fraction in any scale:

e.g. $\cdot\dot{5}7\dot{3}$ in scale 9 $= \dfrac{573}{888}$ in scale 9; and $2\cdot 45\dot{3}$ in scale 6 $= 2\dfrac{453 - 45}{500}$ in scale 6.

EXAMPLE xii.—*In what scale of notation is the common scale number* 4783 *expressed by the figures* 11257?

Here the *units'* figure of the number, when expressed in the scale whose radix is required, is 7.

$\therefore$ 4783 − 7, *i.e.* 4776 must be exactly divisible by the req^d radix.

Also the req^d radix must be a number greater than 7, since this figure is used in the scale.

But the *only small* factors of 4776 are 2, 3, 4, 6, and 8.

$\therefore$ the scale is that of 8. *Ans.*

EXAMPLE xiii.—*How may weights of* 1, 3, 3^2, 3^3, 3^4, *&c., lbs. be used in a balance so as to weigh* 257 *lbs.?*

We express 257 in scale 3.

$$\begin{array}{r l} 3\,) & \underline{257} \\ 3\,) & \underline{85} + 2 \\ 3\,) & \underline{28} + 1 \\ 3\,) & \underline{9} + 1 \\ 3\,) & \underline{3} + 0 \\ & 1 + 0 \end{array}$$

Thus 257 = 100112 in scale 3
$= 3^5 + 3^2 + 3 + 2$
$= 3^5 + 3^2 + 3 + (3 - 1)$

Thus we see that one weight of 3^5 lbs., one of 3^2 lbs., two of 3 lbs. in one scale-pan, and 1 lb. *in the other* scale-pan, will weigh the given amount.

LXV. MISCELLANEOUS EXAMPLES.

MIXTURES.*

A (i).—*If* 9 *lbs. of tea costing* 1*s.* 8*d. per lb.*, 6 *lbs. costing* 1*s.* $10\frac{1}{2}$*d. per lb., and* 5 *lbs. costing* 2*s.* 4*d. per lb. be mixed, what is the cost per lb. of the mixture?*

$$\text{The total cost of the mixture} = \overset{d.}{20} \times 9 + \overset{d.}{22\tfrac{1}{2}} \times 6 + \overset{d.}{28} \times 5 = \mathbf{455}d.$$

$$\text{The total weight of the mixture} = 9 + 6 + 5, \text{ lbs.} = \mathbf{20}\ lbs.$$

$$\therefore \text{the cost of } \mathbf{1} \text{ lb. of the mixture} = \tfrac{455}{20}d. = 22\tfrac{3}{4}d. = 1s.\ 10\tfrac{3}{4}d.\ \ \textit{Ans.}$$

Note.—Hence, when the quantity and cost of each ingredient is given, the cost of a unit of the mixture is found by dividing the *total cost* by the *total* number of units in its *quantity*, *i.e.* we find the *average* cost per unit. (See Ex. D, page 21.)

A (ii).—*In what ratio must teas which cost* 1*s.* 4*d. and* 1*s.* 9*d. per lb. be mixed so that the mixture may cost* 1*s.* $7\frac{1}{2}$*d. per lb.?*

The cheaper tea costs 7 *halfpence* per lb. *less* than the mixture;
the dearer tea costs 3 .... per lb. *more* than the mixture.

[We must therefore so arrange that the gain on the amount of cheaper tea in the mixture *balances* the loss on the amount of the dearer tea.]

∴ with each **3** lbs. of cheaper we must take **7** lbs. of dearer tea. *Ans.*

[For the cost of 3 lbs. of the cheaper is 7 × 3 halfpence *less*, and the cost of 7 lbs. of the dearer is 3 × 7 halfpence *more*, than the corresponding cost of mixture, and these balance each other.]

A (iii).—*How may teas at* 1*s.* 3*d.*, 1*s.* 4*d.*, *and* 1*s.* 9*d. per lb. be mixed so as to form a mixture worth* 1*s.* 7*d. per lb.?*

The first kind costs **4** *pence* per lb. *less* than the mixture,
... second .... **3** .... *less* ....
... third .... **2** .... *more* ....

[We must so arrange the quantities that the *loss* on the third kind, *balances* the *gain* in the *other two kinds*, of tea.]

Now **2** lbs. of the third will thus balance **1** lb. of the first kind,
and **3** lbs. .... **2** lbs. ...... second ......

Hence, if with each **5** lbs. of the third we take **2** lbs. of the second and **1** lb. of the first, the mixture will be of the req[d] value.

Note.—Questions of this kind (in which *three*, or more, ingredients occur) are indeterminate; that is, they admit of a variety of solutions: *e.g.* instead of the above numbers the following will be found to satisfy the question:—7, 2, 2, or 8, 4, 1, &c.

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* Sometimes called *Alligation.* For other examples see Ex. iii, p. 47, and Exs. ix, x, p. 191.

A (iv).—*How may spirits costing* 14*s.*, 16*s.*, 18*s.*, *and* 18*s.* 6*d. per gallon be mixed so that a profit of* 20 *per cent may be made by selling the mixture at* 21*s. per gallon?*

The *cost* of a gallon of mixture = 21*s.* $\times \frac{100}{120}$* = **17***s.* **6***d.*

	sixpences		
Now the first kind costs	7	per gallon *less*	than the mixture,
... second	3	.... *less*	....
... third	1	.... *more*	....
... fourth	2	.... *more*	....

Hence **1** gal. of the *first* balances **7** gals. of the *third*,
and **2** gals. ...... *second* ...... **3** gals. ...... *fourth*.

Ans. 1, 2, 7, 3 gals. respectively.

Note.—There are many other possible solutions, *e.g.* 1, 1, 4, 3, &c.

A (v).—*A mixture of* 35 *gallons of spirit and water contains* 80 *per cent of spirit; how much spirit must be added to raise the percentage of spirit to* 85*?*

As the mixture contains **80**% of spirit, it contains **20**% of water.

∴ the number of gallons of water in it = $\frac{20}{100}$ of 35 gallons
= **7** gallons.

But there is to be **85** per cent of spirit, and ∴ **15** per cent of water, in the new mixture.

Hence 15 per cent of the increased quantity of mixture = 7 gals.
i.e. $\frac{15}{100}$ of the number of gallons in the new mixture = 7.

∴ the number of gallons in the new mixture = 7 $\times \frac{100}{15}$ = $46\frac{2}{3}$ gals.

∴ the number of gallons added = $46\frac{2}{3} - 35 = 11\frac{2}{3}$ gals. *Ans.*

A (vi).—*A* 36-*gallon cask is full of spirit;* 3 *gallons are drawn out, and the cask filled up with water; this process is repeated a second, and a third, time; how many gallons of spirit are then left in the cask?*

After the 1st operation, the cask contains 33 gallons of spirit,
and ∴ $\frac{33}{36}$, or $\frac{11}{12}$, of each gallon of mixture is spirit.

After the 2nd operation, the cask contains $33 - \frac{11}{12}$ of 3
$= 30\frac{1}{4}$, gallons of spirit,

and ∴ $\frac{30\frac{1}{4}}{36}$, or $\frac{121}{144}$, of each gal. of the mixture is spirit.

∴, after the 3rd operation, the cask contains $30\frac{1}{4} - \frac{121}{144}$ of 3
$= 27\frac{35}{48}$, gallons of spirit. *Ans.*

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* See (II), p. 187.

UNIFORM MOTION*: CIRCULAR TRACKS.

B (i).—*If I leave home at a certain time and walk to the railway-station at the rate of* 4 *miles an hour, I shall be* 2 *minutes too late for my train; if at the rate of* $4\frac{1}{2}$ *miles an hour, I shall have* 8 *minutes to spare; how far have I to walk?*

At 4 miles per hour I walk 1 mile in $\frac{1}{4}$ hour

... $4\frac{1}{2}$ .... 1 .... $\frac{2}{9}$ ......

$\therefore$ at the faster rate I save $\frac{1}{4} - \frac{2}{9}$, or $\frac{1}{36}$, hour in each mile.

But .... 2 + 8, min. or $\frac{1}{6}$ hour, in the reqd distance.

$$\therefore \text{ the req}^{\text{d}} \text{ distance} = \overset{\text{mile.}}{1} \times \frac{\frac{1}{6}}{\frac{1}{36}} = \underline{6 \text{ miles. } Ans.}$$

B (ii).—*The road from A to B rises. A man walked from A to B at the rate of* 3 *miles per hour, rested half an hour at B, and returned at the rate of* 5 *miles an hour, reaching A* 3 *hours* 26 *min. after he started. Find the distance from A to B.*

The man takes $\frac{1}{3}$ hour to walk 1 mile in going.

and $\frac{1}{5}$ .... 1 .... returning.

$\therefore$ his total time per mile is $\frac{1}{3} + \frac{1}{5}$, or $\frac{8}{15}$, hour

But his total time for the reqd dist. is $3\frac{13}{30} - \frac{1}{2}$, or $\frac{88}{30}$ hrs.

$$\therefore \text{ the req}^{\text{d}} \text{ dist.} = \overset{\text{mi.}}{1} \times \frac{\frac{88}{30}}{\frac{8}{15}} = \underline{5\tfrac{1}{2} \text{ mi. } Ans.}$$

B (iii).—*A and B travel in the same direction round a circular course* 220 *yards round, starting together from the same point, their rates being* 12 *and* 16 *miles an hour respectively; how soon will they again be together at the starting-point?*

As A goes 12 miles in 60 min.

$\therefore$ A makes a complete circuit, $\frac{1}{8}$ mi., in $\frac{60}{8 \times 12} = \frac{5}{8}$ min.

Similarly B .... $\frac{60}{8 \times 16} = \frac{15}{32}$ min.

[Hence the reqd interval is the smallest number of minutes which *exactly* contains $\frac{5}{8}$ and $\frac{15}{32}$, *i.e.* the L.C.M. of $\frac{5}{8}$ and $\frac{15}{32}$, or of $\frac{20}{32}$ and $\frac{15}{32}$, min.†]

Now the L.C.M. of 20 and 15 is 60,

$\therefore$ the L.C.M. of $\frac{20}{32}$ and $\frac{15}{32}$ is $\frac{60}{32} = 1\frac{7}{8}$. $\underline{Ans.\ 1\frac{7}{8} \text{ min.}}$

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* For other Examples see pages 143, 144. † Compare Ex. D, p. 95, for the method.

B (iv).—*A and B start together from the same point and travel in the same direction round a circular track* 220 *yards in circumference, A at the rate of* 14, *and B at the rate of* 17, *miles an hour; when and where will they be together again for the first time?*

Also, if they travelled in opposite directions, what intervals would elapse between their successive meetings?

(i) B gains 3 miles an hour on A;

$\therefore$ B gains a complete round, $\frac{1}{8}$ mi., on A in $\frac{1}{24}$ hour $= 2\frac{1}{2}$ min.

Now in $\frac{1}{24}$ hour A goes $\frac{1}{24}$ of 14 mi. $= 4\frac{2}{3}$ furlongs, or "laps" of 220 yds.

$\therefore$ when B first overtakes A, A has completed 4 *laps* and $\frac{2}{3}$ of a fifth, and is $\therefore$ $\frac{1}{3}$ of 220 yds. $= 73\frac{1}{3}$ yards from the starting-point.

Ans. (i) in $2\frac{1}{2}$ min.; $73\frac{1}{3}$ yds. from starting-point.

(ii) When they move in opposite directions they separate, or approach each other, at the rate of $14 + 17 = 31$ miles per hour.

$\therefore$ they *jointly* complete a circuit, $\frac{1}{8}$ mi., in $\frac{1}{8 \times 31}$ hour $= 14\frac{16}{31}$ secs.

Hence they meet at intervals of $14\frac{16}{31}$ secs. *Ans.* (ii).

B (v).—*A, B, and C start together from the same point and travel in the same direction round a circular course of* 100 *yards, at rates of* 21, 19, *and* $17\frac{2}{3}$ *feet per second respectively; when and where will all three next be together again?*

A gains 2 feet per sec. on B,

$\therefore$ A gains a complete circuit on B, *i.e.* overtakes him, in $\frac{300}{2}$ secs. $= 150$.

A gains $3\frac{1}{3}$ feet per sec. on C,

$\therefore$ A gains a complete circuit on C, in $\frac{300}{3\frac{1}{3}} = 90$, secs.

Hence all three will next be together in the L.C.M. of 150 and 90, secs., *i.e.* in 450 secs. $= 7\frac{1}{2}$ min.

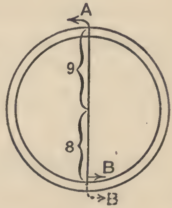
And in 450 secs. A goes 450×7 yds. $= 3150$ yds. $= 31\frac{1}{2}$ circuits.

Hence all three are next together in

$7\frac{1}{2}$ min., and at 50 yards from the starting-point.

N.B.—It would not do in this case to find as in B (iii) the L.C.M. of the times they severally take to complete a circuit, for that would give the interval after which all would be together at the *starting-point*; whereas they are all together, at another point, *sooner*. To find when all are together for the *first* time after the start, we must always, as above, find the *L.C.M. of the times the fastest traveller takes to gain a complete circuit on each one of the others.*

B (vi).—*A and B ride at the same rate,* 18 *miles per hour, in the same direction, round concentric circular tracks, the circumference of A's track being* 440 *yards, and the diameter of B's being to that of A's in the ratio of* 8 : 9. *If they start from opposite points, what interval will elapse before B hides A from the view of a spectator at the centre?*



B's *angular* motion (*i.e.* his apparent motion as viewed by the spectator at the centre 0) will be unchanged, if we suppose him *transferred to A's track* and moving on it at a rate *increased in the ratio of the diameters.*

The problem, then, is to find how long B will take to catch A, who has a start of 220 yards, A moving at the rate of 18 mi. per hour, and B supposed to move on the same track at the rate of $18 \times \frac{9}{8} = 20\frac{1}{4}$, mi. per hour.

	mi.		min.
Now B will gain	$2\frac{1}{4}$	in	60
∴ B	$\frac{1}{8}$	...	$60 \times \dfrac{\frac{1}{8}}{2\frac{1}{4}} = 3\frac{1}{3}$ min. *Ans.*

Work: Uniform Growth, &c.

C (i).—*If* 8 *men with* 6 *boys can dig a piece of ground in* 2 *days, and* 6 *men with* 2 *boys can dig it in* 3 *days; how long would* 5 *men with* 3 *boys take to dig it?*

	men.		boys.				days.	
	8	+	6	do	the whole work	in	2	
∴	8	+	6	do	$\frac{1}{2}$ of the work	...	1	...............(i)
Again	6	+	2	do	the whole work	in	3	
∴	6	+	2	do	$\frac{1}{3}$ of the work	...	1	...............(ii)

[We now take such multiples of the statements (i) and (ii) as to obtain *the same number* of men (or boys) in both. In this case, then, we multiply (i) by 3 [*i.e.* three times as many men + three times as many boys would do three times as much work in a day as is stated in (i)], and (ii) by 4.]

	men.		boys.			day.
Hence, from (i),	24	+	18	do $\frac{3}{2}$	of the work in	1
and from (ii),	24	+	8	do $\frac{4}{3}$	....	1

[Now in these two new statements there is the same number of men, so the extra amount of work in the former must be due to the extra number of boys.]

∴, by subtraction, 10 boys do $\frac{3}{2} - \frac{4}{3}$, or $\frac{1}{6}$, of the work in 1 day

∴ 1 boy does $\frac{1}{60}$ .... 1 ...

But from (ii) 6 men + 2 boys do $\frac{1}{3}$ of the work 1 day.

$\therefore$ 6 men alone do $\frac{1}{3} - \frac{2}{60}$, or $\frac{3}{10}$, ... 1 ...

$\therefore$ 1 man ...... does $\frac{1}{20}$ .... 1 ...

Hence 5 men + 3 boys do $\frac{5}{20} + \frac{3}{60}$, or $\frac{3}{10}$, ... 1 ...

$\therefore$ 5 men + 3 boys .... the whole ...... $1 \times \frac{10}{3}$ days

$= 3\frac{1}{3}$ days. *Ans.*

C (ii).—*If* 10 *oxen were put into a field, which has remained empty for a time, the grass would last them for* 20 *days; but it would last* 8 *oxen for* 35 *days; how long would it last* 12 *oxen, the grass being supposed to grow at a uniform rate?*

[Here we must distinguish between the daily growth of grass while the oxen are in the field, and the previously accumulated growth during the time the field was empty; this latter we shall call the *original* grass.]

20 days' growth + the orig. grass feeds 10 oxen for 20 days.
$\therefore$ 20 .... + ... 10×20, or 200, ... 1 day ... (i).

Again,
35 days' growth + the orig. grass feeds 8 for 35 days.
$\therefore$ 35 .... + ... 8×35, or 280, ... 1 day ...(ii).

[Now in the statements (i) and (ii) the amount of *original* grass is the same in both, so the extra oxen in (ii) are provided for by the extra *daily* growth.]

Hence from (i) and (ii) by subtraction,

15 days' growth feeds 80 oxen for 1 day,

$\therefore$ 1 day's growth ...... $\frac{80}{15}$, or $\frac{16}{3}$, ... 1 ....(iii).

Hence in the req[d] case

$12 - \frac{16}{3}$, or $\frac{20}{3}$, oxen *must be provided for* by the *original* grass.

But, by (i), 20 days' growth + the original grass feeds 200 oxen for 1 day.

And, from (iii), 20 days' growth alone feeds $\frac{16}{3} \times 20$, or $\frac{320}{3}$, ... 1

$\therefore$ the original grass feeds $200 - \frac{320}{3}$, or $\frac{280}{3}$, ... 1

$\therefore$ .... $\frac{20}{3}$ ... $1 \times \frac{280}{20}$ dys.

$= 14$ days *Ans.*

Note.—The principle of the above question may, of course, be clothed in words in various ways; for instance, the *figures* involved in the solution of the following question are the same as those given above, the *words* alone needing to be changed:—

A cistern partly full, into which a steady stream of water is flowing, has

a number of equal holes in the bottom, which can be opened or closed at will. If 10 *were opened the cistern would be emptied in* 20 *minutes; if* 8 *were opened it would be emptied in* 35 *minutes. Twelve are opened: how soon will it be emptied?*

EQUATION OF PAYMENTS.

The *Equated time of Payment* of several sums of money due at different dates is the interval at the end of which all might fairly be paid together.

For instance, if £100 is due in 3 months and £200 is due in 8 months, the equated time of payment is the number of months (between 3 and 8) at the end of which a payment of £300 would balance the two debts.

Now as £300 at the end of this interval is equivalent to £100 in 3 mo. + £200 in 8 mo., the *Present Value** of the £300 must be equal to the *sum of the present values* of the £100 and the £200.

If, then, we suppose 5 per cent to be the rate at which discount is calculated, we have

$$£300 - £300 \times \frac{\text{req}^{\text{d}}\text{ no. of months}}{12} \times \frac{5}{100}$$
$$= £100 - £100 \times \frac{3}{12} \times \frac{5}{100} + £200 - £200 \times \frac{8}{12} \times \frac{5}{100},$$

which reduces to

$$£300 \times \text{req}^{\text{d}}\text{ no. of months} = £100 \times 3 + £200 \times 8.$$
$$\therefore \text{req}^{\text{d}}\text{ no. of months} = \frac{£100 \times 3 + £200 \times 8}{£300}.$$

N.B.—The *rate per cent* does not enter into the result.

Hence the following practical rule for finding the Equated time of Payment (or *Average* term of Credit as it is sometimes called):

Multiply each debt by the number of months (*or days*) *which elapse before it is due, and divide the sum of these products by the sum of the debts.*

D.—*The equated time of payment of* £200 *cash, a bill of exchange for* £350 *due in* 60 *days, and another bill due in* 90 *days, is* 45 *days; find the face-value of the second bill.*

By the above rule,

$$\text{the equated time} = \frac{£200 \times 0^{\dagger} + £350 \times 60 + req^d\ sum \times 90}{£200 + £350 + req^d\ sum}.$$

Hence from the question

$$\frac{£350 \times 60 + req^d\ sum \times 90}{£550 + req^d\ sum} = 45,$$

$$\text{or } £2100 + 90 \text{ times } req^d\ sum = (£550 + req^d\ sum) \times 45$$
$$= £24750 + 45 \text{ times } req^d\ sum.$$
$$\therefore 45 \text{ times } req^d\ sum = £24750 - £2100 = £3750.$$
$$\therefore req^d\ sum = \frac{£3750}{45} = \underline{£833,\ 6s.\ 8d.\ Ans.}$$

* *Bankers'* discount. † As £200 is due in *ready money* its multiplier is 0.

E.—*The sum of* 103 *consecutive numbers is* 105472; *find the least of them.*

[The *middle* number of any odd number of consecutive numbers is the *average** of them; *e.g.*:

$$\frac{3+4+5+6+7+8+9}{7} = \frac{42}{7} = 6, \text{ the middle number. Hence—]}$$

$$\text{The req}^{\text{d}} \textit{ middle } \text{number} = \frac{105472}{103} = 1024,$$

and there are 51 of the *consecutive* numbers below, and 51 above, this.

$\therefore$ the least is $1024 - 51 = \underline{973}$ *Ans.*

Note.—The same principle applies in the case of a series of numbers formed by the successive addition of any other constant number instead of unity; *e.g.* such a series as 17, 20, 23, 26, &c. Series so formed are said to be in Arithmetic Progression. Moreover, as the sums of the first and last terms; 2nd and last but one; and so on, are all equal, *the sum of any number of terms* of such a series is found by *multiplying the sum of the* 1st *and last terms by half the* number *of terms:*

e.g. To find the sum of 86 terms of the series, 1, 3, 5, 7, &c.

Here the **2**nd term = the 1st + 2; the **3**rd term = the 1st + *twice* 2; the **4**th term = the 1st + *thrice* 2; and so on.
Hence the **86**th term = the 1st + **85** times 2;
and the sum of 86 terms = $(1 + 1 + 85 \times 2) \times 43 = 7396$.

F.—*The product of two numbers, whose difference is* 5, *is* 54516; *find them.*

[We perform the work of finding the square root of 54516 until we reach the units' figure, when by trial we choose a figure small enough to leave a final remainder which is equal to five times the "quotient".]

G.—*The dimensions of a box are as* 3 : 4 : 5; *the difference in cost between covering it all over at* 6*d. and* $6\frac{1}{2}$*d. per square foot is* 13*s.*; *find its length.*

13*s.* = 312 halfpence.

Hence the area of the entire surface of the box is 312 sq. ft.

Now the area of the whole surface of a similar box of dimension 3, 4, and 5 feet = $3 \times 4 \times 2 + (4 + 5) \times 2 \times 3$, sq. ft. = 78 sq. ft.; and 312 = 4 times 78.

i.e. the surface of the box whose length is req$^{\text{d}}$
= 4 times the surface of this one.

[But the *areas* of similar figures are in the ratio of the *squares* of their *lengths*.†]

$$\text{Hence req}^{\text{d}} \text{ length} = \sqrt{4} \times 5, \text{ ft.}$$
$$= \underline{10 \text{ feet}} \textit{ Ans.}$$

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* See p. 21. † Euclid, Book VI. Prop. 20.

APPENDIX.

A. Complementary Methods.—The Arithmetical *Complement* of a given number is the number which, when added to it, makes up the unit of next higher order, *e.g.* 3 is the *complement* of 7, since $7 + 3 = 10$; 14 is the *complement* of 986, since $986 + 14 = 1000$.

In Subtraction the complementary method, exemplified below, might well replace any other form of "wording" in the teaching of beginners.

EXAMPLE.—*Subtract* 2653 *from* 8129.

"*Wording.*"	3 and **6** (set down 6) make 9;	8129
	5 and **7** (set down 7) make 12; carry 1,	2653
(1 and 6, 7),	7 and **4** (set down 4) make 11; carry 1,	5476 *Ans.*
(1 and 2, 3),	3 and **5** (set down 5) make 8.	

Note.—This method of subtracting is convenient in such cases as those of the Ex. on p. 8, Ex. iii on p. 39, and in "Abridged" Division.

The use of "complementary" methods often effects considerable saving of labour. *e.g.* See Ex. A (v), p. 19; A (viii), p. 20 (called *Synthetic* Division); Ex. x, p. 42; Ex. ix, p. 77: Exs. vii, viii, p. 98; Ex. ii, p. 101; Note (iii), p. 104.

B. General Definition of Multiplication, applicable to a *fractional* as well as to an integral multiplier.

To multiply one number by a second is to do to the first *that which is done to* unity *to obtain the second.*

Integers: e.g., 8×6,

To obtain 6 we repeat unity *six* times, *i.e.* $1 + 1 + 1 + 1 + 1 + 1, = 6$

$\therefore$, by the above definition, $8 \times 6 = 8 + 8 + 8 + 8 + 8 + 8 = 48$.

Fractions: e.g., $\frac{3}{4} \times \frac{5}{7}$,

To obtain $\frac{5}{7}$, unity is divided into *seven* equal parts, of which *five* are taken;

$$i.e.\ \frac{5}{7} = \frac{1}{7} + \frac{1}{7} + \frac{1}{7} + \frac{1}{7} + \frac{1}{7};$$

$$\therefore\text{, by the above definition, } \frac{3}{4} \times \frac{5}{7} = \frac{\frac{3}{4}}{7} + \frac{\frac{3}{4}}{7} + \frac{\frac{3}{4}}{7} + \frac{\frac{3}{4}}{7} + \frac{\frac{3}{4}}{7}.$$

$$\text{But } \frac{3}{4} \div 7 \text{ is } \frac{3}{28},^{*}\ \therefore \frac{3}{4} \times \frac{5}{7} = \frac{3}{28} + \frac{3}{28} + \frac{3}{28} + \frac{3}{28} + \frac{3}{28} = \frac{15}{28}, \text{ or } \frac{3 \times 5}{4 \times 7}.$$

C. Prime Numbers.—All the primes below any given number may be found by the following method, known as the "*Sieve of Eratosthenes*".†

Write down the numbers in natural order as far as required, thus:—

1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, &c.

Mark every	*second*	number after	2;	these are the multiples of		2;
"	" *third*	"	" 3;	"	" "	3;
"	" *fifth*	"	" 5;	"	" "	5;
"	" *seventh*	"	" 7;	"	" "	7;
"	" *eleventh*	"	" 11;	"	" "	11;

and so on, taking each of the unmarked numbers in turn.

Then the numbers which finally remain *unmarked* are *primes*, since no number left unmarked will be divisible by any number, except unity, less than itself.

* See p. 78. † A Greek of Alexandria, who lived about 200 B.C.

D. Proofs of the Tests of Divisibility. (See page 52.)

(I) Any number may be regarded as a multiple of *ten* + some *units*;

e.g. 4756 is 475 *tens* + 6 *units*,

and, as *ten* is divisible by 2, ∴ any no. of *tens* is also divisible by 2;

∴ any number is divisible by 2, if its units' digit is divisible by 2, *i.e.* if *its units' digit is even.*

(II) Proof exactly similar to that of (I).

(III) Any number may be regarded as a multiple of a *hundred* + the no. of *units* represented by the *two right-hand* digits;

e.g. 83475 is 834 *hundreds* + 75 *units*,

and, as 100 is divisible by 4, or by 25, ∴ any number of *hundreds* is divisible by 4, or by 25;

∴ any number is divisible by 4, or by 25, *if the number represented by the two right-hand digits is divisible by* 4, *or by* 25.

(IV) Consider any number, *e.g.* 23587.

$$23587 = 20000 + 3000 + 500 + 80 + 7.$$

Now, as $\mathbf{10000} = \mathbf{9999} + 1$, ∴ $20000 = 9999 \times 2 + 2 =$ a multiple of $9 + 2$,
as $\mathbf{1000} = \mathbf{999} + 1$, ∴ $3000 = 999 \times 3 + 3 =$ a multiple of $9 + 3$,
as $\mathbf{100} = \mathbf{99} + 1$, ∴ $500 = 99 \times 5 + 5 =$ a multiple of $9 + 5$,
and as $\mathbf{10} = \mathbf{9} + 1$, ∴ $80 = 9 \times 8 + 8 =$ a multiple of $9 + 8$,
also $7 = 7$.

Hence, by addition, $23587 =$ some mult. of $9, + 7 + 8 + 5 + 3 + 2$.

∴ 23587 is divisible by 9 *if the sum of its digits is divisible by* 9.

And any other number might have been dealt with in the same way.

Note.—The proof of the test of divisibility by 3 is identical with this.

(V) Consider any no., *e.g.* $564328 = 500000 + 60000 + 4000 + 300 + 20 + 8$.

Now, as any number represented by an *even* no. of *nines* is divisible by 11,
and as $\mathbf{100000} = \mathbf{99990} + \mathbf{11} - 1$, ∴ $500000 =$ a multiple of $11 - 5$;
as $\mathbf{10000} = \mathbf{9999} + 1$, ∴ $60000 =$ a multiple of $11 + 6$;
as $\mathbf{1000} = \mathbf{990} + \mathbf{11} - 1$, ∴ $4000 =$ a multiple of $11 - 4$;
as $\mathbf{100} = \mathbf{99} + 1$, ∴ $300 =$ a multiple of $11 + 3$;
and as $\mathbf{10} = \mathbf{11} - 1$, ∴ $20 =$ a multiple of $11 - 2$;
also $8 = 8$.

Hence, by addition, $564328 =$ some multiple of $11 + 8 + 3 + 6 - (2 + 4 + 5)$,

∴ 564328 is divisible by 11, *if the difference between the sum of the units', hundreds', &c., digits, and the sum of the tens', thousands', &c., digits is divisible by* 11.

And any other number might have been dealt with in the same way.

The Tests of Divisibility may be summarized thus:—

1. *If the divisor is a factor of*
   (i) the *radix* (*e.g.* 2, 5, factors of 10), the criterion is *the* **units'** *figure*;
   (ii) the *square of the radix* (*e.g.* 4, 25, factors of 100), the criterion is *the number represented by the* **two** *right-hand figures*;
   (iii) the *cube of the radix* (*e.g.* 8, 125, factors of 1000) the criterion is *the number represented by the* **three** *right-hand figures.* And so on.
2. *If the divisor is* **one less** *than the radix*, the criterion is *the sum of the digits.*
3. *If the divisor is* **one more** than the radix, the criterion is *the difference between the sums of the alternate digits.*

Hence Tests corresponding to those of the Decimal Scale of Notation may be applied in any scale; *e.g.*:—

A no. *in Scale* 6 is divisible by 5, if the *sum of its digits* is divisible by 5:
..................12 .... 3, ... *units' figure* is divisible by 3; &c.

E. Casting out Nines.—The reason of this process, applied as a test of multiplication, may be shown thus:—

Any number may be regarded as a multiple of 9 + a remainder consisting of one of the digits (including 0) less than 9. Hence

The multiplicand = a multiple of 9 + some remr, say 7............(i),
and the multiplier = a multiple of 9 + some remr, say 4............(ii)
∴ the product = (i) × (ii).
= a mult. of 81 + 7 times a mult. of 9
+ 4 times a mult. of 9 + 4 × 7*
= some mult. of 9, + 4 × 7.

Hence the remr when the product is divided by 9 = the remr obtained by dividing 4 × 7 by 9.

And these remrs we know, by **D** (IV), are obtained by dividing the *sum of the digits* of each number by 9.

Note 1.—Casting out Nines may be applied as a test of Addition and Subtraction; *e.g.* in Addition: Cast out 9's from each of the numbers added; add the resulting remrs, and cast out nines from their sum, and the final remr should be the same as the remr obtained by casting out 9's from the sum *of the given numbers.*

Note 2.—It is *possible* that a result may satisfy the test of casting out Nines and yet not be correct, since *an error in it amounting to* 9 *or a multiple of* 9 would not affect the *remainder* obtained by dividing it by nine.

Note 3.—Results may be tested in a similar way by *Casting out Elevens.* The way in which this would be done is indicated in D (V).

F. Some General Properties of Numbers.—Besides those already considered the following are noteworthy:—

(i) The sum (or diff.) of *two even*, or of *two odd*, numbers must be *even.*
(ii) The sum (or diff.) of *an even, and an odd*, number must be *odd.*
(iii) The sum of any *even* number of numbers must be *even.*
(iv) The sum of any *odd* number of *odd* numbers must be *odd.*
(v) The product of *even* nos., or of *even and odd* nos., must be *even.*
(vi) The product of *two*, or more, *odd* nos. must be *odd.* And conversely, in *exact* division, if the *dividend* is *odd* the *quotient* is *odd* also.

Note.—The above are evident from the results of the Addn and Multn Tables, since they depend only on the *units'* figures of the numbers.

(vii) *The product of the sum and difference of two numbers is equal to the difference of their squares.*

Take, *e.g.*, any two numbers, say 23 and 7.
Then, the product of their sum and diff

$$\begin{aligned} &= (23 - 7) \times (23 + 7) \\ &= (23 - 7) \times 23 \quad + (23 - 7) \times 7^* \\ &= 23 \times 23 - 7 \times 23 + 23 \times 7 - 7 \times 7^* \\ &= 23^2 - 7^2 \\ &= \text{the diff. of their squares.} \end{aligned}$$

Similarly we might deal with *any other* two numbers.

EXAMPLE 1.—*Simplify* $\dfrac{37{\cdot}8 \times 37{\cdot}8 - 62{\cdot}2 \times 62{\cdot}2}{37{\cdot}8 - 62{\cdot}2}$ *with as little labour as possible.*

By converse of (vii), $\dfrac{(37{\cdot}8)^2 - (62{\cdot}2)^2}{37{\cdot}8 - 62{\cdot}2} = 37{\cdot}8 + 62{\cdot}2 = \underline{100 \textit{ Ans.}}$

---

* By (II), p. 10.

EXAMPLE 2.—*Show that if 2^{20} be decreased by unity the result is a multiple of* 11.

By (vii), $2^{20} - 1 = (2^{10} + 1) \times (2^{10} - 1)$
$= (2^{10} + 1) \times (2^5 + 1) \times (2^5 - 1)$.

Now $2^5 = 32$, $\therefore 2^5 + 1 = 33$, a multiple of 11.

EXAMPLE 3.—*Prove that the difference of the squares of any two odd numbers is a multiple of* 4.

By (viii), the diff. of the squares of the two odd nos.
= (the sum of the nos.) × (their diff.)
= (an *even* no.) × (an *even* no.).........By (i)
= (an odd no. × 2) × (an odd no. × 2)
= a multiple of 4.

EXAMPLE 4.—*If any prime number greater than* 3 *be increased, and also decreased, by unity, one or other of the results is a multiple of* 6.

For every prime (except 2) is *odd*, $\therefore$ both results are *even* and divisible by 2. But the prime − 1, the prime, and the prime + 1 are three consecutive nos., and when the numbers are arranged in natural order every 3rd no. is divisible by 3. $\therefore$ one of *any three consecutive* nos. must be divisible by 3. But this is not the case with the prime.

Hence one of the two "results" is divisible by 3, and is also even.
$\therefore$ it is a multiple of 6.

EXAMPLE 5.—*If the sum of an odd number of fractions, whose numerators and denominators are all odd, is an integer, that integer must be odd.*

Consider, *e.g.*, the *three* fractions $\frac{5}{7}$, $\frac{3}{11}$, $\frac{9}{13}$;

$$\text{their sum} = \frac{5 \times 11 \times 3 + 3 \times 7 \times 13 + 9 \times 7 \times 11}{7 \times 11 \times 13} \text{.........(Q)}$$

Now, by (vi), each of the products $5 \times 11 \times 3$, $3 \times 7 \times 13$, $9 \times 7 \times 11$ is *odd*;
and $\therefore$, by (iv) their *sum* is *odd*.

Also $7 \times 11 \times 3$ is *odd*. $\therefore$, by (vi), the quotient (Q) is *odd*.

Similarly the theorem may be proved in the case of any other set of fractions fulfilling the given conditions.

EXAMPLE 6.—*Show that every perfect cube is either divisible by* 7, *or when divided by* 7 *gives remainder* 1 *or* 6.

Every no. = a mult. of 7 +, or −, 0, 1, 2 or 3.
$\therefore$ Every *cube* = a mult. of 7 +, or −, 0, 1, 8 or 27.*
= some mult. of 7 +, or −, 0, 1, 1 or 6.

EXAMPLE 7.—*If a perfect square end in an odd digit, the last figure but one is even.*

For any odd no. may be regarded as a mult. of *ten* + an odd digit, say 3.

Then its square = (a mult. of 10 + 3)^2
= a mult. of 100 + *twice* a mult. of 10 × 3 + 3^2†
= some *even* mult. of 10 + 9.

Hence the last figure but one is even.

---

* This is evident if we expand, say (mult. of 7 + 3)^3; compare p. 255.
† By Theorem (III), p. 246.

EXAMPLE 8.—*Show that* 1331 *is a perfect cube in any scale of notation whose radix is greater than* 3.

We have seen, on p. 255, that the cube of any number in the Decimal scale, *e.g.*, 47^3, is of the form $4 \times 10^3 + 3 \times 10^2 \times 7 + 3 \times 10 \times 7^2 + 7^3$

Hence, $1 \times 10^3 + 3 \times 10^2 \times 1 + 3 \times 10 \times 1^2 + 1^3 = 1331$, in scale 10,

$1 \times 6^3 + 3 \times 6^2 \times 1 + 3 \times 6 \times 1^2 + 1^3 = 1331$, in scale 6,

$1 \times 5^3 + 3 \times 5^2 \times 1 + 3 \times 5 \times 1^2 + 1^3 = 1331$, in sc. 5, &c.

Thus 1331 is a perfect cube in any scale whose radix is greater than 3.

G. Recurring Decimals. (See p. 122.)

Note 1.—When the recurring period of one of a set of fractions, which yield pure recurring decimals with the maximum no. of figures in their periods, is known, the periods of all others of the set can be written down at once.

e.g. $\frac{1}{17}$ yields a decimal with the maximum number of figures, 16, in its period; hence, if $\frac{1}{17}$ be found as a decimal, and the figures of its period arranged in a ring (as is done for *sevenths* on p. 122), then $\frac{2}{17}$, $\frac{3}{17}$, &c., can at once be read off.

Note 2.—As the figures of the quotient *must* recur when all possible remainders have appeared in the division, but *may* recur sooner, it follows that, in the latter case, the number of figures in the actual period is a *factor* of the number in the maximum;

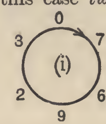
e.g. $\frac{1}{13} = \cdot\dot{0}7692\dot{3}$, a *six*-figure period, where 6 is a factor of 12, the *maximum* in the case of *thirteenths*.

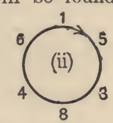
In such cases, as many rings would be needed as the shorter period is contained in the maximum; in this case *two*. It will be found that Ring (i) gives

$\frac{1}{13}, \frac{3}{13}, \frac{4}{13}, \frac{9}{13}, \frac{10}{13}, \frac{12}{13}$;

and Ring (ii),

$\frac{2}{13}, \frac{5}{13}, \frac{6}{13}, \frac{7}{13}, \frac{8}{13}, \frac{11}{13}$.





H. Continued Fractions.—On p. 89 is given a method of reducing a Continued Fraction to a Simple Fraction. We here consider the converse process, which will be evident from the following example.—

Reduce $\frac{274}{79}$ *to a Continued Fraction.*

$$\tfrac{274}{79} = 3 + \tfrac{37}{79} = 3 + \cfrac{1}{\tfrac{79}{37}} = 3 + \cfrac{1}{2 + \tfrac{5}{37}} = 3 + \cfrac{1}{2 + \cfrac{1}{\tfrac{37}{5}}} = 3 + \cfrac{1}{2 + \cfrac{1}{7 + \tfrac{2}{5}}} \text{ \&c.}$$

Now in this operation it is evident that the successive remainders resulting from the divisions, viz., 37, 5, 2, &c., become less and less, so that we must at last arrive at the remainder 1, when the operation ends.

Hence every *rational* fraction (vulgar or decimal) can be expressed as a *terminating* continued fraction.

Note.—The above process exactly corresponds to the ordinary method of finding the G.C.F. of 274 and 79, the numbers 3, 2, 7, &c., being the successive *quotients*.

The fraction which results from stopping at any stage, in the above process, and discarding the remainder at that stage, is called a *Convergent* to the continued fraction.

Thus, in the above example, the *first* convergent is $3 = \frac{3}{1}$,

... *second* .... $3 + \frac{1}{2} = \frac{7}{2}$,

... *third* .... $3 + \cfrac{1}{2 + \frac{1}{7}} = \frac{52}{15}$.

The following properties of convergents are noteworthy.

(1.) *The convergents are alternately less and greater than the complete fraction.*

e.g., in the above ex., the 1*st* convergent is *too small*, since the part $\frac{1}{6}$, &c., is omitted;
the 2*nd* .... *too great*, since the *den*[r] 2 is *less* than $2+\frac{1}{7}$;
the 3*rd* .... *too small*, since the *den*[r] $2+\frac{1}{7}$ is too great, &c.

(2.) *The difference between any two successive convergents is a fraction whose numerator is unity, and denominator the product of their den*[rs].

$$e.g., \frac{7}{2}-\frac{52}{15}=\frac{105-104}{2\times 15}=\frac{1}{2\times 15};\text{ and }\frac{111}{32}-\frac{52}{15}=\frac{1665-1664}{32\times 15}=\frac{1}{32\times 15}.$$

From (1) and (2) it follows that

(3.) *Any convergent is nearer to the complete fraction than the preceding convergent; that it differs from the complete fraction by less than half of* $\frac{1}{\text{prod. of den}^{\text{rs}}\text{ of convergents}}$; *and that it is in lowest terms.*

Hence the successive convergents are successive **approximations** to the value of the continued fraction.

To convert a Surd *into a Continued Fraction, e.g.,* $\sqrt{10}$.

Observing that the greatest perfect square below 10 is 9, we proceed thus—

$$\sqrt{10}=3+\sqrt{10}-3=3+\frac{\sqrt{10}-3}{1}=3+\frac{(\sqrt{10}-3)\times(\sqrt{10}+3)}{1\times(\sqrt{10}+3)}\cdots$$ See p. 252.

$$=3+\frac{1}{\sqrt{10}+3}=3+\frac{1}{6+\sqrt{10}-3}=3+\cfrac{1}{6+\cfrac{1}{\sqrt{10}+3}};\text{ and so on.}$$

$$\therefore \sqrt{10}=3+\cfrac{1}{6+\cfrac{1}{6+\cfrac{1}{6+\&c.}}}$$

Thus a surd produces a *recurring* continued fraction.

Note 1.—By the definition of a surd it is evident that it *cannot* produce a *terminating* continued fraction, for *that* we have seen can be reduced to a *rational* simple fraction, whereas a *surd* is *incommensurable, i.e.* cannot be *exactly expressed in figures.*

Note 2.—A Continued Fraction is often written in the form :—$3+\frac{1}{6}+\frac{1}{6}+\frac{1}{6}+\cdots$

K. Proof of Theorem (IV), Square Root. (See p. 250.)

By Theorem (III), page 246,

$$365^2=360^2+2\times 360\times 5+5^2,\quad \therefore\ \frac{365^2-360^2}{2\times 360}=5+\frac{5^2}{2\times 360}.$$

Hence, in the case of any *three*-figure square root, if when *two* figures of the root have been obtained, we divide the *remainder* (*i.e.* the diff. between the given number and the square of the part of the root already obtained) by the last complete divisor (*i.e.* by double the part of the root already obtained), the

$$\text{quotient} = \text{the remaining fig.} + \frac{(\text{rem}^{\text{g}}\text{ fig.})^2}{\text{double the part of the root already obtained}},$$

and this latter portion of the quotient must be a *proper* fraction, since the num[r] can never exceed 9^2, and the den[r] can never be less than 200.

$\therefore$ the *integral* part of the above quotient = the last fig. of the root.

Thus the theorem holds good in the case of a *three*-figure root.

Similarly it may be shown to hold good in any other case.

L. Cube Root by Horner's Method.—The *reason* of the method belongs to the Theory of Equations; the method is, however, easy to *apply*.

The work is arranged in four columns; in the first we place **1**; in the second and third, **0**; and in the fourth, the number (whose cube root is required) marked off into "periods".

EXAMPLE 1.—*Find the cube root of* 24389.

The integral part of the cube root of the first period is 2. Set 2 on the right.

1	0	0	24'389(29 *Ans.*
	2	4	8
	2	4	16 389
	2	8	16 389
	4	**1200**	
	2	621	
	60	1821	
	9		
	69		

1st stage.

Multiply **1** by the root-fig. 2, and add to 2nd column.

Multiply the result by the root-fig. 2, and add to 3rd column.

Multiply this result by the root-fig. 2, and subtract from 1st period.

Trial Divisor stage.

Multiply **1** by the root-fig. 2, and add to 2nd column.

Multiply the result by the root-fig. 2, and add to 3rd column, afterwards appending *two* ciphers.*

Again multiply **1** by the root-fig. 2, and add to 2nd column, afterwards appending *one* cipher.*

Bring down the next period, and obtain the next fig. of root, 9, by help of the *trial divisor* **1200**.

2nd stage. (We now repeat with the new root-fig. 9 the process of stage 1.)

Multiply **1** by 9 and add to 2nd column.

Multiply the result by 9 and add to 3rd column.

Multiply this result by 9 and subtract from 4th column.

Note.—In forming the 2nd and 3rd columns the multiplications and *additions*; and, in the 4th column, the multiplications and *subtractions*, may easily be combined (see Ex., p. 12).

EXAMPLE 2.—*Find the cube root of* 3 *to three places of decimals.*

Written work in full as in Ex. 1.

1	0	0	3·'000'000'000(1·442
	1	1	1
	1	1	2 000
	1	2	1 714
	2	**300**	256 000
	1	136	241 984
	30	436	14 016 000
	4	152	
	34	**58800**	
	4	1696	
	38	60496	*Ans.* 1·442
	4	1712	
	420	**6220800**	
	4		
	424		
	4		
	428		

Condensed, as suggested in the Note to Ex. 1.

1	0	0	3·' (1·442
	1	1	2 000
	2	**300**	256 000
	30	436	14 016 000
	34	**58800**	
	38	60496	
	420	**6220800**	
	424		
	428		

Ans. 1·442

Note.—If this cube root had been reqd to *five* places of decs. the two extra figs. could be obtained by *Contracted* division. (See p. 258.)

*The reason for these ciphers will be seen by comparing with the example on p. 257.

EXERCISES.—PART I.

I. NUMERATION, NOTATION.

Read, or write in *words*,

1. 30800.
2. 51020.
3. 18200.
4. 70020.
5. 384610.
6. 423654.
7. 2000000.
8. 3003000.
9. 700100.
10. 160011.
11. 503010.
12. 675032.
13. 3503071.
14. 4004004.
15. 32005701.
16. 60600752.
17. 10101010.
18. 122012.
19. 33333333.
20. 50000000.
21. 18800008.
22. 19099090.
23. 567802005.
24. 10007063.
25. 3756421871.
26. 8300235007.
27. 308056300072.
28. 1000000000000.
29. 236750846374.
30. 3008700050000.

Write in *figures*,

31. Seventeen thousand and twenty.
32. One hundred and two thousand, seven hundred.
33. Six hundred and fifty thousand.
34. Five hundred and forty-three thousand and eleven.
35. Three million seven hundred thousand and seventy.
36. Thirteen million, one thousand three hundred.
37. Five million, six hundred and ninety-four thousand, three hundred and eighty-seven.
38. Twenty-three million, one hundred and ten.
39. Four hundred and one million, forty thousand, four hundred and forty.
40. Seven hundred and sixty-eight million six hundred and seventy-five thousand five hundred and eighty-six.
41. Eighty million, eighteen thousand.
42. Seven thousand and eleven million sixty thousand, three hundred.
43. One thousand and three million, one thousand and three.
44. Forty-nine thousand two hundred and eighty-one million, five hundred and fifty thousand, six hundred and sixty.

Write in *figures*,

45. One hundred thousand and seventy million, fourteen thousand and eight.

46. Three hundred and sixty thousand million, three hundred and six thousand, and thirty-six.

47. Two hundred thousand and twenty million, twenty thousand, two hundred.

48. One billion, one million, one thousand and one.

49. Seventeen billion eight hundred and ninety thousand and forty-five million, seven hundred and ninety-six thousand and sixteen.

50. Nineteen billion, one thousand and ninety million, one hundred thousand, nine hundred.

Read, or write in Arabic figures,

51. XIX.	61. LXXXIX.	71. DCCXC.
52. XXIV.	62. XCV.	72. DCLXXIX.
53. LVIII.	63. CXII.	73. MDCCLXIV.
54. XLV.	64. CCIV.	74. MDCXLVII.
55. LXXVI.	65. CLXVI.	75. MDCCCLXXXV.
56. LXVIII.	66. CXLIV.	76. MDLXVI.
57. XLIX.	67. CCLXXXIV	77. MDLXXXVIII.
58. LXXXIII.	68. DXL.	78. MDCLXVI.
59. CIV.	69. CCCXC.	79. MDCCLXXXIX.
60. CLX.	70. DLV.	80. MDCCCXCIX.

Express in Roman numerals,

81. 34.	86. 203.	91. 1241.	96. 1100.
82. 46.	87. 555.	92. 1336.	97. 1413.
83. 73.	88. 423.	93. 1759.	98. 1689.
84. 87.	89. 324.	94. 1867.	99. 1888.
85. 99.	90. 560.	95. 1891.	100. 5000.

II. THE SIMPLE RULES.

Addition.

1.	2.	3.	4.	5.
973	59432	237	240738	348170
49	917	46	59618	91754
61358	6480	32752	485	2760
876	753	47	79	96287
56	96075	187246	4328	462
208	99	7609	19576	5720
1095	2160	9642	275843	18234
283	84317	35132	3629	29216
94	908	23109	73	4056
9048	7563	36	69384	80099

6.	7.	8.	9.	10.
13455	6704	50462	246	59386
6124	78534	4000	10988	370
448	3460	31078	8537	12
90653	79473	2053	54561	2694
7127	36324	275	4123	49057
48	5968	5268	40345	371804
67422	5374	63516	67087	96287
5100	63239	44363	564	1590
589	41258	5086	62742	329105
3694	48485	65045	56653	40462
87318	47256	54678	62583	708
3256	23245	56743	51646	402917
939	9172	6726	96	348170
16285	365	93623	88219	13465
6903	1335	7036	8136	150918

Find the *sum* of—

11. 56841, 7345, 860, 7843217, 14, 500109, 6234, 503, 8040, and 172.

12. 13721, 179, 1270, 430920, 1275, 7079, 32, 7700, 14030211, and 71.

13. 3204, 8571, 396, 14, 9763, 2842, 971, 8309, 2135, 614, 73, 428, 1867, 3905, and 428.

14. 910, 4016, 18791, 20645, 17453, 46, 2712, 406, 57916, 71865 4262, 90014, 370, 12, and 2694.

15. 24910, 4016, 18791, 20645, 17453, 46, 2712, 40689, 57916, 71865, 4262, 90014, 370, 12, 2694, and 49057.

16. 3204, 8571, 396, 14, 9763, 2842, 971, 8309, 2135, 614, 73, 428, 1867, 3905, 428, 6054, 378, 4915, 3284, and 418.

Find the *sum* of—

17. One million, fifteen thousand and eighty; four hundred and nine thousand, seven hundred and ninety; two hundred and forty-two thousand, six hundred and thirty-nine; seventeen thousand, one hundred; and seven thousand, one hundred and seven.

18. Sixty thousand and seventy-six; one hundred and twelve thousand, nine hundred and four; eighty-seven thousand, two hundred and ninety; seven hundred thousand, six hundred and twenty-seven; and eight thousand, eight hundred and eighty-eight.

19. Three hundred and forty thousand and fifty; five millions, nine hundred and twenty-two thousand and nine; seven hundred and four thousand, three hundred and four; twenty thousand and five; sixty-five thousand, six hundred.

20. Seventy million, seventy thousand, eight hundred and nine; eight million and eight; nine hundred and eighty-five thousand, nine hundred and eighty; sixty thousand, eight hundred and five; and nine million, eight thousand, four hundred and forty-four.

Add *across**—

21. $21+43+109+8$.
22. $32+5+91+210$.
23. $180+15+2000+11$.
24. $47+8+967+150$.
25. $17+93+8+154+62$.
26. $9+142+16+11+140$.
27. $42+31+102+9+67$.
28. $750+110+38+70+3$.
29. $92+201+859+4121+7$.
30. $45+309+70+257+8008$.
31. $37+201+1073+16+4$.
32. $1035+67+138+200+19$.
33. $9+33+801+7605+80+7$.
34. $84+307+666+3041+28+1$.
35. $235+25+76+972+270+5$.
36. 9378, 927, 70, 4837, 67, 234.
37. 23, 3916, 204, 1795, 3070, 69.
38. 590617, 475, 180, 106, 25, 2.
39. 201, 384, 17, 6001, 889, 7801.
40. 678, 294, 5, 4291, 328, 55, 1.
41. 56741, 37, 414, 2190, 125, 2013, 4191.
42. 31129, 1341, 1014, 758, 9429, 4711, 10.
43. 2120, 10672, 2923, 89, 19098, 3058, 6120.
44. 31280, 645, 39811, 3064, 11421, 3125, 849.
45. 2152, 3205, 220, 930, 4073, 11909, 386, 2072.
46. 487, 6590, 7803, 2499, 16075, 925, 769, 6291.
47. 876, 1096, 5018, 830, 6537, 1070, 673, 856, 210, 845.
48. 13721, 2356, 2076, 4323, 13276, 74, 7079, 320, 486, 1405.
49. 14245, 819, 9195, 2573, 2406, 1275, 7495, 197, 770, 892.
50. 451, 6003, 876429, 790, 8035, 42, 8, 70312, 170, 8764.

* These Exercises should be worked as they stand; the *answer only* being *written*.

SUBTRACTION.

51. Subtract 4378 from 10649.

52. Subtract 6847 from 9203.

53. From 12007 subtract 11438.

54. From 30201 subtract 7745.

55. Take 14730 from 25000.

56. Take 50687 from 67127.

57. From 123456 take 65432.

58. From 987654 take 123456.

59. Find $750420-93874$.

60. Find $165207-158709$.

61. Find $857142-142857$.

62. Find $987654-321098$.

63. Find $3080715-2991032$.

64. Find $4743021-4446666$.

65. Find the difference between 8975 and 9857.

66. Find the difference between 30405 and 15069.

67. How much greater is 10000 than 9875?

68. How much greater is 40871 than 3900?

69. How much less is 3085 than 12713?

70. How much less is 5893 than 17000?

71. What number must be added to 13708 to make 38190?

72. What number must be added to 120781 to make 138005?

73. What number must be taken from 4708 to leave 1809?

74. What number must be subtracted from 84701 to leave 896?

75. By what number does 12341 exceed 9087?

76. Find the excess of 44444 over 8797.

77. Find the remainder when 4731 is decreased by 4097.

78. Find the remainder when 803 is deducted from 10071.

79. The sum of two numbers is 55555; the greater is 34567: find the less.

80. The sum of two numbers is 10,000; the greater is 8203: find the less.

81. The sum of two numbers is 76804; the less is 1992: find the greater.

82. The sum of two numbers is 14004; the less is 399: find the greater.

83. The difference between two numbers is 589; the greater is 42876: find the less.

84. The difference between two numbers is 2003; the greater is 40010: find the less.

85. By how much does the sum of 36052 and 27581 exceed their difference?

86. By how much does the sum of 60572 and 534891 exceed their difference?

87. From three hundred and two thousand and eleven take eighty-seven thousand seven hundred and eight.

88. Take thirty-seven thousand from one million five thousand.

89. What number must be added to seven hundred million, four hundred and six thousand, five hundred and four to produce eight hundred million?

90. Find the difference between three thousand million, seven hundred and six thousand; and one million, eight hundred and six thousand and seven.

Find the value of—

91. $281-179+325$.
92. $78+432-493$.
93. $46-8-17+3-22$.
94. $23-16+41-19-27$.
95. $72-4+13-23-7-49$.
96. $8+171-29-102+16$.
97. $63-21+17-24+8-19$.
98. $379+801-978-175+2$.
99. $8+19-23+74-89+17$.
100. $176-23-84-15+4-56$.
101. $14-27+122-86$.
102. $37-73+85-41$.
103. $2-71-801+1230-97$.
104. $14-23+2-13-9+38$.
105. $3-11-17-86+45-1+80$.
106. $301-482+65-139+297$.
107. $223-4851+1764+3020$.
108. $102-7689+57+8008$.
109. $67-18973-351+24531-3$.
110. $681-3451+8795-870+76$.

111. $6-7+8-9+1-2+3-4+5+1-12+8-17+20$.
112. $67-76-89+98-54+43+12-41-53+99$.

Find, in *one* operation,

113. $4369-875-654-1207$.
114. $5001-967-856-2330$.
115. $738962-7482-371806-359$.
116. $57431-3209-7698-24807$.

117. Take the sum of 367, 8298, 774207, and 999 from a million.

118. What number must be added to the sum of 82760, 509, 7360 and 93, to make up a total of one hundred thousand?

119. Take the sum of thirty-five millions six hundred and twenty-nine thousand and seventy-three; eight millions two thousand and six; and five millions seven hundred and four; from forty-nine millions fifty thousand six hundred and eleven.

120. The populations of the different parishes of a large town are 16640, 321, 3750, 3906, 5144, 2684, 13360, 391, and 5797 respectively; by how many does the population of the town fall short of sixty thousand?

MULTIPLICATION.

Read off (or write), *at sight*, the product of—

121. 87×10.
122. 56×100.
123. 19×1000.
124. 175×10.
125. 30×100.
126. 700×100.
127. 150×100.
128. 308×10.
129. 430×100.
130. 1002×10.
131. 17×10000.
132. 403×100.
133. 1074×10.
134. 7030×100.
135. 500×100.
136. 710×1000.
137. 270×10000.
138. 800×10000.

Multiply—

139. 123456789 by 9.
140. 987654321 by 9.
141. 857142 by 14.
142. 142857 by 49.
143. 123456789 by 63.
144. 987654321 by 36.
145. 97302 by 7016.
146. 85762 by 3109.
147. 2895 by 35700.
148. 82653 by 19800.
149. 20356740 by 3960.
150. 4205970 by 6390.

Find, using *not more than three* lines in multiplying,

151. 179430×507900.
152. 80410×479030.
153. 30700×54603.
154. 60080×124071.
155. 96800×203750.
156. 380700×765043.

157. Find the product of 867490 and 80900.
158. Find the product of 806010 and 100102.
159. Multiply thirty million and thirty thousand by four hundred thousand and eighty.
160. Multiply one million nine thousand and eighty-seven by six hundred thousand five hundred and forty-three.
161. Multiply 2468 by 3057.
Hence, without any further work, write down the product of 2468 and 500.
162. Multiply 129847 by 468.
Hence, without further *multiplication*, obtain the product of 129847 and 4008.
163. Multiply the sum of fourteen thousand two hundred and thirty-one, and thirteen thousand two hundred and twelve by their difference.
164. Find the product of the sum and difference of two hundred and thirty-eight thousand four hundred and twenty-eight, and one hundred and five thousand and sixty-nine.

Read off (or write), *at sight*, the continued product of—

165. $17 \times 10 \times 10$.
166. $24 \times 10 \times 100$.
167. $8 \times 1000 \times 10$.
168. $75 \times 10 \times 10 \times 10$.
169. $300 \times 10 \times 10$.
170. $810 \times 10 \times 100$.
171. $47 \times 2 \times 5$.
172. $89 \times 5 \times 2$.
173. $79 \times 2 \times 50$.
174. $2 \times 2 \times 5 \times 5$.
175. $7 \times 5 \times 20$.
176. $23 \times 25 \times 4$.

Find the continued product of—

177. 17, 26, and 54.
178. 19, 35, and 87.
179. 95, 462, and 932.
180. 6363, 5252, and 4141.
181. 7070, 6006, and 5342.
182. 19, 28, 307, 460, and 5000.

Find the *square* of—

183. 19.
184. 55.
185. 103.
186. 210.
187. 161.
188. 235.
189. 3108.
190. 4690.
191. 5432.
192. 7777.

Find the *cube* of—

193. 17.
194. 21.
195. 64.
196. 98.
197. 215.
198. 444.
199. 6280.
200. 1111.

Division.

Read off (or write), *at sight*, the quotient and remainder of—

201. $423 \div 10$.
202. $7681 \div 100$.
203. $4865 \div 1000$.
204. $2030 \div 10$.
205. $5140 \div 100$.
206. $4700 \div 10$.
207. $35000 \div 100$.
208. $7006 \div 10$.
209. $8046 \div 1000$.
210. $76008 \div 100$.
211. $36001 \div 100$.
212. $23400 \div 10$.
213. $30101 \div 100$.
214. $27070 \div 100$.
215. $580000 \div 10$.
216. $34050 \div 1000$.
217. $46321 \div 10000$.
218. $120008 \div 1000$.

Find, by *one short* division, the quotient and remainder of—

219. $436251 \div 7$.
220. $780575 \div 9$.
221. $2360513 \div 8$.
222. $680507 \div 11$.
223. $8465307 \div 12$.
224. $668800795 \div 12$.
225. $86431 \div 20$.
226. $57942 \div 30$.
227. $85767 \div 60$.
228. $670432 \div 110$.
229. $8763251 \div 900$.
230. $3805407 \div 500$.
231. $37091 \div 20$.
232. $761053 \div 40$.
233. $558710 \div 70$.
234. $146357 \div 120$.
235. $4605643 \div 7000$.
236. $77635081 \div 8000$.

Divide—

237. 87401 by 23.

238. 226314 by 31.

239. 9516156 by 52.

240. 5942718 by 74.

241. 1719035 by 87.

242. 5973467 by 243.

243. 3597840 by 789.

244. 16122344 by 536.

245. 7855568 by 4173.

246. 56831974 by 3278.

247. 4280960342 by 15003.

248. 237000675 by 63041.

249. 25600160001 by 159601.

250. 888888888888 by 63492.

Divide, using *no unnecessary ciphers*—

251. 386451 by 230.

252. 77513 by 4300.

253. 32541700 by 79000.

254. 687524 by 53000.

255. 393051 by 13100.

256. 34287000 by 17000.

257. 7654302 by 310000.

258. 8463200 by 6700.

259. 7674528 by 43100.

260. 77777777 by 98700.

Find, by using two *short* divisions, the quotient and the complete remainder of—

261. $9315261 \div 16$.

262. $2910141 \div 18$.

263. $917551 \div 24$.

264. $61385 \div 28$.

265. $921124 \div 35$.

266. $571882 \div 36$.

267. $9178854 \div 49$.

268. $106513 \div 48$.

269. $2494614 \div 63$.

270. $1911200 \div 66$.

271. $72295 \div 42$.

272. $478752 \div 81$.

273. $607191 \div 72$.

274. $7997152 \div 64$.

275. $18051231 \div 96$.

276. $22519 \div 84$.

277. $111725 \div 99$.

278. $232181 \div 132$.

279. $4234172762 \div 144$.

280. $814359509 \div 121$.

Divide, using *short* divisions to obtain the quotient and complete remainder,

281. 21011 by $3 \times 5 \times 7$.

282. 43387 by $4 \times 6 \times 7$.

283. 199916 by $7 \times 8 \times 9$.

284. 392252 by $8 \times 9 \times 5$.

285. 92828 by $9 \times 9 \times 9$.

286. 62432 by $11 \times 11 \times 11$.

287. 710818 by $3 \times 5 \times 7 \times 9$.

288. 51327 by $2 \times 7 \times 8 \times 11$.

289. 401807 by $12 \times 12 \times 12$.

290. 603992 by $7 \times 7 \times 7 \times 7$.

291. 30721 by 490.

292. 86203 by 560.

293. 14730687 by 8100.

294. 15630293 by 10800.

295. How many *twenty-threes* make 1771?

296. How many *thirty-sevens* make 11248?

297. Find the remainder when 784135 is divided by 47.

298. Find the remainder when 4096371 is divided by 83.

299. How often can 61 be subtracted from 19032?

300. How many times can 430 be taken from 146200?

301. The dividend is 3452164, the quotient 1858; find the divisor.

302. The dividend is 833382, the quotient 4026; find the divisor.

303. How often is 6704 contained in 302129168?

304. How many times is 9897 contained in 823479885?

305. What remains when 97 has been taken as often as possible from 49381?

306. What is the final remainder after 371 has been subtracted as many times as possible from 93075?

307. By what number must 3094 be multiplied to produce 48288058?

308. What number, multiplied by 32643, produces 218773386?

309. The product of two numbers is 501000500; one of them is 3850; find the other.

310. The product of two numbers is 57380625; one of them is 7575; find the other.

311. Divide two million four hundred and ninety thousand two hundred and one by four thousand and eighty-nine.

312. Divide three thousand five hundred and ninety-five millions five hundred and twenty-one thousand six hundred and forty-five by fifty thousand seven hundred and nine.

313. The dividend is 582167, the quotient is 762, and the remainder is 761; find the divisor.

314. The dividend is 3954007, the quotient is 7060, and the remainder is 407; find the divisor.

315. By what number must 2765 be divided in order that the quotient may be 32 and the remainder 13?

316. The quotient resulting from the division of 2338425 by a certain number is 346, and the remainder is 157. Find the divisor.

317. The quotient is 231, the divisor 321, and the remainder 123; what is the dividend?

318. The divisor is 243, the quotient is 1306, and the remainder is 92; find the dividend.

319. The divisor is 1890, the quotient 365, and the remainder 52; find the dividend.

320. The divisor is 90213, the quotient 738, and the remainder 1857. Find the dividend.

III. MISCELLANEOUS EXERCISES.

1. Write 7707070 in words.
2. Write Thirteen million fifty thousand and eleven in figures.
3. Add together 23, 4217, 806, 1785, 149, 30781, and 6034.
4. Subtract 170492 from 203510.
5. Multiply 5009 by 4100.
6. Divide 1004561024 by 711.
7. Divide 706234 by 63 using short divisions.
8. Simplify $43 - 17 - 21 - 25 + 29 + 3$.
9. In the three volumes of a book there are 1256 pages; the first volume contains 432 pages; the second 396 pages; how many pages are there in the third volume?
10. If, from a sack containing 8900 nuts, 36 nuts were given to each of 245 children, how many nuts would be left in the sack?

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11. Write in words 41305087.
12. Write in figures Seventeen million four hundred and fifty thousand three hundred and two.
13. Add 142857, 285714, 428571, 571428, 714285, and 857142.
14. From 312750 take 17096.
15. Multiply 94100608 by 37009.
16. Divide 5640832 by 4064.
17. Find the complete remainder when 55555 is divided by the factors of 88.
18. Simplify $4751 - 235 - 178 - 45 - 86 - 1937$.
19. William I. died in 1087. Queen Victoria began to reign in 1837 at the age of 18. How many years after the death of William I. was Queen Victoria born?
20. In a page of a newspaper there are 8 columns; in each column 190 lines; and in each line 42 letters. How many letters are there in the page?

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21. Write in words MDCCLXXVII.
22. Write 1851 in Roman numerals.
23. Find the sum of seventy-nine, ninety, nineteen, eleven hundred, seven thousand and ten, ninety-seven thousand, eighteen hundred and sixty, and twelve.
24. Find the difference between two thousand and two hundred thousand.
25. Find the product of 5807 and 2479.

26. How many times is 101 contained in 707707?

27. Cube 61.

28. What is that number which when multiplied by 13 gives the product 59514?

29. A man aged 42 is three times as old as his son; how old was he when his son was born?

30. A train consisted of 17 carriages, each capable of seating 22 persons. There were 293 passengers in the train. How many seats were vacant?

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31. Write in figures MCCCCXIII.

32. Express 1603 in Roman numerals.

33. Add together eight hundred and seventy, twenty-one, one thousand six hundred and twelve, thirteen thousand nine hundred and eighty, fifty-three, and ten thousand and nine.

34. From five hundred and eighty millions five hundred and three thousand and ninety-nine, subtract thirty millions sixty-three thousand and sixty-six.

35. Find the continued product of 47, 407, and 4007.

36. How many times can 111 be subtracted from 333333?

37. Square 7063.

38. The multiplier is 17 and the product is 511377. Find the multiplicand.

39. Seventeen years ago a man was four times as old as his son, whose present age is 29. What is the father's present age?

40. In a certain town there are 117 more females than males. The number of males is 4232. Find the population of the town.

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41. Write down separately in words the value of each 5 in the number 523517.

42. Find $148500 \div 12375$.

43. Find $1230401 - 487920$.

44. Find 70209×302050.

45. Find $87365 + 968476$.

46. Write down *at sight* the value of $74 \times 10 \times 2 \times 10 \times 5$.

47. Simplify $42 - 13 + 24 - 35 + 26 - 47 + 6$.

48. Find, without waste of labour, the quotient and remainder of $47506803 \div 73000$.

49. Find, shortly, $357 + 357 + 357 + 357 + 357 + 357 + 357$.

50. The sum of two numbers is 1872; the less is 746; find their difference.

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51. Write down separately in words the value of each 3 in the number 230731.

52. Find $2345538 \div 90213$.

53. Find $60728 + 13409$.

54. Find 70083×43572.

55. Find $176305 - 84250$.

56. Write down *at sight* the value of $460000 \div 2000$.

57. Simplify $41 - 35 + 8 - 56 + 23 - 17 + 41$.

58. Find, without waste of labour, the value of $2396 \times 2 \times 7 \times 50$.

59. Find, shortly, $384 - 47 - 47 - 47 - 47 - 47 - 47 - 47 - 47$.

60. The difference between two numbers is 953; the greater is 4687; find their sum.

---

61. Express 2300025015 in words.

62. Take the sum of 55, 666, and 7777 from 8888.

63. Multiply 4283 by 987.
Hence, without further work, write down the product of 4283 and 90; also the product of 4283 and 8000.

64. Divide 7398758 by 121, using short divisions.

65. Find, by inspection, the continued product $79030 \times 5 \times 2 \times 2 \times 5$.

66. Find the value of $856 - 394 + 289 + 436 - 752 - 123$.

67. How many times must 312 be added to 4321 that the sum may be 49561?

68. Show that the square of 40 together with the square of 9 is equal to the square of 41.

69. How many figures are made in writing down all the numbers from 1 to 99?

70. A has 63 marbles and B has 29. How many must A give B that they may each have the same number?

---

71. Express in figures, Thirteen million four hundred thousand and fifty-seven.

72. Simplify $3835 - 943 - 807 - 1279 - 56 - 644$.

73. Find the product of the sum and difference of 6713 and 6371.

74. By how much does the square of 31 exceed the square of 29?

75. Find, by inspection, the quotient and remainder of $37053 \div 100$.

76. Divide 2369360 by 7 and the result by 8.
Hence, without further division, write down the quotient of $2369360 \div 560$; also the quotient of $2369360 \div 70$.

77. How many times must 79 be taken from 3716 in order to leave 319?

78. Take any three consecutive numbers and show that the sum of the greatest and least is equal to twice the middle number.

79. Write down the greatest possible number of four figures.

80. Find a number which is as much less than 375 as it is greater than 213.

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81. Express 874 in Roman numerals.

82. Write three and a half millions in figures.

83. What number must be added to seven millions four hundred and six thousand five hundred and four that the sum may be eight millions?

84. Divide the product of 8576453 and 6839 by 10000.

85. Divide 678413 by 17.
Hence, without further work, write down the quotient and remainder of $678413 \div 1700$.

86. Simplify $13 - 27 + 8 - 41 + 23 - 14 + 53$.

87. What number, divided by 71, gives quotient 701 and remainder 17?

88. By how many does the square of 31 exceed the product of 29 and 33?

89. Seven years ago a man was six times as old as his son who will be 21 eight years hence. What is the father's present age?

90. How far would a boy have to walk to bring 10 oranges, placed in a row and 12 yards apart, one by one to a basket close to the first orange?

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91. Express in figures: Twenty-three million thirty thousand and five.

92. Find the difference between three score and ten, and two and a half dozen.

93. Take the sum of 1307, 587, 9630, and 23 from 22222.

94. Find the value of $730 \times 62 \times 51 - 4$.

95. Divide 570021 by 770, using short division.

96. By what number must 594 be multiplied that the result may be equal to the product of 702 and 429?

97. Divide 359907 by 789, and prove the result by casting out nines.

98. Find the value of $5^2 + 6^2 + 7^2 + 8^2$.

99. A chapter of a book began at the top of page 979 and ended at the bottom of page 1010: how many pages were in the chapter?

100. A farmer has 13 horses, twice as many pigs, five times as many oxen, and sixteen times as many sheep; how many animals does he possess?

101. Express 7306002015 in words.

102. Divide 675432 by 43000.

103. By how much does the product of 567 and 809 exceed their sum?

104. If the divisor, quotient, and dividend be respectively 123, 312, and 38389, what is the remainder?

105. If a man were 7 years older than he is, he would be twice as old as his son, who was 38 last year. How old is the father?

106. Prove that the cube of 41 is equal to the sum of the squares of 236 and 115.

107. Find, shortly, $4631 + 247 + 247 + 247 + 247 + 247 + 247$.

108. Simplify $8 \times 7 + 5 \times 9 + 3 \times 2$.

109. Multiply 8763 by 99 with as little labour as possible.

110. Find, shortly, the product of 37173 and 25.

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111. Write in words the number $\overline{\text{X}}$DCCCLXI.

112. How many figures are there in all the numbers from 1 to 200?

113. In a division sum the divisor is eight times, and the quotient seven times, the remainder. The remainder is 452. Find the dividend.

114. How many boxes, each capable of holding 173, would be required for 100000 oranges, supposing that a few were allowed to remain unpacked?

115. A book of 356 pages has on the average 40 lines on a page, and 52 letters in a line. How many letters are in the book?

116. Edward has 41 marbles, Frank has 29; how many must Edward give Frank that they may each have the same number?

117. Six years hence a man will be three times as old as his son whose present age is 13. How old was the father when the son was born?

118. Simplify $3 \times 15 - 8 \times 13 + 5 \times 12$.

119. Multiply 27864 by 11 with as little labour as possible.

120. Find, shortly, the product of 8643 and 125.

---

121. What do you understand by 3^4?

122. Write in words 70007007077.

123. Add together the sum, difference, product, and quotient of 537152 and 763.

124. What number subtracted 73 times in succession from 1497 will leave 37 remaining?

125. How old is a man who, 28 years ago, was 29 times as old as his son whose present age is 30?

126. Divide 240 into two parts, one of which shall be seven times as great as the other.

127. Find, by inspection, the value of $2 \times 2 \times 3 \times 7 \times 5 \times 5$.

128. Find a number which exceeds the square of half 42 by unity.

129. Multiply, as shortly as you can, 7864 by 999.

130. Multiply 3612 by 497 by a short method.

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131. In the number 658457, by how many does the value of one of the 5's exceed that of the other?

132. Divide 143 into two parts, the difference of which shall be 7.

133. What number multiplied by 119 gives the same product as 187 multiplied by 133?

134. The total number of guests at a ball was 163: there were 17 more ladies than gentlemen; how many ladies were there?

135. The continued product of 35, 36, and a third number is 165060; find the third number.

136. Take any seven consecutive numbers, and show that their sum is seven times the middle number.

137. Divide 1554584788 by 2197.

138. Find the value of $7 + 3 \times (12 + 5)$.

139. Divide 97 into two parts whose difference shall be 13.

140. Multiply 2781 by 357 in two lines.

---

141. Write $\overline{\text{X}}$DCCXLIX in words.

142. Find the sum of the greatest and least numbers each of five digits.

143. By how much does the cube of 73 exceed the square of 623?

144. The dividend is 18732510, and the quotient is 2643; find the divisor and remainder.

145. The product of 79 and 86 is 6794. Find, without multiplication, the product of 78 and 86.

146. Find, shortly, $3876 - 237 - 237 - 237 - 237 - 237 - 237$.

147. James had 14 more marbles than William; he gave William 6; how many more than William had he then?

148. Simplify $73 \times 13 - 12 \times 72 + 1$.

149. Multiply 387654 by 98 as shortly as possible.

150. Find $47091000 \div 125$ without actually dividing.

---

151. Subtract seventeen times three thousand and nine from half a million.

152. Find the value of $78351 - 89463 - 16231 + 30005$.

153. The sum of two numbers is 1767; the less is 572; find their difference.

154. How many bags, each holding 7316 pebbles, must be thrown together to make a heap of more than a million?

154. The sum of two numbers, one of which is treble of the other, is 7228; find them.

155. What number is contained as many times in 15929 as 881 is contained in 14977?

156. Write down *at sight* the value of $93 \times 8 \times 125$.

157. What number is that to the double of which if 9 be added the result is 55?

158. Divide 2716 into two parts, one of which shall be three times as great as the other.

159. Simplify $(7+3) \times 12+5$.

160. Find, shortly, 932571×125.

---

161. Find the difference between the greatest and least numbers which can be expressed by the digits 1, 5, 7, 3, 9.

162. Divide 823479885 by 9897.

163. Find, by inspection, $5 \times 10 \times 2 \times 25 \times 2 \times 5$.

164. The total number of boys and masters in a school is 140. There are 7 masters. What is the average number of boys for each master?

165. The total number of votes polled for two candidates at an election was 7397; the successful candidate's majority was 551; how many votes did each receive?

166. Ten years hence a man will be twice as old as his son who was 13 twenty years ago. Find their present ages.

167. Divide 6507 into two parts, the difference of which shall be 723.

168. Find the value of $19-(12+3)$.

169. Multiply 36165 by 287 in two lines.

170. Find, shortly, $47091375 \div 125$.

---

171. Find the sum of the four largest numbers that can be expressed by the four figures 3, 5, 2, 7.

172. Find the product of the sum and difference of 735 and 357.

173. By what number must 27658 be divided that the quotient may be 285 and the remainder 13?

174. In a force of 1000 volunteers 40 are officers: what is the average number of men to each officer?

175. The difference between two numbers is 36; their sum is 110; find them.

176. If from the square of a certain number we subtract 5 the result when divided by 4 yields quotient 11; find the number.

177. The sum of the ages of two boys is 23 years; seven years ago one of them was 6; how old are they?

178. Simplify $8 \times 5 - 7 \times (6 - 3)$.

179. Find 9368×125 without multiplying.

180. Multiply 2357 by 568 in two lines.

---

181. Write in figures Two billion twenty million two hundred.

182. What divisor of 145291 yields quotient 481 and remainder 29?

183. Find the complete remainder when 84359752 is divided by $7 \times 5 \times 11 \times 3$ by successive short divisions.

184. Find, shortly, the product of 99999 and 23758.

185. Find the sum of the seven consecutive numbers the least of which is 351.

186. In a match at chess the winner won three games out of every four which he played; he won 18 games altogether; how many did he lose?

187. A cricketer made the following scores during a cricket week:— 17, 9, 0, 41, 3, 26. What was his average for the week?

188. If from a certain number we subtract 3, divide the remainder by 3, add 3 to the quotient, and then multiply the resulting sum by 3, the product is 342. Find the number.

189. Simplify $8 \times (5 + 7) - 49 \div 7$.

190. Multiply 47653 by 512648 in three lines.

---

191. Express 6666 in Roman characters.

192. Multiply 810901 by 809101, and prove the result by casting out nines.

193. Divide the difference between 80000 and 57735 by 365.

194. A certain number when divided by 5, 7, and 11 in succession yielded remainders 2, 4, and 9 respectively; what was the complete remainder?

195. I distributed 79 oranges among 21 children, giving each boy 3 and each girl 5; how many girls were there?

196. The sum of two numbers is 5115, their difference 953; find them.

197. A, B, and C have 89 marbles between them; A and B have 43; B and C have 53; how many has each?

198. Find, by inspection, $2 \times 2 \times 5 \times 2 \times 5 \times 7$.

199. Find, shortly, the product of 2367 and 998.

200. Find the value of $(14 - 5)(13 - 6)$.

---

201. What number must be added to 4037 that the result may be equal to the sum of 19, 505, 650, and 9003?

202. Find, by inspection, $7 \times 11 \times 25 \times 2 \times 2$.

203. Divide 19094867 by 4009. Hence write down the quotient of $3058867 \div 4009$; also the quotient of $252567 \div 4009$.

204. The product of 79 and 86 is 6794: hence find, without multiplication, the product of 79 and 85.

205. A farmer one season lost one out of every six lambs born; 235 lived; how many died?

206. A class of 17 boys in an hour worked correctly 12, 16, 14, 9, 6, 3, 11, 8, 13, 13, 5, 0, 12, 8, 17, 6, 0, sums respectively; what was their average?

207. At an election there were two candidates and 4501 votes were recorded. The successful candidate's majority was 397. How many persons voted for him?

208. Find, shortly, the product of 3467 and 9990.

209. Simplify $79 - 2 \times (19 + 17 - 18)$.

210. Find, shortly, $566327 \div 25$.

---

211. Find the difference between the sum and the continued product of 22, 444, and 5555.

212. Divide 7891242387 by 3493, and prove the result by casting out nines.

213. The quotient is seven times the divisor, the divisor is seven times the remainder, and the sum of all three is 741; find the dividend.

214. Find the sum of 20 consecutive numbers beginning with 32514.

215. The difference between two numbers is 4321; their sum is 8765; find them.

216. Ten years ago the sum of the ages of a man and a boy was 31 years. The man is 15 years older than the boy. Find the present age of each.

217. Simplify $5 + (16 - 3) \times 4 - 1$.

218. Find, shortly, 44678×98.

219. Find, shortly, $938357 \div 2500$.

220. Multiply 380652 by 378546 in three lines.

---

221. If the base of our system of notation were 6 instead of 10, how many units would be represented by 423?

222. Find the product of the sum and difference of the greatest and least numbers of four digits.

223. Divide the sum of the numbers 34651, 96327, 65412, and 61810 by the sum of their digits.

224. A sum in division was correctly worked on a slate, and then partly rubbed out, when all that remained was what is here given. Restore the twelve missing figures, whose places are indicated by the asterisks.

```
4 *)* * 8 *(* 7
    * 8 *
    -----
      * 6 *
      * * 9
      -----
        * 0
```

225. The sum of two numbers is 999999; their difference is 714285; find them.

226. A man aged 50 will, seven years hence, be twice as old as his son will then be; how old is his son now?

227. At an election the total number of votes polled for three candidates was 9452; the successful candidate received 561 more votes than one, and 1702 more votes than the other unsuccessful candidate. What was the exact state of the poll?

228. Find, shortly, the product of 36493 and 9999.

229. Divide, shortly, 463285 by 99.

230. Multiply 234678 by 378426 in three steps.

---

231. What would 1234 represent if 5 were the base of our system of notation?

232. Find the product of the greatest and least numbers which can be represented by the four digits 1, 3, 5, 7.

233. Divide 3422 by 29, and explain the process.

234. The quotient is five times the divisor, and the divisor is three times the remainder. The difference between the quotient and remainder is 868. Find the dividend.

235. A multiplication sum, having been correctly worked on a slate, was partly rubbed out, and all that remained was what is shown here. Restore the twelve missing figures whose places are shown by the asterisks.

```
    4 * *
      3 *
  -------
  3 6 * *
* * 7 *
---------
* * 3 * *
```

236. A bag contains 450 nuts. The nuts are distributed among 69 children, each boy receiving 5 and each girl 7, and there is then one nut left in the bag. How many boys and how many girls are there?

237. Seven planks are laid side by side; three are placed across them; and so on, in successive layers of 7 and 3, the last being the 23rd layer. How many planks are there in the stack?

238. If the sum of 83, 176, 871, and 120 be added to a certain number, if the result be divided by 43, and the difference between the quotient and 308 be multiplied by 5, the final result is 1335. Find the number.

239. Multiply 182357 by 192648 as shortly as possible.

240. Find, as shortly as possible, the difference between 495×753 and 495×387.

IV. REDUCTION.

MONEY.

Reduce—

1. £113, 17*s.* 10$\frac{3}{4}$*d.* to farthings.
2. £819, 14*s.* 3$\frac{1}{2}$*d.* to farthings.
3. £1111, 11*s.* 11$\frac{1}{4}$*d.* to farthings.
4. £999, 19*s.* 9$\frac{3}{4}$*d.* to farthings.
5. £36, 3*s.* 10$\frac{1}{2}$*d.* to halfpence.
6. £47, 10*s.* 8$\frac{1}{2}$*d.* to halfpence.
7. £201, 15*s.* 4$\frac{1}{2}$*d.* to halfpence.
8. £222, 2*s.* 2$\frac{1}{2}$*d.* to halfpence.
9. £29, 19*s.* 9*d.* to threepences.
10. £43, 13*s.* 6*d.* to threepences.
11. £108, 17*s.* 3*d.* to threepences.
12. £441, 6*s.* 9*d.* to threepences.
13. £17, 17*s.* 8*d.* to fourpences.
14. £88, 8*s.* 8*d.* to fourpences.
15. £145, 19*s.* 4*d.* to fourpences.
16. £302, 2*s.* 4*d.* to fourpences.
17. £51, 15*s.* 6*d.* to sixpences.
18. £85, 18*s.* 6*d.* to sixpences.
19. £257, 11*s.* 6*d.* to sixpences.
20. £666, 6*s.* 6*d.* to sixpences.
21. £572, 18*s.* to florins.
22. £631, 14*s.* to florins.
23. £3051, 16*s.* to florins.
24. £4235, 8*s.* to florins.
25. £25, 15*s.* to crowns.
26. £68, 5*s.* to crowns.
27. £472, 10*s.* to crowns.
28. £833, 10*s.* to crowns.
29. £235, 7*s.* 6*d.* to half-crowns.
30. £643, 2*s.* 6*d.* to half-crowns.
31. £457, 12*s.* 6*d.* to half-crowns.
32. £326, 5*s.* to half-crowns.
33. £721, 17*s.* 6*d.* to half-crowns.
34. £830, 10*s.* to half-crowns.
35. £21, 2*s.* 4$\frac{1}{2}$*d.* to three-halfpences.
36. £42, 14*s.* 9*d.* to threepences.
37. £77431, 10*s.* to half-sovereigns.
38. £963, 16*s.* to double-florins.

Reduce—

39. 42035 pence to £, *s. d.*
40. 36941 pence to £, *s. d.*
41. 100000 farthings to £, *s. d.*
42. 444444 farthings to £, *s. d.*
43. 5000 halfpence to £, *s. d.*
44. 7673 halfpence to £, *s. d.*
45. 60301 halfpence to £, *s. d.*
46. 57003 halfpence to £, *s. d.*
47. 6845 twopences to £, *s. d.*
48. 9719 fourpences to £, *s. d.*
49. 22222 threepences to £, *s. d.*
50. 77777 threepences to £, *s. d.*
51. 37545 sixpences to £, *s. d.*
52. 8603 sixpences to £, *s. d.*
53. 12371 florins to £, *s. d.*
54. 17659 florins to £, *s. d.*
55. 487 crowns to £, *s. d.*
56. 555 crowns to £, *s. d.*
57. 1234 half-crowns to £, *s. d.*
58. 4321 half-crowns to £, *s. d.*
59. 7638 half-crowns to £, *s. d.*
60. 9708 half-crowns to £, *s. d.*
61. 10207 half-crowns to £, *s. d.*
62. 20005 half-crowns to £, *s. d.*
63. 4741 double florins to £, *s. d.*
64. 30821 half-sovereigns to £, *s. d.*
65. 2771 three-halfpences to £, *s. d.*
66. 56307 halfpence to £, *s. d.*

Reduce—

67. 1781 guineas to shillings.
68. 4631 guineas to shillings.
69. 473 guineas to pence.
70. 175 guineas to farthings.
71. 981 florins to twopences.
72. 1276 florins to sixpences.
73. 487 half-crowns to pence.
74. 867 crowns to halfpence.
75. 7413 crowns to threepences.
76. 3295 hf.-crowns to sixpences.
77. 4224 sixpences to half-crowns.
78. 311 half-crowns to farthings.
79. 7563 halfpence to florins.
80. 11234 pence to half-crowns.
81. 4872 sixpences to guineas.
82. 3875 pence to hf.-sovereigns.
83. 77240 threepences to florins.
84. 2761 fourpences to crowns.
85. 100000 farthings to guineas.
86. 8888 halfpence to guineas.

Reduce—

87. 4440 guineas to pounds.
88. 8340 guineas to pounds.
89. 2873 guineas to pounds.
90. 12407 guineas to pounds.
91. £13986 to guineas.
92. £16317 to guineas.
93. £40631 to guineas.
94. £36003 to guineas.
95. 8762 sixpences to fourpences.
96. 5083 fourpences to sixpences.
97. 7031 half-crowns to shillings.
98. 8888 florins to half-crowns.
99. 998 shillings to half-guineas.
100. 1032 hf.-crns. to fourpences.
101. 674512 half-crowns to florins.
102. 63021 guineas to half-crowns.
103. 15351 crowns to half-guineas.
104. 14196 florins to half-guineas.
105. 189550 hf.-guineas to hf.-crns.
106. 463252 half-crowns to florins

107. What would penny stamps for 500 letters cost?
108. What would halfpenny stamps for 3000 circulars cost?
109. How many penny stamps can be bought for £5?
110. How many halfpenny stamps can be bought for 3 half-crowns?
111. What would be the cost of a dinner for 412 persons at two shillings a head?
112. How many people could receive half a crown a-piece from a fund of £142, 12*s.* 6*d.*?
113. How many five-pound notes should be given in exchange for 1560 half-crowns?
114. How many pencils at $1\frac{1}{2}d.$ each could be bought for 9*s.* 6*d.*?
115. What income-tax, at 6*d.* in the pound, is paid by a man who has £550 a year?
116. How much income-tax, at 4*d.* in the pound, would be paid by a man whose income was £953?

TIME.

Reduce—

117. 19 hrs. 33 min. 42 sec. to seconds.
118. 23 hrs. 41 min. 5 sec. to seconds.
119. 17 days 13 hrs. 47 min. to minutes.
120. 23 days 11 hrs. 39 min. to minutes.
121. 6 wks. 2 days 18 hrs. to hours.
122. 5 wks. 3 days 15 hrs. to hours.
123. 3 com. yrs. 217 days 17 hrs. to hours.
124. 2 com. yrs. 189 days 13 hrs. to hours.
125. 2 com. yrs. 37 days 1 hr. 47 min. to minutes.
126. 3 wks. 6 days 10 hrs. 13 secs. to seconds.

Reduce—

127. 31423 secs. to hours &c.
128. 57029 secs. to hours &c.
129. 17962 mins. to days &c.
130. 20765 mins. to days &c.
131. 320841 secs. to days &c.
132. 479073 secs. to days &c.
133. 475905 mins. to weeks &c.
134. 27630820 secs. to weeks &c.
135. 1372573 mins. to yrs., dys. &c.
136. 2007603 mins. to yrs., dys. &c.

137. How many hours are there altogether in the months of April and May?
138. How many minutes are there in the three months September, October, and November?
139. How many hours are there between 11 A.M. on Monday and 7 P.M. on the following Wednesday?
140. How many hours are there between 9 A.M. on Tuesday and 9 P.M. on the following Saturday?
141. How many minutes are there between 7.30 A.M. and 5.15 P.M. on the same day?
142. How many seconds are there between 5 minutes past 2 o'clock P.M. and a quarter to 5 P.M. on the same day?
143. How many days were there from the beginning of the year till the end of August 1890?
144. How many days were there in the first 9 months of the year 1884?
145. How many days were there altogether in the five years 1887, 1888, 1889, 1890, and 1891?
146. How many hours were there altogether in the years 1891 and 1892?

Avoirdupois Weight.

Reduce—

147. 17 cwt. 1 qr. 13 lbs. to lbs.
148. 9 cwt. 3 qrs. 24 lbs. to lbs.
149. 1 qr. 11 lbs. 7 ozs. to ozs.
150. 2 qrs. 7 lbs. 13 ozs. to ozs.
151. 13 cwts. 13 lbs. 13 ozs. to ozs.
152. 17 cwt. 8 lbs. 12 ozs. to ozs.
153. 17 tons 3 cwts. 20 lbs. to lbs.
154. 23 tons 17 cwt. 15 lbs. to lbs.
155. 3 qrs. 10 lbs. 7 ozs. to ozs.
156. 19 lbs. 11 ozs. 7 drs. to drams.
157. 2 tons 13 cwt. 1 qr. 15 lbs. 7 ozs. to ounces.
158. 7 tons 11 cwt. 3 qrs. 25 lbs. 11 ozs. to ounces.
159. 13 tons 6 cwt. 3 qrs. 24 lbs. 10 ozs. 7 drs. to drams.
160. 52 tons 6 cwt. 2 qrs. 17 lbs. 15 drs. to drams.
161. 3 cwt. 3 qrs. 1 st. to stones.
162. 7 tons 15 cwts. 1 qr. to stones.
163. 1 ton 11 cwt. 111 lbs. to lbs.
164. 4 tons 3 cwt. 73 lbs. to lbs.
165. 5 cwt. 93 lbs. 6 ozs. to ounces.
166. 18 cwt. 89 lbs. 11 ozs. to ozs.
167. 2 cwt. 1 qr. 5 lbs. to grains.
168. 5 cwt. 2 qr. 17 lbs. to grains.

Reduce—

169. 1111 ozs. to quarters &c.
170. 1503 ozs. to quarters &c.
171. 4371 drams. to pounds &c.
172. 5029 drams to pounds &c.
173. 10729 lbs. to tons, cwts. &c.
174. 47568 lbs. to tons, cwts. &c.
175. 16417 ozs. to cwts. qrs. &c.
176. 19382 ozs. to cwts. qrs. &c.
177. 568430 drs. to cwts. qrs. &c.
178. 780451 drs. to cwts. qrs. &c.
179. 23075 ozs. to cwts. qrs. &c.
180. 33333 ozs. to cwts. qrs. &c.
181. 43970 ozs. to tons, cwts. &c.
182. 80307 ozs. to tons, cwts. &c.
183. 648327 ozs. to tons &c.
184. 4765183 ozs. to tons &c.
185. 7657938 ozs. to tons &c.
186. 1147274 drams to tons &c.
187. 5555555 drams to tons &c.
188. 9179065 drams to tons &c.

Troy Weight.

Reduce—

189. 3 lbs. 7 ozs. 14 dwt. 5 grs. to grains.
190. 7 lbs. 7 ozs. 7 dwt. 7 grs. to grains.
191. 5 lbs. 11 ozs. 18 dwt. to grs.
192. 29 ozs. 235 grs. to grains.
193. 37 ozs. 340 grs. to grains.
194. 45 ozs. 417 grs. to grains.

Reduce—

195. 7525 grs. to lbs. ozs. dwt. &c.
196. 25525 grs. to lbs. ozs. dwt. &c.
197. 87863 grs. to lbs. ozs. dwt. &c.
198. 87776 grs. to ozs. Troy.
199. 63745 grs. to ozs. Troy.
200. 90671 grs. to ozs. Troy.

Long Measure.

Reduce—

201. 17 yds. 2 ft. 11 in. to inches.
202. 29 yds. 1 ft. 7 in. to inches.
203. 56 yds. 1 ft. 9 in. to inches.
204. 88 yds. 2 ft. 10 in. to inches.
205. 4 mi. 380 yds. to yards.
206. 13 mi. 1056 yds. to yards.
207. 3 mi. 931 yds. 2 ft. to feet.
208. 5 mi. 1125 yds. 1 ft. to feet.
209. 7 mi. 1601 yds. 1 ft. to feet.
210. 10 mi. 1066 yds. 2 ft. to feet.

Reduce—

211. 441 in. to yds. &c.
212. 693 in. to yds. &c.
213. 1305 in. to yds. &c.
214. 2071 in. to yds. &c.
215. 76541 yds. to mi. and yds.
216. 84563 yds. to mi. and yds.
217. 36581 ft. to mi., yds. &c.
218. 47891 ft. to mi., yds. &c.
219. 358323 in. to mi., yds. &c.
220. 765045 in. to mi., yds. &c.

Reduce—

221. 24 po. to yards.
222. 38 po. to yards.
223. 27 po. $2\frac{1}{2}$ yds. to yards.
224. 39 po. $4\frac{1}{2}$ yds. to yards.
225. 18 po. 3 yds. 1 ft. to ft
226. 22 po. 5 yds. 2 ft. to ft.
227. 31 po. to half-yards.
228. 37 po. to half-yards.
229. 19 po. 4 yds. to half-yds.
230. 28 po. 3 yds. to half-yds.
231. 36 po. to inches.
232. 38 po. 3 yds. to inches.
233. 23 po. 2 yds. 9 in. to in.
234. 37 po. 1 yd. 11 in. to in.
235. 17 po. 4 yds. 1 ft. 7 in. to in.
236. 29 po. 5 yds. 2 ft. 8 in. to in.
237. 7 po. 3 yds. 1 ft. to feet.
238. 9 po. 4 yds. 2 ft. to feet.
239. 13 po. 4 yds. 1 ft. 6 in. to ft.
240. 17 po. 3 yds. 2 ft. 6 in to ft.
241. 8 mi. 5 fur. 103 yds. to yards.
242. 11 mi. 1 fur. 1 ft. to feet.
243. 17 mi. 7 fur. 1 ft. to feet.
244. 19 mi. 1 fur. 119 yds. to yds.
245. 5 mi. 2 ft. 3 in. to in.
246. 7 mi. 4 fur. 78 yds. to yds.

247. 7 mi. 5 fur. 32 po. 4 yds. to yards.
248. 6 mi. 7 fur. 24 po. 5 yds. to feet.
249. 10 mi. 1 fur. 10 po. 1 yd. 1 ft. to inches.
250. 22 mi. 2 fur. 22 po. 2 yds. 2 ft. to feet.
251. 3 mi. 5 fur. 17 po. 1 yd. 1 ft. to inches.
252. 1 mi. 3 fur. 37 po. 3 yds. 2 ft. to inches.
253. 3 mi. 1 fur. 39 po. 3 yds. 2 ft. 8 in. to inches.
254. 17 mi. 3 fur. 19 po. 4 yds. to inches.
255. 25 mi. 6 fur. 17 po. 4 yds. 3 in. to inches.
256. 43 mi. 5 fur. 23 po. 11 in. to inches.

Reduce—

257. 1033 half-yards to poles.
258. 986 half-yards to poles.
259. 3587 half-yards to poles.
260. 2634 half-yards to poles.
261. 242 yards to half-yds.
262. 642 yards to half-yds.
263. 660 yards to half-yds.
264. 1634 yards to half-yds.
265. 1023 yards to poles.
266. 693 yards to poles.
267. 735 yards to poles.
268. 1617 yards to poles.
269. 856 yards to poles &c.
270. 1234 yards to poles &c.
271. 1763 inches to poles &c.
272. 2904 inches to poles &c.
273. 3759 inches to poles &c.
274. 6888 inches to poles &c.
275. 273 feet to poles &c.
276. 381 feet to poles &c.
277. 496 feet to poles &c.
278. 503 feet to poles &c.
279. 571 feet to poles &c.
280. 687 feet to poles &c.

Reduce to miles, furlongs, poles &c.

281. 154737 inches.
282. 241985 inches.
283. 268543 inches.
284. 527895 inches.
285. 674381 inches.
286. 3126749 inches.
287. 9367875 inches.
288. 3744576 inches.
289. 10000 yards.
290. 44444 yards.
291. 57383 yards.
292. 1234567 yards.
293. 7777777 yards.
294. 112566 feet.
295. 1000000 feet.
296. 103962 feet.
297. 1847638 feet.
298. 6754321 feet.
299. 101010101 inches.
300. 100000000 inches.

Square Measure.

Reduce—

301. 3 sq. yds. 7 ft. 19 in. to sq. in.
302. 17 sq. yds. 8 ft. 79 in. to sq. in.
303. 2 sq. yds. 1 ft. 128 in. to sq. in.
304. 15 sq. yds. 3 ft. 141 in. to sq. in.
305. 72 ac. 3 ro. 19 po. to sq. po.
306. 131 ac. 1 ro. 37 po. to sq. po.
307. 143 ac. 1 ro. 25 po. to sq. po.
308. 89 ac. 3 ro. 21 po. to sq. po.

Reduce—

309. 6453 sq. in. to sq. yds. &c.
310. 8507 sq. in. to sq. yds. &c.
311. 70538 sq. in. to sq. yds. &c.
312. 123756 sq. in. to sq. yds. &c.
313. 3651 sq. po. to acres &c.
314. 7408 sq. po. to acres &c.
315. 73217 sq. po. to acres &c.
316. 90573 sq. po. to acres &c.

Reduce—

317. 24 sq. po. to sq. yds.
318. 36 sq. po. to sq. yds.
319. 20 sq. po. 18 sq. yds. to sq. yds.
320. 32 sq. po. 27 sq. yds. to sq. yds.
321. 18 sq. po. to sq. yds.
322. 22 sq. po. to sq. yds.
323. 17 sq. po. to sq. yds.
324. 29 sq. po. to sq. yds.
325. 14 sq. po. $17\frac{1}{2}$ sq. yds. to sq. yds.
326. 21 sq. po. 13 sq. yds. to sq. yds.
327. 4 sq. po. 23 sq. yds. to sq. ft.
328. 8 sq. po. 30 sq. yds. to sq. ft.
329. 6 sq. po. $4\frac{1}{2}$ sq. ft. to sq. ft.
330. 10 sq. po. 27 sq. yds. to sq. ft.
331. 17 sq. po. $5\frac{3}{4}$ sq. yds. to sq. ft.
332. 19 sq. po. $12\frac{1}{4}$ sq. yds. to sq. ft.
333. 30 sq. po. to sq. in.
334. 21 sq. po. to sq. in.
335. 27 sq. po. to sq. in.
336. 39 sq. po. to sq. in.
337. 5 sq. po. 17 sq. yds. to sq. in.
338. 7 sq. po. 28 sq. yds. to sq. in.
339. 7 ac. 2 ro. 32 po. to sq. yds.
340. 4 ac. 3 ro. 36 po. to sq. yds.
341. 2 ac. 19 po. to sq. yds.
342. 4 ac. 37 po. to sq. yds.
343. 5 ac. 870 sq. yds. to sq. yds.
344. 9 ac. 2003 sq. yds. to sq. yds.
345. 17 ac. 138 sq. yds. to sq. yds.
346. 27 ac. 450 sq. yds. to sq. yds.

347. 3 ac. 2 ro. 27 po. 27 yds. 7 ft. 25 in. to square inches.
348. 8 ac. 2 ro. 34 po. 3 ft. 87 in. to square inches.
349. 53 ac. 21 po. 8 ft. 125 in. to square inches.
350. 3 ac. 3 ro. 33 po. 3 yds. 3 ft. 33 in. to square inches.

Reduce—

351. 616 sq. yds. to sq. po.
352. 851 sq. yds. to sq. po.
353. 267 sq. yds. to sq. po. &c.
354. 353 sq. yds. to sq. po. &c.
355. 917 sq. yds. to sq. po. &c.
356. 1061 sq. yds. to sq. po. &c.
357. 882 sq. ft. to sq. po. &c.
358. 1234 sq. ft. to sq. po. &c.
359. 180123 sq. in. to sq. po. &c.
360. 246843 sq. in. to sq. po. &c.

Reduce to acres, roods, poles &c.

361. 11495 sq. yds.
362. 83061 sq. yds.
363. 37495 sq. yds.
364. 657345 sq. ft.
365. 562936 sq. ft.
366. 1000000 sq. ft.
367. 7865432 sq. in.
368. 16019400 sq. in.
369. 25607809 sq. in.
370. 54650895 sq. in.
371. 895487 sq. yds.
372. 3333333 sq. ft.
373. 333274481 sq. in.
374. 99597888 sq. in.

Cubic Measure.

Reduce to cubic inches—

375. 17 cub. ft. 1335 in.

376. 18 cub. ft. 1073 in.

377. 1 cub. yd. 24 ft. 760 in.

378. 1 cub. yd. 18 ft. 631 in.

379. 4 cub. yds. 13 ft. 5 in.

380. 9 cub. yds. 21 ft. 875 in.

Reduce to cubic yards &c.

381. 143562 cub. in.

382. 764591 cub. in.

383. 847325 cub. in.

384. 987407 cub. in.

385. 4831850 cub. in.

386. 8001304 cub. in.

Capacity.

387. 13 gal. 3 qt. 1 pt. to pints.

388. 19 gal. 1 qt. 1 pt. to pints.

389. 18 gal. 1 qt. to pints.

390. 23 gal. 1 pt. to pints.

391. 875 pints to gallons &c.

392. 678 pints to gallons &c.

393. 1232 pints to gallons &c.

394. 1805 pints to gallons &c.

395. 7 qrs. 3 bush. 2 pks. to pecks.

396. 9 qrs. 6 bush. 2 pks. to pecks.

397. 121 qrs. 1 bush. 1 pk. 1 qt. to quarts.

398. 89 qrs. 3 bush. 3 pks. 1 gal. 1 pt. to pints.

399. 2559 pints to qrs., bush. &c.

400. 33333 qts. to qrs. &c.

Miscellaneous.

401. Express 225 ozs. Troy, in lbs. Avoirdupois.

402. Reduce 2 miles 71 chains 36 links to links.

403. Reduce 3 yds. 3 qrs. 3 nails to inches.

404. Reduce 7 square miles to square yards.

405. In 3 barrels of beer how many pints?

406. Reduce 17 reams 13 quires 11 sheets to sheets.

407. Reduce 31° 47′ 29″ to seconds.

408. How many knots are equivalent to 38 miles?

409. Reduce 14520 square chains to acres.

410. How many Troy ozs. are equal to 6 cwts.?

V. THE COMPOUND RULES.

Compound Addition.

	£	s.	d.
1.	71	3	2
	32	17	$1\frac{1}{2}$
	361	8	$9\frac{1}{4}$
	917	10	8
	2752	15	$3\frac{3}{4}$
	526	1	$1\frac{1}{2}$
	45	2	$11\frac{3}{4}$
	52	1	3
	3041	2	$7\frac{1}{4}$

	£	s.	d.
2.	408	13	$10\frac{1}{2}$
	176	13	7
	109	19	$10\frac{1}{2}$
	36	11	4
	79	8	$6\frac{3}{4}$
	8	13	$10\frac{1}{4}$
	5	8	2
	21	18	$9\frac{1}{2}$
	40	15	$6\frac{3}{4}$

	£	s.	d.
3.	245	6	$2\frac{1}{4}$
	8139	7	$4\frac{1}{4}$
	94	3	$4\frac{1}{2}$
	9065	13	$8\frac{3}{4}$
	1067	13	8
	2935	8	$1\frac{3}{4}$
	239	2	5
	163	9	$11\frac{3}{4}$
	752	15	3

	£	s.	d.
4.	416	9	8
	63	9	11
	385	13	6
	65	13	8
	7	13	8
	612	3	2
	46	3	6
	1130	1	4
	79	12	9
	287	6	8
	2307	4	2
	784	9	11

	£	s.	d.
5.	287	14	7
	19	3	8
	21	18	9
	139	7	4
	5	8	1
	39	2	5
	6904	7	3
	752	15	3
	7245	6	2
	85	2	2
	109	19	10
	2015	1	2

	£	s.	d.
6.	207	4	6
	94	3	4
	211	6	8
	9035	2	10
	409	12	8
	61	10	8
	492	13	7
	56	4	10
	287	9	11
	9603	2	5
	408	13	10
	8043	7	9

	£	s.	d.
7.	9416	9	8
	215	9	5
	691	13	9
	239	2	5
	134	6	6
	9374	5	7
	752	15	3
	2935	8	1
	4321	18	9
	196	3	2
	651	4	3
	360	5	5
	4201	6	8
	476	13	4
	2376	0	9

	£	s.	d.
8.	8204	15	2
	476	3	9
	1067	13	8
	734	5	1
	4614	3	3
	3198	10	8
	57	2	4
	1316	9	8
	4917	10	8
	6917	11	9
	976	2	5
	8416	9	10
	5618	7	6
	9314	2	9
	1618	2	5

	£	s.	d.
9.	4321	18	9
	8139	7	4
	2935	8	1
	239	2	5
	6904	7	3
	752	15	3
	7245	6	2
	485	2	2
	109	19	10
	2015	1	2
	210	14	8
	6218	12	8
	783	8	7
	1397	7	1
	691	13	9

	£	s.	d.
10.	643	17	6
	485	2	2
	8139	7	4
	6409	12	8
	239	2	5
	4561	10	8
	612	3	2
	2935	8	1
	6218	12	8
	109	19	10
	46	3	6
	4201	6	8
	94	3	4
	2015	1	2

	£	s.	d.
11.	817	12	6
	6127	9	10
	408	13	10
	1067	13	8
	8385	13	6
	631	7	7
	8279	12	9
	360	5	5
	9532	8	7
	1130	1	4
	3	2	5
	6904	7	3
	563	4	1
	163	9	11

	£	s.	d.
12.	4391	16	9
	4123	17	6
	211	6	8
	9035	2	10
	7245	6	2
	207	4	6
	9065	13	8
	56	4	10
	3276	5	9
	691	13	9
	2307	4	2
	362	3	4
	1287	14	7
	5976	2	5

	£	s.	d.
13.	827516	3	9
	734321	18	9
	16391	1	4
	76899	13	7
	897	15	3
	11835	0	3
	403750	5	8
	274032	12	1
	528636	4	11
	675940	18	2
	94636	1	1
	16391	1	4
	691040	3	7
	153748	2	9
	95483	17	10
	765491	9	5
	673159	0	1
	31140	15	6
	310397	8	7

	£	s.	d.
14.	29215	9	5
	326408	13	10
	639875	12	3
	810732	17	1
	106747	18	3
	74321	18	9
	42361	8	9
	23109	19	10
	6679	14	5
	810327	4	4
	97243	17	11
	239763	2	1
	112935	8	1
	93198	10	8
	310397	8	7
	228416	9	10
	20242	0	7
	299653	1	8
	7187	7	7

	£	s.	d.
15.	16391	1	4
	6679	14	5
	23109	19	10
	376514	10	5
	528636	4	11
	47983	19	3
	810732	17	1
	274032	12	1
	94636	1	1
	291414	12	4
	17218	9	6
	714961	13	11
	93198	10	8
	153748	2	9
	34728	12	6
	765491	9	5
	673159	0	1
	31140	15	6
	299653	1	8

16. Add together £8385, 13*s.* 6*d.*, £8019, 3*s.* 8$\frac{1}{4}$*d.*, £9416, 2*s.* 8*d.*, £207, 4*s.* 6$\frac{1}{4}$*d.*, £612, 3*s.* 2$\frac{3}{4}$*d.*, £46, 3*s.* 6*d.*, £1130, 1*s.* 4*d.*, £8279, 12*s.* 9$\frac{1}{2}$*d.*, and £4321, 18*s.* 9$\frac{1}{2}$*d.*

17. Find the sum of £3241, 11*s.* 6*d.*, £163, 9*s.* 11*d.*, £5, 13*s.* 6*d.*, £9065, 13*s.* 8*d.*, £1067, 13*s.* 8*d.*, £612, 3*s.* 2*d.*, £46, 3*s.* 6*d.*, £30, 1*s.* 4*d.*, £8279, 12*s.* 9*d.*, £87, 6*s.* 8*d.*, £307, 4*s.* 2*d.*, £2784, 9*s.* 11*d.*, £5, 16*s.* 4*d.*, £27, 9*s.* 10*d.*, and £201, 6*s.* 8*d.*

18. Find the sum of £106747, 18*s.* 3*d.*, £29215, 9*s.* 5*d.*, £76899, 13*s.* 7*d.*, £321, 18*s.* 9*d.*, £750, 5*s.* 8*d.*, £6, 3*s.* 9*d.*, £61, 8*s.* 9*d.*, £97, 11*s.* 7*d.*, £243, 17*s.* 11*d.*, £408, 13*s.* 10*d.*, £321, 18*s.* 9*d.*, £374, 5*s.* 7*d.*, £9763, 2*s.* 1*d.*, £9642, 5*s.* 2*d.*, £83, 17*s.* 10*d.*, £263, 9*s.* 5*d.*, £397, 8*s.* 7*d.*, £97, 8*s.* 7*d.*, and £649603, 2*s.* 5*d.*

19.

days.	hrs.	min.	secs.
17	21	36	51
11	20	52	57
1	14	39	25
22	19	49	46
38	7	17	38

20.

days.	hrs.	min.	secs.
4	17	25	39
3	21	55	17
1	19	48	56
2	16	51	3
	3	7	42

21.

yrs.	days.	hrs.
4	223	17
5	317	12
1	47	23
19	199	7
2	13	15

22.

yrs.	days.	hrs.
17	201	13
9	333	21
7	149	11
27	73	9
14	351	19

23.

cwts.	qrs.	lbs.	ozs.
1	2	13	5
7	1	15	11
	3	14	2
11	0	17	9
1	1	5	14

24.

qrs.	lbs.	ozs.	drs.
1	17	13	2
	27	11	15
3	9	7	3
2	26	14	13
	19	12	12

25.

tons.	cwts.	qrs.	lbs.
5	13	2	19
1	16	1	23
18	9	3	10
4	16	0	27
12	7	1	9

26.

tons.	cwts.	qrs.	lbs.	ozs.
3	17	2	25	13
1	1	3	11	7
14	10	0	20	2
7	15	1	8	11
	5	2	19	15

27.

cwts.	lbs.	ozs.
1	102	7
2	89	10
5	41	15
11	98	5
	81	1

28.

tons.	cwts.	lbs.	ozs.
3	17	89	8
1	12	29	11
5	19	98	6
	2	23	15
12	14	88	3

29.

lbs. Tr.	ozs.	dwts.	grs.
2	1	7	15
7	11	13	1
19	2	7	19
3	5	2	5
4	0	17	23

30.

ozs. Tr.	grs.
27	412
30	389
1	89
2	461
15	123

31.

yds.	ft.	in.
3	1	7
1	2	11
7	0	9
	2	10
2	1	8

32.

yds.	ft.	in.
4	2	6
13	1	10
1	1	11
8	2	5
3	2	11

33.

mi.	yds.
4	225
1	1026
2	1320
3	486
5	1237

34.

mi.	yds.	ft.	in.
7	85	1	7
1	180	2	3
2	1100	1	8
11	1625	0	2
2	977	1	11

35.

po.	yds.	ft.	in.
31	5	2	11
2	3	1	7
17	4	1	8
8	3	2	10
29	1	1	9

36.

mi.	fur.	po.	yds.	ft.
7	7	27	5	2
3	6	15	2	1
9	1	7	1	0
11	4	29	2	2
	5	38	4	2

37.

fur.	po.	yds.	ft.	in.
1	31	3	1	7
2	33	4	2	10
3	17	3	2	8
7	12	2	2	9
4	5	1	0	11

38.

mi.	chains.	links.
3	39	55
1	68	82
2	27	41
10	74	93
5	59	70

39.

sq. yds.	sq. ft.	sq. in.
7	8	79
1	3	112
11	1	45
5	0	131
4	7	68

40.

sq. yds.	sq. ft.	sq. in.
1	5	88
2	8	112
1	6	109
3	2	93
10	4	125

41

ac.	ro.	sq. po.
79	2	31
2	1	28
1	3	5
221	0	37
2	2	21

42.

ac.	ro.	sq. po.
13	1	27
5	3	29
41	2	16
9	0	38
52	1	8

43.

ac.	ro.	sq. po.	sq. yds.
1	2	27	20
2	1	18	25
	3	31	16
5	0	7	12
	1	16	8

44.

ac.	ro.	sq. po.	sq. yds.
7	1	25	20
1	3	28	29
2	3	35	17
3	1	32	28
	2	21	13

45.

ac.	ro.	sq. po.	sq. yds.	sq. ft.
4	3	29	29	8
7	1	34	17	5
11	2	17	18	1
	1	24	23	6
	3	12	14	1

46.

sq. po.	sq. yds.	sq. ft.	sq. in.
31	28	7	112
7	16	5	84
28	25	3	56
19	27	8	138
23	30	1	79

47.

cub. yds.	cub. ft.	cub. in.
14	5	107
2	17	1234
7	24	389
21	12	43
1	18	497

48.

cub. yds.	cub. ft.	cub. in.
22	22	222
173	15	1191
25	19	985
17	5	637
204	26	1680

49.

gals.	qts.	pts.
3	1	1
17	3	0
5	2	1
13	1	1
7	3	0

50.

qrs.	bush.	pecks.	gals.
13	5	3	1
5	7	2	1
18	2	2	0
29	6	1	1
16	3	0	1

Compound Subtraction.

51. Subtract £1, 11*s.* $11\frac{3}{4}$*d.* from £10, 1*s.* $2\frac{1}{2}$*d.*

52. Subtract £21, 11*s.* $11\frac{3}{4}$*d.* from £25, 16*s.* 4*d.*

53. From £4586, 7*s.* $9\frac{1}{4}$*d.* take £1397, 18*s.* $10\frac{1}{2}$*d.*

54. From £8133, 11*s.* $6\frac{1}{2}$*d.* take £533, 13*s.* $8\frac{3}{4}$*d.*

55. Find the difference between £3, 17*s.* $10\frac{1}{2}$*d.* and £5.

56. Find the difference between £113, 9*s.* 2*d.* and £97, 17*s.* $8\frac{3}{4}$*d.*

57. What must be added to £29, 13*s.* $5\frac{1}{2}$*d.* to make £31, 12*s.* 2*d.*?

58. What must be taken from £87, 6*s.* $2\frac{1}{4}$*d.* to leave £79, 9*s.* $4\frac{1}{2}$*d.*?

59. By how much does £1001, 1*s.* 1*d.* exceed £909, 19*s.* 9*d.*?

60. By how much does £978, 19*s.* 2*d.* fall short of £1000?

Subtract—

61. 2 hrs. 40 min. 25 secs. from 11 hrs. 8 min. 15 secs.
62. 12 hrs. 37 min. 51 secs. from 19 hrs. 5 min. 35 secs.
63. 8 days 17 hrs. 45 min. from 12 days 3 hrs. 21 min.
64. 2 days 13 hrs. 35 min. 47 secs. from 5 days 7 hrs.

65. 8 lbs. 13 ozs. 7 drams from 13 lbs. 5 ozs. 1 dram.
66. 1 qr. 23 lbs. 11 ozs. from 3 qrs. 11 lbs. 3 ozs.
67. 17 tons 14 cwts. 2 qrs. from 25 tons 6 cwts. 1 qr.
68. 23 tons 17 cwts. 3 qrs. 19 lbs. from 35 tons 13 cwts. 1 qr.
69. 5 tons 17 cwts. 2 qrs. 15 lbs. from 7 tons 14 cwts. 2 qrs. 10 lbs.
70. 3 cwts. 1 qr. 27 lbs. 15 ozs. from 1 ton 1 qr. 11 lbs. 11 ozs.
71. 17 tons 15 cwts. 108 lbs. 10 ozs. from 23 tons 56 lbs.
72. 13 tons 17 cwts. 1 qr. 19 lbs. 9 ozs. 13 drs. from 20 tons.

73. 3 lbs. Tr. 5 ozs. 17 dwts. 19 grs. from 11 lbs. Tr. 1 oz. 14 dwts.
74. 13 ozs. Tr. 256 grs. from 27 ozs. Tr. 120 grs.

75. 3 yds. 2 ft. 9 in. from 17 yds. 1 ft. 4 in.
76. 14 yds. 1 ft. 10 in. from 23 yds. 0 ft. 7 in.
77. 5 mi. 967 yds. 2 ft. from 13 mi. 241 yds. 1 ft.
78. 10 mi. 1380 yds. 7 in. from 32 mi. 47 yds. 2 ft.
79. 156 mi. 4 fur. 216 yds. from 191 mi. 2 fur. 2 yds.
80. 3 fur. 33 po. 3 yds. from 1 mi. 17 po. 5 yds.
81. 7 mi. 7 fur. 37 po. 3 yds. 1 ft. 7 in. from 8 mi. 26 po.
82. 2 mi. 22 po. 2 ft. from 5 mi. 5 fur. 5 yds. 5 in.

83. 3 sq. yds. 7 sq. ft. 117 sq. in. from 8 sq. yds. 2 sq. ft.
84. 13 sq. yds. 8 sq. ft. 132 sq. in. from 21 sq. yds. 89 sq. in.
85. 23 ac. 3 ro. 29 po. from 71 ac. 2 ro. 13 po.
86. 198 ac. 2 ro. 37 po. from 305 ac. 2 ro. 14 po.
87. 2 ro. 27 po. 28 sq. yds. from 3 ro. 25 po. 18 sq. yds.
88. 3 ro. 19 po. 30 sq. yds. from 3 ac. 2 ro. 1 po.
89. 17 ac. 3130 sq. yds. from 23 ac. 1760 sq. yds.
90. 99 ac. 1873 sq. yds. from 1 sq. mi. 230 ac.
91. 3 ro. 29 po. 27 sq. yds. 8 sq. ft. from 2 ac. 2 ro.
92. 39 po. 17 sq. yds. 98 sq. in. from 1 ac. 1 ro. 1 sq. ft.
93. 2 ro. 32 po. 12 sq. yds. 6 sq. ft. 10 sq. in. from 2 acres.
94. 3 ac. 3 ro. 33 po. 3 sq. yds. 3 sq. ft. 33 sq. in. from 5 acres.

95. 12 cub. yds. 22 ft. 1222 in. from 21 cub. yds. 2 ft. 221 in.
96. 276 cub. yds. 987 in. from 456 cub. yds. 17 ft.

97. 19 gal. 3 qts. 1 pt. from 39 gal. 1 qt.
98. 3 bush. 2 pks. 1 gal. from 7 bush. 1 pk.
99. 357 qrs. 7 bush. from 402 qrs. 3 bush.
100. 89 qrs. 3 bush. 1 pk. from 157 qrs. 1 bush.

Find the value of—

101. $2s.\ 6d. + 8s.\ 7\frac{1}{2}d. - 4s.\ 3d. + 9s.\ 11\frac{1}{4}d. - 11s.\ 7\frac{3}{4}d.$
102. $14s.\ 2d. - 19s.\ 8\frac{1}{2}d. - 17s.\ 11\frac{3}{4}d. - 10s.\ 10\frac{1}{2}d. + 18s.\ 1\frac{1}{4}d. + 18s.$
103. £1, 12s. − £1, 4s. 7d. + £2, 15s. 4d. − £1, 3s. 11d. + £3, 19s. 5d.
104. £2, 3s. 4d. − £5, 17s. + £3, 1s. 6d. − £4, 15s. 10d. + £6, 6s.
105. £24781, 13s. $9\frac{1}{4}d.$ − £31265, 8s. $3\frac{1}{2}d.$ + £8503, 11s. 3d.
106. £85608, 17s. $2\frac{1}{2}d.$ + £459, 12s. $11\frac{3}{4}d.$ − £77953, 18s. 6d.
107. $4s.\ 5d. - 9s.\ 8\frac{1}{2}d. + 11s.\ 2\frac{1}{4}d. - 13s.\ 11\frac{3}{4}d. + 8s.\ 6d. - 2s.\ 10d. + 15s\ 1\frac{3}{4}d. - 7s.\ 7d. + 5s.\ 1\frac{1}{2}d.$
108. $13s.\ 9d. - 8s.\ 11d. - 14s.\ 1d. + 2s.\ 6\frac{1}{2}d. + 17s.\ 8d. - 4s.\ 9\frac{1}{4}d. - 18s.\ 5\frac{1}{2}d. + 16s.\ 11d. - 1s.\ 10\frac{1}{4}d. + 19s.\ 2\frac{3}{4}d.$

Find, in one operation, the value of—

109. £20 − £1, 17s. 6d. − £2, 15s. 11d. − £7, 13s. 8d. − £5, 11s. 3d.
110. £7, 7s. 7d. − £1, 13s. $4\frac{1}{2}d.$ − £2, 11s. $10\frac{3}{4}d.$ − £1, 8s. $7\frac{1}{2}d.$ − 14s. 9d.
111. £31, 5s. 3d. − £11, 7s. $9\frac{1}{2}d.$ − £4, 13s. 3d. − £1, 15s. $1\frac{1}{4}d.$ − £9, 12s.
112. £100 − £21, 10s. − £5, 3s. 6d. − £17, 19s. 8d. − £49, 19s. 2d.
113. £5 − (£1, 2s. 6d. + 14s. 5d. + £1, 17s. $11\frac{1}{2}d.$ + 13s. $9\frac{1}{4}d.$).
114. £13, 3s. 3d. − (£2, 14s. 7d. + £1, 12s. 9d. + 17s. $5\frac{3}{4}d.$ + 16s. $8\frac{1}{2}d.$).

115. Take the sum of £2, 11s. 8d., £11, 19s. 2d., £4, 13s. 9d., and £3, 8s. 11d. from £25, 2s. 6d.

116. From £37, 15s. 3d. take the sum of £1, 17s. $5\frac{1}{2}d.$, 13s. $8\frac{1}{4}d.$, £13, 16s. 9d., and £5, 19s. $7\frac{1}{2}d.$

117. By how much does the sum of £1, 13s. $6\frac{1}{2}d.$, £2, 4s. 10d., 18s. $8\frac{1}{2}d.$, £2, 12s. 6d., 15s. 9d., and £10, 18s. fall short of £20?

118. Subtract the sum of £2, 13s. $3\frac{1}{2}d.$, £5, 7s. $11\frac{1}{4}d.$, £18, 19s. $3\frac{3}{4}d.$, £2, 12s. 6d., and £31, 19s. $10\frac{1}{2}d.$ from £86, 4s. 2d.

119. How much is left out of £10 after paying bills of 13s. 6d., £2, 19s. 7d., £1, 14s. 8d., £3, 12s. 11d. and 4s. 10d.?

120. What is left out of £50 after paying bills of £5, 3s. 8d., £14, 16s. 7d., £19, 7s. 10d., £4, 18s. 9d., £1, 11s., and £2, 14s. 1d.?

Compound Multiplication.

Multiply—

12 . £17, 13*s.* $5\frac{1}{2}$*d.* by 9.

122. £18, 15*s.* $10\frac{1}{4}$*d.* by 11.

123. £154, 16*s.* $8\frac{3}{4}$*d.* by 7.

124. £62, 17*s.* $9\frac{1}{2}$*d.* by 12.

125. £93, 9*s.* $7\frac{1}{2}$*d.* by 15.

126. £149, 14*s.* $6\frac{3}{4}$*d.* by 18.

127. £43, 12*s.* $4\frac{3}{4}$*d.* by 24.

128. £71, 18*s.* $7\frac{3}{4}$*d.* by 30.

129. £2, 16*s.* $8\frac{1}{2}$*d.* by 63.

130. £5, 11*s.* $6\frac{1}{2}$*d.* by 96.

131. £250, 13*s.* 4*d.* by 66.

132. £247, 13*s.* $5\frac{1}{4}$*d.* by 64.

133. £22, 15*s.* $5\frac{1}{2}$*d.* by 100.

134. £16, 14*s.* 11*d.* by 144.

135. £12, 9*s.* 11*d.* by 200.

136. £8, 10*s.* 9*d.* by 700.

137. £7, 13*s.* 9*d.* by 1000.

138. £2, 11*s.* 3*d.* by 3000.

139. £53, 7*s.* 11*d.* by 17.

140. £36, 12*s.* $1\frac{1}{2}$*d.* by 23.

141. £4, 13*s.* $8\frac{1}{2}$*d.* by 43.

142. £2, 16*s.* $3\frac{1}{4}$*d.* by 51.

143. £72, 14*s.* 5*d.* by 76.

144. £60, 18*s.* 10*d.* by 86.

145. £14, 6*s.* $7\frac{1}{4}$*d.* by 68.

146. £11, 8*s.* $2\frac{1}{2}$*d.* by 94.

147. £231, 10*s.* $4\frac{1}{2}$*d.* by 85.

148. £748, 11*s.* $3\frac{1}{4}$*d.* by 92.

149. £19, 15*s.* 1*d.* by 130.

150. £25, 8*s.* $8\frac{1}{2}$*d.* by 170.

151. £3, 13*s.* 10*d.* by 340.

152. £24, 3*s.* 7*d.* by 360.

153. £5, 12*s.* 4*d.* by 1007.

154. £8, 14*s.* 5*d.* by 2050.

155. £124, 7*s.* $11\frac{1}{2}$*d.* by 19.

156. £3, 16*s.* $2\frac{1}{4}$*d.* by 69.

157. £388, 12*s.* $9\frac{1}{2}$*d.* by 29.

158. £2017, 17*s.* $5\frac{1}{2}$*d.* by 39.

159. £76, 3*s.* 11*d.* by 59.

160. £18, 9*s.* 8*d.* by 89.

161. £237, 4*s.* 9*d.* by 119.

162. £74, 17*s.* $8\frac{1}{2}$*d.* by 297.

163. £1, 15*s.* 4*d.* by 253.

164. £1, 17*s.* $3\frac{1}{2}$*d.* by 322.

165. £2, 13*s.* 5*d.* by 365.

166. £3, 8*s.* 9*d.* by 365.

167. £1084, 7*s.* 6*d.* by 672.

168. £67, 4*s.* 2*d.* by 1335.

169. £1, 10*s.* 8*d.* by 2037.

170. £3, 9*s.* 5*d.* by 4708.

Find, shortly,

171. 191 times £49, 19*s.* 6*d.*

172. 201 times £59, 19*s.* 9*d.*

173. 279 times 4*s.* $11\frac{1}{2}$*d.*

174. 187 times 9*s.* $10\frac{3}{4}$*d.*

175. 311 times 19*s.* $11\frac{3}{4}$*d.*

176. 253 times £1, 19*s.* $10\frac{1}{2}$*d.*

177. 1023 times £7, 18*s.* 8*d.*

178. 3008 times £21, 18*s.* 9*d.*

179. 2457 times £1, 17*s.* 6*d.*

180. 1897 times £2, 17*s.*

Find, without separating the multiplier into parts,

181. 873 times £2, 0*s.* 2*d.*
182. 769 times £7, 0*s.* 4*d.*
183. 2813 times 16*s.* 2*d.*
184. 3597 times 12*s.* 1*d.*
185. 716 times £5, 0*s.* 7½*d.*
186. 987 times £13, 0*s.* 7*d.*
187. 777 times £7, 7*s.* 7*d.*
188. 1111 times £11, 11*s.* 11*d.*
189. 2573 times £17, 1*s.* 8*d.*
190. 3011 times £12, 3*s.* 3½*d.*

Multiply—

191. 2 hrs. 33 min. 49 secs. by 51.
192. 1 hr. 17 min. 15 secs. by 107.

193. 12 tons 17 cwts. 1 qr. by 12.
194. 1 qr. 12 lbs. 14 ozs. by 10.
195. 2 lbs. 7 ozs. 11 drs. by 42.
196. 9 cwts. 0 qr. 16 lbs. by 24.

197. 5 tons 12 cwts. 2 qrs. 13 lbs. by 13.
198. 1 cwt. 27 lbs. 15 ozs. by 17.
199. 18 tons 3 cwts. 2 qrs. 9 ozs. by 23.
200. 11 cwts. 3 qrs. 15 lbs. 4 ozs. by 46.
201. 4 tons 15 cwts. 2 qrs. 27 lbs. by 195.
202. 6 cwts. 1 qr. 21 lbs. 5 ozs. by 321.

203. 13 yds. 1 ft. 7 in. by 9.
204. 5 mi. 3 fur. 27 po. by 12.
205. 1 mi. 7 fur. 39 po. by 41.
206. 7 yds. 2 ft. 10 in. by 53.

207. 7 mi. 6 fur. 22 po. 4 yds. by 27.
208. 2 po. 3 yds. 2 ft. 3 in. by 54.
209. 53 mi. 5 fur. 23 po. 4 yds. by 19.
210. 5 mi. 3 fur. 17 po. 4 yds. 2 ft. by 29.
211. 3 mi. 7 fur. 31 po. 3 yds. by 103.
212. 17 po. 4 yds. 2 ft. 9 in. by 114.

213. 7 A. 3 R. 20 P. by 12.
214. 12 A. 1 R. 32 P. by 9.
215. 8 sq. ft. 117 sq. in. by 14.
216. 6 sq. ft. 94 sq. in. by 18.

217. 1 ac. 2 ro. 3 sq. po. 4 sq. yds. 5 sq. ft. by 11.
218. 2 ac. 3 ro. 31 sq. po. 23 sq. yds. 7 sq. ft. by 8.
219. 1 ac. 3 ro. 7 sq. po. 4 sq. yds. by 17.
220. 209 ac. 3 ro. 25 sq. po. 3 sq. yds. by 13.
221. 4 ac. 1 ro. 27 sq. po. 19 sq. yds. 4 sq. ft. 72 sq. in. by 27.
222. 19 sq. po. 11 sq. yds. 5 sq. ft. 128 sq. in. by 84.

223. 3 cub. yds. 25 cub. ft. 1118 cub. in. by 30.
224. 13 qrs. 5 bush. by 1144.

COMPOUND DIVISION.

Divide, using short division—

225. £6, 9*s.* 5¾*d.* by 11.
226. £214, 11*s.* 3*d.* by 12.
227. £271, 15*s.* 9¼*d.* by 8.
228. £2031, 17*s.* 11½*d.* by 9.
229. £3475, 2*s.* 3*d.* by 6.
230. £56831, 0*s.* 8*d.* by 7.
231. £23471 by 9.
232. £695031 by 8.

233. £75, 19*s.* 6*d.* by 24.
234. £118, 1*s.* 11*d.* by 28.
235. £786, 4*s.* 7*d.* by 60.
236. £308, 15*s.* 10*d.* by 72.
237. £2105, 18*s.* 3½*d.* by 99.
238. £4021, 10*s.* 5¾*d.* by 144.
239. £3050, 9*s.* 10½*d.* by 81.
240. £6595, 4*s.* 8*d.* by 44.
241. £9652, 6*s.* 0*d.* by 96.
242. £15942, 16*s.* 6*d.* by 108.
243. £145845, 17*s.* 9¾*d.* by 15.
244. £560127, 17*s.* 9¾*d.* by 45.

Divide—

245. £39, 11*s.* 3¼*d.* by 19.
246. £379, 19*s.* 7¼*d.* by 19.
247. £208, 2*s.* 6¼*d.* by 23.
248. £2065, 19*s.* 6*d.* by 23.
249. £153, 1*s.* 3¾*d.* by 29.
250. £183, 12*s.* 1½*d.* by 58.
251. £146, 14*s.* 10½*d.* by 53.
252. £22912, 9*s.* 0½*d.* by 53.
253. £29, 11*s.* 4½*d.* by 83.
254. £68, 9*s.* 11¼*d.* by 69.
255. £14736, 19*s.* 7*d.* by 73.
256. £78791, 11*s.* 9*d.* by 73.
257. £21544, 14*s.* 2½*d.* by 97.
258. £102354, 14*s.* 8¼*d.* by 93.
259. £9418, 12*s.* 3*d.* by 118.
260. £85071, 19*s.* 4*d.* by 136.
261. £203745, 12*s.* 1½*d.* by 147.
262. £317235, 1*s.* 6¾*d.* by 145.
263. £118332, 19*s.* 5¼*d.* by 321.
264. £98703, 19*s.* 11*d.* by 359.
265. £1412, 1*s.* 10½*d.* by 365.
266. £10839, 7*s.* 2¼*d.* by 365.
267. £266780 by 365.
268. £1850321 by 730.
269. £34161, 17*s.* 11*d.* by 2705.
270. £84304, 5*s.* 3½*d.* by 2801.
271. £1459, 13*s.* by 4723.
272. £68493, 2*s.* by 7891.
273. £1624270, 6*s.* 3*d.* by 935.
274. £226125587, 3*s.* 5½*d.* by 602.
275. £12801 by 7300.
276. £86107, 10*s.* 8*d.* by 7300.

Divide—

277. £1998, 17*s.* 1*d.* by 100.
278. £6521, 9*s.* 2*d.* by 100.
279. £11705, 12*s.* 6*d.* by 100.
280. £32065, 14*s.* 2½*d.* by 100.
281. £2371, 17*s.* 9½*d.* by 100.
282. £40312, 13*s.* 4¼*d.* by 100.
283. £113, 17*s.* 11*d.* by 100.
284. £97, 16*s.* 0½*d.* by 1000.
285. £876, 11*s.* 10½*d.* by 1000.
286. £27906, 13*s.* 4*d.* by 100.

Divide—

287. 365 days 5 hrs. 48 min. 48 secs. by 12.
288. 47 days 15 hrs. 23 min. 20 secs. by 20.
289. 1146 days 14 hrs. 38 min. 20 secs. by 100.
290. 146097 days 0 hrs. 5 min. 30 secs. by 365.

291. 13 tons 14 cwts. 1 qr. 12 lbs. 4 ozs. by 6.
292. 7 tons 6 cwts. 3 qrs. 17 lbs. 2 ozs. by 9.
293. 201 tons 3 cwts. 1 qr. by 56.
294. 201 tons 13 cwts. 1 qr. 4 lbs. by 64.
295. 12 tons 3 cwts. 1 qr. 25 lbs. by 37.
296. 309 tons 12 cwts. 2 qrs. 19 lbs. by 47.
297. 10 tons 8 cwts. 3 qrs. 11 lbs. 12 ozs. 15 drs. by 69.
298. 30 tons 6 cwts. 1 qr. 24 lbs. 8 ozs. by 168.
299. 56 tons 6 cwts. 1 qr. 13 lbs. by 321.
300. 756 tons 13 cwts. 3 qrs. 17 lbs. 11 ozs. 8 drs. by 251.

301. 247 yds. 1 ft. 6 in. by 33.
302. 3 mi. 102 yds. 2 ft. by 24.
303. 8 mi. 3 fur. 25 po. 3 yds. 1 ft. 6 in. by 6.
304. 17 mi. 5 fur. 21 po. 4 yds. 1 ft. 3 in. by 11.
305. 1 mi. 2 fur. 35 po. 4 yds. 2 ft. 11 in. by 17.
306. 18 mi. 3 fur. 23 po. 1 yd. 2 ft. 6 in. by 35.
307. 75 mi. 1 fur. 39 po. 3 yds. by 75.
308. 756 mi. 634 yds. 0 ft. 8 in. by 208.
309. 1693 mi. 1660 yds. by 737.
310. 3 mi. 6 fur. 35 po. 3 yds. 6 in. by 551.

311. 29 sq. yds. 7 sq. ft. 72 sq. in. by 18.
312. 23 sq. yds. 6 sq. ft. 48 sq. in. by 96.
313. 24 ac 1 ro. 33 sq. po. 18 sq. yds. 2 sq. ft. 42 sq. in. by 10.
314. 12 ac. 3 ro. 27 sq. po. 21 sq. yds. 6 sq. ft. 1 sq. in. by 7.
315. 4851 ac. 3 ro. 36 sq. po. 28 sq. yds. by 94.
316. 198 ac. 3 ro. 16 sq. po. 3 yds. 0 ft. 72 in. by 187.
317. 1 ac. 2 ro. 8 sq. po. 11 yds. 2 ft. 108 in. by 367.
318. 40301 ac. 0 ro. 38 sq. po. 9 sq. yds. by 251.

319. 944 cub. yds. 1 cub. ft. 1104 cub. in. by 59.
320. 53 qrs. 6 bush. 2 pks. 3 qts. 1 pt. by 217.

Find—

321. £48, 6s. ÷ 11s. 6d.
322. £44, 19s. ÷ 9s. 8d.
323. £3294, 4s. ÷ £18, 2s.
324. £289, 17s. ÷ 4s. 3d.
325. £437, 18s. 9d. ÷ 8s. 9d.
326. £8485 ÷ £565, 13s. 4d.
327. £152, 7s. $7\frac{1}{2}d.$ ÷ 3s. $4\frac{1}{2}d.$
328. £38, 0s. 5d. ÷ 1s. $7\frac{1}{4}d.$
329. £43, 6s. 8d. ÷ £3, 6s. 8d.
330. £28, 2s. 6d. ÷ £1, 17s. 6d.
331. £103, 14s. 9d. ÷ £17, 5s. $9\frac{1}{2}d.$
332. £203, 10s. ÷ £12, 14s. $4\frac{1}{2}d.$
333. 7 hrs. 20 min. ÷ 7 min. 20 secs.
334. 17 hrs. 30 min. ÷ 1 hr. 15 min.
335. 8 yds. 2 ft. ÷ 1 ft. 1 in.
336. 15 yds. 1 ft. 8 in. ÷ 2 ft. 4 in.
337. 39 ac. 8 po. ÷ 2 ro. 8 po.
338. 30 bush. 3 gal. ÷ 3 gal. 3 pts.

Divide—

339. £904, 11s. 4d. by £56, 10s. $8\frac{1}{2}d.$
340. £500, 1s. $7\frac{1}{2}d.$ by £27, 15s. $7\frac{3}{4}d.$
341. £5647, 18s. $11\frac{1}{4}d.$ by £3, 4s. $5\frac{1}{4}d.$
342. £3226, 0s. 10d. by £3, 4s. $6\frac{1}{4}d.$
343. 10 tons 7 cwts. by 12 cwts. 3 qrs. 21 lbs.
344. 6 tons 8 cwts. 8 lbs. by 2 cwts. 2 ozs.
345. 57 tons 5 cwts. 1 qr. 7 lbs. by 1 ton 12 cwts. 2 qrs. 25 lbs.
346. 23 ozs. 15 dwts. 7 grs. by 7 dwts. 19 grs.

How many times is—

347. £4, 7s. 6d. contained in £161, 17s. 6d.?
348. £3, 17s. 6d. contained in £399, 2s. 6d.?
349. 195 yds. 1 ft. 8 in. contained in 1 mile?
350. 8 po. 1 yd. contained in 1 mile?
351. 2 yds. 2 ft. contained in 27 mi. 551 yds. 2 ft.?
352. 1 rood 24 poles contained in 320 acres?
353. 1 sq. ft. 127 in. contained in 7 sq. yds. 8 ft. 77 in.?
354. 5 cub. ft. 1037 in. contained in 1 cub. yd. 1 ft. 1 in.?

355. How many payments of 14s. $11\frac{3}{4}d.$ amount to £89, 2s. $6\frac{1}{4}d.$?
356. How many lbs. of tea at 1s. $10\frac{1}{2}d.$ a lb. can be bought for £45, 5s. $7\frac{1}{2}d.$?
357. How many tons of coal at 13s. 9d. per ton could be bought for £413, 3s. 9d.?
358. How many persons could each be supplied with a pint and a half of milk from a vessel containing 7 gallons 2 quarts?
359. How many bits each $4\frac{1}{2}$ inches long could be cut from 12 yards of tape?
360. How many plots of 2 ro. 16 po. 8 yds. are there in 10 ac. 36 po. 15 yds.?

VI. MISCELLANEOUS EXERCISES.

1. Reduce £237, 4*s.* 9¼*d.* to farthings, and prove the result.
2. How many more seconds are there in July than in June?
3. Add £119, 6*s.* 8½*d.*, £34, 17*s.* 2*d.*, £33, 11*s.* 0¾*d.*, £21, 0*s.* 10½*d.*, £17, 16*s.* 4¼*d.*, £9, 18*s.* 7¾*d.*, and £2, 16*s.* 2½*d.*
4. Take the sum of £13, 4*s.* 7½*d.* and £25, 19*s.* 10¾*d.* from £42, 4*s.* 6*d.*
5. Multiply £4628, 9*s.* 8¼*d.* by 9.
6. Divide £85307, 18*s.* 0*d.* by 48.
7. What was the income of a man who spent on the average £2, 3*s.* 6*d.* a day, and saved £50 in the year?
8. Divide the difference between £24299, 14*s.* 5*d.* and £14487, 16*s.* 11*d.* by 100.
9. How many twopence-halfpenny stamps can be bought for ten shillings?
10. How many times is 3 cwts. 3 qrs. 15 lbs. contained in 1 ton 7 cwts. 21 lbs.?

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11. Reduce 12 tons 3 cwts. 2 qrs. 11 lbs. to pounds, and prove the result.
12. How many minutes are there in the last six months of the year?
13. Find the sum of £201, 3*s.* 5½*d.*, £18, 0*s.* 9¾*d.*, £178, 6*s.* 11¼*d.*, £125, 17*s.* 1½*d.*, £574, 4*s.* 0*d.*, and £13, 19*s.* 7¼*d.*
14. After paying bills of £20, 13*s.* 4½*d.*, £30, 15*s.* 6½*d.*, £46, 13*s.* 11½*d.* what is left out of £100?
15. Multiply £608, 13*s.* 7¼*d.* by 72.
16. Divide £74, 15*s.* 7½*d.* by 37.
17. What sum of money when divided by 17 gives quotient 3*s.* 4½*d.* and remainder 2*s.* 5¾*d.*?
18. Find half the difference between 111111 farthings and £111, 11*s.* 11¼*d.*
19. If eggs are sold at 7 for sixpence, how many will three half-crowns purchase?
20. How many times is 2 ft. 3 in. contained in 19 yds. 2 ft. 5 in.?

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21. Show that there are as many farthings in £59, 12*s.* 1¾*d.* as there are pounds in 25 tons 10 cwts. 3 qrs. 19 lbs.
22. How many pence are there in £19 + 19 half-sovereigns + 19 half-crowns + 19 shillings + 19 sixpences?
23. From 300 half-guineas take 300 half-crowns.
24. Multiply 2 tons 9 cwts. 15 lbs. 9 ozs. by 27.
25. Divide £7777, 17*s.* 1*d.* by 17, and prove the result.

26. Find, shortly, the value of 648 things at 19*s.* 2*d.* each.

27. If 2 cwts. 1 lb. cost £116, 19*s.* 0$\frac{3}{4}$*d.*, what is the cost of 1 lb.?

28. If a sovereign were distributed equally among 320 children what would each receive?

29. How many cricket-balls at 5*s.* 6*d.* each could be bought for £166, 7*s.* 6*d.*?

30. How many times does a wheel, which is 7 ft. 4 in. in circumference, turn round in going a distance of 3 miles?

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31. How many weeks, days, &c., are there in as many seconds as there are ounces in fifty tons?

32. Express in £, *s.* *d.* the sum of 1000 sovereigns, 1000 shillings, 1000 pence, and 1000 farthings.

33. Reduce 5346 half-guineas to farthings.

34. Multiply 8 tons 14 cwts. 2 qrs. 14 lbs. 12 ozs. by 38.

35. If £99, 2*s.* 9$\frac{3}{4}$*d.* is divided equally among 47 people, what does each receive?

36. Find, as shortly as you can, the value of 731 articles at 9*s.* 11$\frac{1}{2}$*d.* each.

37. A leg of mutton, at 9*d.* per lb., cost 5*s.* 7$\frac{1}{2}$*d.*; what did it weigh?

38. What would be the cost of giving a penny bun and a halfpenny orange to each of a thousand school-children?

39. Divide 16*s.* 9$\frac{1}{2}$*d.* by 1*s.* 3$\frac{1}{2}$*d.*, and prove the result.

40. How many bottles, each holding 3 pints, could be filled from a cask containing 36 gallons?

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41. How many half-crowns would be a fair exchange for 1560 florins?

42. What is the value of the contents of a purse which contains a five-pound note, a cheque for £3, 8*s.* 7*d.*, 3 sovereigns, 3 half-sovereigns, a half-crown, a florin, 3 shillings, 3 sixpences, a threepenny bit, 7 penny stamps, and 7 halfpenny stamps?

43. Find the cost of 1183 slates at 4*d.* each?

44. If a man's income is £685, and he saves £274, 7*s.* 6*d.*, what is his average daily expenditure?

45. Find the sum of money nearest to £7 which could be equally divided among 93 persons.

46. Take the sum of £25, 16*s.* 1$\frac{3}{4}$*d.*, £2, 17*s.* 11$\frac{1}{2}$*d.*, £9, 5*s.* 3$\frac{1}{4}$*d.*, and 7*s.* 7$\frac{1}{2}$*d.* from £52.

47. What sum of money would be spent in paying £149, 17*s.* 10*d.* to each of 23 persons?

48. A man received £4, 0*s.* 6*d.* as wages for 28 days: how much did he earn per day?

49. Postcards are sold at 6 for $3\frac{1}{2}d.$: how many can be bought for $10s.\ 6d.$?

50. A draper bought 240 yards of cloth at $14s.\ 9d.$ a yard, and sold it at $15s.\ 7d.$ a yard: how much did he gain?

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51. Which is greater 17 half-crowns or 21 florins?

52. A man left home having in his purse two ten-pound notes, 3 sovereigns, 2 half-sovereigns, 7 shillings, and 3 pence. He paid bills of £4, $3s.\ 5d.$, £10, $7s.\ 6d.$, £2, $17s.\ 4d.$, and £1, $4s.$, and he bought a dozen halfpenny stamps. How much money did he bring back?

53. Find the cost of 2465 articles at $6d.$ each.

54. A man's income is £550 a year: he spends on the average £1, $7s.\ 3d.$ daily. What does he save annually?

55. Divide one thousand five hundred and forty-nine pounds nineteen shillings and fourpence farthing into thirty-one equal parts.

56. From £1000, $3s.\ 4\frac{3}{4}d.$ take the sum of £17, $8s.\ 6\frac{1}{4}d.$, £25, $14s.\ 6\frac{3}{4}d.$, and £12, $19s.\ 11\frac{3}{4}d.$

57. Find, as shortly as you can, 311 times £39, $19s.\ 11\frac{1}{4}d.$

58. If £465 $12s.\ 6d.$ be divided equally among 100 persons, what does each receive?

59. How many persons could each be paid $7s.\ 7d.$ out of £294, $12s.\ 3d.$?

60. Find the value of 70 articles all alike, if 5 of them are worth £1, $19s.\ 2d.$

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61. How many hours are there in a leap-year?

62. How many half-ounces are there in 6 tons 12 cwts. 1 qr. 17 lbs. 5 ozs. 8 drams?

63. Reduce 4972165 seconds to days, hrs. &c.

64. Add 176 cwts. 3 qrs. 15 lbs.; 48 cwts. 2 qrs. 11 lbs.; 35 cwts. 3 qrs. 7 lbs.; 7 cwts. 2 qrs. 21 lbs., and 4 cwts. 1 qr. 23 lbs.

65. Subtract 235 yds. 2 ft. 7 in. from 1 mile.

66. Multiply 2 dwts. 2 grs. by 101.

67. Divide 526 tons 11 cwts. 1 qr. 25 lbs. 3 ozs. by 197.

68. If 3 pennies weigh an ounce Avoirdupois, how many will balance a four-stone weight?

69. The ages of the 9 boys in a class are 11 yrs. 3 mo., 12 yrs. 3 mo., 9 yrs. 10 mo., 10 yrs., 12 yrs. 1 mo., 11 yrs. 7 mo., 9 yrs. 7 mo., 13 yrs., and 12 yrs. 5 mo. Find the average age.

70. If the length of a man's step be $27\frac{1}{2}$ inches, how many steps will he take in walking 4 miles?

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71. How many cwts. qrs. &c., are there in 10192000 grains?

72. How many half-minutes are there in 1 day 5 hrs. 36 min. 30 secs.?

73. Express 364713 feet in miles and yards.

74. Add 47 A. 3 R. 17 P., 19 A. 1 R. 37 P., 5 A. 2 R. 19 P., 13 A. 1 R. 25 P., and 43 A. 3 R. 22 P.

75. Subtract 2 gross 8 dozen and 10 from 4 gross 5 doz. and 7.

76. Multiply 1 yd. 1 ft. 1 in. by 101.

77. If 20 railway wheels weigh 2 tons 15 cwts. 1 qr. 2 lbs., what is the weight of one wheel?

78. If 3 pennies weigh an ounce Avoirdupois, what would 240000 pence weigh?

79. The heights of 7 boys are 5 ft. 3 in., 5 ft. 2 in., 4 ft. 10 in., 4 ft. $9\frac{1}{2}$ in., 5 ft. $1\frac{1}{2}$ in., 5 ft. 5 in., 5 ft.; what is the average height?

80. How many medals, each weighing 1 oz. 8 dwts. could be made from 100 ozs. 16 dwts. of metal?

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81. How many guineas are there in 9730 crowns?

82. Add together 17 halfpence, 17 pence, 17 threepences, 17 sixpences, 17 shillings, 17 florins, 17 half-crowns, 17 double-florins, 17 crowns, 17 half-sovereigns, and 17 sovereigns.

83. Multiply £427, 17*s.* $5\frac{1}{2}$*d.* by 91.

84. Multiply £29, 19*s.* $10\frac{3}{4}$*d.* by 192, as shortly as you can.

85. Multiply $3\frac{3}{4}$*d.* by fifty millions five thousand and six.

86. Find the profit on 841 sheep bought at £1, 19*s.* 9*d.* a head, and sold at £2, 0*s.* 3*d.* a head.

87. Divide £387, 13*s.* $9\frac{1}{2}$*d.* by 60, using short divisions.

88. Divide £376540583, 3*s.* 6*d.* by 972.

89. How many persons could each be paid £17, 17*s.* 9*d.* out of a fund which amounted to £554, 10*s.* 3*d.*?

90. If two oranges cost three-halfpence, how many can be bought for 5*s.* 6*d.*?

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91. Which is the greater, the number of farthings in £3,549,596, 14*s.* $6\frac{3}{4}$*d.*, or the number of two-shilling pieces in three hundred and forty millions, seven hundred and sixty-one thousand, two hundred and eighty-six pounds?

92. What is left out of a ten-pound note after paying bills of £2, 11*s.* $7\frac{1}{2}$*d.*, £1, 3*s.* 11*d.*, £1, 13*s.* $3\frac{1}{2}$*d.*, 17*s.* $5\frac{1}{2}$*d.*, and £2, 7*s.* 9*d.*?

93. Find the value of 10000 herrings when two are worth $1\frac{1}{2}$*d.*

94. Multiply £3, 5*s.* $11\frac{3}{4}$*d.* by 752.

95. Find, as shortly as you can, 999 times £5, 5*s.* 5*d.*

96. Find the profit on 1479 lbs. of tea bought at 1*s.* 10½*d.* per lb., and sold at 2*s.* 2*d.* per lb.

97. Divide £49396, 16*s.* 6¾*d.* by 531.

98. Find the sum of money which, when multiplied by 1000, amounts to £10542, 14*s.* 2*d.*

99. Among how many persons must £641, 14*s.* 11¼*d.* be divided that the share of each may be £2, 15*s.* 6¾*d.*?

100. A sum of £12, 2*s.* 7*d.* was divided equally among a certain number of persons, and the shares of three of them amounted to 17*s.* 9*d.* How many persons were there?

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101. Reduce 2 mi. 6 fur. 4 yds. to feet.

102. How far do I travel third class at a penny a mile if I pay £2, 3*s.* 7*d.* for my ticket?

103. How long would it take to count a billion at the rate of 100 per minute?

104. How much coal would supply 12 fires for 27 weeks, each fire consuming 1 cwt. 1 qr. 14 lbs. weekly?

105. A person's annual income is £555, 15*s.* 5*d.*; he saves £171 a year: what is his average daily expenditure?

106. Find the cost of 3 tons 2 cwts. 6 lbs. of sugar at 2½*d.* per lb.

107. Divide 42 mi. 7 fur. 7 po. 2 yds. 8 in. by 19.

108. Divide 5 ac. 3 ro. 24 po. by 1 ro. 19 po.

109. Divide £2 between George and John, so that George may have seven times as much as John.

110. If there are 34 lines on each side of a sheet of ruled foolscap paper, how many lines are there in a ream of such paper?

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111. Reduce 15 mi. 3 fur. 29 po. 4½ yds. to inches.

112. Multiply 1 mi. 1 fur. 1 po. 1 yd. 1 ft. 1 in. by 19.

113. How much would 3 tons 18 cwts. 1 qr. of butter cost at 14*d.* per lb.?

114. If it took a man, working 9 hours a day and counting 120 a minute, 17 days to count a heap of pins, how many were there in the heap?

115. Divide £337426, 5*s.* 4*d.* by 832.

116. If a man spends on the average 9*s.* 7*d.* daily and saves £25 yearly, what is his income?

117. Find the cost of 3 gross of note-books at 8*d.* per dozen.

118. A field of 18 ac. 36 po. is divided into allotments, each containing 1 rood 14 poles: how many allotments are there?

119. Divide £34, 16*s.* 9*d.* between two persons so that one may have eight times as much as the other.

120. If a silver tankard weighs 175 ozs. Troy, what is its weight Avoirdupois?

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121. In 1956742 ozs., how many tons, cwts., &c.?

122. Which is greater: 250000 farthings, or 2083 half-crowns?

123. What is the size of an estate which consists of 142 ac. 2 ro. of arable land, 113 ac. 1 ro. of pasture, 71 ac. of wood, and 89 ac. 1 ro. of moor?

124. If a man with a salary of 25 guineas per quarter spends at the rate of 5*s.* 3*d.* per day, how much does he save in a year?

125. Divide 69 mi. 7 fur. 39 po. 2 ft. by 492.

126. A flock of 170 sheep were bought at £2, 18*s.* 6*d.* each; half of them were sold at £2, 19*s.* 9*d.* each, and the rest at £3, 2*s.* 0*d.* each; what profit was made?

127. How many times is 1 cub. ft. 117 cub. in. contained in 3 cub. yds. 25 cub. ft. 1332 cub. in.?

128. Divide £4, 4*s.* 4*d.* between A and B, so that A may have £1, 19*s.* 10*d.* more than B.

129. A sum of money was divided between three persons, A, B, and C, in such a manner that the shares of A and B together amounted to 13*s.*, those of A and C to 14*s.*, and those of B and C to 15*s.* How much had each?

130. Three acres and a cow cost £170. The land costs seven times as much as the animal. Find the cost of the land per acre.

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131. Show that there are as many halfpence in £3, 11*s.* 7*d.* as there are half-inches in 23 yds. 2 ft. 7 in.

132. Reduce 3241985 inches to miles, furlongs, poles, &c.

133. Divide the sum of £17, 3*s.* 2*d.*, £12, 19*s.* 5½*d.*, £21, 17*s.* 11*d.*, and £24, 12*s.* 10*d.* by 47.

134. If a man's income be £37, 15*s.* per calendar month, and he spend at the rate of £1, 4*s.* 7½*d.* per day, how much will he save in a year?

135. Divide 13 tons 17 cwts. 3 qrs. 20 lbs. into two parts, one of which shall be double of the other.

136. A green-grocer bought 420 oranges for £1, 1*s.*; he sold half of them at 9*d.* a dozen, and the rest at 7 for sixpence; what did he gain?

137. How many allotments of 2 ro. 4 po. each can be made from a field of 7 ac. 3 ro. 24 po.?

138. Divide £18, 0*s.* 2*d.* between 11 men, giving one of them 10*s.* more than each of the others.

139. The total cost of a coat, hat, and umbrella was £6. The coat and hat alone would have cost £5, 1*s.*; the hat and umbrella alone, £1, 16*s.* Find the cost of each article.

140. Divide £24, 6*s.* 6*d.* into two parts, one of which shall contain as many shillings as there are half-crowns in the other.

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141. Find in pounds the difference between one million eight hundred and three thousand eight hundred and forty farthings, and one hundred and fifty thousand four hundred threepenny pieces.

142. Reduce 11 ro. 11 po. 11 sq. yds. to square inches, and *prove* the result.

143. Find, as shortly as you can, the value of 8632 articles at £2, 18*s.* 11½*d.* each.

144. How much a day is £4651, 9*s.* 4½*d.* a year?

145. Divide 15 cwts. 1 qr. 2 lbs. into two parts, one of which shall be nine times as large as the other.

146. Two friends set out on a tour with £12, 16*s.* 9*d.* between them; one had £1, 7*s.* 5*d.* more than the other; what had each?

147. A grocer buys 15 cwts. of rice for £24, 10*s.* At what price per lb. must he sell it to gain £7?

148. The circuit of a racing path is 137½ yards: how many times round it will make 5 miles?

149. A postman whose pay is 15*s.* a week is fined 1*s.* 6*d.* whenever he is late, and at the end of 13 weeks he has received £8, 15*s.* 6*d.* How often had he been late?

150. Divide £10 among 16 men, 16 women, and 16 boys, giving each woman 2*s.* less, and each boy 5*s.* less than each man.

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151. If a sheet of notepaper contains 54 square inches, how many sheets would cover an acre of ground? Also, what would they weigh if 11 sheets weigh 1 ounce?

152. If the Mint buys sufficient silver to coin seventy-four million eight hundred and sixty-seven thousand five hundred florins, but when half of the florins are finished, coins the rest of the silver into half-crowns, how many of these will there be?

153. Find, with as little labour as possible, the smallest yearly income which permits of an average daily expenditure of £1, 19*s.* 8¼*d.*

154. What sum of money must be subscribed by each of 100 persons to raise a fund of £251, 2*s.* 11*d.*?

155. Divide £82, 4*s.* 6*d.* between A and B, so that A may have 77 times as much as B.

156. Twelve chairs and a table cost £12, 7*s.* 6*d.*, the chairs cost 12*s.* 6*d.* each; find the cost of the table.

157. The circumferences of the wheels of a bicycle are 6 ft. 5 in. and 7 ft. 4 in.; how many more turns does the small wheel make than the large one in running 7 miles?

158. A grocer buys a cwt. of sugar for £1, 13*s.* 4*d.* and sells it for £2, 2*s.* 8*d.*; what does he gain per lb.?

159. A bill of £9, 8*s.* 10½*d.* was paid in equal shares by a number of persons, of whom 14 together paid £3, 11*s.* 5½*d.*: how many persons were there altogether?

160. Divide £9, 1*s.* 6*d.* among 11 men, 22 women, and 33 boys, giving each man three times as much as each woman, and each woman three times as much as each boy.

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161. How many days are there in seven consecutive years, beginning with 1892?

162. Reduce 1234 lbs. Avoirdupois to ounces and grains Troy.

163. Find, by inspection, the difference between £43, 10*s.* and 48,000 farthings.

164. What is the weight of a mile of wire if 3 inches of it weigh an ounce Av.?

165. February 23rd, 1892, was Tuesday; what day of the week was August 19th, 1891?

166. Divide £16, 16*s.* between 35 persons, giving to one of them twice as much as to each of the others.

167. If with every 6 lbs. of black tea, which I buy at 1*s.* 8*d.* per lb., I mix half a pound of green tea at 2*s.* 9*d.* per lb., what does the mixture cost me per lb.?

168. If thirty-three telegraph posts, placed at equal distances, extend a mile, how far apart are they?

169. Divide £33, 5*s.* among 30 men, 30 women, and 30 boys, giving every two men as much as 3 women, and every two women as much as 3 boys.

170. A sum of £28, 5*s.* 0*d.* was divided among 71 persons, some of whom received 8*s.* 4*d.* each, and the rest 7*s.* 6*d.* each. How many received 8*s.* 4*d.*?

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171. How many days are there from January 13th, 1880 to July 7th, 1893?

172. How many times does the hammer of a clock, which strikes the hours only, hit the bell in a week?

173. Write down, at sight, 241 times £3, 0*s.* 2*d.*

174. February 19th, 1892 fell on a Friday; what day of the month was the second Wednesday in May that year?

175. Divide £45 between two persons, so that for every sovereign one has the other may have half-a-crown.

176. An innkeeper buys 10 gallons of spirit at 12*s.* a gallon, 15 at 4*s.* 6*d.*, and 18 at 5*s.* 9*d.* How does he sell the mixture per gallon, if he gains £2, 2*s.* 3*d.* altogether?

177. If $1\frac{1}{2}$ inches of wire make a pin, how many yards of wire would be used in making 80 gross of pins?

178. If £5 be divided among 6 men, 12 women, and 17 boys, so that two men receive as much as five boys, and two women as much as three boys; how much will each receive?

179. A sum of £24, 7*s.* 11*d.* was divided among a number of men and boys, the boys each receiving 3*s.* 1*d.* and the men 10*s.*; how many men were there, the total number of persons being 73?

180. The *Saturday Review* of Feb. 6, 1892, was numbered 1893; what was the number on a copy bearing the date Dec. 27, 1890?

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181. What is the cost of the *Times* daily newspaper for the year 1893 at 3*d.* per copy? The year began on a Sunday.

182. Two pieces of cloth of the same length cost £11, 3*s.* 6*d.* and £14, 8*s.* 0*d.* respectively, the price of the first was 6*s.* $2\frac{1}{2}$*d.* per yard; what was the price of the second per yard?

183. Find the cost of a steel hammer weighing 225 tons, at the rate of 7 lbs. for 1*s.*

184. Divide £92, 16*s.* among 19 men and 17 women, giving each woman £1 less than each man.

185. How many shillings, each weighing 3 dwts. 12 grs., could be made from 37 lbs. of silver mixed with 3 lbs. of alloy?

186. A takes 1980 steps in walking a mile, and B takes 1920 steps. How much longer is B's stride than A's?

187. If a dealer bought eggs, to the value of £3, at 11 for a shilling and sold them at 13*d.* a dozen, how much would he gain or lose?

188. A person bought equal quantities of sugar at $3\frac{1}{2}$*d.* per lb., tea at 3*s.* 9*d.* per lb., butter at 1*s.* $11\frac{1}{2}$*d.* per lb., and coffee at 1*s.* 4*d.* per lb. The whole cost £1, 16*s.* 8*d.* How much of each did he buy?

189. A wine merchant bottles off a 36-gallon cask of claret into an equal number of reputed quart bottles and half-bottles; how many dozen of each has he?

190. January 9, 1892, fell on a Saturday; what day of the week was June 20, 1837?

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191. What was paid for the *Standard* penny daily newspaper by a person who began to buy it on January 1st, and ceased to do so after March 26th in the year 1892?

192. If 6 dozen table-spoons weigh 11 lbs. 6 ozs. 12 dwts. and the same number of tea-spoons weigh 3 lbs. 7 ozs. 10 dwts., by how much does the weight of a table-spoon exceed that of a tea-spoon?

193. If 11 thalers, 6 florins, 3 guineas, and 5 dollars amount to £6, 0*s*. 7*d*., and a dollar is worth 4*s*. 2*d*., what is the value of a thaler?

194. A wine-merchant bought a 36-gallon cask of brandy at 27*s*. 6*d*. per gallon, diluted it with water and then sold it at 30*s*. per gallon, making a profit of £10, 10*s*. How much water did he add?

195. Two boys start for a quarter-mile race; one takes four strides of 40 inches each while the other takes five of 33 inches; which wins and by how much?

196. The live stock on a farm consists of a certain number of horses worth 60 guineas each, an equal number of pigs worth £2, 10*s*. each, three times as many cows worth £18, 10*s*. each, and fifteen times as many sheep worth £1, 15*s*. each. The whole value of the live stock is £1030, 15*s*. How many are there of each kind?

197. The *Journal of Education* is published monthly; the issue of February, 1892, was numbered 271; when was No. 186 published?

198. A wine-merchant bottles off 42 gallons of beer into an equal number of imperial quart, pint, and half-pint bottles; how many dozen of each has he?

199. A man bought 374 eggs at 2 a penny, and some others at 3 for twopence. He paid altogether £1, 9*s*. 11*d*. for them. How many eggs did he buy?

200. On April 14, 1892, a man had lived 14610 days; find the date of his birth.

VII. FACTORS, MULTIPLES, PRIMES.

Without actually dividing, find by which of the numbers 2, 5, 10: 4, 25; 3, 9, each of the following numbers is exactly divisible :—

1. 2016.
2. 6750.
3. 5638.
4. 5625.
5. 7109.
6. 31452.
7. 17348.
8. 20520.
9. 73818.
10. 19875.
11. 66312.
12. 24680.
13. 13579.
14. 71799.
15. 138600.
16. 17265.
17. 321000.
18. 444444.
19. 853050.
20. 714051.

Which of the following numbers is a multiple of 11?—

21. 38964035 and 76057183.
22. 413780495 and 709082649.

Find, without actually dividing, the remainder of—

23. $73578 \div 4$, $597361 \div 25$, $84127 \div 3$ and $267488 \div 9$.
24. $30857 \div 4$, $8978377 \div 25$, $76426 \div 3$ and $789635 \div 9$.

Find, by inspection, the prime factors of—

25. 12.
26. 18.
27. 20.
28. 24.
29. 27.
30. 34.
31. 39.
32. 42.
33. 51.
34. 57.
35. 63.
36. 65.
37. 70.
38. 88.
39. 90.
40. 100.
41. 28.
42. 36.
43. 40.
44. 45.
45. 52.
46. 69.
47. 81.
48. 91.
49. 99.
50. 110.
51. 121.
52. 125.
53. 130.
54. 104.
55. 108.
56. 144.

Resolve into prime factors—

57. 112.
58. 120.
59. 111.
60. 128.
61. 136.
62. 148.
63. 161.
64. 171.
65. 252.
66. 255.
67. 369.
68. 448.
69. 441.
70. 567.
71. 693.
72. 882.
73. 999.
74. 1000.
75. 1331.
76. 1760.
77. 1728.
78. 2240.
79. 4840.
80. 1892.
81. 1893.
82. 5760.
83. 3024.
84. 6435.
85. 2871.
86. 7777.
87. 56430.
88. 21970.
89. 13013.
90. 14641.
91. 111111.
92. 21978.
93. 14847.
94. 873873.
95. 123456.
96. 999999.

97. How many primes are there between 1 and 100?
98. How many primes are there between 100 and 200?
99. Is 1009 a prime number?
100. Is 1513 a prime number?

VIII. GREATEST COMMON FACTOR.*

Find, *by inspection*, the G.C.F. of—

1. 35 and 42.
2. 70 and 90.
3. 25 and 65.
4. 50 and 75.
5. 12 and 16.
6. 16 and 24.
7. 24 and 32.
8. 36 and 48.
9. 34 and 51.
10. 39 and 65.
11. 80 and 100.
12. 45 and 30.
13. 41 and 82.
14. 23 and 69.
15. 38 and 57.
16. 28 and 42.
17. 99 and 66.
18. 66 and 88.
19. 56 and 96.
20. 600 and 800.
21. 34 and 43.
22. 51 and 15.
23. 69 and 96.
24. 62 and 93.
25. 101 and 202.
26. 102 and 201.
27. 777 and 999.
28. 606 and 808.
29. 1313 and 1919.
30. 2346 and 3162.

Find the G.C.F. of—

31. 315 and 385.
32. 255 and 935.
33. 428 and 824.
34. 432 and 324.
35. 432 and 3888.
36. 1056 and 576.
37. 837 and 999.
38. 849 and 1132.
39. 847 and 1001.
40. 677 and 8801.
41. 949 and 1387.
42. 1081 and 1311.
43. 1008 and 1036.
44. 2240 and 5760.
45. 1760 and 4840.
46. 1547 and 5712.
47. 3465 and 4655.
48. 1869 and 3471.
49. 2187 and 8019.
50. 7623 and 8316.
51. 5763 and 5198.
52. 6741 and 2289.
53. 4611 and 5307.
54. 4833 and 7533.
55. 6468 and 2009.
56. 7701 and 9231.
57. 8401 and 8374.
58. 9805 and 8374.
59. 15933 and 10011.
60. 28101 and 92017.

61. 185625 and 38115.
62. 929181 and 1012891.
63. 87318 and 206910.
64. 437248 and 489159.
65. 6545253 and 12151659.
66. 214258289 and 262833917.

67. 86, 215 and 387.
68. 126, 198 and 270.
69. 196, 308 and 420.
70. 85, 153 and 595.
71. 252, 264 and 444.
72. 1221, 2109 and 5402.
73. 6003, 3519 and 145107.
74. 8487, 39123 and 46782.
75. 581, 83083 and 146993.
76. 84411, 98649 and 102717.
77. 108, 288, 360 and 924.
78. 102, 187, 357 and 680.
79. 384, 1280, 752 and 352.
80. 713, 1219, 1817 and 2001.

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* Often called Greatest Common *Measure* (G.C.*M.*).

IX. LEAST COMMON MULTIPLE.

Find, *by inspection*, the L.C.M. of—

1. 4 and 6.
2. 8 and 12.
3. 10 and 15.
4. 9 and 18.
5. 8 and 16.
6. 12 and 36.
7. 20 and 30.
8. 15 and 20.
9. 20 and 25.
10. 22 and 33.
11. 2, 4 and 6.
12. 2, 4 and 8.
13. 3, 4 and 5.
14. 4, 5 and 6.
15. 4, 5 and 8.
16. 3, 6 and 9.
17. 7, 21 and 42.
18. 4, 6 and 8.

Find the L.C.M. of—

19. 35 and 77.
20. 55 and 88.
21. 15 and 35.
22. 63 and 99.
23. 84 and 96.
24. 49 and 84.
25. 26 and 65.
26. 34 and 51.
27. 38 and 57.
28. 74 and 111.
29. 85 and 119.
30. 23 and 29.
31. 17 and 71.
32. 13 and 91.
33. 112 and 120.
34. 112 and 128.
35. 18, 45 and 63.
36. 22, 33 and 55.
37. 32, 6 and 27.
38. 42, 63 and 105.
39. 15, 27 and 39.
40. 15, 21 and 35.
41. 14, 16 and 112.
42. 16, 17 and 18.
43. 6, 8, 16 and 21.
44. 10, 15, 20 and 25.
45. 4, 10, 18 and 24.
46. 15, 20, 33 and 110.
47. 6, 8, 9, 10 and 12.
48. 3, 6, 9, 12 and 15.
49. 63, 12, 84 and 14.
50. 18, 28, 30 and 42.
51. 12, 16, 18, 24 and 36.
52. 10, 12, 14, 36 and 42.
53. 14, 28, 40 and 48.
54. 12, 16, 21 and 70.
55. 7, 11, 21, 33 and 42.
56. 36, 60, 96 and 108.
57. 42, 64, 70 and 112.
58. 18, 24, 30 and 56.
59. 65, 91 and 104.
60. 121, 143 and 169.
61. 6, 8, 10, 12, 14 and 16.
62. 6, 8, 10, 12, 16 and 20.
63. 7, 14, 28, 49, 140 and 210.
64. 300, 400, 500, 600 and 800.
65. 6, 10, 12, 15, 20, 24, 25.
66. 12, 16, 20, 24, 28, 32, 36.
67. 30, 32, 36, 42, 44 and 50.
68. 63, 84, 105, 126 and 252.
69. 121, 132, 143, 154, 165.
70. 34, 51, 68, 85, 102, 119.
71. 885 and 5900.
72. 1763 and 2021.
73. 1728 and 1760.
74. 973 and 12649.
75. 29181 and 16851.
76. 187, 781 and 497.
77. 323, 493 and 551.
78. 1078, 1617 and 1386.
79. 690, 713, 736 and 759.
80. 2733, 4555, 6377 and 10021.

X. MISCELLANEOUS EXERCISES.

1. Express 31416 as the product of its prime factors.
2. Show that 811 is a prime number.
3. Find, *by inspection*, the G.C.M. of 59 and 5959.
4. Find the least number of which both 168 and 560 are factors.
5. What number is the same multiple of 13 that 6579 is of 17?

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6. Separate 132288 into its elementary factors.
7. Prove that 449 is a prime number.
8. Find, *by inspection*, the G.C.M. of 707 and 1212.
9. Find the greatest number which divides 33495 and 106260.
10. Prove 16327 to be the same multiple of 29 that 113163 is of 201.

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11. Find the sum of all the prime numbers between 75 and 115.
12. Find the G.C.M. of 104, 286 and 663.
13. Find the L.C.M. of 77, 707 and 7007.
14. Show that there are six numbers, besides unity and the number itself, by which 455 is exactly divisible.
15. What is the smallest sum of money which must be subtracted from £102, 10*s.* 9*d.* in order that the remainder may exactly contain £1, 13*s.* 7*d.*?

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16. Find the sum of all the prime numbers between 300 and 400.
17. Find the G.C.M. of 798, 5016 and 8664.
18. Find the L.C.M. of 200, 500, 800, 1100 and 1700.
19. Find a common multiple of 15, 25 and 35, which is less than 2000 and greater than 1500.
20. Find the least sum of money which added to £243, 5*s.* 7½*d.* makes the result exactly divisible by 431.

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21. Express 693693 as the continued product of its prime factors.
22. Prove that 115991 and 118009 are prime to each other.
23. Find the largest divisor of 201, 501, 801, 1101 and 1701.
24. Find the least number which exactly contains both 451 and 533.
25. Find, without division, the remainder of $432561 \div 9$.

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26. Resolve 114660 into its prime factors.
27. Write down at sight the G.C.F. of $11 \times 17 \times 23$, $13 \times 19 \times 17$ and $17 \times 11 \times 19$.
28. Find the smallest number which is exactly divisible by all the numbers between 1 and 13.

29. Find the largest number which is exactly contained in each of the numbers 798, 5016 and 8645.

30. Find, without dividing, the remainder of $73894251 \div 4$.

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31. Find the complete remainder when 32547631 is divided by the prime factors of 1155.

32. Is 70608050304 a multiple of 11?

33. How many times does the L.C.M. of 42, 56, 84 and 98 contain their G.C.M.?

34. Find the L.C.M. of 1*s.* 2*d.*, 1*s.* 4*d.*, 1*s.* 6*d.* and 1*s.* 8*d.*

35. Find the number nearest to 76478 which is divisible by 103.

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36. Prove that 425 and 1062 are prime to each other.

37. Is 9 a factor of 476583207?

38. Find the difference between the G.C.M. and L.C.M. of 111, 185 and 259.

39. Find the L.C.M. of 2 tons 1 cwt. 2 qrs. and 7 tons 5 cwts. 1 qr.

40. What is the least number which must be added to a million that the result may be exactly divisible by 365?

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41. Find the multiple of 37 which is nearest to 1500.

42. Find the G.C.M. of 2 days 23 hours 40 mins., and 3 days 10 hours 25 mins.

43. Find the smallest number which when divided by 2, 3, 4, 5, 6, 8, 9, 10, 11, or 12, gives remainder 1 in each case.

44. Find, shortly, the sum of 567×36 and 567×64.

45. The G.C.M. of 493 and another number is 17; their L.C.M. is 18241; find the other number.

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46. Find the least number of three digits which exactly contains 41.

47. Find the G.C.M. of 5 cwts. 3 qrs. 24 lbs. 7 ozs. and 4 cwts. 1 qr. 18 lbs. 1 oz.

48. Find the least number which can be divided by 9, 12, 15, or 20, with remainder 7 in each case.

49. Find, shortly, the difference between $90424 \div 73$ and $83051 \div 73$.

50. The G.C.F. of two numbers, each of four digits, is 431, their L.C.M. is 15085. Find the numbers.

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51. Find the number nearest to 1000 which exactly contains 37.

52. Find the sum of all multiples of 8 greater than 300 and less than 400.

53. Find the greatest number which divides 3243 with remainder 3, and 4255 with remainder 7.

54. Three bells toll at intervals of 7, 12 and 15 seconds respectively. If they all begin to toll together, after how many minutes will they again toll together?

55. Show that 108900 is a perfect square.

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56. Find the greatest number of three figures which is divisible by 41 without remainder.

57. Find the sum of all the multiples of 9 which are greater than 150 and less than 200.

58. What number divides 10224 with remainder 621, and 11886 with remainder 537?

59. If a franc is worth $9\frac{1}{2}d.$ and a rupee $1s.\ 2\frac{1}{4}d.$, what is the smallest number of francs which contains an exact number of rupees?

60. Find the greatest number of grains which is exactly contained in both a pound Troy and a pound Avoirdupois.

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61. Find the smallest number of five digits which is a multiple of 473.

62. Find the greatest length which exactly measures either 106 yds. 1 ft., or 135 yds. 2 ft.

63. Find the least number which when divided by 77 gives remainder 76, when divided by 88 gives remainder 87, and when divided by 99 gives remainder 98.

64. Find two numbers which have G.C.M. 91, and L.C.M. 1911.

65. The product of two numbers, one of which is double the other, is 3528; find them.

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66. Find the greatest and least numbers each of six digits which are exactly divisible by 789.

67. Find the shortest space of time which exactly contains either 1 day 12 hrs. 17 min., or 1 day 17 hrs. 28 min.

68. What is the least number which when divided by 26, 65, 91, or 143 leaves remainder 1 in each case?

69. The G.C.M. of two numbers is 137; the sum of the two numbers is 959; find them. How many correct answers are there?

70. By what must 40768 be multiplied to form a perfect square?

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71. The numbers 390507 and 391337 are not prime to each other; find, without division, their G.C.F.

72. Find *all* the common factors of 37037 and 23023.

73. Two heaps of the same kind of shot weigh 180 tons 4 cwts. 2 qrs., and 124 tons 8 cwts. 2 qrs., respectively: what is the greatest possible weight of each shot?

74. The product of two numbers is 76614; their G.C.F. is 113; find their L.C.M.

75. The product of three consecutive numbers is 10626; find them.

XI. FRACTIONS.

NOTATION.

Read, or write in words—

1. $\frac{1}{7}$; $\frac{2}{5}$; $\frac{3}{8}$; $\frac{5}{9}$; $\frac{1}{11}$; $\frac{4}{13}$.
2. $\frac{1}{20}$; $\frac{3}{7}$; $\frac{9}{10}$; $\frac{4}{19}$; $\frac{3}{40}$; $\frac{17}{30}$.
3. $\frac{2}{15}$; $\frac{10}{11}$; $\frac{19}{20}$; $\frac{3}{1000}$; $\frac{19}{800}$.
4. $\frac{12}{13}$; $\frac{1}{100}$; $\frac{69}{70}$; $\frac{7}{19}$; $\frac{211}{350}$.

Write in figures—

5. One-tenth; One-twelfth; Seven-twentieths; Thirteen-sixteenths.
6. Five-ninths; Twelve-nineteenths; Twenty-three hundredths.
7. Two-fifths; Seventy-three seven-hundredths; Three thousandths.
8. Three-sevenths; Nineteen ninety-ninths; Eight eighty-oneths.

Read off (or write down) at sight the number of—

9. shillings in $\frac{1}{2}$; $\frac{1}{4}$; $\frac{1}{5}$; $\frac{1}{10}$; $\frac{1}{20}$; of a sovereign.
10. pence in $\frac{1}{2}$; $\frac{1}{3}$; $\frac{1}{4}$; $\frac{1}{6}$; $\frac{1}{12}$; of a shilling.
11. ounces in $\frac{1}{2}$; $\frac{1}{4}$; $\frac{1}{8}$; $\frac{1}{16}$; of a pound Avoirdupois.
12. lbs. in $\frac{1}{2}$; $\frac{1}{7}$; $\frac{1}{14}$; $\frac{1}{28}$; of a quarter.
13. shillings in $\frac{3}{20}$; $\frac{7}{20}$; $\frac{11}{20}$; $\frac{13}{20}$; $\frac{19}{20}$; of £1.
14. pence in $\frac{5}{12}$; $\frac{7}{12}$; $\frac{3}{12}$; $\frac{11}{12}$; $\frac{8}{12}$; of a shilling.
15. ounces in $\frac{3}{16}$; $\frac{5}{16}$; $\frac{7}{16}$; $\frac{13}{16}$; $\frac{11}{16}$; of a lb. Av.
16. lbs. in $\frac{3}{28}$; $\frac{11}{28}$; $\frac{19}{28}$; $\frac{5}{28}$; $\frac{23}{28}$; of a quarter.
17. hours in $\frac{1}{2}$; $\frac{1}{3}$; $\frac{1}{4}$; $\frac{1}{6}$; $\frac{1}{8}$; $\frac{1}{12}$; $\frac{1}{24}$; of a day.
18. minutes in $\frac{1}{2}$; $\frac{1}{3}$; $\frac{1}{4}$; $\frac{1}{5}$; $\frac{1}{6}$; $\frac{1}{10}$; $\frac{1}{12}$; $\frac{1}{15}$; $\frac{1}{20}$; $\frac{1}{30}$; of an hour.
19. hours in $\frac{7}{24}$; $\frac{11}{24}$; $\frac{9}{24}$; $\frac{13}{24}$; $\frac{23}{24}$; of a day.
20. minutes in $\frac{17}{60}$; $\frac{19}{60}$; $\frac{13}{60}$; $\frac{29}{60}$; $\frac{51}{60}$; of an hour.

Find the value of—

21. Half of 68; One-third of 93; Two-fifths of 365.
22. One-fifth of 80; One-seventh of 343; Five-elevenths of 187.
23. Three-fourths of a ton; Two-thirds of £24; One-tenth of 2*s.* 6*d.*
24. Three-quarters of an hour; Nine-tenths of £30; One-fifth of 7*s.* 6*d.*
25. £$\frac{3}{10}$; £$\frac{7}{10}$; £$\frac{9}{10}$; £$\frac{2}{5}$; £$\frac{4}{5}$; £$\frac{3}{5}$; £$\frac{1}{240}$; £$\frac{7}{240}$; £$\frac{23}{240}$.
26. £$\frac{3}{4}$; £$\frac{1}{10}$; £$\frac{11}{10}$; £$\frac{9}{20}$; £$\frac{18}{20}$; £$\frac{1}{40}$; £$\frac{11}{240}$; £$\frac{19}{240}$; £$\frac{29}{240}$.
27. $\frac{3}{4}$*s.*; $\frac{2}{3}$*s.*; $\frac{5}{6}$*s.*; $\frac{1}{24}$*s.*; $\frac{7}{24}$*s.*; $\frac{11}{24}$*s.*; $\frac{7}{48}$*s.*; $\frac{11}{48}$*s.*; $\frac{1}{8}$*s.*; $\frac{5}{8}$*s.*
28. $\frac{3}{4}$ min.; $\frac{5}{12}$ day; $\frac{3}{10}$ ton; $\frac{3}{10}$ hour; $\frac{7}{36}$ yard; $\frac{5}{18}$ sq. ft.
29. £$\frac{13}{60}$; $\frac{5}{16}$*s.*; $\frac{17}{30}$ hour; $\frac{13}{40}$ ton; $\frac{5}{6}$ foot; $\frac{3}{10}$ of an acre.
30. £$\frac{19}{30}$; $\frac{13}{16}$*s.*; $\frac{2}{5}$ of a year; $\frac{5}{14}$ cwt.; $\frac{5}{11}$ mile; $\frac{3}{16}$ of a gallon.

XII. FRACTIONS.

Improper Fractions and Mixed Numbers.

Read, or write in words—

1. $5\frac{3}{8}$; $7\frac{4}{5}$; $9\frac{1}{11}$; $12\frac{3}{13}$; $15\frac{7}{20}$.
2. $1\frac{2}{33}$; $12\frac{17}{50}$; $10\frac{5}{9}$; $28\frac{7}{25}$.

Write in figures—

3. Two and three-fifths; Seven and one-third.
4. Eight and seven-tenths; One and one-twelfth.
5. Eleven and three-fourths; Seventeen and a quarter.
6. Twenty and three-twentieths; Nine and eight-ninths.

Turn, mentally, into integers—

7. $\frac{77}{7}$; $\frac{24}{8}$; $\frac{132}{11}$; $\frac{51}{3}$; $\frac{38}{19}$.
8. $\frac{100}{5}$; $\frac{100}{25}$; $\frac{808}{101}$; $\frac{81}{3}$; $\frac{56}{14}$.

Turn, mentally, into mixed numbers—

9. $\frac{5}{3}$; $\frac{8}{5}$; $\frac{11}{2}$; $\frac{7}{6}$; $\frac{10}{7}$; $\frac{17}{4}$.
10. $\frac{7}{2}$; $\frac{9}{5}$; $\frac{14}{11}$; $\frac{21}{8}$; $\frac{16}{7}$; $\frac{13}{3}$.
11. $\frac{22}{7}$; $\frac{18}{5}$; $\frac{30}{11}$; $\frac{41}{12}$; $\frac{80}{9}$.
12. $\frac{37}{6}$; $\frac{48}{11}$; $\frac{91}{12}$; $\frac{100}{9}$; $\frac{123}{10}$.

Turn, mentally, into improper fractions—

13. $1\frac{1}{5}$; $2\frac{1}{3}$; $3\frac{3}{4}$; $5\frac{4}{7}$; $6\frac{5}{8}$.
14. $1\frac{1}{9}$; $3\frac{2}{5}$; $4\frac{5}{6}$; $5\frac{1}{2}$; $2\frac{5}{12}$.
15. $4\frac{3}{4}$; $5\frac{5}{6}$; $3\frac{2}{7}$; $11\frac{1}{2}$; $10\frac{5}{8}$.
16. $7\frac{1}{7}$; $4\frac{2}{9}$; $1\frac{7}{20}$; $21\frac{1}{2}$; $9\frac{7}{9}$.

Express as improper fractions—

17. $3\frac{13}{25}$.	25. $9\frac{7}{15}$.	33. $9\frac{1}{33}$.	41. $100\frac{3}{8}$.
18. $2\frac{5}{38}$.	26. $15\frac{7}{9}$.	34. $21\frac{7}{13}$.	42. $5\frac{103}{108}$.
19. $2\frac{17}{24}$.	27. $12\frac{11}{12}$.	35. $2\frac{51}{53}$.	43. $97\frac{1}{51}$.
20. $3\frac{19}{20}$.	28. $4\frac{7}{19}$.	36. $27\frac{7}{9}$.	44. $27\frac{7}{19}$.
21. $4\frac{20}{27}$.	29. $5\frac{16}{17}$.	37. $11\frac{23}{41}$.	45. $53\frac{179}{200}$.
22. $4\frac{9}{47}$.	30. $11\frac{10}{11}$.	38. $35\frac{5}{12}$.	46. $50\frac{721}{831}$.
23. $12\frac{11}{13}$.	31. $13\frac{12}{13}$.	39. $12\frac{29}{50}$.	47. $5\frac{301}{1197}$.
24. $2\frac{2}{71}$.	32. $7\frac{5}{29}$.	40. $30\frac{17}{40}$.	48. $5\frac{2194}{2359}$.

Express as mixed numbers—

49. $\frac{1000}{7}$.	57. $\frac{4231}{100}$.	65. $\frac{673}{35}$.	73. $\frac{3797}{50}$.
50. $\frac{1001}{9}$.	58. $\frac{53267}{1000}$.	66. $\frac{827}{45}$.	74. $\frac{5811}{40}$.
51. $\frac{1010}{3}$.	59. $\frac{603}{13}$.	67. $\frac{7423}{300}$.	75. $\frac{11325}{41}$.
52. $\frac{1000}{11}$.	60. $\frac{215}{14}$.	68. $\frac{1973}{700}$.	76. $\frac{42003}{31}$.
53. $\frac{851}{12}$.	61. $\frac{701}{15}$.	69. $\frac{873}{110}$.	77. $\frac{3622}{147}$.
54. $\frac{777}{8}$.	62. $\frac{843}{17}$.	70. $\frac{8573}{5000}$.	78. $\frac{7960}{207}$.
55. $\frac{2361}{10}$.	63. $\frac{789}{23}$.	71. $\frac{817}{552}$.	79. $\frac{42653}{558}$.
56. $\frac{579}{20}$.	64. $\frac{1325}{29}$.	72. $\frac{605}{324}$.	80. $\frac{18579}{844}$.

XIII. FRACTIONS.

Reduction.

Reduce, mentally—

1. Two *thirds* to *ninths*.
2. One *fourth* to *twelfths*.
3. Seven *tenths* to *fiftieths*.
4. Three *sevenths* to *sixty-thirds*.
5. Seven *eighths* to *fortieths*.
6. 4 *elevenths* to *fifty-fifths*.
7. 3 *fifths* to *fifty-fifths*.
8. 5 *sixths* to *thirtieths*.
9. 13 *fourteenths* to *seventieths*.
10. 23 *twentieths* to *sixtieths*.
11. Six *eighths* to *fourths*.
12. Twelve *twentieths* to *fifths*.
13. Four *twenty-fourths* to *sixths*.
14. Six *thirtieths* to *tenths*.
15. Twelve *sixtieths* to *fifteenths*.
16. 9 *twenty-oneths* to *sevenths*.
17. 15 *twenty-fifths* to *fifths*.
18. 8 *twentieths* to *fifths*.
19. 18 *thirtieths* to *tenths*.
20. 25 *fortieths* to *eighths*.

Supply the proper figures in place of the asterisks in—

21. $\frac{2}{3} = \frac{*}{6}$.
22. $\frac{3}{4} = \frac{*}{12}$.
23. $\frac{2}{5} = \frac{*}{25}$.
24. $\frac{5}{6} = \frac{*}{24}$.
25. $\frac{9}{10} = \frac{*}{50}$.
26. $\frac{7}{11} = \frac{*}{44}$.
27. $\frac{1}{12} = \frac{*}{84}$.
28. $\frac{5}{9} = \frac{*}{63}$.
29. $\frac{11}{13} = \frac{*}{39}$.
30. $\frac{2}{5} = \frac{*}{100}$.
31. $\frac{1}{4} = \frac{21}{*}$.
32. $\frac{3}{14} = \frac{15}{*}$.
33. $\frac{3}{17} = \frac{6}{*}$.
34. $\frac{5}{11} = \frac{20}{*}$.
35. $\frac{8}{9} = \frac{64}{*}$.
36. $\frac{5}{19} = \frac{15}{*}$.
37. $\frac{19}{25} = \frac{76}{*}$.
38. $\frac{20}{31} = \frac{60}{*}$.
39. $\frac{9}{17} = \frac{45}{*}$.
40. $\frac{4}{19} = \frac{12}{*}$.
41. $\frac{10}{12} = \frac{5}{*}$.
42. $\frac{24}{28} = \frac{6}{*}$.
43. $\frac{35}{42} = \frac{5}{*}$.
44. $\frac{40}{55} = \frac{8}{*}$.
45. $\frac{12}{20} = \frac{3}{*}$.
46. $\frac{16}{24} = \frac{2}{*}$.
47. $\frac{40}{60} = \frac{2}{*}$.
48. $\frac{75}{100} = \frac{3}{*}$.
49. $\frac{28}{32} = \frac{7}{*}$.
50. $\frac{21}{49} = \frac{3}{*}$.
51. $\frac{120}{132} = \frac{*}{11}$.
52. $\frac{63}{72} = \frac{*}{8}$.
53. $\frac{15}{35} = \frac{*}{7}$.
54. $\frac{24}{64} = \frac{*}{16}$.
55. $\frac{33}{99} = \frac{*}{33}$.
56. $\frac{48}{84} = \frac{*}{14}$.
57. $\frac{16}{56} = \frac{*}{14}$.
58. $\frac{57}{75} = \frac{*}{25}$.
59. $\frac{80}{144} = \frac{*}{9}$.
60. $\frac{95}{200} = \frac{*}{40}$.

Express—

61. $\frac{7}{11}$ with denominator 55.
62. $\frac{9}{13}$ „ „ 65.
63. $\frac{17}{21}$ „ „ 84.
64. $\frac{5}{7}$ „ „ 119.
65. $\frac{7}{12}$ „ „ 156.
66. $\frac{41}{53}$ „ „ 159.
67. $\frac{5}{7}$ with numerator 40.
68. $\frac{9}{11}$ „ „ 108.
69. $\frac{2}{17}$ „ „ 100.
70. $\frac{13}{14}$ „ „ 169.
71. $\frac{12}{16}$ with denominator 8.
72. $\frac{24}{32}$ „ „ 4.
73. $\frac{40}{60}$ „ „ 3.
74. $\frac{56}{84}$ „ „ 12.
75. $\frac{75}{125}$ „ „ 25.
76. $\frac{110}{120}$ „ „ 24.
77. $\frac{18}{20}$ with numerator 9.
78. $\frac{42}{63}$ „ „ 6.
79. $\frac{91}{169}$ „ „ 7.
80. $\frac{220}{330}$ „ „ 2.

XIV. FRACTIONS.

Reduction to Lowest Terms.

Reduce to lowest terms—

1. $\frac{6}{12}$.	2. $\frac{8}{12}$.	3. $\frac{9}{12}$.	4. $\frac{10}{12}$.
5. $\frac{15}{20}$.	6. $\frac{18}{20}$.	7. $\frac{12}{20}$.	8. $\frac{16}{20}$.
9. $\frac{8}{24}$.	10. $\frac{9}{24}$.	11. $\frac{15}{24}$.	12. $\frac{16}{24}$.
13. $\frac{6}{21}$.	14. $\frac{14}{21}$.	15. $\frac{12}{30}$.	16. $\frac{18}{30}$.
17. $\frac{28}{38}$.	18. $\frac{27}{36}$.	19. $\frac{25}{35}$.	20. $\frac{22}{33}$.
21. $\frac{24}{36}$.	22. $\frac{18}{36}$.	23. $\frac{30}{36}$.	24. $\frac{27}{36}$.
25. $\frac{19}{38}$.	26. $\frac{26}{39}$.	27. $\frac{34}{51}$.	28. $\frac{14}{42}$.
29. $\frac{16}{48}$.	30. $\frac{32}{48}$.	31. $\frac{21}{48}$.	32. $\frac{36}{48}$.
33. $\frac{75}{100}$.	34. $\frac{80}{100}$.	35. $\frac{125}{500}$.	36. $\frac{625}{1000}$.
37. $\frac{49}{63}$.	38. $\frac{88}{121}$.	39. $\frac{81}{99}$.	40. $\frac{54}{66}$.
41. $\frac{27}{72}$.	42. $\frac{60}{108}$.	43. $\frac{64}{96}$.	44. $\frac{45}{120}$.
45. $\frac{39}{65}$.	46. $\frac{50}{75}$.	47. $\frac{51}{85}$.	48. $\frac{57}{95}$.
49. $\frac{900}{1200}$.	50. $\frac{1500}{3500}$.	51. $\frac{260}{390}$.	52. $\frac{3800}{5700}$.
53. $\frac{101}{707}$.	54. $\frac{111}{444}$.	55. $\frac{1001}{7007}$.	56. $\frac{2002}{3003}$.
57. $\frac{303}{808}$.	58. $\frac{777}{999}$.	59. $\frac{1313}{1717}$.	60. $\frac{3333}{5555}$.
61. $\frac{119}{153}$.	62. $\frac{117}{169}$.	63. $\frac{133}{209}$.	64. $\frac{141}{329}$.
65. $\frac{273}{364}$.	66. $\frac{292}{365}$.	67. $\frac{305}{366}$.	68. $\frac{384}{672}$.
69. $\frac{392}{441}$.	70. $\frac{299}{391}$.	71. $\frac{377}{403}$.	72. $\frac{299}{377}$.
73. $\frac{215}{440}$.	74. $\frac{414}{441}$.	75. $\frac{713}{943}$.	76. $\frac{501}{2171}$.
77. $\frac{595}{1071}$.	78. $\frac{501}{1837}$.	79. $\frac{594}{2079}$.	80. $\frac{949}{1387}$.
81. $\frac{261}{3103}$.	82. $\frac{111}{1101}$.	83. $\frac{3330}{5328}$.	84. $\frac{4844}{5536}$.
85. $\frac{1160}{1624}$.	86. $\frac{7623}{8316}$.	87. $\frac{1133}{1339}$.	88. $\frac{1331}{1430}$.
89. $\frac{1633}{1863}$.	90. $\frac{2592}{3456}$.	91. $\frac{1892}{1936}$.	92. $\frac{2108}{3813}$.
93. $\frac{9471}{9922}$.	94. $\frac{1919}{2223}$.	95. $\frac{2007}{12265}$.	96. $\frac{24389}{26071}$.
97. $\frac{21189}{36414}$.	98. $\frac{64961}{68419}$.	99. $\frac{237655}{367285}$.	100. $\frac{114135}{220661}$.

Express as mixed numbers in simplest form—

101. $\frac{802}{14}$.	102. $\frac{402}{9}$.	103. $\frac{725}{15}$.	104. $\frac{747}{18}$.
105. $\frac{441}{72}$.	106. $\frac{495}{65}$.	107. $\frac{924}{172}$.	108. $\frac{987}{105}$.
109. $\frac{1332}{33}$.	110. $\frac{1967}{28}$.	111. $\frac{1440}{84}$.	112. $\frac{1313}{91}$.
113. $\frac{417}{411}$.	114. $\frac{1407}{1400}$.	115. $\frac{4191}{771}$.	116. $\frac{3553}{779}$.
117. $\frac{3894}{385}$.	118. $\frac{6123}{598}$.	119. $\frac{27474}{20485}$.	120. $\frac{100110}{31866}$.

XV. FRACTIONS.

Reduction to Least Common Denominator.

Reduce to their least common denominator—

1. $\frac{5}{6}$ and $\frac{7}{8}$.
2. $\frac{7}{12}$ and $\frac{11}{18}$.
3. $\frac{3}{20}$ and $\frac{7}{30}$.
4. $\frac{11}{15}$ and $\frac{13}{20}$.
5. $\frac{5}{16}$ and $\frac{11}{24}$.
6. $\frac{9}{50}$ and $\frac{13}{75}$.
7. $\frac{17}{48}$ and $\frac{21}{64}$.
8. $\frac{16}{63}$ and $\frac{19}{84}$.
9. $\frac{13}{24}$ and $\frac{17}{26}$.
10. $\frac{7}{39}$ and $\frac{4}{51}$.
11. $\frac{11}{36}$ and $\frac{17}{54}$.
12. $\frac{22}{57}$ and $\frac{41}{95}$.
13. $\frac{1}{6}$, $\frac{3}{8}$ and $\frac{5}{12}$.
14. $\frac{3}{4}$, $\frac{5}{6}$ and $\frac{7}{8}$.
15. $\frac{3}{8}$, $\frac{7}{12}$ and $\frac{9}{16}$.
16. $\frac{2}{3}$, $\frac{3}{4}$ and $\frac{4}{5}$.
17. $\frac{5}{14}$, $\frac{8}{21}$, $\frac{11}{28}$.
18. $\frac{11}{24}$, $\frac{15}{36}$, $\frac{19}{48}$.
19. $\frac{2}{15}$, $\frac{3}{20}$, $\frac{4}{25}$.
20. $\frac{7}{30}$, $\frac{9}{40}$, $\frac{11}{50}$.
21. $\frac{41}{48}$, $\frac{51}{56}$, $\frac{61}{72}$.
22. $\frac{11}{42}$, $\frac{11}{63}$, $\frac{11}{84}$.
23. $\frac{1}{68}$, $\frac{1}{85}$, $\frac{1}{102}$.
24. $\frac{4}{39}$, $\frac{4}{65}$, $\frac{4}{91}$.

Reduce to their least common denominator—

25. $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$ and $\frac{1}{6}$.
26. $\frac{7}{8}$, $\frac{5}{16}$, $\frac{3}{32}$ and $\frac{1}{64}$.
27. $\frac{1}{10}$, $\frac{2}{15}$, $\frac{3}{20}$ and $\frac{4}{25}$.
28. $\frac{21}{22}$, $\frac{29}{33}$, $\frac{19}{44}$ and $\frac{18}{55}$.
29. $\frac{5}{12}$, $\frac{8}{21}$, $\frac{11}{28}$ and $\frac{17}{42}$.
30. $\frac{5}{14}$, $\frac{4}{15}$, $\frac{14}{45}$ and $\frac{15}{56}$.
31. $\frac{71}{202}$, $\frac{141}{404}$ and $\frac{281}{808}$.
32. $\frac{83}{333}$, $\frac{167}{666}$ and $\frac{251}{999}$.
33. $\frac{3}{25}$, $\frac{3}{40}$, $\frac{3}{55}$ and $\frac{3}{70}$.
34. $\frac{3}{55}$, $\frac{4}{65}$, $\frac{7}{121}$ and $\frac{8}{143}$.
35. $\frac{307}{1760}$ and $\frac{301}{1728}$.
36. $\frac{13}{1212}$, $\frac{16}{1515}$ and $\frac{19}{1818}$.

Which is greater—

37. $\frac{5}{7}$ or $\frac{7}{9}$?
38. $\frac{9}{14}$ or $\frac{13}{20}$?
39. $\frac{7}{8}$ or $\frac{93}{104}$?
40. $\frac{17}{24}$ or $\frac{83}{108}$?
41. $\frac{31}{36}$ or $\frac{79}{90}$?
42. $\frac{23}{84}$ or $\frac{37}{144}$?
43. $\frac{1}{9}$ or $\frac{88}{801}$?
44. $\frac{5}{7}$ or $\frac{51}{71}$?
45. $\frac{10}{143}$ or $\frac{11}{156}$?
46. $\frac{16}{19}$ or $\frac{14}{17}$?
47. $\frac{25}{37}$ or $\frac{52}{73}$?
48. $\frac{17}{117}$ or $\frac{23}{153}$?

Arrange in *ascending* order of magnitude—

49. $\frac{2}{3}$, $\frac{5}{6}$, $\frac{8}{11}$ and $\frac{17}{22}$.
50. $\frac{3}{7}$, $\frac{5}{12}$, $\frac{10}{21}$ and $\frac{19}{42}$.
51. $\frac{13}{28}$, $\frac{16}{35}$ and $\frac{19}{42}$.
52. $\frac{7}{10}$, $\frac{8}{11}$, $\frac{15}{22}$ and $\frac{39}{55}$.
53. $\frac{3}{16}$, $\frac{5}{27}$, $\frac{7}{36}$ and $\frac{25}{144}$.
54. $\frac{3}{68}$, $\frac{4}{85}$ and $\frac{5}{119}$.

Arrange in *descending* order of magnitude—

55. $\frac{26}{33}$, $\frac{53}{66}$ and $\frac{80}{99}$.
56. $\frac{7}{12}$, $\frac{9}{16}$, $\frac{11}{18}$ and $\frac{13}{24}$.
57. $\frac{4}{5}$, $\frac{11}{15}$, $\frac{19}{25}$ and $\frac{27}{35}$.
58. $\frac{17}{26}$, $\frac{33}{52}$, $\frac{42}{65}$ and $\frac{87}{130}$.

59. Compare $\frac{19}{50}$, $\frac{23}{70}$, $\frac{31}{90}$

60. Compare $\frac{1}{11}$, $\frac{11}{111}$, $\frac{111}{1111}$.

XVI. ADDITION OF FRACTIONS.

Read off, or write down, the sum of—

1. 3 *elevenths* and 5 *elevenths*.
2. 5 *twelfths* and 6 *twelfths*.
3. 7 *thirtieths* and 16 *thirtieths*.
4. 19 *ninetieths* and 13 *ninetieths*.
5. $\frac{5}{24}$, $\frac{7}{24}$ and $\frac{1}{24}$.
6. $\frac{3}{17}$, $\frac{5}{17}$ and $\frac{4}{17}$.
7. $\frac{3}{29}$, $\frac{5}{29}$ and $\frac{11}{29}$.
8. $\frac{7}{43}$, $\frac{9}{43}$ and $\frac{10}{43}$.
9. $\frac{9}{100}$, $\frac{11}{100}$ and $\frac{13}{100}$.
10. $\frac{5}{27}$, $\frac{7}{27}$, $\frac{8}{27}$ and $\frac{2}{27}$.

Find, mentally, the sum of—

11. $\frac{1}{2}$ and $\frac{1}{3}$.
12. $\frac{1}{3}$ and $\frac{1}{4}$.
13. $\frac{2}{3}$ and $\frac{1}{4}$.
14. $\frac{1}{4}$ and $\frac{2}{5}$.
15. $\frac{1}{2}$ and $\frac{5}{12}$.
16. $\frac{2}{3}$ and $\frac{1}{6}$.
17. $\frac{1}{3}$ and $\frac{7}{12}$.
18. $\frac{3}{4}$ and $\frac{1}{6}$.
19. $\frac{1}{3}$, $\frac{1}{4}$ and $\frac{5}{12}$.
20. $\frac{1}{4}$, $\frac{1}{5}$ and $\frac{11}{20}$.
21. $\frac{1}{2}$, $\frac{1}{6}$ and $\frac{1}{12}$.
22. $\frac{1}{3}$, $\frac{1}{4}$ and $\frac{1}{6}$.

Add together—

23. $\frac{3}{8}$ and $\frac{7}{12}$.
24. $\frac{4}{21}$ and $\frac{5}{28}$.
25. $\frac{5}{9}$ and $\frac{7}{11}$.
26. $\frac{5}{8}$ and $\frac{5}{7}$.
27. $\frac{11}{18}$ and $\frac{17}{27}$.
28. $\frac{19}{21}$ and $\frac{12}{35}$.
29. $\frac{1}{4}$ and $\frac{5}{28}$.
30. $\frac{3}{8}$ and $\frac{1}{24}$.
31. $\frac{3}{5}$ and $\frac{4}{35}$.
32. $\frac{5}{12}$ and $\frac{7}{60}$.
33. $\frac{6}{7}$ and $\frac{10}{21}$.
34. $\frac{5}{9}$ and $\frac{25}{36}$.
35. $\frac{1}{6}$, $\frac{3}{7}$ and $\frac{5}{21}$.
36. $\frac{2}{5}$, $\frac{1}{6}$ and $\frac{1}{60}$.
37. $\frac{1}{8}$, $\frac{1}{9}$ and $\frac{5}{144}$.
38. $\frac{1}{16}$, $\frac{1}{24}$ and $\frac{41}{48}$.
39. $\frac{5}{8}$, $\frac{1}{40}$ and $\frac{23}{80}$.
40. $\frac{8}{15}$, $\frac{11}{40}$ and $\frac{41}{120}$.

Find, in simplest form, the value of—

41. $\frac{2}{3} + \frac{5}{12} + \frac{1}{4}$.
42. $\frac{3}{4} + \frac{4}{5} + \frac{5}{6}$.
43. $\frac{3}{10} + \frac{7}{20} + \frac{11}{30}$.
44. $\frac{8}{9} + \frac{1}{27} + \frac{5}{18}$.
45. $\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5}$.
46. $\frac{1}{2} + \frac{3}{8} + \frac{4}{15} + \frac{7}{24}$.
47. $\frac{1}{10} + \frac{11}{100} + \frac{17}{1000}$.
48. $\frac{3}{10} + \frac{3}{100} + \frac{3}{1000} + \frac{3}{10000}$.
49. $\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6}$.
50. $\frac{1}{2} + \frac{1}{3} + \frac{1}{8} + \frac{1}{24}$.
51. $\frac{3}{4} + \frac{13}{16} + \frac{13}{32} + \frac{7}{480}$.
52. $\frac{1}{3} + \frac{3}{8} + \frac{2}{5} + \frac{5}{12} + \frac{3}{7}$.
53. $\frac{3}{4} + \frac{5}{6} + \frac{7}{8} + \frac{9}{10} + \frac{11}{12}$.
54. $\frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \frac{4}{5} + \frac{5}{6} + \frac{6}{7}$.
55. $\frac{7}{15} + \frac{13}{25} + \frac{23}{24}$.
56. $\frac{13}{42} + \frac{5}{72} + \frac{17}{63} + \frac{7}{24} + \frac{9}{56}$.
57. $\frac{12}{17} + \frac{8}{51} + \frac{2}{3}$.
58. $\frac{5}{17} + \frac{2}{51} + \frac{3}{34}$.
59. $\frac{7}{13} + \frac{5}{26} + \frac{11}{39}$.
60. $\frac{7}{65} + \frac{17}{39} + \frac{4}{15}$.

61. $2\frac{1}{3} + 3\frac{1}{4} + 4\frac{1}{5}$.
62. $1\frac{2}{3} + 1\frac{3}{4} + 1\frac{4}{5}$.
63. $17\frac{1}{2} + 12\frac{5}{6} + 9\frac{1}{3}$.
64. $2\frac{2}{5} + 13\frac{3}{7} + 4\frac{4}{21}$.
65. $\frac{7}{16} + \frac{2}{3} + \frac{5}{48} + 3\frac{19}{24}$.
66. $2\frac{3}{7} + 5\frac{1}{4} + \frac{8}{21} + 1\frac{5}{12}$.
67. $120\frac{1}{2} + 791\frac{2}{3} + 12\frac{7}{12}$.
68. $785\frac{1}{8} + 81\frac{5}{6} + 133\frac{1}{3}$.
69. $90\frac{7}{30} + 90\frac{7}{40} + 90\frac{7}{50}$.
70. $17\frac{1}{3} + 12\frac{2}{5} + \frac{11}{100} + 99\frac{1}{2}$.

71. $5\frac{4}{9} + \frac{3}{8} + \frac{7}{12} + 2\frac{5}{24}$.

72. $3\frac{4}{15} + 17\frac{1}{12} + 9\frac{1}{2} + 15\frac{2}{3}$.

73. $1\frac{1}{3} + 2\frac{2}{9} + 3\frac{4}{27} + 4\frac{5}{81}$.

74. $6\frac{1}{2} + 5\frac{3}{4} + 4\frac{5}{8} + 3\frac{7}{16} + 2\frac{9}{32}$.

75. $13\frac{7}{38} + 14\frac{8}{57}$.

76. $1\frac{3}{28} + 2\frac{5}{42} + 3\frac{13}{154}$.

77. $3\frac{1}{3} + 4\frac{3}{8} + 2\frac{1}{9} + \frac{19}{72}$.

78. $3\frac{1}{7} + 3\frac{3}{10} + 2\frac{3}{11} + 2\frac{5}{22} + \frac{2}{35}$.

79. $1\frac{1}{4} + 2\frac{5}{12} + \frac{4}{15} + \frac{7}{25} + \frac{11}{30}$.

80. $7\frac{9}{20} + 12\frac{4}{15} + 4\frac{7}{12} + \frac{9}{40} + 3\frac{7}{10}$.

81. $200\frac{1}{400} + 300\frac{1}{600} + 400\frac{1}{800}$.

82. $404\frac{2}{15} + 2\frac{1}{25} + 1002\frac{7}{45}$.

83. $46231\frac{7}{110} + 2063\frac{11}{70}$.

84. $35\frac{3}{4} + 762\frac{7}{8} + 8456\frac{2}{3}$.

85. $5\frac{1}{3} + \frac{15}{7} + \frac{5}{6} + 3\frac{1}{14}$.

86. $\frac{2}{5} + 6\frac{5}{7} + \frac{13}{9} + \frac{8}{21}$.

87. $\frac{3}{7} + \frac{5}{8} + \frac{23}{14} + 4\frac{1}{4}$.

88. $\frac{4}{15} + \frac{5}{8} + \frac{17}{10} + 4\frac{5}{6}$.

89. $\frac{14}{3} + \frac{26}{4} + 5\frac{1}{5} + 2061\frac{11}{15} + 7\frac{2}{25}$.

90. $\frac{145}{225} + \frac{39}{90} + 7 + 5\frac{45}{45} + 3\frac{12}{5}$.

91. $\frac{5}{144} + \frac{5}{156} + 3\frac{5}{168}$.

92. $1\frac{3}{25} + 11\frac{33}{125} + 111\frac{333}{625}$.

93. $1\frac{7}{1225} + 2\frac{5}{49} + 3\frac{3}{245}$.

94. $1\frac{19}{219} + 1\frac{18}{365} + 1\frac{19}{292} + 1\frac{17}{146}$.

95. $17\frac{17}{16} + 18\frac{18}{17}$.

96. $10\frac{5}{1763} + 20\frac{5}{2173}$.

97. $3\frac{324}{432} + 7\frac{252}{288}$.

98. $11\frac{2163}{5047} + 17\frac{868}{1953}$.

99. $425913\frac{175}{1434} + 796753\frac{96}{2151}$.

100. $7\frac{1}{2542} + 8\frac{1}{3813} + 9\frac{1}{5084}$.

XVII. SUBTRACTION OF FRACTIONS.

Find, mentally, the difference between—

1. $\frac{8}{11}$ and $\frac{3}{11}$.
2. $\frac{9}{17}$ and $\frac{5}{17}$.
3. $\frac{17}{25}$ and $\frac{4}{25}$.
4. $\frac{30}{49}$ and $\frac{25}{49}$.
5. $\frac{7}{8}$ and $\frac{3}{4}$.
6. $\frac{5}{12}$ and $\frac{5}{24}$.
7. $\frac{3}{8}$ and $\frac{5}{16}$.
8. $\frac{7}{15}$ and $\frac{7}{30}$.
9. $\frac{7}{8}$ and $\frac{1}{32}$.
10. $\frac{5}{7}$ and $\frac{9}{56}$.
11. $\frac{17}{28}$ and $\frac{2}{7}$.
12. $\frac{19}{64}$ and $\frac{3}{16}$.
13. $\frac{17}{18}$ and $\frac{4}{9}$.
14. $\frac{23}{48}$ and $\frac{3}{8}$.
15. $\frac{101}{112}$ and $\frac{1}{56}$.
16. $\frac{79}{80}$ and $\frac{3}{40}$.
17. $13\frac{1}{3}$ and 7.
18. $12\frac{3}{8}$ and 9.
19. $7\frac{5}{6}$ and $2\frac{5}{6}$.
20. $11\frac{5}{21}$ and $3\frac{5}{21}$.
21. $2\frac{5}{6}$ and $\frac{2}{3}$.

Find, mentally, the value of—

22. $3 - \frac{7}{12}$.
23. $5 - \frac{4}{7}$.
24. $7 - \frac{3}{8}$.
25. $11 - \frac{3}{16}$.
26. $15 - \frac{9}{20}$.
27. $12 - \frac{11}{14}$.
28. $17 - \frac{19}{25}$.
29. $8 - 3\frac{2}{5}$.
30. $7 - 1\frac{5}{6}$.
31. $12 - 6\frac{1}{7}$.
32. $14 - 12\frac{2}{9}$.
33. $17 - 7\frac{7}{17}$.
34. $19 - 1\frac{1}{19}$.
35. $23 - 7\frac{15}{28}$.
36. $41 - 32\frac{9}{40}$.
37. $12\frac{1}{2} - 7\frac{1}{4}$.
38. $8\frac{3}{4} - 1\frac{5}{8}$.
39. $7\frac{3}{5} - 2$.
40. $12\frac{17}{28} - 7$.
41. $5\frac{3}{4} - 2\frac{2}{3}$.
42. $7\frac{2}{5} - 3\frac{1}{4}$.

Find the difference between—

43. $\frac{7}{30}$ and $\frac{17}{36}$.

44. $\frac{23}{63}$ and $\frac{17}{42}$.

45. $8\frac{5}{24}$ and $12\frac{7}{32}$.

46. $12\frac{13}{42}$ and $17\frac{9}{28}$.

47. $\frac{7}{12}$ and $\frac{19}{32}$.

48. $\frac{13}{40}$ and $\frac{19}{56}$.

49. $\frac{5}{39}$ and $\frac{7}{52}$.

50. $\frac{7}{65}$ and $\frac{11}{91}$.

51. $21\frac{7}{9}$ and $21\frac{8}{11}$.

52. $67\frac{4}{21}$ and $67\frac{5}{28}$.

53. $79\frac{5}{8}$ and $78\frac{5}{9}$.

54. $110\frac{7}{9}$ and $109\frac{7}{10}$.

Find, in simplest form, the value of—

55. $3\frac{1}{7} - 1\frac{1}{5}$.

56. $7\frac{3}{8} - 2\frac{3}{5}$.

57. $12\frac{2}{5} - 8\frac{3}{4}$.

58. $14\frac{3}{7} - 5\frac{2}{3}$.

59. $20\frac{1}{4} - 6\frac{5}{8}$.

60. $23\frac{1}{2} - 10\frac{7}{8}$.

61. $8\frac{4}{5} - 4\frac{9}{11}$.

62. $31\frac{5}{8} - 22\frac{5}{7}$.

63. $75\frac{5}{21} - \frac{9}{28}$.

64. $33\frac{11}{36} - 29\frac{29}{45}$.

65. $11\frac{11}{78} - 7\frac{10}{13}$.

66. $16\frac{2}{15} - 11\frac{7}{18}$.

67. $100\frac{1}{100} - 4\frac{57}{140}$.

68. $472\frac{7}{130} - \frac{19}{39}$.

69. $34\frac{3}{77} - 33\frac{3}{14}$.

70. $13\frac{9}{280} - 12\frac{6}{35}$.

71. $19\frac{19}{84} - 14\frac{15}{28}$.

72. $120\frac{7}{60} - 9\frac{43}{220}$.

73. $63\frac{5}{76} - 34\frac{46}{57}$.

74. $17\frac{11}{153} - 15\frac{19}{34}$.

75. $111\frac{12}{13} - 7\frac{30}{31}$.

76. $19\frac{5}{432} - \frac{11}{54}$.

77. $15\frac{1}{17} - \frac{32}{33}$.

78. $17\frac{7}{707} - 7\frac{7}{777}$.

79. $5\frac{17}{30} - \frac{443}{400}$.

80. $578\frac{3}{14} - \frac{4931}{70}$.

81. $\frac{23}{286} - \frac{51}{637}$.

82. $\frac{170}{171} - \frac{207}{209}$.

83. $112\frac{20}{1210} - \frac{130}{1100}$.

84. $8569\frac{25}{750} - \frac{875}{1000}$.

85. $9876 - \frac{9874}{9875}$.

86. $12345 - \frac{12344}{12345}$.

87. $\frac{8}{2717} - \frac{7}{2431}$.

88. $\frac{231}{264} - \frac{594}{693}$.

89. $\frac{1001}{1386} - \frac{935}{2295}$.

90. $\frac{1065}{38766} - \frac{165}{45045}$.

Simplify—

91. $\frac{13}{60} - \frac{13}{42} + \frac{13}{90}$.

92. $\frac{19}{35} - \frac{38}{45} + \frac{19}{63}$.

93. $\frac{5}{14} - \frac{5}{16} - \frac{1}{112}$.

94. $2\frac{9}{10} - 1\frac{7}{16} + 1\frac{17}{40}$.

95. $1\frac{7}{102} + \frac{5}{34} - 1\frac{18}{85}$.

96. $\frac{4}{9} - \frac{5}{18} + \frac{7}{36} - \frac{11}{72}$.

97. $\frac{11}{180} - \frac{7}{12} + \frac{5}{9} - \frac{1}{30}$.

98. $\frac{1}{3} - \frac{4}{21} + \frac{1}{2} - \frac{7}{11}$.

99. $\frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \frac{1}{5} + \frac{47}{60}$.

100. $\frac{3}{8} - \frac{5}{12} + \frac{8}{15} - \frac{3}{20} - \frac{1}{120}$.

101. $3\frac{1}{2} - 1\frac{7}{8} + 4\frac{5}{12} - 2\frac{11}{30}$.

102. $3\frac{10}{11} + 5\frac{7}{15} - 2\frac{9}{22} - 4\frac{9}{10}$.

103. $2\frac{3}{4} + 1\frac{5}{8} + 2\frac{3}{10} - 3\frac{5}{24} + 1\frac{8}{15}$.

104. $\frac{1}{6} + 2\frac{1}{7} + 13\frac{3}{10} + \frac{4}{21} - 4\frac{33}{35}$.

105. $1\frac{3}{8} + 3\frac{1}{3} - 5\frac{5}{6} + 2\frac{1}{4} - 1\frac{1}{12}$.

106. $2\frac{1}{5} + 1\frac{4}{7} - 4\frac{1}{2} - \frac{3}{5} + 2\frac{8}{35}$.

107. $3\frac{1}{2} - 4\frac{1}{3} + 5\frac{1}{4} - 6\frac{1}{5} + 7\frac{1}{6}$.

108. $12\frac{1}{12} - 13\frac{1}{13} + 26\frac{1}{26} - 24\frac{1}{24}$.

109. $3\frac{1}{4} - 4\frac{3}{8} + 5\frac{5}{12} - 6\frac{7}{16} + 3\frac{1}{6}$.

110. $7\frac{5}{7} - 8\frac{4}{5} - 9\frac{11}{21} + 10\frac{7}{15} + 1\frac{3}{35}$

111. $20 - \frac{2}{3} - \frac{3}{4} - \frac{4}{5} - \frac{5}{6}$.

112. $10 - \frac{3}{5} - \frac{5}{7} - \frac{7}{10} - \frac{9}{14}$.

113. $\frac{1}{8} - \frac{1}{64} - \frac{1}{512} - \frac{1}{4096}$.

114. $\frac{1}{3} - \frac{1}{9} - \frac{1}{27} - \frac{1}{81} - \frac{1}{243}$.

115. $1\frac{1}{4} - 2\frac{7}{8} - 3\frac{7}{9} - 4\frac{7}{12} + 11\frac{1}{6}$.

116. $2\frac{2}{5} - 3\frac{3}{10} - 4\frac{4}{15} - 7\frac{7}{25} + 15\frac{1}{30}$.

117. $34\frac{1}{4} + 22\frac{1}{6} - 16\frac{7}{8} - 14\frac{7}{9} - 12\frac{11}{12}$.

118. $4\frac{1}{4} - 5\frac{3}{16} - 6\frac{5}{64} - 7\frac{7}{256} + 18\frac{1}{512}$.

119. $\frac{311}{7} - \frac{211}{5} + \frac{111}{3} - \frac{11}{2} + 1$.

120. $\frac{2400}{7} - \frac{342}{49} + \frac{48}{343} - \frac{14}{2401}$.

XVIII. FRACTIONS.

MULTIPLICATION AND DIVISION BY AN INTEGER.

Read off, or write down, in lowest terms—

1. $\frac{3}{7} \times 2$.
2. $\frac{5}{11} \times 2$.
3. $\frac{7}{19} \times 2$.
4. $\frac{8}{17} \times 2$.
5. $\frac{2}{15} \times 4$.
6. $\frac{3}{16} \times 3$.
7. $\frac{1}{11} \times 7$.
8. $\frac{5}{23} \times 4$.
9. $\frac{12}{97} \times 8$.
10. $\frac{13}{100} \times 7$.
11. $\frac{3}{8} \times 2$.
12. $\frac{5}{12} \times 2$.
13. $\frac{1}{10} \times 2$.
14. $\frac{3}{14} \times 2$.
15. $\frac{5}{21} \times 3$.
16. $\frac{2}{15} \times 3$.
17. $\frac{7}{100} \times 4$.
18. $\frac{11}{100} \times 5$.
19. $\frac{2}{35} \times 7$.
20. $\frac{3}{32} \times 8$.
21. $\frac{6}{11} \div 2$.
22. $\frac{8}{15} \div 2$.
23. $\frac{10}{17} \div 2$.
24. $\frac{14}{19} \div 2$.
25. $\frac{12}{13} \div 3$.
26. $\frac{15}{16} \div 3$.
27. $\frac{16}{21} \div 4$.
28. $\frac{28}{45} \div 4$.
29. $\frac{25}{26} \div 5$
30. $\frac{14}{41} \div 7$.
31. $\frac{7}{16} \div 2$.
32. $\frac{3}{10} \div 2$.
33. $\frac{5}{26} \div 2$.
34. $\frac{11}{34} \div 2$.
35. $\frac{7}{16} \div 4$.
36. $\frac{23}{25} \div 3$.
37. $\frac{5}{12} \div 8$.
38. $\frac{19}{24} \div 4$.
39. $\frac{11}{20} \div 6$.
40. $\frac{10}{11} \div 7$.

Multiply—

41. $\frac{5}{21}$ by 14.
42. $\frac{4}{39}$ by 26.
43. $\frac{7}{60}$ by 48.
44. $\frac{11}{48}$ by 64.
45. $\frac{22}{57}$ by 38.
46. $\frac{23}{84}$ by 36.
47. $\frac{7}{19}$ by 21.
48. $\frac{13}{17}$ by 40.
49. $\frac{15}{51}$ by 34.
50. $\frac{27}{46}$ by 69.
51. $6\frac{3}{4}$ by 2.
52. $7\frac{5}{9}$ by 3.
53. $4\frac{5}{12}$ by 9.
54. $3\frac{7}{15}$ by 10.
55. $8\frac{2}{3}$ by 11.
56. $7\frac{7}{9}$ by 15.
57. $8\frac{11}{30}$ by 45.
58. $10\frac{13}{21}$ by 28.
59. $12\frac{17}{18}$ by 12.
60. $14\frac{5}{14}$ by 49.
61. $\frac{127}{146}$ by 365.
62. $\frac{258}{365}$ by 219.
63. $58\frac{17}{26}$ by 390.
64. $79\frac{83}{144}$ by 168.

Divide—

65. $\frac{16}{17}$ by 12.
66. $\frac{6}{7}$ by 27.
67. $\frac{14}{25}$ by 35.
68. $\frac{28}{33}$ by 42.
69. $\frac{20}{21}$ by 3.
70. $\frac{17}{20}$ by 5.
71. $\frac{22}{31}$ by 77.
72. $\frac{84}{95}$ by 63.
73. $\frac{17}{50}$ by 13.
74. $\frac{23}{25}$ by 25.
75. $1\frac{1}{2}$ by 9.
76. $3\frac{3}{7}$ by 12.
77. $5\frac{5}{8}$ by 18.
78. $7\frac{1}{3}$ by 33.
79. $6\frac{3}{8}$ by 34.
80. $12\frac{2}{3}$ by 8.
81. $10\frac{5}{7}$ by 25.
82. $7\frac{5}{13}$ by 24.
83. $6\frac{5}{12}$ by 42.
84. $3\frac{7}{9}$ by 51.
85. $240\frac{2}{3}$ by 209.
86. $91\frac{1}{4}$ by 292.
87. $461\frac{1}{9}$ by 332.
88. $488\frac{1}{7}$ by 323.

89. Simplify $1347\frac{3}{4} \times 12$.

90. Simplify $1347\frac{3}{4} \div 12$.

XIX. MULTIPLICATION OF FRACTIONS.

Find the value of—

1. $\frac{3}{8}$ of $\frac{7}{9}$.
2. $\frac{4}{7}$ of $\frac{3}{10}$.
3. $\frac{8}{15}$ of $\frac{9}{28}$.
4. $\frac{14}{25}$ of $\frac{45}{56}$.
5. $\frac{9}{11}$ of $\frac{13}{17}$.
6. $\frac{16}{21}$ of $\frac{8}{9}$.
7. $\frac{8}{9}$ of $\frac{9}{8}$.
8. $\frac{8}{11}$ of $\frac{11}{16}$.
9. $\frac{13}{39}$ of 3.
10. $\frac{1}{12}$ of $\frac{17}{34}$.
11. $\frac{8}{9}$ of $3\frac{3}{4}$.
12. $\frac{21}{40}$ of $1\frac{1}{9}$.
13. $\frac{4}{5}$ of $1\frac{1}{39}$.
14. $\frac{11}{38}$ of $8\frac{1}{7}$.
15. $\frac{35}{36}$ of $10\frac{2}{7}$.
16. $\frac{21}{28}$ of $1\frac{1}{3}$.
17. $\frac{26}{91}$ of $6\frac{1}{2}$.
18. $\frac{7}{8}$ of $60\frac{4}{7}$.
19. $\frac{112}{365}$ of $2\frac{3}{35}$.
20. $\frac{95}{219}$ of $19\frac{4}{19}$.

Multiply—

21. $\frac{11}{42}$ by $\frac{28}{55}$.
22. $\frac{33}{56}$ by $\frac{35}{88}$.
23. $\frac{21}{48}$ by $\frac{32}{63}$.
24. $\frac{44}{81}$ by $\frac{36}{77}$.
25. $\frac{96}{108}$ by $\frac{60}{80}$.
26. $\frac{84}{144}$ by $\frac{64}{112}$.
27. $\frac{51}{85}$ by $\frac{25}{27}$.
28. $\frac{65}{100}$ by $\frac{52}{169}$.
29. $\frac{45}{70}$ by $\frac{28}{18}$.
30. $\frac{35}{84}$ by $\frac{36}{15}$.
31. $1\frac{7}{8}$ by $1\frac{1}{3}$.
32. $2\frac{2}{5}$ by $2\frac{1}{7}$.
33. $10\frac{2}{7}$ by $2\frac{11}{12}$.
34. $12\frac{1}{12}$ by $1\frac{1}{15}$.
35. $4\frac{3}{13}$ by $3\frac{6}{11}$.
36. $15\frac{4}{5}$ by $3\frac{8}{9}$.
37. $3\frac{4}{5}$ by $3\frac{4}{5}$.
38. $7\frac{1}{7}$ by $7\frac{1}{7}$.
39. $6\frac{16}{23}$ by $6\frac{3}{11}$.
40. $6\frac{38}{45}$ by $1\frac{58}{77}$.

Simplify—

41. $255\frac{1}{2} \times 1\frac{1}{7} \times 10$.
42. $132\frac{4}{5} \times 3\frac{1}{8} \times \frac{1}{10}$.
43. $12\frac{1}{12} \times 7\frac{1}{5} \times \frac{1}{2}$.
44. $7\frac{7}{44} \times 2\frac{17}{63} \times 2$.
45. $\frac{36}{65} \times \frac{52}{99} \times \frac{11}{80}$.
46. $\frac{39}{46} \times \frac{85}{91} \times \frac{69}{102}$.
47. $\frac{50}{161} \times \frac{11}{400} \times \frac{7}{24}$.
48. $\frac{84}{85} \times \frac{51}{132} \times \frac{91}{117}$.
49. $\frac{1}{12}$ of $11\frac{1}{3}$ of $\frac{54}{85}$.
50. $\frac{5}{11}$ of $25\frac{1}{3}$ of $\frac{77}{95}$.
51. $\frac{2}{5}$ of $7\frac{1}{2} \times \frac{1}{8}$ of $11\frac{1}{5}$.
52. $\frac{3}{4}$ of $1\frac{5}{11} \times \frac{2}{7}$ of $8\frac{7}{9}$.
53. $3\frac{2}{3} \times 3\frac{3}{5} \times \frac{5}{11} \times \frac{7}{18}$.
54. $2\frac{1}{7} \times 1\frac{1}{3} \times 2\frac{1}{4} \times 14$.
55. $5\frac{1}{6} \times 9\frac{3}{62} \times \frac{5}{9} \times 5\frac{2}{5}$.
56. $9\frac{4}{5} \times 2\frac{6}{7} \times 5\frac{1}{3} \times 2\frac{5}{8}$.
57. $2\frac{14}{27}$ of $1\frac{11}{15} \times 2\frac{7}{34} \times 1\frac{5}{13}$.
58. $3\frac{1}{7}$ of $3\frac{3}{10} \times 2\frac{3}{11} \times 2\frac{5}{22} \times \frac{2}{35}$.
59. $\frac{15}{86}$ of $2\frac{11}{16}$ of $5\frac{1}{24} \times 11\frac{7}{11}$.
60. $\frac{34}{57}$ of $\frac{33}{85}$ of $\frac{76}{117} \times 1\frac{31}{104}$.
61. $\frac{32}{99} \times 1\frac{11}{31} \times 1\frac{23}{32} \times 1\frac{23}{70}$.
62. $\frac{35}{132} \times 7\frac{5}{13} \times 2\frac{10}{21} \times \frac{11}{30}$.
63. $2\frac{21}{37} \times 5\frac{16}{19} \times 2\frac{11}{20} \times 1\frac{19}{85}$.
64. $5\frac{4}{23} \times 4\frac{16}{25} \times 2\frac{13}{51} \times 1\frac{17}{58}$.
65. $10\frac{9}{16} \times \frac{9}{143} \times 1\frac{7}{13} \times \frac{44}{45}$.
66. $\frac{63}{88} \times 36\frac{41}{56} \times \frac{30}{221} \times 2\frac{14}{45}$.
67. $\frac{51}{91}$ of $1\frac{27}{68} \times \frac{77}{171}$ of $2\frac{46}{55}$.
68. $\frac{19}{255}$ of $1\frac{6}{29}$ of $1\frac{13}{38}$ of $2\frac{18}{49}$.
69. $\frac{117}{441}$ of $\frac{111}{1001}$ of $\frac{343}{407}$.
70. $\frac{209}{338}$ of $\frac{91}{187}$ of $\frac{130}{133}$ of $\frac{17}{60}$.

Find the continued product of—

71. $\frac{93}{128}$, $1\frac{163}{187}$, $\frac{22}{35}$ and $2\frac{6}{31}$.
72. $4\frac{3}{7}$, $\frac{16}{35}$, $1\frac{289}{341}$ and $1\frac{5}{72}$.
73. $\frac{65}{123}$, $1\frac{5}{6}$, $\frac{75}{121}$ and $1\frac{43}{325}$.
74. $5\frac{14}{23}$, $1\frac{13}{56}$, $3\frac{15}{43}$ and $\frac{49}{243}$.
75. $\frac{18}{19}$, $182\frac{1}{2}$, $\frac{11}{219}$ and $4\frac{29}{45}$.
76. $1\frac{7}{26}$, $8\frac{13}{14}$, $\frac{7}{198}$ and $1\frac{14}{25}$.
77. $1\frac{67}{186}$, $\frac{144}{161}$, $2\frac{55}{112}$ and $1\frac{97}{99}$.
78. $\frac{5}{21}$, $\frac{13}{15}$, $\frac{48}{109}$, $\frac{105}{208}$ and $21\frac{4}{5}$.
79. $\frac{91}{1440}$, $\frac{15}{17}$, $\frac{51}{77}$, $\frac{99}{100}$ and $\frac{256}{585}$.
80. $\frac{440}{441}$, $\frac{407}{507}$, $\frac{338}{341}$ and $\frac{147}{148}$.

XX. DIVISION OF FRACTIONS.

Divide—

1. 9 by $\frac{3}{4}$.
2. 6 by $\frac{3}{5}$.
3. 12 by $\frac{4}{5}$.
4. 36 by $\frac{12}{13}$.
5. 77 by $\frac{11}{12}$.
6. 128 by $\frac{4}{5}$.
7. 972 by $\frac{9}{10}$.
8. 324 by $\frac{6}{7}$.
9. 560 by $\frac{14}{15}$.
10. 910 by $\frac{26}{29}$.
11. $\frac{2}{3}$ by $\frac{3}{4}$.
12. $\frac{2}{5}$ by $\frac{5}{8}$.
13. $\frac{7}{9}$ by $\frac{2}{3}$.
14. $\frac{11}{12}$ by $\frac{3}{8}$.
15. $\frac{6}{7}$ by $\frac{9}{14}$.
16. $\frac{11}{12}$ by $\frac{1}{24}$.
17. $9\frac{1}{11}$ by $9\frac{1}{11}$.
18. $10\frac{2}{7}$ by $10\frac{2}{7}$.
19. $\frac{87}{112}$ by $\frac{29}{42}$.
20. $1\frac{29}{55}$ by $\frac{108}{121}$.

Find the value of—

21. $7 \div \frac{3}{4}$.
22. $10 \div \frac{3}{5}$.
23. $14 \div \frac{4}{5}$.
24. $16 \div \frac{12}{13}$.
25. $38 \div \frac{19}{20}$.
26. $\frac{1}{64} \div \frac{5}{8}$.
27. $\frac{13}{96} \div \frac{65}{84}$.
28. $\frac{17}{84} \div \frac{85}{96}$.
29. $133 \div 1\frac{2}{5}$.
30. $155 \div 8\frac{6}{7}$.
31. $11\frac{3}{5} \div 4\frac{1}{7}$.
32. $2\frac{4}{7} \div 1\frac{13}{14}$.
33. $14\frac{4}{9} \div 11\frac{2}{3}$.
34. $27\frac{4}{5} \div 3\frac{4}{5}$.
35. $5\frac{3}{4} \div 15\frac{1}{3}$.
36. $11\frac{2}{3} \div 1\frac{3}{4}$.
37. $20\frac{1}{4} \div 4\frac{1}{2}$.
38. $20\frac{1}{4} \div 2\frac{2}{9}$.
39. $\frac{33}{52} \div 5\frac{29}{39}$.
40. $\frac{29}{36} \div 3\frac{65}{84}$.

Simplify—

41. $\dfrac{\frac{3}{4}}{\frac{2}{7}}$.
42. $\dfrac{\frac{7}{8}}{\frac{4}{5}}$.
43. $\dfrac{\frac{7}{12}}{\frac{8}{9}}$.
44. $\dfrac{\frac{5}{11}}{\frac{15}{22}}$.
45. $\dfrac{\frac{13}{14}}{\frac{8}{21}}$.
46. $\dfrac{\frac{3}{17}}{\frac{2}{51}}$.
47. $\dfrac{\frac{24}{49}}{\frac{9}{14}}$.
48. $\dfrac{\frac{15}{16}}{\frac{25}{36}}$.
49. $\dfrac{3\frac{3}{8}}{\frac{15}{16}}$.
50. $\dfrac{4\frac{1}{6}}{\frac{20}{27}}$.
51. $\dfrac{\frac{7}{9}}{2\frac{4}{5}}$.
52. $\dfrac{\frac{3}{8}}{7\frac{1}{5}}$.
53. $\dfrac{\frac{3}{7}}{5}$.
54. $\dfrac{\frac{2}{5}}{9}$.
55. $\dfrac{\frac{11}{12}}{22}$.
56. $\dfrac{\frac{8}{11}}{24}$.
57. $\dfrac{9}{\frac{4}{5}}$.
58. $\dfrac{7}{\frac{3}{11}}$.
59. $\dfrac{12}{\frac{9}{10}}$.
60. $\dfrac{18}{\frac{4}{11}}$.
61. $\dfrac{7\frac{1}{2}}{5}$.
62. $\dfrac{2\frac{5}{8}}{3}$.
63. $\dfrac{10\frac{1}{9}}{14}$.
64. $\dfrac{10\frac{5}{8}}{34}$.
65. $\dfrac{15}{2\frac{3}{4}}$.
66. $\dfrac{27}{2\frac{1}{4}}$.
67. $\dfrac{21}{5\frac{5}{8}}$.
68. $\dfrac{18}{6\frac{6}{7}}$.
69. $\dfrac{2\frac{4}{9}}{2\frac{2}{3}}$.
70. $\dfrac{7\frac{1}{9}}{2\frac{2}{7}}$.
71. $\dfrac{9\frac{3}{8}}{2\frac{4}{5}}$.
72. $\dfrac{11\frac{1}{5}}{2\frac{6}{11}}$.
73. $\dfrac{17\frac{2}{3}}{22\frac{5}{7}}$.
74. $\dfrac{\frac{23}{24}}{12\frac{15}{16}}$.
75. $\dfrac{31\frac{13}{27}}{\frac{34}{81}}$.
76. $\dfrac{8\frac{44}{53}}{4\frac{8}{11}}$.
77. $5\frac{44}{47} \div \frac{107}{141}$.
78. $5\frac{11}{17} \div 1\frac{59}{85}$.
79. $1\frac{7}{110} \div 4\frac{14}{55}$.
80. $1\frac{25}{171} \div 5\frac{16}{57}$.
81. $2\frac{17}{44} \div 2\frac{73}{121}$.
82. $3\frac{17}{23} \div 1\frac{11}{161}$.
83. $3\frac{9}{14} \div \frac{221}{661}$.
84. $33\frac{3}{25} \div 2\frac{91}{265}$.
85. $706\frac{3}{4} \div 110\frac{1}{7}$.
86. $17\frac{48}{115} \div 26\frac{29}{230}$.
87. $15\frac{163}{171} \div 33\frac{44}{133}$.
88. $106\frac{79}{215} \div 9\frac{261}{301}$.
89. $\frac{38}{39}$ of $12\frac{1}{7}$ of $2\frac{1}{19} \div \frac{5}{14}$.
90. $\frac{1}{9}$ of $3\frac{1}{125}$ of $3\frac{4}{7} \div \frac{47}{345}$.

XXI. FRACTIONS.

SIMPLIFICATION OF FRACTIONAL EXPRESSIONS.

Simplify—

1. $42 \div \frac{4}{7}$ of $9\frac{4}{5}$.
2. $64 \div \frac{2}{9}$ of $2\frac{2}{5}$.
3. $12\frac{9}{10} \div \frac{1}{3}$ of $6\frac{3}{4}$.
4. $\frac{2}{3}$ of $4\frac{1}{2} \div \frac{4}{13}$ of $3\frac{1}{4}$.
5. $\frac{7}{55}$ of $\frac{13}{20} \div \frac{52}{33}$ of $1\frac{1}{20}$.
6. $5\frac{3}{5}$ of $4\frac{8}{9} \div \frac{1}{6}$ of $3\frac{2}{3}$.
7. $6\frac{3}{7}$ of $4\frac{1}{5} \div 1\frac{1}{8}$ of $3\frac{1}{6}$.
8. $9\frac{4}{5}$ of $3\frac{1}{8} \div 1\frac{7}{8}$ of $8\frac{5}{6}$.
9. $1\frac{3}{5} \div \frac{3}{4}$ of $2\frac{2}{3}$ of $2\frac{1}{7}$.
10. $7\frac{7}{12} \div \frac{1}{2}$ of $4\frac{1}{3}$ of $3\frac{3}{4}$ of $2\frac{4}{5}$.

11. $1\frac{1}{4} + 3\frac{1}{5}$ of $2\frac{5}{8}$.
12. $3\frac{3}{8}$ of $2\frac{2}{9} + 4\frac{1}{2}$.
13. $\frac{3}{7}$ of $\frac{5}{12} + \frac{1}{2}$ of $1\frac{9}{14}$.
14. $\frac{9}{10}$ of $\frac{21}{50} - \frac{9}{10}$ of $\frac{1}{50}$.
15. $1\frac{1}{4}$ of $4\frac{1}{6} - 2\frac{1}{12}$ of $2\frac{1}{10}$.
16. $2\frac{1}{3}$ of $5\frac{1}{6} - 1\frac{7}{8}$ of $6\frac{1}{15}$.
17. $\frac{6}{7} + 6\frac{1}{7} + \frac{6}{7}$ of $6\frac{1}{7}$.
18. $\frac{8}{9} + 7\frac{1}{9} + \frac{8}{9}$ of $8\frac{1}{9}$.
19. $7\frac{7}{8} + 6\frac{3}{5}$ of $5\frac{2}{3} - 4\frac{1}{2}$.
20. $\frac{2}{3}$ of $\frac{1}{7}$ of $8\frac{3}{4} + \frac{1}{4}$ of $\frac{2}{5}$.
21. $20 - \frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$ of $\frac{5}{6}$.
22. $50 - 5\frac{1}{2}$ of $5\frac{1}{7}$ of $1\frac{3}{11}$ of $1\frac{1}{12}$.
23. $17\frac{4}{9} + \frac{5}{27} + 4\frac{2}{3} + 3\frac{1}{3}$ of $\frac{1}{9}$.
24. $3\frac{5}{14} + 5\frac{4}{21} + \frac{11}{24} + 1\frac{1}{3}$ of $\frac{5}{56}$.
25. $6\frac{1}{2}$ of $5\frac{3}{4} + 4\frac{5}{8}$ of $3\frac{7}{16} + 2\frac{9}{32}$.
26. $\frac{3}{4}$ of $9\frac{1}{3} + \frac{7}{8} + 8\frac{4}{5}$ of $\frac{25}{26}$ of $8\frac{10}{11}$.
27. $8\frac{1}{2} - 4\frac{2}{3} + \frac{3}{5} - 1\frac{1}{6}$ of $\frac{2}{5}$.
28. $12\frac{1}{2} - 14\frac{3}{4} + 5\frac{7}{8} - \frac{5}{6}$ of $2\frac{2}{5}$.
29. $\frac{3}{4}$ of $\frac{5}{6} + \frac{5}{6}$ of $\frac{7}{8} + \frac{7}{8}$ of $\frac{9}{10}$.
30. $\frac{3}{8}$ of $\frac{6}{11} - \frac{9}{14}$ of $2\frac{6}{11} + \frac{1}{4}$ of $6\frac{1}{11}$.

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31. $2\frac{2}{3} + 7\frac{1}{2} \times \frac{3}{5}$.
32. $\frac{5}{6} \times 1\frac{1}{5} + 5\frac{3}{4}$.
33. $1\frac{1}{16} \times 1\frac{1}{17} + 1\frac{1}{15} \times 1\frac{1}{14}$.
34. $5\frac{1}{3} \times 6\frac{1}{4} \times 7\frac{1}{5} - 8\frac{1}{6}$.
35. $8\frac{1}{2} + 2\frac{3}{4} \div \frac{5}{11}$.
36. $\frac{8}{15} \div 5\frac{3}{5} + 2\frac{1}{21}$.
37. $17\frac{1}{9} \div 3\frac{3}{10} - 1\frac{1}{6} \times \frac{6}{7}$.
38. $\frac{7}{12} + 2\frac{1}{6} + 5\frac{1}{2} \div 4\frac{2}{5}$.
39. $\frac{11}{17} \times 21\frac{6}{7} - \frac{11}{19} \times 10\frac{5}{9} + \frac{11}{21} \div \frac{33}{61}$.
40. $\frac{3}{7} - \frac{2}{49} \times \frac{1}{3} - \frac{1}{9} \div \frac{2}{3} - \frac{2}{9} \times \frac{1}{7} - \frac{1}{49}$.
41. $\dfrac{3\frac{1}{3}}{3\frac{3}{4}} + \frac{7}{8} \times \frac{4}{21} + 17$.
42. $1\frac{1}{3} + \frac{8}{3} \times \frac{41}{34} + \dfrac{4}{5\frac{1}{10}}$.
43. $\dfrac{1\frac{3}{4}}{3\frac{4}{5}} - \dfrac{2\frac{2}{3}}{4\frac{3}{4}} + \dfrac{2\frac{2}{9}}{6\frac{1}{3}}$.
44. $\dfrac{1\frac{3}{4}}{3\frac{4}{5}} + \dfrac{2\frac{2}{3}}{4\frac{3}{4}} - \dfrac{2\frac{2}{9}}{6\frac{1}{3}}$.
45. $\dfrac{2\frac{1}{2}}{7} + \dfrac{2}{7\frac{1}{2}} + \dfrac{\frac{1}{9}}{3\frac{1}{3}} + \dfrac{1\frac{1}{2}}{4\frac{3}{8}}$.
46. $\dfrac{1}{11\frac{2}{3}} + \dfrac{1}{13\frac{1}{2}} + \dfrac{1}{15\frac{3}{4}} + \dfrac{1}{17\frac{1}{2}}$.
47. $3\frac{1}{2} \times \dfrac{4\frac{1}{3}}{5\frac{1}{4}} + 8\frac{1}{9}$.
48. $7\frac{3}{4} + \frac{5}{7} \times \dfrac{4\frac{2}{3}}{2\frac{1}{2}} - 1\frac{2}{5}$.
49. $\frac{3}{8} \div 6 + \frac{63}{128} \times \frac{1}{3} \times 2\frac{2}{3} + \dfrac{1}{6\frac{6}{7}} \div \frac{7}{8} + \frac{27}{81}$.
50. $\dfrac{2\frac{5}{7}}{5} - 10\frac{5}{6} \div 1\frac{12}{13} + \frac{3}{7} \times 3\frac{3}{4} \times 3\frac{4}{5} - 1\frac{1}{60}$.

BRACKETS.

51. $\frac{3}{13}$ of $(2\frac{2}{3} + 2\frac{3}{4})$.

52. $(4\frac{2}{5} + 3\frac{1}{2}) \times 1\frac{2}{7}$.

53. $(3\frac{3}{8} - 2\frac{2}{5}) \div 1\frac{1}{25}$.

54. $(\frac{5}{6} + \frac{2}{9} - \frac{11}{15}) \div 2\frac{5}{12}$.

55. $(3\frac{1}{3} + 7\frac{1}{7}) \times (\frac{2}{5} + 9\frac{3}{4})$.

56. $(2\frac{1}{2} + \frac{1}{6}) \div (3\frac{2}{3} - \frac{1}{8})$.

57. $5\frac{2}{3}$ of $\frac{1}{15} \div (6\frac{1}{5} - \frac{2}{25})$.

58. $\frac{1}{8}$ of $1\frac{1}{3} \div (1\frac{1}{6} + 6\frac{2}{3})$.

59. $(4\frac{5}{8} + 3\frac{3}{8})(4\frac{5}{8} - 3\frac{3}{8})$.

60. $(8\frac{3}{14} - 7\frac{1}{7})(8\frac{7}{15} - 7\frac{7}{10})$.

61. $4\frac{5}{8} - (3\frac{5}{6} - 1\frac{3}{4})$.

62. $11 - (3\frac{3}{8} + 5\frac{5}{6}) - (5\frac{3}{8} - 3\frac{5}{8})$.

63. $(\frac{2}{5} - \frac{3}{10}) \times (\frac{3}{10} - \frac{4}{15}) \times (\frac{4}{15} - \frac{5}{20})$.

64. $(\frac{1}{3} + \frac{1}{4}) \times (\frac{1}{4} + \frac{1}{5}) \times (\frac{1}{5} + \frac{1}{7})$.

65. $\frac{1}{5}$ of $(\frac{3}{5} + \frac{6}{7}) + \frac{1}{7}$ of $(\frac{2}{5} + \frac{4}{7})$.

66. $2\frac{11}{12} \times (1 - \frac{25}{49}) + 1\frac{3}{11} \times (1 - \frac{16}{49})$.

67. $(\frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \frac{1}{5}) \div (\frac{1}{2} + \frac{1}{3} - \frac{1}{4} - \frac{1}{5})$.

68. $(\frac{8}{15} + \frac{7}{18} - \frac{17}{105} + \frac{1}{3}) \div (\frac{11}{30} - \frac{8}{35} + \frac{11}{15})$.

69. $(\frac{2}{7} - \frac{2}{49})$ of $(\frac{1}{3} - \frac{1}{9}) \div (\frac{2}{3} - \frac{2}{9})$ of $(\frac{1}{7} - \frac{1}{49})$.

70. $(\frac{3}{13} - \frac{5}{26})$ of $(\frac{7}{26} - \frac{9}{39}) \div (\frac{11}{52} - \frac{13}{65})$ of $(\frac{9}{39} - \frac{11}{52})$.

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71. $\dfrac{3\frac{3}{5} \text{ of } 4\frac{4}{5}}{2\frac{1}{4} \text{ of } 3\frac{3}{7}}$.

72. $\dfrac{2\frac{2}{3} \text{ of } 3\frac{3}{4} \text{ of } 4\frac{4}{5}}{5\frac{5}{6} \text{ of } 6\frac{6}{7}}$.

73. $\dfrac{5\frac{1}{3} \text{ of } 6\frac{1}{2} \text{ of } 7\frac{1}{5}}{2\frac{1}{13} \text{ of } 3\frac{1}{5} \text{ of } 4\frac{1}{3}}$.

74. $\dfrac{1\frac{7}{9} \times 3\frac{5}{7} \times 5\frac{3}{5}}{3\frac{1}{5} \times 2\frac{1}{6} \times 4\frac{3}{8}}$.

75. $\dfrac{3\frac{1}{2} - 1\frac{7}{8}}{3\frac{1}{4} \times 2\frac{1}{2}}$.

76. $\dfrac{3\frac{1}{7} - \frac{2}{3}}{\frac{4}{49} \times 7\frac{7}{12}}$.

77. $\dfrac{9\frac{1}{4} - 2\frac{7}{10}}{2\frac{3}{4} + 3\frac{4}{5}}$.

78. $\dfrac{4\frac{1}{7} - 2\frac{1}{4}}{6\frac{1}{2} - 2\frac{1}{7}}$.

79. $\dfrac{3\frac{1}{5} - 2\frac{1}{3}}{5\frac{2}{7} - 1\frac{1}{9}}$.

80. $\dfrac{2\frac{1}{2} + 3\frac{1}{3} - 4\frac{1}{4}}{5\frac{1}{5} + 7\frac{1}{12}}$.

81. $\dfrac{3\frac{7}{8} - 4\frac{1}{3} + 2\frac{1}{12}}{9\frac{1}{36} - 7\frac{19}{27}}$.

82. $\dfrac{2\frac{1}{9} - 1\frac{1}{8}}{7\frac{1}{2} - 6\frac{2}{3} + \frac{11}{72}}$.

83. $\dfrac{\frac{1}{2} + \frac{1}{4} + \frac{1}{7} + \frac{1}{14} + \frac{1}{28}}{\frac{1}{2} + \frac{3}{4} + \frac{6}{7} + \frac{13}{14} + \frac{27}{28}}$.

84. $\dfrac{\frac{1}{7} + \frac{1}{14} + \frac{1}{28} + \frac{1}{56}}{\frac{1}{7} - \frac{1}{14} + \frac{1}{28} - \frac{1}{56}}$.

85. $\dfrac{2\frac{2}{3} - 1\frac{1}{2}}{2\frac{2}{3} + 1\frac{1}{2}} + \dfrac{2\frac{3}{5} \text{ of } 9}{2\frac{4}{11} \text{ of } 2\frac{3}{4} \text{ of } 5}$.

86. $\dfrac{\frac{2}{9} \text{ of } 1\frac{2}{3} \text{ of } 2\frac{2}{5}}{\frac{7}{9} + \frac{5}{12} - \frac{3}{4}} + \dfrac{5\frac{2}{5} \text{ of } 7\frac{2}{9}}{8\frac{7}{24} - 3\frac{5}{12}}$.

87. $\dfrac{1\frac{2}{3} + \frac{3}{5}}{\frac{2}{3} \text{ of } 2\frac{1}{8}} \times \dfrac{10 - 7\frac{1}{2}}{3\frac{1}{3} - 1\frac{1}{5}} \times 1\frac{1}{15}$.

88. $\dfrac{\frac{1}{9} \text{ of } 1\frac{1}{4} \text{ of } 4\frac{1}{2}}{\frac{5}{56} \text{ of } 1\frac{1}{3} \text{ of } 3\frac{1}{2}} - \dfrac{3\frac{1}{4} + 4\frac{1}{3}}{6\frac{1}{2} + 1\frac{1}{12}}$.

89. $\dfrac{3\frac{1}{4} - 2\frac{1}{3}}{3\frac{1}{6} + 1\frac{1}{2}} \div 1\frac{4}{7}$.

90. $\dfrac{\frac{1}{2} + \frac{1}{3} - \frac{1}{4}}{\frac{1}{3} + \frac{1}{4} - \frac{1}{5}} \div 1\frac{12}{23}$.

91. $4\frac{2}{9} \times 6\frac{3}{7} \div \dfrac{2\frac{1}{2}}{7}$.

92. $3\frac{3}{4} \times 4\frac{4}{5} \times 5\frac{5}{6} \div \dfrac{9\frac{3}{5}}{6\frac{6}{7}}$.

93. $\dfrac{3\frac{3}{4}}{60} \div \dfrac{\frac{3}{4}}{\frac{1}{2} \text{ of } \frac{1}{6}}$.

94. $3\frac{1}{3} \div \dfrac{1 - \frac{15}{16}}{\frac{1}{5} - \frac{1}{8}}$.

95. $\dfrac{4\frac{5}{6}}{\frac{5}{6} + \frac{3}{8}} \div \dfrac{4\frac{3}{8}}{\frac{5}{6} - \frac{3}{8}}$.

96. $\dfrac{\frac{1}{4} + \frac{11}{13}}{\frac{2}{7} + \frac{9}{11}} \div \dfrac{\frac{1}{5} + \frac{3}{4}}{\frac{1}{3} + \frac{2}{7}}$.

97. $\dfrac{4\frac{1}{7} - 2\frac{1}{4}}{6\frac{1}{2} + 2\frac{1}{7}} \div \dfrac{\frac{3}{7} + \frac{1}{2}}{\frac{5}{7} - \frac{1}{2}}$.

98. $\dfrac{7\frac{1}{5} \times 5\frac{1}{7}}{7\frac{1}{5} - 5\frac{1}{7}} \div \dfrac{5\frac{1}{3} \times 3\frac{1}{5}}{5\frac{1}{3} + 3\frac{1}{5}}$.

99. $\dfrac{3\frac{1}{2} - 1}{15\frac{1}{3} + 5\frac{1}{2} - 3\frac{1}{3}} \div \dfrac{1 - \frac{3}{4}}{1 + \frac{3}{4}}$.

100. $\dfrac{4\frac{1}{2} - 3\frac{3}{4}}{4\frac{1}{2} + 3\frac{3}{4}} \div \dfrac{3\frac{3}{4} + 2\frac{5}{8}}{3\frac{3}{4} - 2\frac{5}{8}}$.

Continued Fractions.

101. $\dfrac{2}{2 + \frac{2}{3}}$.

102. $\dfrac{3}{3 + \frac{3}{4}}$.

103. $\dfrac{1}{2 + \dfrac{1}{3 + \frac{1}{4}}}$.

104. $\dfrac{1}{2 + \dfrac{1}{3 + \frac{1}{5}}}$.

105. $\dfrac{2}{3 + \dfrac{3}{4 + \frac{4}{5}}}$.

106. $\dfrac{2}{3 - \dfrac{3}{4 - \frac{4}{5}}}$.

107. $\dfrac{1}{1 + \dfrac{1}{1 + \dfrac{1}{1 + \frac{1}{2}}}}$.

108. $\dfrac{2}{2 + \dfrac{1}{3 - \dfrac{2}{5 + \frac{1}{2}}}}$.

109. $1 + \dfrac{3}{1 + \dfrac{3}{1 + \frac{1}{2}}}$.

110. $1 - \dfrac{2}{3 + \dfrac{4}{5 - \frac{6}{7}}}$.

Various.

111. $\frac{2}{3} + \frac{3}{4} + \frac{4}{5}$.

112. $\frac{2}{3} \times \frac{3}{4} \times \frac{4}{5}$.

113. $\frac{2}{3} + \frac{3}{4} - \frac{4}{5}$.

114. $\frac{2}{3} - \frac{3}{4} + \frac{4}{5}$.

115. $\frac{2}{3} + \frac{3}{4}$ of $\frac{4}{5}$.

116. $\frac{2}{3}$ of $\frac{3}{4} + \frac{4}{5}$.

117. $\frac{4}{5} - (\frac{3}{4} - \frac{2}{3})$.

118. $\frac{2}{3} - \frac{3}{4} \times \frac{4}{5}$.

119. $\frac{2}{3}$ of $\frac{3}{4} \div \frac{4}{5}$.

120. $\frac{2}{3} \div \frac{3}{4}$ of $\frac{4}{5}$.

121. $\frac{2}{3} + \frac{3}{4} \div \frac{4}{5}$.

122. $\frac{2}{3} \div \frac{3}{4} + \frac{4}{5}$.

123. $\frac{2}{3}$ of $(\frac{3}{4} + \frac{4}{5})$.

124. $(\frac{2}{3} + \frac{3}{4}) \times \frac{4}{5}$.

125. $(\frac{2}{3} + \frac{3}{4}) \div \frac{4}{5}$.

126. $\frac{2}{3} \div (\frac{3}{4} + \frac{4}{5})$.

127. $\frac{2}{3} \div \frac{3}{4} \times \frac{4}{5}$.

128. $\frac{2}{3} \div \frac{3}{4} \div \frac{4}{5}$.

129. $3\frac{1}{2} + 2\frac{1}{12} + 3\frac{3}{16} - 7\frac{19}{24}$.

130. $11\frac{1}{9} - 10\frac{26}{27} + 9\frac{1}{6} - 8\frac{17}{18}$.

131. $17 - 3\frac{1}{3}$ of $4\frac{1}{5} + 5\frac{1}{4}$ of $2\frac{2}{3}$.

132. $\frac{1}{4} \times 6\frac{2}{3} + \frac{7}{8} \div 2\frac{1}{3} - \frac{1}{24}$.

133. $\dfrac{1\frac{1}{2}}{2\frac{1}{2}} \times \dfrac{1\frac{1}{4}}{2\frac{1}{4}} \times \dfrac{1\frac{1}{6}}{2\frac{1}{6}} \times \dfrac{1\frac{1}{8}}{2\frac{1}{8}} \times \dfrac{1\frac{1}{10}}{2\frac{1}{10}}$.

134. $\dfrac{\frac{1}{2} - \frac{1}{4}}{\frac{1}{3} - \frac{1}{4}}$ of $\dfrac{\frac{1}{4} - \frac{1}{6}}{\frac{1}{4} - \frac{1}{5}}$ of $\dfrac{\frac{1}{6} - \frac{1}{8}}{\frac{1}{5} - \frac{1}{6}}$.

135. $\dfrac{2\frac{1}{2}}{3\frac{2}{3}} + \dfrac{3\frac{1}{3} - 1\frac{1}{6}}{5\frac{1}{5} - 1\frac{1}{6}} + \frac{1}{22}$ of $4\frac{9}{11}$.

136. $\dfrac{5\frac{2}{5}}{6\frac{2}{5}} + \dfrac{3\frac{1}{2} + 4\frac{1}{5}}{5\frac{1}{2} + 7\frac{1}{3}} + \frac{9}{16}$ of $\frac{1}{10}$.

137. $\dfrac{12\frac{1}{4}}{1\frac{1}{8}} + \dfrac{12\frac{1}{4} - 3\frac{1}{5}}{12\frac{1}{4} + 3\frac{1}{5}}$ of $34\frac{1}{3} - 15\frac{1}{2}$.

138. $\dfrac{2\frac{5}{9}}{8\frac{1}{9}}$ of $\dfrac{1\frac{3}{5} + \frac{5}{6}}{1\frac{3}{5} - \frac{5}{6}} + \frac{5}{8}$ of $\dfrac{1\frac{5}{6} - \frac{3}{5}}{\frac{5}{6} + \frac{2}{3}}$.

139. $\dfrac{1\frac{1}{2}}{4\frac{1}{3}} + \dfrac{3\frac{1}{3} - 2\frac{5}{6}}{4\frac{1}{2} + 5\frac{9}{10}} + \frac{3}{26}$ of $5\frac{1}{4}$.

140. $1\frac{1}{6}$ of $5\frac{1}{4} + \dfrac{4\frac{1}{2} - \frac{3}{10}}{1\frac{1}{15}} - \dfrac{6\frac{3}{4}}{\frac{1}{2} \text{ of } 2\frac{2}{3}}$.

141. $\dfrac{2\frac{1}{4}}{2\frac{2}{3}} + \dfrac{2\frac{1}{2} + 5\frac{1}{5}}{3\frac{1}{3} + 9\frac{1}{2}} + \frac{1}{2} + \frac{3}{8}$ of $\frac{3}{20}$.

142. $\dfrac{1\frac{1}{2}}{8\frac{1}{2}}$ of $\dfrac{1\frac{1}{2} + 1\frac{1}{3}}{2\frac{3}{8} - 1\frac{1}{6}} + \dfrac{2\frac{1}{5} + 4\frac{1}{3}}{1\frac{1}{5} - \frac{1}{9}}$ of $\frac{1}{9}$.

143. $\frac{1}{4}$ of $\frac{7}{8}$ of $\left(\dfrac{2\frac{1}{3}}{3\frac{1}{2}} + \dfrac{4\frac{1}{2}}{5\frac{1}{4}}\right)$.

144. $\frac{2}{3}$ of $\frac{4}{5}$ of $\frac{6}{7} \div \left(\dfrac{1\frac{2}{3}}{1\frac{4}{5}} + \dfrac{2\frac{1}{3}}{4\frac{1}{5}}\right)$.

145. $\dfrac{3\frac{1}{2} \times 3\frac{1}{2} \times 3\frac{1}{2} - 3}{3\frac{1}{2} \times 3\frac{1}{2} - 3}$.

146. $\dfrac{2\frac{1}{3} \times 2\frac{1}{3} \times 2\frac{1}{3} - 1}{2\frac{1}{3} \times 2\frac{1}{3} - 1}$.

147. $\dfrac{(5\frac{3}{8} - 3\frac{5}{8}) \text{ of } \frac{29}{30}}{5\frac{3}{8} - 3\frac{5}{8} \text{ of } \frac{30}{29}}$.

148. $\dfrac{(5\frac{3}{8} + 3\frac{5}{8}) \text{ of } \frac{29}{30}}{5\frac{3}{8} + 3\frac{5}{8} \text{ of } \frac{30}{29}}$.

149. $\dfrac{1\frac{1}{2} \times 2\frac{1}{4} + 3\frac{1}{8} \times 4\frac{1}{16}}{1\frac{1}{2} + 2\frac{1}{4} \times 3\frac{1}{8} + 4\frac{1}{16}}$.

150. $\dfrac{2\frac{1}{2} + 3\frac{1}{4} \div \frac{4}{5} \text{ of } \frac{13}{20}}{(2\frac{1}{3} + 1\frac{3}{4}) \times 2\frac{4}{7} - 5\frac{1}{4}}$.

151. $\dfrac{4\frac{3}{4} + 4\frac{1}{4}}{4\frac{3}{4} \times 4\frac{1}{4}} \div \dfrac{4\frac{3}{4} - 4\frac{1}{4}}{4\frac{3}{4} \div 4\frac{1}{4}}$.

152. $\dfrac{4\frac{4}{9} \text{ of } 8\frac{1}{3}}{4\frac{1}{4} + \frac{7}{36}} \div \dfrac{3\frac{1}{4} + 5\frac{1}{7}}{1\frac{1}{4} + 3\frac{1}{7}}$.

153. $\dfrac{\frac{13}{18} - \frac{31}{81}}{1 - \frac{31}{81} \times \frac{13}{18}} - \dfrac{\frac{10}{13} - \frac{23}{31}}{1 + \frac{23}{31} \times \frac{10}{13}}$.

154. $\dfrac{\frac{7}{8} + \frac{2}{3} \text{ of } \frac{1}{4}}{(\frac{7}{8} + \frac{2}{3}) \text{ of } \frac{1}{4}} \times \dfrac{\frac{7}{8} \text{ of } (\frac{2}{3} + \frac{1}{4})}{\frac{7}{8} \text{ of } \frac{2}{3} + \frac{1}{4}} \times 37$.

155. $\left\{2\frac{3}{4} + 2\frac{1}{2} \text{ of } \dfrac{7}{3\frac{4}{5}} - \dfrac{1\frac{2}{3}}{2\frac{1}{2}}\right\} \div 1\frac{77}{228} + 5$.

156. $\left\{\dfrac{3\frac{1}{3} - \frac{3}{5} \text{ of } 1\frac{1}{9}}{\frac{1}{3} \text{ of } 2\frac{1}{4} + \frac{5}{8}} - \dfrac{1}{5\frac{1}{2}}\right\} \div \dfrac{2}{1\frac{4}{29}} - 1$.

157. $\dfrac{\frac{2}{3} + \frac{3}{5}}{1 - \frac{2}{3} \text{ of } \frac{3}{5}} + \frac{3}{10}$ of $\dfrac{3\frac{1}{9}}{5\frac{5}{6}}$ of $\dfrac{8\frac{1}{3}}{1\frac{1}{2}} - \dfrac{4\frac{1}{6}}{3\frac{3}{4}} \times 4\frac{1}{2}$ of $\frac{3}{5}$.

158. $\frac{7}{12} \times 4\frac{1}{8} - \dfrac{4\frac{1}{4}}{3} + \dfrac{1\frac{3}{5} - \frac{4}{15}}{\frac{2}{3} - \frac{2}{15}} \times \dfrac{5\frac{3}{5}}{9\frac{1}{3}} + \left(\dfrac{1}{1\frac{4}{11}} + 2\frac{2}{5}\right) \times \frac{5}{6}$.

159. $\dfrac{(5\frac{1}{5}-3\frac{1}{3})\div\frac{3}{5} \text{ of } 1\frac{1}{6}}{18\frac{2}{3}-5\frac{1}{5} \text{ of } 3\frac{1}{3}}$.

160. $\dfrac{5\frac{3}{4}+2\frac{2}{35}\div 1\frac{11}{25}-\frac{3}{7} \text{ of } 15\frac{3}{4}}{\frac{3}{4} \text{ of } 7\frac{3}{7}-5\frac{3}{5}\div 3\frac{4}{15}}$.

161. $\dfrac{2\frac{1}{2}}{2\frac{1}{4}} \text{ of } \dfrac{3\frac{1}{2}}{3\frac{1}{4}} \text{ of } \dfrac{4\frac{1}{2}}{4\frac{1}{4}}\div\dfrac{2\frac{1}{2}}{4\frac{1}{2}} \text{ of } \dfrac{2\frac{1}{3}}{4\frac{1}{3}} \text{ of } \dfrac{2\frac{1}{4}}{4\frac{1}{4}}$.

162. $\dfrac{7\frac{1}{2}-10\frac{1}{3}+11\frac{1}{4}}{7\frac{1}{2} \text{ of } 9\frac{1}{3} \text{ of } 11\frac{1}{4}}\div\dfrac{2\frac{1}{4}-4\frac{1}{6}+6\frac{1}{8}}{6\frac{1}{4} \text{ of } 8\frac{1}{6} \text{ of } 10\frac{1}{8}}$.

163. $4\frac{1}{2}+\dfrac{2\frac{1}{4}}{4\frac{1}{2}}+4\frac{1}{2} \text{ of } 2\frac{1}{4}+\dfrac{4\frac{1}{2}+2\frac{1}{4}}{4\frac{1}{2}-2\frac{1}{4}} \text{ of } \frac{1}{2} \text{ of } \frac{1}{4}$.

164. $2\frac{1}{3}+\dfrac{2\frac{1}{3}}{2\frac{1}{4}}+\dfrac{2\frac{1}{3}+2\frac{1}{4}}{2\frac{1}{3}-2\frac{1}{4}}+2\frac{1}{3} \text{ of } 2\frac{1}{4}-2\frac{1}{4}$.

165. $4\frac{1}{2}+\left(\dfrac{2\frac{1}{4}}{4\frac{1}{2}}+4\frac{1}{2}\right) \text{ of } \left(2\frac{1}{4}+\dfrac{4\frac{1}{2}+2\frac{1}{4}}{4\frac{1}{2}-2\frac{1}{4}}\right) \text{ of } \frac{1}{2} \text{ of } \frac{1}{4}$.

166. $2\frac{1}{4}+\dfrac{2\frac{1}{4}}{2\frac{1}{3}}+\left(\dfrac{2\frac{1}{3}+2\frac{1}{4}}{2\frac{1}{3}-2\frac{1}{4}}+2\frac{1}{3}\right) \text{ of } 2\frac{1}{4}-2\frac{1}{3}$.

167. $\dfrac{1-\dfrac{1}{3\frac{1}{2}}}{1-\dfrac{1}{1\frac{1}{2}}} \text{ of } \dfrac{1-\dfrac{1}{2\frac{1}{2}}}{1-\dfrac{1}{5\frac{1}{2}}} \text{ of } \dfrac{1-\dfrac{1}{4\frac{1}{2}}}{1-\dfrac{1}{6\frac{1}{2}}} \text{ of } \dfrac{1-\dfrac{1}{8\frac{1}{2}}}{1-\dfrac{1}{7\frac{1}{2}}}$.

168. $1\frac{1}{11}-\dfrac{1-\frac{7}{22}}{2-\frac{1}{3}}+\dfrac{1\frac{2}{5}}{3\frac{1}{2}}-\dfrac{5\frac{5}{8}}{6\frac{1}{4}} \text{ of } \left(\frac{1}{5}-\dfrac{\frac{1}{2}-\frac{1}{3}}{4\frac{3}{4}-3\frac{2}{9}}\right)$.

169. $\dfrac{6\frac{1}{2}+6\frac{5}{6}}{1\frac{5}{6}\times 12\frac{7}{11}}+\dfrac{(4\frac{7}{12}-3\frac{5}{24}) \text{ of } 2\frac{2}{11}}{4\frac{7}{12}+3\frac{5}{24} \text{ of } 2\frac{2}{11}}+\dfrac{1\frac{5}{12}}{5\frac{19}{24}}$.

170. $\frac{18}{17} \text{ of } (1-\frac{64}{81})+\frac{8}{11} \text{ of } \frac{1}{6} \text{ of } (\frac{1}{2}+\frac{5}{12})+\dfrac{1\frac{1}{2}\times 9\frac{1}{3}}{2\frac{3}{4}\times 7\frac{7}{11}}$.

171. $\dfrac{\frac{1}{3}+\frac{2}{11}+\frac{7}{24}-\frac{1}{3} \text{ of } \frac{2}{11} \text{ of } \frac{7}{24}}{1-\frac{1}{3} \text{ of } \frac{2}{11}-\frac{2}{11} \text{ of } \frac{7}{24}-\frac{7}{24} \text{ of } \frac{1}{3}}$.

172. $\dfrac{2\frac{3}{25}-\frac{3}{5} \text{ of } \frac{2}{25}+3\frac{2}{5}}{2\frac{2}{5} \text{ of } (\frac{3}{5}+\frac{3}{25})\div 3\frac{3}{5} \text{ of } (\frac{2}{5}-\frac{2}{25})}$.

173. $(\frac{1}{2}+\frac{1}{3}) \text{ of } (\frac{1}{3}+\frac{1}{4})+(\frac{1}{2}+\frac{1}{4}) \text{ of } (\frac{1}{4}+\frac{1}{5})+(\frac{1}{2}+\frac{1}{5}) \text{ of } (\frac{1}{5}+\frac{1}{6})$.

174. $\dfrac{(\frac{1}{2}+\frac{2}{3}) \text{ of } (\frac{3}{4}+\frac{4}{5})+(\frac{1}{2}+\frac{3}{4}) \text{ of } (\frac{2}{3}+\frac{4}{5})+(\frac{1}{2}+\frac{4}{5}) \text{ of } (\frac{2}{3}+\frac{3}{4})}{\frac{1}{2} \text{ of } (\frac{2}{3}+\frac{3}{4}+\frac{4}{5})+(\frac{1}{2}+\frac{2}{3}+\frac{3}{4}) \text{ of } \frac{4}{5}}$.

175. $\dfrac{5}{7-\dfrac{9}{3-\frac{3}{4}}}+\dfrac{5}{16-\dfrac{11}{2-\frac{1}{6}}}$.

176. $\dfrac{3}{8-\dfrac{7}{2-\frac{3}{4}}}+\dfrac{5}{6-\dfrac{5}{2-\frac{5}{6}}}$.

177. $\dfrac{\frac{1}{7}+\frac{1}{3}(\frac{1}{2}+\frac{1}{7})}{1+\dfrac{1}{2-\frac{1}{4}}}$.

178. $3-3\frac{1}{3}$ of $\dfrac{3}{3-\dfrac{3}{3-\frac{1}{3}}}$ of $\dfrac{\frac{1}{3}}{1\frac{1}{4}}\div 3\frac{2}{3}$.

179. $\dfrac{1\frac{2}{3} \text{ of } 1\frac{1}{4}}{3\frac{2}{5}+\dfrac{5}{1\frac{19}{41}}}+\dfrac{4}{5+\dfrac{6}{7\frac{8}{9}}}$.

180. $2\frac{4}{5}-\frac{1}{9}$ of $\dfrac{5\frac{1}{7}-2\frac{1}{4}}{1+\dfrac{1}{3+\frac{1}{6}}}$ of $\dfrac{9\frac{1}{3}}{12\frac{1}{11}}\div 2\frac{1}{5}$.

181. $3\dfrac{3\frac{1}{3}}{3}\div 3\dfrac{3}{3\dfrac{3}{3\frac{1}{3}}}$.

182. $\left\{\dfrac{1}{2-\dfrac{3}{4-\frac{5}{6}}}-\dfrac{1}{2+\dfrac{3}{4+\frac{5}{6}}}\right\}\times 10\frac{5}{9}$.

183. $\dfrac{\dfrac{1}{1\frac{1}{2}}+\dfrac{1}{1\frac{1}{3}}+\dfrac{1}{1\frac{1}{4}}}{3-\dfrac{1}{1-\frac{1}{16}}+\dfrac{1}{4-\frac{8}{17}}}$.

184. $\frac{13}{17}\times\dfrac{15\frac{1}{2}}{15\frac{2}{3}}\times\dfrac{\dfrac{1}{3\frac{1}{2}}+\dfrac{1}{4\frac{1}{3}}}{\dfrac{1\frac{1}{2}}{3}+\dfrac{1\frac{1}{3}}{4}}\times\dfrac{\dfrac{3}{4+\dfrac{1}{2+\frac{1}{3}}}}{\dfrac{3}{4-\dfrac{1}{2-\frac{1}{3}}}}$.

185. $\frac{4}{5}$ of $6\frac{1}{3}-2\frac{1}{13}$ of $\dfrac{1\frac{6}{7}\times\dfrac{\frac{2}{3}}{\frac{3}{4}}\times 1\frac{1}{6}}{1\frac{1}{5}\times\dfrac{\frac{1}{3}}{\frac{3}{5}}\times 1\frac{7}{8}}-2\frac{1}{12}$ of $\frac{4}{5}+\frac{4}{5}$.

186. $7\frac{1}{7}$ of $\dfrac{2\frac{1}{2}+7\frac{1}{7}}{2\frac{1}{7}+7\frac{1}{2}}$ of $\dfrac{\dfrac{2\frac{1}{7}}{7\frac{1}{2}}+\dfrac{2\frac{1}{2}}{7\frac{1}{7}}}{\dfrac{2\frac{1}{7}}{7\frac{1}{7}}+\dfrac{2\frac{1}{2}}{7\frac{1}{2}}}\div\dfrac{7\frac{1}{7}}{2\frac{1}{2}}$ of $4\frac{13}{19}$.

187. $\frac{1}{2}$ of $(\frac{1}{3}+\frac{1}{4}+\frac{1}{5})+7\times(\frac{1}{35}+\frac{1}{36})-\frac{1}{12}\div\left(\dfrac{11\frac{1}{2}}{15\frac{1}{3}}-\frac{1}{4}\right)$.

188. $\dfrac{1\frac{1}{4}-1\frac{1}{6} \text{ of } \frac{9}{14}}{5\frac{1}{2}+\frac{5}{6} \text{ of } \frac{3}{5}}\div\frac{1}{6}+\left(3\frac{1}{3} \text{ of } \frac{1}{7}+\dfrac{2}{10\frac{1}{2}}-\frac{5}{9} \text{ of } \frac{2}{7}\right)\div\frac{4}{7}$.

189. $\dfrac{3\frac{1}{3}-1\frac{1}{2} \text{ of } 1\frac{1}{5}-1\frac{1}{7}}{(3\frac{1}{3}-1\frac{1}{2}) \text{ of } (1\frac{1}{5}-1\frac{1}{7})}\times\dfrac{3\frac{1}{3}-1\frac{1}{2} \text{ of } (1\frac{1}{5}-1\frac{1}{7})}{(3\frac{1}{3}-1\frac{1}{2}) \text{ of } 1\frac{1}{5}-1\frac{1}{7}}$.

190. $\left\{\frac{1}{8} \text{ of } (\frac{1}{10}-\frac{1}{11})\div\dfrac{\frac{1}{7}-\frac{1}{9}\div(\frac{4}{9}+\frac{4}{11})}{(\frac{1}{7}+\frac{1}{9})\div(\frac{4}{9}-\frac{4}{11})}\right\}\times\dfrac{\frac{1}{3}+\frac{1}{7}\div(\frac{1}{7}-\frac{1}{9})}{(\frac{1}{3}+\frac{1}{7})\div\frac{1}{7}-\frac{1}{9}}$.

XXII. FRACTIONS.

VALUE OF A FRACTION OF A CONCRETE QUANTITY.

Find, *mentally*, the value of—

1. $\frac{1}{2}$; $\frac{1}{4}$; $\frac{1}{8}$; $\frac{1}{16}$, of a shilling.
2. $\frac{1}{3}$; $\frac{1}{6}$; $\frac{1}{12}$; $\frac{1}{24}$; $\frac{1}{48}$, of a shilling.
3. $\frac{3}{8}$; $\frac{5}{8}$; $\frac{7}{8}$; $\frac{3}{16}$; $\frac{5}{16}$, of a shilling.
4. $\frac{2}{3}$; $\frac{5}{6}$; $\frac{5}{24}$; $\frac{11}{24}$; $\frac{5}{48}$, of a shilling.
5. $\frac{1}{2}$; $\frac{1}{4}$; $\frac{1}{8}$; $\frac{1}{16}$, of £1.
6. $\frac{1}{3}$; $\frac{2}{3}$; $\frac{1}{6}$, $\frac{5}{6}$; $\frac{1}{12}$; $\frac{11}{12}$, of £1.
7. $\frac{3}{4}$; $\frac{3}{8}$; $\frac{5}{8}$; $\frac{7}{8}$; $\frac{3}{16}$, of £1.
8. $\frac{1}{5}$; $\frac{2}{5}$; $\frac{3}{5}$; $\frac{4}{5}$; $\frac{1}{15}$; $\frac{19}{40}$, of £1.
9. $\frac{1}{240}$; $\frac{17}{240}$; $\frac{1}{120}$; $\frac{1}{60}$; $\frac{7}{60}$; $\frac{1}{80}$; $\frac{9}{80}$; $\frac{1}{960}$; $\frac{17}{960}$; $\frac{1}{480}$, of £1.
10. $\frac{23}{240}$; $\frac{37}{240}$; $\frac{53}{240}$; $\frac{11}{120}$; $\frac{31}{120}$; $\frac{7}{80}$; $\frac{7}{480}$; $\frac{13}{960}$; $\frac{23}{960}$; $\frac{37}{960}$, of £1.

Find the value of—

11. $\frac{1}{4}$ of £7, 12*s.* 5*d.*
12. $\frac{1}{5}$ of £11, 16*s.* 9*d.*
13. $\frac{1}{5}$ of £2, 7*s.* 3*d.*
14. $\frac{1}{8}$ of £12, 13*s.* 9*d.*
15. $\frac{1}{7}$ of £1, 5*s.* 3*d.*
16. $\frac{1}{9}$ of £3, 2*s.* 7*d.*
17. $\frac{1}{5}$ of £8, 12*s.* $7\frac{1}{4}$*d.*
18. $\frac{1}{6}$ of £13, 2*s.* $3\frac{1}{4}$*d.*
19. $\frac{5}{12}$ of £1.
20. $\frac{3}{8}$ of £2.
21. $\frac{2}{5}$ of £7.
22. $\frac{5}{9}$ of £4, 1*s.*
23. $\frac{3}{7}$ of £10.
24. $\frac{5}{11}$ of £12.
25. $\frac{4}{9}$ of 18*s.* 7*d.*
26. $\frac{5}{12}$ of 13*s.* 5*d.*
27. $\frac{11}{15}$ of 1 day.
28. $\frac{17}{36}$ of 5 hours.
29. $\frac{5}{16}$ of 3 acres.
30. $\frac{3}{11}$ of a mile.
31. $\frac{5}{6}$ of 4 yds. 2 ft.
32. $\frac{5}{8}$ of 1 ton 2 cwts.
33. $\frac{5}{7}$ of 12 yds.
34. $\frac{7}{11}$ of 3 sq. yds.
35. $\frac{1}{12}$ of £122, 4*s.* $6\frac{7}{8}$*d.*
36. $\frac{1}{12}$ of £411, 17*s.* $1\frac{5}{6}$*d.*
37. $\frac{2}{7}$ of £16, 13*s.* 4*d.*
38. $\frac{5}{11}$ of £25, 14*s.* 7*d.*
39. $\frac{7}{19}$ of £8, 12*s.* $11\frac{3}{4}$*d.*
40. $\frac{17}{73}$ of £7, 18*s.* 2*d.*
41. $\frac{31}{64}$ of £4523, 14*s.* 8*d.*
42. $\frac{44}{45}$ of £119, 19*s.* $9\frac{3}{4}$*d.*
43. $\frac{7}{9}$ of 11 yds. 2 ft. 7 in.
44. $\frac{5}{8}$ of 3 tons 13 cwts. 2 qrs.
45. $\frac{7}{15}$ of 3 tons 17 cwts. 1 qr. 9 lbs.
46. $\frac{1}{22}$ of 1 ac. 33 po. 22 sq. yds.
47. $\frac{1}{2}$ of $\frac{3}{32}$ of £1.
48. $\frac{3}{10}$ of $\frac{5}{24}$ of £7.
49. $\dfrac{73\frac{1}{2}}{42}$ of $6\frac{3}{4}$*s.*
50. $\dfrac{17}{27\frac{3}{4}}$ of $6\frac{1}{6}$*s.*
51. $\frac{9}{17}$ of 2*s.* $11\frac{1}{2}$*d.*
52. $\frac{5}{19}$ of 7*s.* $8\frac{1}{4}$*d.*
53. £$15\frac{5}{8} \times \dfrac{4\frac{2}{5}}{9\frac{1}{6}}$.
54. £$3\frac{7}{8} \times \dfrac{22\frac{19}{20}}{34\frac{7}{8}}$.
55. $\frac{23}{31}$ of 10 tons.
56. $\frac{17}{29}$ of 17 tons.
57. $\dfrac{30\frac{1}{3}}{10\frac{5}{6}}$ of $32\frac{1}{7}$ qrs.
58. $\dfrac{15\frac{1}{6}}{7\frac{1}{3}}$ of $4\frac{7}{12}$ hrs.

Multiply—

59. £3, 3*s.* $3\frac{3}{4}$*d.* by $3\frac{3}{4}$.
60. £4, 17*s.* $6\frac{3}{4}$*d.* by $2\frac{4}{7}$.
61. £56, 10*s.* $10\frac{1}{2}$*d.* by $71\frac{1}{2}$.
62. £22, 13*s.* 5*d.* by $89\frac{1}{3}$.
63. £7, 12*s.* $4\frac{3}{8}$*d.* by $8\frac{4}{7}$.
64. £5, 17*s.* $6\frac{5}{9}$*d.* by $14\frac{2}{7}$.

Divide—

65. £3, 3*s.* $3\frac{3}{4}$*d.* by $3\frac{3}{4}$.

66. £24, 10*s.* 3*d.* by $17\frac{2}{3}$.

67. £11, 11*s.* 11*d.* by $11\frac{1}{11}$.

68. £23, 13*s.* $7\frac{1}{4}$*d.* by $17\frac{3}{4}$.

69. £219, 9*s.* 7*d.* by $12\frac{2}{7}$.

70. £184, 15*s.* 9*d.* by $18\frac{3}{4}$.

Find the sum of—

71. $\frac{3}{8}$ of 1*s.*, $\frac{2}{7}$ of 21*s.*, $\frac{1}{12}$ of 2*s.* 6*d.* and $\frac{1}{48}$ of £1.

72. $\frac{7}{16}$ of £5, $\frac{2}{7}$ of £9, 13*s.* $2\frac{3}{4}$*d.* and $\frac{5}{12}$ of 2*s.* 6*d.*

73. $\frac{2}{3}$ of 5*s.*, $\frac{13}{126}$ of £1, 1*s.*, $\frac{1}{3}$ of 18*s.* 6*d.* and $\frac{5}{12}$ of £1.

74. $1\frac{1}{3}$ of £1, 1*s.*, $\frac{5}{8}$ of £1, $\frac{5}{16}$ of 2*s.* 6*d.* and $29\frac{19}{20}$ of $7\frac{1}{2}$*d.*

75. £$3\frac{43}{96}$ and $(9\frac{1}{12} - 2\frac{7}{8})$ of 5*s.*

76. £$7\frac{35}{96}$ and $(3\frac{1}{4} - 2\frac{7}{12})$ of 3*s.* 6*d.*

77. $\frac{7}{8}$ po., $\frac{3}{8}$ yd. and $\frac{3}{16}$ ft.

78. $\frac{4}{5}$ ac., $\frac{1}{36}$ ro. and $\frac{8}{9}$ sq. po.

Find the difference between—

79. $\frac{5}{6}$ of 14*s.* $10\frac{1}{2}$*d.* and $\frac{6}{7}$ of the same sum.

80. $2\frac{3}{4}$ of 11*s.* 8*d.* and $1\frac{1}{2}$ of 17*s.* $2\frac{1}{2}$*d.*

81. $\frac{7}{8}$ of £9, 11*s.* 10*d.* and $\frac{5}{6}$ of £10, 3*s.* $10\frac{1}{2}$*d.*

82. $\frac{3}{11}$ of £78, 16*s.* $2\frac{1}{2}$*d.* and $\frac{3}{5}$ of £35, 14*s.* $8\frac{1}{4}$*d.*

83. $\frac{2}{7}$ of $\frac{21}{22}$ of £15, 10*s.* 9*d.* and $\frac{7}{130}$ of $3\frac{1}{3}$ of $2\frac{11}{14}$ of £2, 15*s.* 8*d.*

84. $\dfrac{2\frac{4}{9} \text{ of } 5\frac{1}{8}}{27\frac{1}{2}}$ of £1, 2*s.* 6*d.* and $\dfrac{5\frac{1}{12}}{2\frac{1}{2} + 3\frac{3}{4}}$ of 12*s.* 6*d.*

85. $1\frac{2}{3}$ of 1 ton 12 cwts. 3 qrs. and $\frac{3}{7}$ of 3 tons 13 cwts. 1 qr.

86. $3\frac{1}{27}$ of 2 qrs. 25 lbs. and $\frac{1}{4}$ of 4 cwts. 2 qrs. 20 lbs.

Find the value of—

87. $\frac{1}{4}$ of $\dfrac{4\frac{7}{9} \text{ of } 6\frac{2}{3}}{7\frac{26}{27}}$ of $\dfrac{3\frac{2}{5} - 3\frac{1}{6}}{3\frac{3}{5} + 2\frac{1}{6}}$ of £182, 7*s.* 5*d.*

88. $4\frac{4}{7}$ of $\dfrac{4\frac{1}{10}}{4\frac{5}{9}}$ of $\dfrac{\dfrac{1}{20\frac{1}{2}} + \dfrac{1}{13\frac{2}{3}}}{\dfrac{1}{10\frac{1}{4}} + \dfrac{1}{8\frac{1}{5}}}$ of $\dfrac{6\frac{5}{6}}{5\frac{6}{7}}$ of $5\frac{1}{8}$ guineas.

89. $\left\{\dfrac{\frac{35}{36} - \frac{34}{35}}{\frac{24}{25} - \frac{23}{24}} - \dfrac{\frac{24}{25} - \frac{23}{24}}{\frac{15}{16} - \frac{14}{15}}\right\}$ of 15 cwts.

90. $\left\{1 + \frac{65}{81} \text{ of } \dfrac{1\frac{2}{3} + 1\frac{3}{5} - 1\frac{1}{6}}{1\frac{2}{3} \text{ of } (1\frac{3}{5} - 1\frac{1}{6})} \times \dfrac{(1\frac{2}{3} + 1\frac{3}{5}) \div 1\frac{1}{6}}{1\frac{2}{3} \text{ of } 1\frac{3}{5} \text{ of } 1\frac{1}{6}}\right\}$ of 3 tons.

XXIII. FRACTIONS.

Reduction of one Quantity to the Fraction of Another.

Reduce, *mentally*, each to the fraction of 1*s*.—

1. 6*d*.; 3*d*.; $1\frac{1}{2}d.$; $\frac{3}{4}d.$

2. $4\frac{1}{2}d.$; $7\frac{1}{2}d.$; $10\frac{1}{2}d.$

3. 4*d*.; 2*d*.; 1*d*.; $\frac{1}{2}d.$; $\frac{1}{4}d.$

4. 8*d*.; 10*d*.; 7*d*.; $2\frac{1}{2}d.$; $1\frac{1}{4}d.$

Reduce, *mentally*, each to the fraction of £1—

5. 10*s*.; 5*s*.; 2*s*. 6*d*.; 1*s*. 3*d*.

6. 15*s*.; 7*s*. 6*d*.; 12*s*. 6*d*.; 17*s*. 6*d*.

7. 6*s*. 8*d*.; 13*s*. 4*d*.; 3*s*. 4*d*.; 1*s*. 8*d*.

8. 4*s*.; 8*s*.; 12*s*.; 16*s*.; 1*s*. 4*d*.

9. 1*d*.; 2*d*.; 4*d*.; 3*d*.; $\frac{1}{2}d.$; $\frac{1}{4}d.$

10. 7*d*.; 11*d*.; $3\frac{1}{2}d.$; $4\frac{1}{4}d.$; $7\frac{3}{4}d.$

Express, in lowest terms—

11. 10*d*. as a fraction of 3*s*.

12. 1*s*. 6*d*. as a fraction of 7*s*. 6*d*.

13. $7\frac{1}{2}d.$ .... 2*s*.

14. $4\frac{1}{2}d.$ .... 3*s*. 3*d*.

15. 1*s*. 8*d*. .... 8*s*. 4*d*.

16. 2*s*. 9*d*. .... 9*s*. 3*d*.

17. 4*s*. $2\frac{1}{2}d.$ .... 11*s*. 6*d*.

18. 1*s*. $11\frac{3}{4}d.$ .... 14*s*. $8\frac{1}{4}d.$

19. 3*s*. $5\frac{1}{4}d.$ .... 10*s*.

20. 12*s*. 10*d*. .... 16*s*. 4*d*.

21. 17*s*. 6*d*. .... £2, 12*s*.

22. 13*s*. 4*d*. .... £3, 6*s*.

23. £1, 3*s*. 6*d*. .... £7, 5*s*.

24. £2, 1*s*. 8*d*. .... £3, 16*s*.

25. 9*s*. $7\frac{1}{2}d.$ .... £1.

26. 1*s*. $11\frac{1}{4}d.$ .... £1.

27. $\frac{2}{5}$ of $7\frac{1}{2}d.$ .... 2*s*. 6*d*.

28. $\frac{3}{11}$ of 1*s*. 10*d*. .... £2, 10*s*.

29. $\frac{2}{3}$ of $7\frac{1}{2}d.$ .... 4*s*. 3*d*.

30. $\frac{1}{5}$ of 7*s*. .... $\frac{2}{8}$ of 9*s*. 6*d*.

31. 2 qrs. 7 lbs. .... 1 cwt.

32. 19 lbs. 8 ozs. .... 1 cwt.

33. 2 cwts. 1 qr. .... 2 tons.

34. 14 cwts. 3 qrs. 7 lbs. 1 ton.

35. 17 yds. 1 ft. 6 in..... $\frac{1}{2}$ mi.

36. 5 fur. 18 po. .... 1 mile.

37. $10\frac{1}{2}$ poles .... 1 acre.

38. $5\frac{1}{2}$ in. .... 1 pole.

39. 16 hrs. 40 m. 48 s..... 1 day.

40. 4 oz. 7 dwt. 12 grs.... 5 oz. Tr.

Express—

41. £2, 8*s*. 9*d*. in *shillings*.

42. £5, 11*s*. $7\frac{1}{2}d.$ in *shillings*.

43. £3, 3*s*. $3\frac{3}{4}d.$ in *shillings*.

44. £2, 17*s*. $2\frac{1}{4}d.$ in *shillings*.

45. £14, 17*s*. 4*d*. in *pounds*.

46. £21, 8*s*. $4\frac{1}{2}d.$ in *pounds*.

47. £7, 8*s*. 9*d*. in *pounds*.

48. £7, 1*s*. $10\frac{1}{2}d.$ in *pounds*.

49. 7 cwt. 3 qr. 21 lbs. in *quarters*.

50. 1 ton 13 cwts. 3 qrs. in *cwts*.

51. 9 tons 12 cwts. 16 lbs. in *cwts*.

52. 3 cwts. 1 qr. 7 lbs. 8 ozs. in *qrs*.

53. 17 yds. 2 ft. 9 in. in *yards*.

54. 3 fur. 7 po. $2\frac{3}{4}$ yds. in *poles*.

55. 12 ac. 1 ro. 16 po. in *acres*.

56. 7 ac. 3 ro. 24 po. in *acres*.

57. How many times is £3, 17*s.* 6*d.* contained in £178, 5*s.*?

58. How many times is £1, 18*s.* 9*d.* contained in £120, 2*s.* 6*d.*?

59. What fraction of £50 is £8, 6*s.* 8*d.*?

60. What fraction of £22, 5*s.* 6*d.* is £1, 17*s.* $1\frac{1}{2}$*d.*?

61. What fraction of $9\frac{1}{4}$ guineas is $\frac{7}{16}$ of £9, 17*s.* 4*d.*?

62. What part of £24, 8*s.* $3\frac{3}{4}$*d.* is $\frac{1}{3}$ of £1, 17*s.* $6\frac{3}{4}$*d.*?

63. Reduce $2\frac{1}{8}$ of £5, 11*s.* to the fraction of $2\frac{5}{16}$ of £4, 5*s.*

64. Reduce $\frac{3}{5}\frac{1}{7}$ of £2, 2*s.* 9*d.* to the fraction of £2, 2*s.* $7\frac{1}{2}$*d.*

65. Express the sum of $\frac{5}{7}$ of a guinea, $\frac{3}{5}$ of £1, $\frac{2}{70}$ of 25*s.* and $\frac{1}{4}$ of 1*s.* as the fraction of £$4\frac{7}{20}$.

66. Express the difference between $\frac{2}{3}$ of £38, 12*s.* 6*d.* and $\frac{4}{5}$ of £47, 16*s.* 3*d.* as the fraction of £100.

67. Express 2 ft. 3 in. as the fraction of $4\frac{1}{2}$ yds.

68. What part of 1 mi. 30 po. is 2 yds. 3 in.?

69. Express $1\frac{1}{2}$ feet to the fraction of $\frac{1}{4}$ of a mile.

70. What fraction of 30 miles is 1 fur. 20 poles?

71. Reduce $\frac{3}{4}$ of a yard to the fraction of a mile.

72. Express 25 acres as the fraction of $1\frac{1}{2}$ sq. miles.

73. What fraction of 3 sq. po. is $\frac{1}{5}$ of 1 sq. po. 11 sq. yds.?

74. Reduce 4 ozs. 7 dwts. 12 grs. to the fraction of 5 ozs. Avoirdupois.

75. Reduce 22 days 4 hrs. 35 mins. 42 secs. to the fraction of 34 days 20 hrs. 56 min. 6 secs.

76. Express the sum of $\frac{2}{3}$ of 1 ton 3 cwts. 1 qr. and $\frac{2}{3}$ of 1 ton 1 cwt. 3 qrs. as a fraction of $\frac{4}{5}$ of 3 tons 1 cwt. 1 qr.

77. Reduce $\dfrac{3\frac{5}{6} - 4\frac{5}{8} + 5\frac{5}{12}}{5\frac{7}{10} - 6\frac{7}{15} + 7\frac{7}{20}}$ of 3 tons 19 cwts. to the fraction of 3 tons 14 cwts.

78. Reduce $\dfrac{2\frac{2}{11} + 3\frac{3}{22} - 4\frac{4}{33}}{2\frac{2}{11} - 3\frac{3}{22} + 4\frac{4}{33}}$ of 1 fur. 36 po. to the fraction of 5 fur. 37 po.

79. Reduce $\frac{1664}{1408}$ of $\left\{\frac{7}{10}\text{ of } £3\frac{3}{14} + 6\frac{2}{3}\text{ of } £3,\ 0s.\ 9d. - 4\frac{13}{31}\text{ of } £3,\ 2s.\right\}$ to the fraction of £100.

80. Reduce $\dfrac{16\frac{1}{2} - 8\frac{5}{8}\text{ of } 2\frac{2}{23} + 2\frac{2}{69}}{(16\frac{1}{2} - 8\frac{5}{8})\text{ of } (2\frac{2}{23} + 2\frac{2}{69})}$ of £18, 12*s.* 9*d.* to the fraction of a guinea.

XXIV. MISCELLANEOUS EXERCISES.

1. Write $\frac{19}{98}$ in words.
2. Write Ninety-one one hundred and forty-ninths in figures.
3. Reduce $7\frac{7}{17}$ to an improper fraction.
4. Reduce $\frac{77}{17}$ to a mixed number.
5. Express $\frac{7}{37}$ with denominator 777.
6. Reduce $\frac{1188}{1881}$ to its lowest terms.
7. Reduce $\frac{8}{77}$ and $\frac{7}{88}$ to their least common denominator.
8. Add together $\frac{2}{5}$, $\frac{7}{16}$, $\frac{7}{50}$, and $\frac{3}{2800}$.
9. Subtract $5\frac{5}{63}$ from $13\frac{3}{35}$.
10. A boy spent $\frac{5}{6}$ of his money and then had 4*d.* left. How much had he at first?

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11. Express $\frac{91}{90}$ in words.
12. Express in figures Seven hundred, and nineteen ninety-oneths.
13. Express $99\frac{9}{19}$ as an improper fraction.
14. Express $\frac{999}{19}$ as a mixed number.
15. Express $\frac{7}{17}$ with numerator 63.
16. Reduce $\frac{1818}{8181}$ to its lowest terms.
17. Which is greater, $\frac{17}{38}$ or $\frac{25}{57}$?
18. Add $8\frac{17}{18}$ to $8\frac{71}{81}$.
19. From $99\frac{9}{19}$ take $\frac{999}{19}$.
20. A boy, having spent $\frac{6}{7}$ of his pocket-money, had 9*d.* left. What had he at first?

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21. How many tenths are there in ten and one-tenth?
22. How many fifty-oneths make two-thirds?
23. Express $\frac{825}{2739}$ in its simplest form.
24. Find the value of $3\frac{23}{125} + 4\frac{23}{50}$.
25. Find the value of $23\frac{7}{740} - 19\frac{19}{20}$.
26. Find the value of $\frac{153}{308} \times \frac{77}{255}$.
27. Find the value of $365\frac{1}{4} \div 365$.
28. Simplify $\frac{1}{3} + \frac{1}{5} - \frac{6}{7} + \frac{2}{35} + \frac{4}{15}$.
29. What must be added to $\frac{1}{3}$ of $\frac{4}{7}$ of $\frac{3}{5}$ to make $\frac{1}{6}$?
30. A boy gave a third of his cake to one friend, half of what remained to another, and ate the rest. How much did he eat?

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31. How many units are there in Three-hundred-and-forty-three sevenths?

32. How many ninths are equivalent to thirty-six eighty-oneths?

33. Which is greater, $\frac{3663}{4477}$ or $\frac{9}{10}$?

34. Find the value of $2\frac{26}{75} \div 1\frac{31}{35}$.

35. Find the value of $95\frac{70}{143} - 74\frac{90}{91}$.

36. Find the value of $7\frac{13}{56} + 1\frac{23}{112}$.

37. Find the value of $4\frac{16}{17} \times 2\frac{61}{63}$.

38. Simplify $1\frac{1}{2} + 15\frac{7}{15} - 8\frac{5}{12} - 6\frac{4}{5}$.

39. From $2\frac{1}{3}$ of $4\frac{1}{19}$ take the sum of $2\frac{1}{3}$ and $4\frac{1}{19}$.

40. Half the arable land of a farm is planted with wheat, one-third with barley, one-twelfth with oats, and there are 5 acres of beans. How many acres of wheat are there?

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41. Convert $13\frac{13}{33}$ into an improper fraction.

42. Find a fraction with numerator 19 which shall be equal to $\frac{171}{747}$.

43. Which is greater, $\frac{19}{81}$ or $\frac{56}{243}$?

44. Add together $\frac{10}{17}$, $3\frac{6}{7}$, $5\frac{7}{51}$ and $\frac{9}{14}$.

45. Subtract $19\frac{15}{64}$ from $23\frac{11}{56}$.

46. Multiply together $3\frac{8}{9}$, $\frac{18}{49}$, $2\frac{10}{39}$ and $\frac{91}{165}$.

47. Divide $2\frac{7}{39}$ by $2\frac{31}{78}$.

48. Express in simplest form the sum of $\dfrac{1}{8\frac{1}{7}}$ and $\dfrac{1}{7\frac{1}{8}}$.

49. Simplify $12\frac{1}{2} - 11\frac{5}{16} + \frac{13}{16}$.

50. A mowed $\frac{1}{3}$, B mowed $\frac{1}{4}$, and C mowed $\frac{2}{5}$, of a field. What part of the field then remained unmown?

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51. Reduce $\frac{2018}{101}$ to a mixed number.

52. Express $\frac{41}{77}$ with denominator 231.

53. Reduce $\frac{1881}{8118}$ to its lowest terms.

54. Add together $\frac{14}{39}$, 10 and $4\frac{9}{65}$.

55. Subtract $3\frac{9}{95}$ from $7\frac{2}{57}$.

56. Multiply together $\frac{88}{245}$, $8\frac{55}{64}$, $\frac{80}{121}$ and $2\frac{23}{27}$.

57. Divide $3\frac{71}{156}$ by $1\frac{169}{216}$.

58. Simplify $2\frac{1}{5} + 1\frac{4}{7} - 4\frac{1}{2} - \frac{3}{5} + 2\frac{8}{35}$.

59. How many bits each $1\frac{1}{5}$ inches long can be cut from a yard?

60. A boy after spending $\frac{4}{5}$ of his money had $7\frac{1}{2}d.$ left. How much had he at first?

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61. How many seventieths are there in seven and one-seventh?

62. Reduce $\frac{1188}{8811}$ to its lowest terms.

63. Add $2\frac{7}{45}$, $\frac{13}{165}$, $\frac{17}{60}$ and $5\frac{7}{9}$.

64. What must be taken from $65132\frac{7}{8}$ to leave $51423\frac{11}{12}$?

65. Find the continued product of
$1\frac{1}{2}$, $1\frac{1}{3}$, $1\frac{1}{4}$, $1\frac{1}{5}$, $1\frac{1}{6}$, $1\frac{1}{7}$, $1\frac{1}{8}$ and $1\frac{1}{9}$.

66. Divide the sum of $\frac{7}{8}$ and $\frac{11}{12}$ by $1\frac{19}{24}$.

67. Find, in its simplest form, the value of $\dfrac{6\frac{47}{138}}{6\frac{21}{184}}$.

68. Simplify $1\frac{2}{3}$ of $2\frac{1}{2}$ of $3\frac{3}{5} + 4\frac{1}{5}$ of $5\frac{5}{7} - 6$.

69. Which is greater, three-fourths of 11 yds. 0 ft. 8 in., or half of 16 yds. 2 ft. 7 in.?

70. How long will a journey of 130 miles take, at the rate of $33\frac{1}{3}$ miles per hour?

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71. How many ninetieths are there in nine and one-ninth?

72. Reduce $\frac{1786}{2451}$ to lowest terms.

73. Find the value of $\dfrac{1}{2 \times 3 \times 4} + \dfrac{1}{3 \times 4 \times 5} + \dfrac{1}{4 \times 5 \times 6}$.

74. Take half the sum of the greatest and least of the three fractions $\frac{8}{9}$, $\frac{5}{6}$ and $\frac{13}{15}$ from the other.

75. What is the value of $4 \times (\frac{2}{3} - \frac{3}{4}$ of $\frac{8}{9})$?

76. Divide $23\frac{1}{25}$ by $3\frac{122}{175}$.

77. Simplify $12\frac{3}{5} + 11\frac{2}{5} \times 13\frac{5}{9} - 3\frac{5}{9}$.

78. Simplify $(12\frac{3}{5} + 11\frac{2}{5}) \times (13\frac{5}{9} - 3\frac{5}{9})$.

79. Express £$3\frac{9}{16}$ in £, *s.* *d.*

80. How long will a man who walks $3\frac{3}{4}$ miles per hour, be in walking $35\frac{1}{4}$ miles?

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81. Express $\frac{27}{31}$ with numerator 999.

82. Compare $\frac{5}{6}$, $\frac{5+6}{6+7}$, and $\frac{6}{7}$.

83. What must be added to $9\frac{13}{14}$ that the sum may be $19\frac{1}{2}$?

84. Find the value of $450362 \div 5\frac{1}{2}$.

85. Divide $\frac{1}{82} + \frac{1}{86}$ by $\frac{1}{82} - \frac{1}{86}$.

86. Simplify $1\frac{2}{3} - \frac{6}{8} + \frac{4}{5} - \frac{20}{24} - \frac{2}{5} - \frac{1}{2} + \frac{1}{60}$.

87. Simplify $\frac{5}{12} \times \frac{9}{16} \times 2\frac{2}{11} + \frac{215}{440}$.

88. Find $\frac{3}{7}$ of £5, 18*s.* 5*d.*

89. What fraction of 2 cwts. is 7 ozs.?

90. When 16 pages of a book of 112 pages have been read, what fraction of the book remains to be read?

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91. Express $\frac{1}{3}$ with denominator 777.

92. Which is greatest: $\dfrac{11 \times 9}{12 \times 10}$, $\dfrac{11 + 9}{12 + 10}$, or $\dfrac{11 \div 9}{12 \div 10}$?

93. What proper fraction, added to the sum of $47\frac{3}{5}$, $83\frac{5}{8}$ and $29\frac{7}{12}$, makes the result integral?

94. Divide the sum of $\frac{1}{6}$ and $\frac{2}{9}$ by their difference.

95. Simplify $71\frac{4}{7} - 39\frac{33}{110} + 41\frac{5}{12} - 2\frac{17}{35} - \frac{4}{63} + 11\frac{15}{28}$.

96. Simplify $\dfrac{3\frac{1}{3} \times 3\frac{1}{3} \times 3\frac{1}{3} - 35}{2\frac{1}{2} \times 2\frac{1}{2} \times 2\frac{1}{2} - 8\frac{3}{4}}$.

97. Divide £4, 3*s.* 9*d.* by $2\frac{2}{3}$.

98. From $\frac{2}{7}$ of $\frac{5}{6}$ of £1, 1*s.* take $\frac{2}{5}$ of $\frac{3}{8}$ of £1.

99. Express £1, 4*s.* $0\frac{3}{4}$*d.* as the fraction of £33.

100. A pole 25 feet long is sawn up into an exact number of lengths of 2 ft. 3 in. each: what fraction of the pole becomes saw-dust?

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101. What integer is equal to $\frac{323}{17}$?

102. Simplify $\dfrac{68 \times 27 \times 45}{51 \times 81 \times 100}$.

103. Find, *by inspection*, the sum of $3\frac{3}{8}$, $7\frac{1}{6}$, $14\frac{2}{9}$, $5\frac{5}{8}$, $1\frac{5}{6}$ and $3\frac{7}{9}$.

104. Find the continued product of $\dfrac{5\frac{1}{22}}{6\frac{15}{16}}$, $\dfrac{6\frac{9}{17}}{10\frac{1}{11}}$, $\dfrac{7\frac{13}{14}}{15\frac{6}{7}}$ and $\dfrac{13\frac{7}{8}}{27\frac{3}{4}}$.

105. Divide $3\frac{5}{14} + 2\frac{4}{9} - 4\frac{19}{21}$ by $3\frac{37}{63}$.

106. What number added to $2\frac{1}{3}$ of $\frac{5}{7} + 2\frac{1}{4}$ of $2\frac{2}{3}$ will make $9\frac{2}{3}$?

107. Simplify $\dfrac{(2\frac{1}{2} + 3\frac{1}{3}) \times (4\frac{1}{4} - 1\frac{1}{5})}{(2\frac{1}{2} + 3\frac{1}{3} \text{ of } 4\frac{1}{4}) \div 1\frac{1}{5}}$.

108. Express £$7\frac{47}{960}$ + £$\frac{61}{240}$ in £, *s. d.*

109. Reduce 2 cwts. 22 lbs. 8 ozs. to the fraction of 33 tons 17 cwts. 3 qrs. 14 lbs.

110. Divide £4, 11*s.* 7*d.* between A and B, so that A may have a third as much again as B.

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111. Express $\frac{351}{428}$ with denominator 1712.

112. Divide $63 \times 64 \times 65$ by $26 \times 27 \times 28$.

113. Add $6\frac{91}{198}$, $5\frac{63}{286}$, $19\frac{23}{156}$ and $27\frac{22}{39}$.

114. Find the value of $\frac{3}{10}$ of $\frac{5}{18} - \frac{2}{9}$ of $\frac{3}{8}$.

115. Divide $1\frac{59}{60}$ by the difference between $\frac{11}{24}$ and $\frac{7}{40}$.

116. What must be taken from 10 that the result may be equal to $3\frac{1}{2} - 2\frac{1}{5} + 1\frac{1}{6}$?

117. Simplify $\left(\dfrac{97}{194} + \dfrac{291}{388}\right) \div \left(\dfrac{485}{582} - \dfrac{291}{388}\right)$.

118. Express $£11\frac{97}{120} - £6\frac{107}{430}$ in £, *s.* *d.*

119. Reduce 10 yds. 2 ft. $7\frac{1}{9}$ ins. to the fraction of $\frac{1}{6}$ of a mile.

120. Divide £19, 19*s.* $4\frac{1}{2}$*d.* between A and B, so that A may have half as much again as B.

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121. Find the value of 420 lbs. of sugar at $2\frac{3}{8}$*d.* per lb.

122. Which is greater, $\frac{3}{5}$ of $\dfrac{2\frac{1}{3}}{4\frac{1}{4}}$, or $\frac{5}{7}$ of $\dfrac{1\frac{1}{3}}{2\frac{5}{6}}$?

123. What must be subtracted from the sum of $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$ and $\frac{5}{6}$ that an even integer may be left?

124. How many times does the sum of $63\frac{1}{8}$ and $72\frac{1}{7}$ contain their difference?

125. How many times can $\frac{2}{7}$ be taken from $4\frac{3}{5}$, and what remains?

126. Find the value of $120 \times (\frac{1}{21} + \frac{1}{35} + \frac{1}{49} - \frac{1}{28} - \frac{1}{42} - \frac{1}{56})$.

127. Simplify $\dfrac{2\frac{1}{13} \text{ of } \frac{5}{9} - 1\frac{1}{8} \text{ of } \frac{2}{3}}{1\frac{1}{21} \div 5\frac{4}{7}}$.

128. By what fraction of a penny does $\frac{3}{100}$ of £1 exceed $\frac{3}{103}$ of £1, 0*s.* 3*d.*?

129. James has 70 marbles; he gives $\frac{3}{7}$ of them to George, and $\frac{4}{5}$ of those which remain to Charles; how many has he left then?

130. A can do a piece of work in $2\frac{1}{2}$ days, and B in $3\frac{1}{3}$ days; what portion of the work will they together do in one day?

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131. Find the cost of $43052\frac{1}{2}$ lbs. at 2*s.* 6*d.* per lb.

132. A walks $20\frac{1}{2}$ miles in 6 hours, and B $16\frac{3}{4}$ miles in $4\frac{1}{2}$ hours. Which walks at the faster rate?

133. Find the value of $9\frac{4}{13} + 14\frac{23}{132} + 3\frac{17}{24} + 7\frac{9}{143}$.

134. By how much does $43\frac{44}{51}$ exceed $37\frac{33}{34}$?

135. Divide $1\frac{1}{2}$ of $1\frac{1}{2}$ of $1\frac{1}{2}$ by $1\frac{1}{2} + 1\frac{1}{2} + 1\frac{1}{2}$.

136. Find the value of $7\frac{1}{7} + \dfrac{7\frac{1}{7}}{7} + \dfrac{7}{7\frac{1}{7}} + \dfrac{7\frac{1}{7} + 7}{7\frac{1}{7} - 7} + 7 \times 7\frac{1}{7}$.

137. Simplify $\dfrac{5\frac{3}{4} - \frac{2}{7} \text{ of } 15\frac{3}{4} + 2\frac{2}{35} \div 1\frac{11}{25}}{\frac{3}{4} \text{ of } 7\frac{3}{7} - 5\frac{3}{5} \div 3\frac{4}{15}}$.

138. Find the value of $6\frac{2}{5}$ of $\dfrac{6\frac{3}{7} \text{ of } 6\frac{1}{8}}{6\frac{2}{9} \text{ of } 6\frac{3}{10}}$ of $6\frac{1}{15}$ of a guinea.

139. A man left $\frac{1}{4}$ of his property to his wife, $\frac{1}{10}$ to each of his 3 daughters, and the residue to his only son. What part of the property did the son receive?

140. A alone can reap a field in $2\frac{1}{2}$ days, B alone in $2\frac{1}{3}$ days. If they work together for one day, what fraction of the field will then remain unreaped?

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141. Multiply £7, 11*s.* 4*d.* by $37\frac{1}{7}$.

142. Find both the G.C.M. and the L.C.M. of $\frac{3}{4}$, $\frac{5}{8}$ and $\frac{7}{12}$.

143. Add $\dfrac{1}{10\frac{1}{11}}$ to $\dfrac{1}{11\frac{1}{10}}$.

144. Find the value of $(\frac{1}{20}+\frac{1}{30})+(\frac{1}{20}+\frac{1}{30})^2+(\frac{1}{20}+\frac{1}{30})^3$.

145. Simplify $\dfrac{5\frac{1}{5}+3\frac{1}{3}}{5\frac{1}{5}\times 3\frac{1}{3}} \div \dfrac{5\frac{1}{5}-3\frac{1}{3}}{5\frac{1}{5}\div 3\frac{1}{3}}$.

146. Write down at sight the number of days, hrs. &c., in $\dfrac{21\dfrac{3\frac{1}{2}}{24}}{365}$ of a year.

147. If 40 shillings weigh 7 ozs. 5 dwts. and contain 10 dwts. 21 grs. of alloy, how much pure silver is there in a shilling?

148. A surveyor's chain had one link bent, and was consequently $\frac{3}{8}$ of an inch too short; what was the error in measuring a mile?

149. If a man went $\frac{16}{19}$ of his journey by train, $\frac{5}{57}$ of it by tram, and walked the rest, what fraction of his journey did he walk?

150. Divide £11, 3*s.* $11\frac{1}{2}$*d.* among 44 men, giving one of them $\frac{3}{5}$ of the whole and the rest equal shares.

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151. Multiply £2, 19*s.* 9*d.* by $1\frac{81}{239}$.

152. Find the G.C.M. and the L.C.M. of $2\frac{2}{5}$ and $3\frac{3}{7}$.

153. Take $\dfrac{7\frac{7}{8}}{17\frac{17}{18}}$ from $\dfrac{14\frac{2}{3}}{28\frac{2}{23}}$.

154. Find the value of $\dfrac{1}{5\times7\times11}+\dfrac{1}{3\times7\times11}+\dfrac{1}{3\times5\times11}+\dfrac{1}{3\times5\times7}$.

155. Express the product of $\frac{221}{629}$ and $\frac{72816}{8528}$ in simplest form.

156. Simplify $\dfrac{(11\frac{3}{7}-7\frac{3}{11})\div 11\frac{3}{7} \text{ of } 7\frac{3}{11}}{7\frac{3}{7} \text{ of } 11\frac{3}{11}\div(7\frac{3}{7}+11\frac{3}{11})}$.

157. Write down the number of cwts., qrs., &c., in $\dfrac{17\dfrac{3\frac{5}{14}}{4}}{20}$ of a ton.

158. If a penny stamp is $\frac{15}{16}$ of an inch long, how many placed in a row would extend a mile?

159. If $\frac{7}{15}$ of a guinea be taken from $\frac{3}{10}$ of $\frac{5}{9}$ of £5, what fraction of £3, 8*s.* 8*d.* will remain?

160. A and B start together on bicycles for a place 45 miles distant: A goes $7\frac{1}{2}$, and B $6\frac{1}{4}$ miles an hour; how much sooner will A arrive than B?

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161. Express $\frac{22}{35}$ as a complex fraction having numerator 1.

162. Multiply 17 tons 10 cwts. 1 qr. 5 lbs. by $13\frac{1}{3}$.

163. How many times does the square of 1001 contain the product of 91 and 143?

164. Compare $\frac{7}{11}$, $\frac{7-4}{11-4}$ and $\frac{7+4}{11+4}$.

165. Find, as shortly as possible, the product of $999\frac{358}{359}$ and 287.

166. What must be added to $\dfrac{4\frac{1}{5}-3\frac{1}{6}}{4\frac{1}{5}+3\frac{1}{6}} - 3 \div 24\frac{5}{9}$ that the sum may be 2?

167. Simplify $1 - \dfrac{2}{7-\dfrac{3}{4-\frac{1}{2}}} + \left(\dfrac{3\frac{1}{3}}{7} + \dfrac{2}{10\frac{1}{2}} - \frac{5}{18} \times \frac{4}{7}\right) \times \frac{13}{43}$ of $1\frac{31}{32}$.

168. Find the average of the fractions $\frac{2}{3}$, $\frac{4}{5}$, $\frac{5}{6}$, $\frac{9}{10}$ and $\frac{7}{15}$.

169. By how many inches is 3 fur. 22 po. $1\frac{1}{9}$ yds. less than $\frac{4}{9}$ of a mile?

170. What fraction of the day is the sun above the horizon when it rises at 7 h. 3 m. and sets at 5 h. 24 m.?

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171. Express $\frac{1}{7}$ with denominator consisting entirely of *threes*.

172. Divide 3964 tons 17 cwts. 1 qr. 11 lbs. by $27\frac{1}{2}$.

173. How many times does the continued product of 72, 73, 74 and 75 contain 555?

174. Arrange $\frac{31}{91}$, $\frac{41}{119}$ and $\frac{51}{143}$ in ascending order of magnitude.

175. Multiply $99\frac{273}{274}$ by 167, shortly.

176. In a book on Arithmetic a question was printed thus:—
"Add $3\frac{7}{8}$, $4\frac{\ }{12}$, $5\frac{8}{9}$"; the answer given was $14\frac{13}{72}$; find the missing figure.

177. Take $\dfrac{1}{1+\dfrac{1}{1+\dfrac{1}{1\frac{1}{5}}}}$ of $\dfrac{1\frac{1}{11}}{11\frac{1}{5}-5\frac{1}{11}}$ from $1+\frac{1}{11}$ of $1\frac{1}{4}$ of $2\frac{3}{7}$.

178. Find the product of $\dfrac{\frac{108}{111}+\frac{1}{37}}{1\frac{46}{1113}-\frac{53}{371}}$ and $\dfrac{\frac{97}{101}+\frac{4}{101}}{1\frac{97}{1001}-\frac{14}{143}}$.

179. How many feet are there in
$\frac{3}{35}$ of a mile $+ \frac{16}{21}$ of a furlong $- \frac{2}{77}$ of a pole?

180. During what fraction of the day was there sunshine, if the sun rose at 6.46, was obscured by clouds from 11.30 to 1.17, and set at 5.39?

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181. The numerators of two equal fractions are 323 and 901; the denominator of the former is 437; find the denominator of the latter.

182. Find the smallest integer which exactly contains each of the following mixed numbers: $9\frac{6}{7}$, $8\frac{3}{14}$, $7\frac{11}{28}$.

183. Arrange the fractions $\frac{7}{22}$, $\frac{8}{25}$, $\frac{9}{28}$, $\frac{10}{31}$ in descending order of magnitude.

184. Add together $\frac{1}{25\frac{1}{2}}$, $\frac{1}{12\frac{3}{4}}$, $\frac{1}{10\frac{1}{5}}$, $\frac{1}{7\frac{2}{7}}$, $\frac{1}{6\frac{3}{8}}$, $\frac{1}{5\frac{1}{10}}$, $\frac{1}{4\frac{7}{11}}$ and $\frac{1}{3\frac{12}{13}}$.

185. Find, with as little work as possible, the sum of $9\frac{1}{3} \times 94\frac{3}{8}$ and $7\frac{1}{2} \times 83\frac{8}{9}$.

186. How many lengths of $2\frac{2}{3}$ feet can be cut from 1001 yards, and how many inches will be left over?

187. Simplify $\dfrac{(\frac{1}{2}+\frac{1}{5})(\frac{1}{2}+\frac{1}{6})}{(\frac{1}{2}-\frac{1}{3})(\frac{1}{2}-\frac{1}{4})} - \dfrac{(\frac{1}{3}+\frac{1}{5})(\frac{1}{3}+\frac{1}{6})}{(\frac{1}{2}-\frac{1}{3})(\frac{1}{3}-\frac{1}{4})} + \dfrac{(\frac{1}{4}+\frac{1}{5})(\frac{1}{4}+\frac{1}{6})}{(\frac{1}{2}-\frac{1}{4})(\frac{1}{3}-\frac{1}{4})}$.

188. What fraction is that from which if $\frac{2}{3}$ of $\dfrac{3-1\frac{1}{4}}{2\frac{2}{5}}$ be taken and the remainder be divided by $\dfrac{5\frac{1}{8}}{13\frac{2}{3}}$, the result is $\frac{1}{3}$?

189. Express the difference between 1 oz. Troy, and 1 oz. Avoirdupois, as the fraction of their sum.

190. A boy, instead of dividing a certain fraction by $\frac{5}{7}$, multiplied it by $\frac{5}{7}$ in mistake, and obtained the result $\frac{20}{29}$. What was the correct answer?

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191. The denominators of two equal fractions are 11907 and 7497: the numerator of the first is 5103; find the numerator of the second.

192. Find the smallest integer which contains $19\frac{1}{4}$, $19\frac{4}{5}$ and $19\frac{5}{9}$ each an exact number of times.

193. Arrange the fractions $\frac{5}{41}$, $\frac{2}{17}$, $\frac{3}{25}$, $\frac{4}{33}$ in ascending order of magnitude.

194. Find the sum of $\frac{4141}{5151}$, $\frac{3131}{3434}$ and $\frac{5757}{6868}$.

195. Find, with as little labour as possible, the difference between $53\frac{2}{3} \times 2\frac{1}{3}$ and $32\frac{1}{5} \times 3\frac{1}{2}$.

196. How many times can $3\frac{1}{5}$ be taken from $29\frac{7}{8}$, and what remains?

197. Find the value of $\left\{1 + \dfrac{1}{\frac{1}{2\frac{1}{2}} + \frac{1}{3\frac{1}{2}} + \frac{1}{4\frac{1}{2}}} \div \left[\left(\frac{1}{11}+\frac{1}{13}\right)\left(\frac{1}{3}+\frac{1}{4}\right)\right]\right\}$ of £4.

198. What fraction is that from which if $\dfrac{3\frac{3}{4}}{4\frac{4}{5}}$ be taken, and the remainder be divided by $\dfrac{4\frac{3}{4}}{3\frac{4}{5}}$, the result is $\frac{3}{40}$?

199. Find the number of grains in 1 cwt. 1 qr. 2 lbs. $11\frac{3}{7}$ ozs.

200. If there is three times as much sea as land on the surface of the globe, and if three-fourths of all the land is in the northern hemisphere, what fraction of the surface of the southern hemisphere is water?

XXV. SIMPLE PRACTICE.

Write down the value of the following aliquot parts—

1. $\frac{1}{2}$; $\frac{1}{3}$; $\frac{1}{4}$; $\frac{1}{6}$; $\frac{1}{8}$; $\frac{1}{12}$; $\frac{1}{16}$; of a shilling.
2. $\frac{1}{2}$; $\frac{1}{3}$; $\frac{1}{4}$; $\frac{1}{5}$; $\frac{1}{6}$; $\frac{1}{8}$; $\frac{1}{10}$; $\frac{1}{12}$; $\frac{1}{15}$; $\frac{1}{16}$; $\frac{1}{20}$; $\frac{1}{30}$; $\frac{1}{40}$; $\frac{1}{60}$; $\frac{1}{80}$; of £1

What aliquot part

3. of £1 is 5*s.*; 4*s.*; 2*s.*; 6*s.* 8*d.*; 3*s.* 4*d.*; 1*s.* 8*d.*; 2*s.* 6*d.*; 1*s.* 3*d.*?
4. of 10*s.* is 5*s.*; 2*s.* 6*d.*; 1*s.* 3*d.*; 3*s.* 4*d.*; 1*s.* 8*d.*; 10*d.*; 2*s.*; 1*s.*; 6*d.*?
5. of 5*s.* is 2*s.* 6*d.*; 1*s.* 3*d.*; 7½*d.*; 1*s.*; 6*d.*; 3*d.*; 5*d.*; 4*d.*; 2*d.*; 1*d.*?
6. of 2*s.* 6*d.* is 1*s.* 3*d.*; 7½*d.*; 3¾*d.*; 6*d.*; 3*d.*; 1½*d.*; 10*d.*; 5*d.*; 2½*d.*?
7. of 1*s.* 3*d.* is 7½*d.*; 3¾*d.*; 5*d.*; 2½*d.*; 1¼*d.*; 3*d.*; 1½*d.*; ¾*d.*?
8. of a shilling is 6*d.*; 3*d.*; 1½*d.*; ¾*d.*; 4*d.*; 2*d.*; 1*d.*?
9. of 6*s.* 8*d.* is 3*s.* 4*d.*; 1*s.* 8*d.*; 10*d.*; 5*d.*; 1*s.* 4*d.*; 8*d.*; 4*d.*; 2*d.*; 1*d.*?
10. of 3*s.* 4*d.* is 1*s.* 8*d.*; 10*d.*; 5*d.*; 2½*d.*; 8*d.*; 4*d.*; 2*d.*; 1*d.*?

Find, using only *one* aliquot part, the cost of—

11. 462 at 5*s.*	17. 545 at 2*s.* 6*d.*	23. 1541 at 6*s.* 8*d.*
12. 387 at 5*s.*	18. 673 at 2*s.* 6*d.*	24. 1253 at 6*s.* 8*d.*
13. 742 at 4*s.*	19. 1751 at 2*s.* 6*d.*	25. 89 at 3*s.* 4*d.*
14. 635 at 4*s.*	20. 1903 at 2*s.* 6*d.*	26. 97 at 3*s.* 4*d.*
15. 1423 at 2*s.*	21. 430 at 6*s.* 8*d.*	27. 385 at 1*s.* 8*d.*
16. 1978 at 2*s.*	22. 532 at 6*s.* 8*d.*	28. 769 at 1*s.* 8*d.*
29. 1705 at 6*d.*	33. 871 at 4*d.*	37. 1432 at 1½*d.*
30. 2371 at 6*d.*	34. 473 at 4*d.*	38. 2453 at 1½*d.*
31. 234 at 3*d.*	35. 1267 at 2*d.*	39. 978 at 1½*d.*
32. 179 at 3*d.*	36. 2578 at 2*d.*	40. 1023 at 1½*d.*

Find, using only *two* aliquot parts, the cost of—

41. 782 at 11*s.*	45. 93 at 12*s.*	49. 4266 at 12*s.* 6*d.*
42. 1013 at 11*s.*	46. 107 at 12*s.*	50. 5711 at 12*s.* 6*d.*
43. 987 at 6*s.*	47. 809 at 7*s.* 6*d.*	51. 87 at 3*s.* 9*d.*
44. 1761 at 6*s.*	48. 1205 at 7*s.* 6*d.*	52. 191 at 3*s.* 9*d.*
53. 8432 at 7*d.*	59. 999 at 7½*d.*	65. 2431 at 6½*d.*
54. 7065 at 7*d.*	60. 1111 at 7½*d.*	66. 4315 at 6½*d.*
55. 1653 at 5*d.*	61. 523 at 1¼*d.*	67. 237 at 3¾*d.*
56. 2089 at 5*d.*	62. 684 at 1¼*d.*	68. 549 at 3¾*d.*
57. 345 at 4½*d.*	63. 133 at 2¼*d.*	69. 810 at 6¾*d.*
58. 543 at 4½*d.*	64. 267 at 2¼*d.*	70. 1103 at 6¾*d.*

Find, using only *one* aliquot part, the cost of—

71. 622 at £1, 10*s.*
72. 903 at £1, 5*s.*
73. 733 at £1, 4*s.*
74. 271 at £1, 2*s.*
75. 423 at £1, 2*s.* 6*d.*
76. 547 at £1, 2*s.* 6*d.*
77. 94 at £1, 6*s.* 8*d.*
78. 83 at £1, 3*s.* 4*d.*
79. 152 at £1, 1*s.* 8*d.*
80. 205 at £1, 1*s.* 8*d.*
81. 47 at £3, 5*s.*
82. 68 at £2, 4*s.*
83. 156 at £5, 2*s.* 6*d.*
84. 171 at £11, 2*s.* 6*d.*
85. 344 at £7, 2*s.*
86. 287 at £6, 10*s.*
87. 1631 at £2, 6*s.* 8*d.*
88. 2350 at £4, 3*s.* 4*d.*

Find, using only *one* aliquot part, the cost of—

89. 7353 at 1*s.* 6*d.*
90. 8649 at 1*s.* 3*d.*
91. 5732 at 1*s.* 4*d.*
92. 4642 at 1*s.* 2*d.*
93. 784 at 1*s.* 1½*d.*
94. 809 at 1*s.* 1½*d.*
95. 40863 at 1*s.* 1*d.*
96. 59791 at 1*s.* 1*d.*
97. 144 at 6*s.* 3*d.*
98. 288 at 5*s.* 4*d.*
99. 96 at 11*s.* 1½*d.*
100. 435 at 9*s.* 1½*d.*

Find, using only *one* aliquot part, the cost of—

101. 137 at 15*s.*
102. 215 at 15*s.*
103. 78 at 16*s.*
104. 82 at 18*s.*
105. 429 at 18*s.*
106. 683 at 17*s.* 6*d.*
107. 514 at 17*s.* 6*d.*
108. 2103 at 13*s.* 4*d.*
109. 1635 at 17*s.* 6*d.*
110. 831 at 16*s.* 8*d.*
111. 34 at £1, 17*s.* 6*d.*
112. 129 at £1, 18*s.*
113. 65 at £1, 16*s.* 8*d.*
114. 73 at £1, 18*s.* 4*d.*
115. 138 at £3, 15*s.*
116. 182 at £4, 17*s.* 6*d.*
117. 29 at £9, 16*s.*
118. 45 at £9, 13*s.* 4*d.*

Find, using only *one* aliquot part, the cost of—

119. 1434 at 9*d.*
120. 2067 at 8*d.*
121. 536 at 10*d.*
122. 487 at 10½*d.*
123. 158 at 11*d.*
124. 87 at 1*s.* 11*d.*
125. 105 at 1*s.* 9*d.*
126. 235 at 1*s.* 10*d.*
127. 653 at 1*s.* 10½*d.*
128. 809 at 1*s.* 10½*d.*
129. 444 at 3*s.* 11*d.*
130. 156 at 9*s.* 11*d.*
131. 72 at 11*s.* 10*d.*
132. 98 at 7*s.* 8*d.*
133. 126 at 4*s.* 9*d.*
134. 261 at 2*s.* 11*d.*
135. 1234 at 9*s.* 10½*d.*
136. 2431 at 7*s.* 10½*d.*

Find the cost of—

137. 279 at £1, 6*s.*
138. 217 at £1, 11*s.*
139. 825 at £1, 12*s.* 6*d.*
140. 1473 at £1, 7*s.* 6*d.*
141. 1503 at £1, 13*s.*
142. 2387 at £1, 11*s.* 3*d.*
143. 163 at £1, 13*s.* 7*d.*
144. 218 at £1, 7*s.* 9½*d.*
145. 132 at £2, 12*s.* 9*d.*
146. 147 at £5, 7*s.* 10*d.*
147. 253 at £3, 4*s.* 8*d.*
148. 183 at £4, 11*s.* 8*d.*
149. 710 at £12, 6*s.* 4½*d.*
150. 520 at £14, 13*s.* 2½*d.*
151. 1162 at £21, 3*s.* 1*d.*
152. 1044 at £36, 11*s.* 1*d.*
153. 1430 at £1, 3*s.* 2¼*d.*
154. 1560 at £1, 2*s.* 7¾*d.*
155. 376 at £51, 12*s.* 5½*d.*
156. 264 at £73, 1*s.* 9¼*d.*

Find the cost of—

157. 7931 at $2\frac{3}{4}$d.
158. 2107 at $5\frac{1}{4}$d.
159. 842 at $6\frac{3}{4}$d.
160. 956 at $5\frac{3}{4}$d.
161. 214 at 1s. $3\frac{1}{4}$d.
162. 184 at 1s. $5\frac{1}{2}$d.
163. 137 at 1s. $7\frac{1}{4}$d.
164. 175 at 1s. $4\frac{3}{4}$d.
165. 84 at 3s. $1\frac{1}{4}$d.
166. 96 at 5s. $2\frac{1}{4}$d.
167. 55 at 8s. $4\frac{1}{2}$d.
168. 34 at 11s. $7\frac{1}{2}$d.
169. 103 at 9s. $1\frac{1}{2}$d.
170. 142 at 12s. $1\frac{1}{4}$d.
171. 76 at 14s. $5\frac{1}{2}$d.
172. 94 at 10s. $3\frac{3}{4}$d.
173. 462 at 13s. $6\frac{1}{4}$d.
174. 895 at 7s. $7\frac{3}{4}$d.

Find the cost of—

175. 975 at £42, 5s. $8\frac{1}{4}$d.
176. 857 at £56, 2s. $7\frac{1}{4}$d.
177. 624 at £13, 11s. $5\frac{1}{2}$d.
178. 432 at £23, 5s. $7\frac{3}{4}$d.
179. 1358 at £26, 10s. $9\frac{1}{4}$d.
180. 1023 at £38, 11s. $7\frac{1}{4}$d.
181. 356 at £29, 5s. $2\frac{1}{2}$d.
182. 269 at £19, 7s. $5\frac{1}{4}$d.
183. 634 at £1, 8s. 6d.
184. 657 at £1, 8s. 8d.
185. 325 at £2, 9s. 4d.
186. 317 at £2, 9s. 5d.
187. 327 at £1, 14s. $6\frac{1}{2}$d.
188. 123 at £3, 14s. 7d.
189. 1871 at £1, 11s. $0\frac{1}{4}$d.
190. 4035 at £2, 4s. $0\frac{1}{2}$d.
191. 753 at £1, 7s. $0\frac{1}{4}$d.
192. 888 at £1, 9s. $0\frac{1}{2}$d.
193. 257 at £5, 5s. $0\frac{3}{4}$d.
194. 183 at £6, 14s. $0\frac{1}{2}$d.

Find, as shortly as you can, the cost of—

195. 4657 at 19s.
196. 4214 at 19s. 6d.
197. 964 at 17s. 9d.
198. 1022 at 18s. 10d.
199. 558 at £2, 16s. $10\frac{1}{2}$d.
200. 427 at £3, 15s. 8d.
201. 2345 at 18s. $7\frac{3}{4}$d.
202. 3333 at 17s. $9\frac{3}{4}$d.
203. 226 at £2, 18s. $8\frac{1}{2}$d.
204. 144 at £3, 19s. 2d.
205. 413 at £4, 16s. 10d.
206. 285 at £9, 17s. $7\frac{3}{4}$d.
207. 567 at 2s. $9\frac{3}{4}$d.
208. 286 at 4s. $10\frac{1}{4}$d.
209. 1032 at 9s. $11\frac{1}{4}$d.
210. 1796 at 9s. $8\frac{1}{4}$d.

Find the cost of—

211. $149\frac{1}{2}$ at £2, 6s. 6d.
212. $303\frac{1}{2}$ at £1, 4s. 7d.
213. $73\frac{1}{4}$ at £2, 2s. 4d.
214. $87\frac{1}{4}$ at £3, 7s. 3d.
215. $2132\frac{1}{4}$ at £2, 12s. 5d.
216. $1467\frac{1}{2}$ at £2, 17s. $9\frac{1}{2}$d.
217. $38\frac{3}{4}$ at £1, 3s. 1d.
218. $42\frac{3}{8}$ at £1, 10s. 10d.
219. $153\frac{1}{3}$ at £2, 2s. 3d.
220. $137\frac{2}{3}$ at £2, 3s. 3d.
221. $38\frac{3}{4}$ at £1, 5s. 5d.
222. $97\frac{1}{6}$ at £1, 18s. $10\frac{1}{2}$d.
223. $245\frac{3}{10}$ at £1, 13s. 9d.
224. $743\frac{1}{12}$ at £1, 9s. 9d.
225. $721\frac{13}{20}$ at £1, 11s. 8d.
226. $531\frac{5}{6}$ at £2, 14s. 9d.
227. $423\frac{1}{2}$ at 17s. $10\frac{1}{2}$d.
228. $532\frac{3}{5}$ at 18s. $10\frac{1}{4}$d.

229. $49\frac{1}{2}$ at 3*s*. $5\frac{1}{2}$*d*.

230. $73\frac{1}{4}$ at 2*s*. 4*d*.

231. $423\frac{1}{2}$ at 1*s*. $2\frac{1}{2}$*d*.

232. $752\frac{1}{3}$ at 1*s*. $1\frac{1}{2}$*d*.

233. $142\frac{5}{8}$ at 4*s*. 6*d*.

234. $235\frac{1}{12}$ at 4*s*. 3*d*.

235. $743\frac{1}{12}$ at 9*s*. 9*d*.

236. $1531\frac{5}{6}$ at 4*s*. 9*d*.

237. $2489\frac{1}{2}$ at 1*s*. $1\frac{1}{2}$*d*.

238. $1487\frac{1}{2}$ at 1*s*. $3\frac{1}{2}$*d*.

239. $17463\frac{3}{4}$ at 7*s*. 3*d*.

240. $26974\frac{7}{8}$ at 5*s*. 4*d*.

Find, in £, *s*. *d*. and a fraction of a penny, the value of—

241. $134\frac{1}{4}$ at 2*s*. $7\frac{1}{4}$*d*.

242. $129\frac{1}{4}$ at 15*s*. $9\frac{1}{2}$*d*.

243. $434\frac{5}{8}$ at 6*s*. 9*d*.

244. $447\frac{3}{8}$ at 13*s*. $11\frac{1}{2}$*d*.

245. $812\frac{7}{8}$ at 2*s*. 7*d*.

246. $401\frac{7}{10}$ at 16*s*. $9\frac{3}{4}$*d*.

247. $420\frac{7}{12}$ at 6*s*. $1\frac{1}{2}$*d*.

248. $986\frac{11}{15}$ at 13*s*. $3\frac{3}{4}$*d*.

249. 37417 at $5\frac{5}{8}$*d*.

250. 38643 at $4\frac{7}{8}$*d*.

251. $417\frac{3}{7}$ at 4*s*. $2\frac{2}{5}$*d*.

252. $333\frac{3}{13}$ at 3*s*. $3\frac{1}{3}$*d*.

Find the dividend on—

253. £5725 at 4*s*. 7*d*. in the £.

254. £8790 at 8*s*. 6*d*. in the £.

255. £4736, 5*s*. at 2*s*. 8*d*. „

256. £1342, 12*s*. at 2*s*. $3\frac{1}{2}$*d*. „

257. £856 at 13*s*. 8*d*. in the £.

258. £3469 at 15*s*. $6\frac{1}{2}$*d*. in the £.

259. £473, 18*s*. at 8*s*. 4*d*. in the £.

260. £1723, 16*s*. 8*d*. at 7*s*. 3*d*. in the £.

Find the cost of—

261. 456 tons 12 cwts. at 15*s*. 6*d*. per ton.

262. 981 qrs. 7 lbs. at 8*s*. $4\frac{1}{2}$*d*. per qr.

263. 73 yds. 1 ft. 6 in. at 3*s*. 2d. per yd.

264. 87 yds. 0 ft. 9 in. at 2*s*. 10*d*. per yd.

265. 13 tons 13 cwts. 3 qrs. of sugar at 28*s*. per cwt.

266. 2 tons 3 cwts. 1 qr. 17 lbs. at 1*s*. $7\frac{1}{4}$*d*. per lb.

267. 2 ro. 13 po. 24 sq. yds. of land at 3*s*. $8\frac{1}{2}$*d*. per sq. yd.

268. 4 ac. 3 ro. 27 sq. po. of land at 3*s*. $7\frac{5}{8}$*d*. per sq. yd.

269. 47 ozs. 5 dwts. of silver at 3*s*. $10\frac{1}{2}$*d*. per oz.

270. 3 cwts. 2 qrs. 16 lbs. 8 ozs. at 1*s*. $10\frac{1}{2}$*d*. per lb.

271. 56472 young trees at £1, 2*s*. 9*d*. per dozen.

272. 4787 things at £11, 8*s*. 4*d*. per score.

273. Find, by Practice, 1437 times 8 lbs. 10 ozs. 7 drs.

274. Find, by Practice, 6748 times 9 yds. 2 ft. $7\frac{1}{2}$ in.

275. Find the rent of 317 ac. 2 ro. 16 sq. po. at 25*s*. per acre.

276. Find the rent of 212 ac. 3 ro. 30 sq. po. at £2, 3*s*. 8*d*. per acre.

277. Find the rent of 230 ac. 1 ro. 10 sq. po. at £1, 15*s*. 6*d*. per acre.

278. Find the rent of 1635 A. 2 R. 24 P. at £1, 12*s*. 8*d*. per acre.

279. Find the weight of 132 hogsheads of tallow averaging 12 cwts. 2 qrs. 17 lbs. each.

280. Find the weight of 2468 bales of goods averaging 4 cwts 2 qrs. 6 lbs. per bale.

XXVI. COMPOUND PRACTICE.

Write down the value of the following aliquot parts:—

1. $\frac{1}{2}$; $\frac{1}{4}$; $\frac{1}{5}$; $\frac{1}{10}$ of a ton; $\frac{1}{2}$; $\frac{1}{4}$; $\frac{1}{7}$; $\frac{1}{8}$ of a cwt.
2. $\frac{1}{2}$; $\frac{1}{4}$; $\frac{1}{7}$ of a qr.; $\frac{1}{2}$; $\frac{1}{4}$; $\frac{1}{8}$; $\frac{1}{16}$ of a lb.
3. $\frac{1}{2}$; $\frac{1}{4}$; $\frac{1}{8}$; $\frac{1}{16}$ of an acre; $\frac{1}{2}$; $\frac{1}{4}$; $\frac{1}{5}$; $\frac{1}{8}$ of a rood.
4. $\frac{1}{2}$; $\frac{1}{3}$; $\frac{1}{4}$; $\frac{1}{6}$; $\frac{1}{9}$; $\frac{1}{12}$ of a yd.; $\frac{1}{2}$; $\frac{1}{3}$; $\frac{1}{4}$; $\frac{1}{6}$; $\frac{1}{12}$ of a foot.

What aliquot part

5. of a ton is 10 cwts.; 5 cwts.; 4 cwts.; 2 cwts.?
   of 1 cwt. is 2 qrs.; 1 qr.; 14 lbs.; 16 lbs.?
6. of 10 cwts. is 5 cwts.; 1 cwt.?
   of 2 qrs. is 14 lbs.; 7 lbs.; 8 lbs.; 4 lbs.?
7. of 1 acre is 2 ro.; 1 ro.; 20 po.; 32 po.; 16 po.?
   of 2 roods is 1 ro.; 20 po.; 16 po.; 8 po.?
8. 1 oz. Tr. is 10 dwts.; 5 dwts.; 4 dwts.; 2 dwts.?
   of 1 dwt. is 12 grs.; 8 grs.; 6 grs.; 4 grs.; 3 grs.; 2 grs.?
9. of 1 yd. is 1 ft. 6 in.; 1 ft.; 9 in.; 6 in.; 4 in.; 3 in.?
   of 1 ft. 6 in. is 9 in.; 6 in.; 3 in.; $1\frac{1}{2}$ in.?
10. of 1 fur. is 20 po.; 10 po.; 8 po.; 5 po.? of 2 fur. is 10 po.; 8 po.?
    of 1 pole is 2 yds. 2 ft. 3 in.; 1 ft. 6 in.?

Find the cost of—

11. 1 cwt. 3 qrs. 14 lbs. at £3, 12*s.* 6*d.* per cwt.
12. 1 cwt. 3 qrs. 7 lbs. at £2, 17*s.* 8*d.* per cwt.
13. 2 cwts. 1 qr. 21 lbs. at £66, 4*s.* per cwt.
14. 3 cwts. 3 qrs. 21 lbs. at £44, 2*s.* per cwt.
15. 13 cwts. 3 qrs. 4 lbs. at £2, 18*s.* 4*d.* per cwt.
16. 17 cwts 1 qr. 5 lbs. at £2, 13*s.* 8*d.* per cwt.
17. 1 ton 6 cwts. 2 qrs. 14 lbs. at £3, 13*s.* 4*d.* per ton.
18. 1 ton 16 cwts. 1 qr. 7 lbs. at £2, 6*s.* 8*d.* per ton.
19. 18 galls. 2 qts. 1 pint at £1, 6*s.* 4*d.* per gallon.
20. 21 galls. 3 qts. 1 pt. at £1, 2*s.* 8*d.* per gallon.
21. 8 ac. 2 ro. 30 sq. po. at £65 per acre.
22. 3 ac. 3 ro. 25 sq. po. at £120 per acre.
23. 17 ac. 2 ro. 25 sq. po. at £76, 10*s.* per acre.
24. 19 ac. 2 ro. 24 sq. po. at £60, 5*s.* per acre.
25. 7 cwts. 1 qr. 12 lbs. at £1, 19*s.* 8*d.* per cwt.
26. 12 cwts. 3 qrs. 12 lbs. at £1, 10*s.* 4*d.* per cwt.
27. 5 cwts. 2 qrs. 16 lbs. at £113, 3*s.* 4*d.* per ton.
28. 1 ton 10 cwts. 2 qrs. 22 lbs. at £1, 13*s.* 10*d.* per cwt.

29. 18 yds. 1 ft. 9 in. at £1, 16*s.* per yard.
30. 17 yds. 2 ft. 6 in. at £5, 7*s.* 6*d.* per yard.
31. 7 yds. 1 ft. 11 in. at 9*s.* 9*d.* a yard.
32. 11 yds. 2 ft. 9 in. at 7*s.* 3*d.* a yard.
33. 16 ozs. 6 dwts. 12 grs. of gold at £3, 17*s.* 6*d.* per oz.
34. 15 ozs. 11 dwts. 8 grs. of gold at £3, 17*s.* 6*d.* per oz.
35. 6 lbs. 13 ozs. 11 drams at £1, 1*s.* 4*d.* per lb.
36. 5 lbs. 11 ozs. 12 drams at £1, 2*s.* 8*d.* per lb.
37. 7 cwts. 3 qrs. 26 lbs. at £1, 10*s.* 4*d.* per cwt.
38. 7 cwts. 2 qrs. 24 lbs. at £1, 7*s.* 5*d.* per cwt.
39. 3 qrs. 18 lbs. 8 ozs. at £8, 12*s.* 8*d.* per cwt.
40. 3 qrs. 9 lbs. 4 ozs. at £10, 5*s.* 4*d.* per cwt.
41. 63 tons 15 cwts. 40 lbs. at £1, 12*s.* 8*d.* per ton.
42. 56 tons 15 cwts. 105 lbs. at £1, 5*s.* 4*d.* per ton.
43. 1 ton 17 cwts. 49 lbs. 12 ozs. at £65, 6*s.* 8*d.* per ton.
44. 1 ton 13 cwts. 38 lbs. 8 ozs. at £52, 13*s.* 4*d.* per ton.
45. 9 cwts. 3 qrs. 14 lbs. at 30*s.* a ton.
46. 3 cwts. 3 qrs. 14 lbs. at 16*s.* 8*d.* per ton.
47. 2 cwts. 1 qr. 20 lbs. 4 ozs. at £1, 17*s.* 4*d.* per quarter.
48. 1 cwt. 3 qrs. 19 lbs. 8 ozs. at £1, 18*s.* 6*d.* per quarter.
49. 1 ton 14 cwts. 1 qr. 11 lbs. 12 ozs. at £4, 13*s.* 4*d.* per cwt.
50. 1 ton 18 cwts. 1 qr. 11 lbs. 6 ozs. at £6, 10*s.* 8*d.* per cwt.
51. 5 yds. $22\frac{1}{2}$ in. at £2, 1*s.* 2*d.* per yd.
52. 3 yds. $20\frac{1}{4}$ in. at £3, 9*s.* 8*d.* per yd.
53. 3 lbs. 11 ozs. 18 dwts. 4 grs. at £3, 17*s.* 6*d.* per oz.
54. 6 lbs. 8 ozs. 12 dwts. 12 grs. at £4, 4*s.* 6*d.* per oz.

Find, in £, *s.* *d.* and a fraction of a penny, the value of—

55. 3 cwts. 2 qrs. 21 lbs. at £55, 10*s.* 6*d.* per cwt.
56. 11 cwts. 3 qrs. 8 lbs. at £3, 4*s.* 6*d.* per cwt.
57. 17 cwts. 2 qrs. 22 lbs. at £4, 6*s.* $7\frac{1}{2}d.$ per cwt.
58. 25 cwts. 3 qrs. 8 lbs. at 9*s.* $5\frac{1}{2}d.$ per cwt.
59. 7 yds. 2 ft. 7 in. at 7*s.* 9*d.* a yd.
60. 15 yds. 2 ft. 11 in. at 10*s.* per yard.
61. 19 yds. 2 ft. $4\frac{1}{2}$ in. at £2, 15*s.* 4*d.* per yard.
62. 17 yds. 1 foot 7 in. at 6*s.* $9\frac{1}{2}d.$ per yard.
63. 4 ac. 2 ro. 25 po. at £111, 13*s.* 3*d.* per acre.
64. 44 ac. 2 ro. 25 po. at £55, 16*s.* $7\frac{1}{2}d.$ per acre.

65. 3 qrs. 14 lbs. 12 ozs. at £12, 16*s*. 8*d*. per cwt.

66. 14 cwts. 2 qrs. 24 lbs. at £4, 1*s*. per cwt.

67. 24 cwts. 3 qrs. 16 lbs. 10 ozs. at £2, 0*s*. 8*d*. per cwt.

68. 21 cwts. 2 qrs. 9 lbs. 4 ozs. at £2, 1*s*. 10*d*. per cwt.

69. 80 lbs. at £5, 15*s*. per cwt. 70. 66 lbs. at £1, 5*s*. per cwt.

71. 3 tons 5 cwts. 1 qr. 16 lbs. at £2, 15*s*. per cwt.

72. 1 ton 12 cwts. 0 qrs. 22 lbs. at £4, 16*s*. 8*d*. per cwt.

73. 8 mi. 6 fur. 8 po. at £15, 6*s*. per mile.

74. 7 mi. 7 fur. 7 po. at £16, 5*s*. per mile.

75. 2 mi. 3 fur. 26 po. $2\frac{3}{4}$ yds. at £10 per mile.

76. 1 mi. 5 fur. 32 po. 3 yds. at £12 per mile.

77. 2 ac. 3 ro. 24 po. 11 sq. yds. at £50 per acre.

78. 7 ac. 1 ro. 31 po. $5\frac{1}{2}$ sq. yds. at £60, 12*s*. per acre.

79. 11 cub. yds. 4 cub. ft. 144 cub. in. at 8*s*. 6*d*. per cub. yd.

80. 9 cub. yds. 7 cub. ft. 216 cub. in. at 7*s*. 9*d*. per cub. yd.

Find, *by Practice*, as shortly as you can, the value of—

81. 973 tons 16 cwts. 1 qr. 8 lbs. at £1, 15*s*. per ton.

82. 1452 tons 13 cwts. 3 qrs. 5 lbs. at £1, 8*s*. per ton.

83. 19 tons 19 cwts. 2 qrs. 13 lbs. at 7*s*. per cwt.

84. 23 tons 17 cwts. 2 qrs. 23 lbs. at 10*s*. 6*d*. per cwt.

85. 289 qrs. 7 bush. 3 pks. at 17*s*. 9*d*. per qr.

86. 372 qrs. 7 bush. 2 pks. at 31*s*. 6*d*. per qr.

87. 465 ac. 3 ro. 10 po. at £33, 6*s*. 8*d*. per acre.

88. 434 ac. 2 ro. 32 po. $2\frac{3}{4}$ sq. yds. at £44 per acre.

89. 216 tons 12 cwts. 2 qrs. at £32, 17*s*. 2*d*. per ton.

90. 319 tons 3 cwts. 2 qrs. 15 lbs. 8 ozs. at £9, 6*s*. 8*d*. per ton.

91. 54 tons 17 cwts. 3 qrs. 18 lbs. at 16*s*. 4*d*. per cwt.

92. 65 tons 3 cwts. 1 qr. 20 lbs. 9 ozs. at £1, 12*s*. per cwt.

93. 1213 ozs. 6 dwts. 17 grs. at 5*s*. 10*d*. per oz.

94. 35 lbs. 16 dwts. 12 grs. at 2*s*. $1\frac{1}{2}$*d*. per oz.

95. 99 mi. 7 fur. 70 yds. at 3*s*. 8*d*. per furlong.

96. 9 mi. 7 fur. 121 yds. at £671 per mile.

97. 345 ac. 2 ro. 15 per. at 42*s*. 6*d*. an acre.

98. 779 ac. 3 ro. 32 po. at £4, 3*s*. 8*d*. per acre.

99. 1759 ac. 2 ro. 31 po. at £1, 8*s*. 6*d*. per acre.

100. 1215 ac. 1 ro. $27\frac{1}{2}$ po. at £19, 6*s*. 8*d*. per acre.

XXVII. INVOICES.

Make out bills with names, dates, &c., for—

1. 6 lbs. of mutton at $8\frac{1}{2}d.$ per lb.; $10\frac{1}{2}$ lbs. of beef at $8d.$ per lb.; and $1\frac{1}{4}$ lbs. of suet at $8d.$ per lb.
2. $8\frac{1}{2}$ lbs. of mutton at $9d.$ per lb.; 7 lbs. of mutton at $9\frac{1}{2}d.$ per lb.; and $\frac{1}{2}$ lb. of suet at $8\frac{1}{2}d.$ per lb.
3. 3 lbs. of tea at 1*s.* 8*d.* per lb.; 3 lbs. of coffee at 1*s.* 6d. per lb.; and 7 lbs. of sugar at $2\frac{1}{2}d.$ per lb.
4. 4 lbs. of tea at 1*s.* 10*d.* per lb.; 14 lbs. of sugar at $2\frac{1}{2}d.$ per lb.; and 6 lbs. of candles at $8d.$ per lb.
5. 12 yds. of calico at $8\frac{1}{4}d.$ per yard; 2 doz. reels of cotton at $2\frac{1}{2}d.$ per reel; and 4 packets of needles at $1\frac{3}{4}d.$ per packet.
6. 12 yds. of muslin at $7\frac{1}{2}d.$ per yard; 3 doz. skeins of thread at $1\frac{3}{4}d.$ per skein; and 5 knots of braid at $2\frac{1}{4}d.$ per knot.
7. 4 packets of note-paper at $9\frac{1}{2}d.$ per pkt.; 3 quires of blotting-paper at 1*s.* 9*d.* per qre.; and 500 envelopes at $1\frac{1}{2}d.$ per pkt. of 25.
8. 3 reams of foolscap paper at 4*s.* 9*d.* per ream; 2 boxes of pens at $10\frac{1}{2}d.$ per box; and $\frac{1}{2}$ gallon of ink at 5*s.* 9*d.* per gallon.
9. 5 lbs. of American cheese at $8\frac{1}{2}d.$ per lb.; 6 lbs. of butter at 1*s.* 5*d.* per lb.; and $8\frac{1}{2}$ lbs. of bacon at $10\frac{1}{2}d.$ per lb.
10. 8 lbs. of lard at $9\frac{1}{2}d.$ per lb.; 4 lbs. of butter at 1*s.* 2*d.* per lb.; and $3\frac{1}{2}$ lbs. of Gorgonzola cheese at 1*s.* 1*d.* per lb.
11. $7\frac{1}{2}$ lbs. of salmon at 1*s.* 8*d.* per lb.; 1 doz. kippered herrings at 2 for $1\frac{1}{2}d.$; and 2 tins of preserved lobster at $10\frac{3}{4}d.$ per tin.
12. 13 lbs. of cod at $5\frac{1}{4}d.$ per lb.; $\frac{1}{2}$ a score of oysters at 3*s.* 9*d.* per score; and $3\frac{1}{2}$ lbs. of dried haddock at $5\frac{1}{2}d.$ per lb.
13. 5 loaves at $4\frac{1}{2}d.$ each; $\frac{1}{2}$ a stone of flour at 2*s.* $10\frac{1}{2}d.$ per stone; and 8 lbs. of biscuits at $7\frac{1}{2}d.$ per lb.
14. 7 loaves at $4\frac{1}{4}d.$ each; $3\frac{1}{2}$ lbs. of flour at 2*s.* 10*d.* per stone; and 12 lbs. of biscuits at $6\frac{3}{4}d.$ per lb.
15. 2 bottles of cod-liver oil at 1*s.* 2*d.* per bottle; 3 ozs. of jujubes at $3\frac{1}{2}d.$ per oz.; and 2 doz. tablets of soap at $2\frac{3}{4}d.$ per tablet.
16. 4 ozs. of cough lozenges at $2\frac{1}{2}d.$ per oz.; $\frac{1}{4}$ oz. of quinine at 2*s.* 3*d.* per oz.; and 5 jars of meat extract at 1*s.* $10\frac{1}{2}d.$ per jar.
17. 3 doz. bottles of sherry at 28*s.* 6*d.* per doz.; 6 doz. of bottled beer at 3*s.* 3*d.* per doz.; and $\frac{1}{2}$ doz. of whisky at 39*s.* per doz.
18. 4 doz. pints of bottled stout at 3*s.* 9*d.* per dozen; 4 bottles of brandy at 63*s.* per doz.; and 6 doz. of claret at 18*s.* 6*d.* per doz.
19. $5\frac{1}{2}$ lbs. of cake at $10\frac{1}{2}d.$ per lb.; 1 doz. pots of marmalade at $6\frac{3}{4}d.$ per pot; and 1 pork-pie, weighing $2\frac{1}{2}$ lbs., at $9\frac{1}{2}d.$ per lb.
20. $7\frac{1}{4}$ lbs. of cake at 11*d.* per lb.; 3 doz. ices at $4\frac{1}{2}d.$ each; and $3\frac{1}{2}$ lbs. of cooked ham at 1*s.* $2\frac{1}{2}d.$ per lb.

21. Leg of mutton, $9\frac{1}{2}$ lbs., at $9\frac{1}{2}$*d.* per lb.; quarter of lamb, $15\frac{3}{4}$ lbs., at 11*d.* per lb.; steak, 5 lbs., at $11\frac{1}{2}$*d.* per lb.; and chops, $3\frac{1}{2}$ lbs., at $10\frac{1}{2}$*d.* per lb.

22. Beef, $16\frac{1}{2}$ lbs., at $8\frac{1}{2}$*d.* per lb.; mutton, $6\frac{3}{4}$ lbs., at 8*d.* per lb.; suet, $2\frac{1}{4}$ lbs., at 8*d.* per lb.; and pork, $8\frac{1}{2}$ lbs., at $7\frac{1}{2}$*d.* per lb.

23. $14\frac{1}{2}$ yds. of calico at $10\frac{1}{2}$*d.* per yd.; $9\frac{1}{4}$ yds. of flannel at 1*s.* 10*d.* per yd.; 16 reels of cotton at 2*s.* 3*d.* per doz. reels; and 8 yds. of ribbon at $5\frac{1}{4}$*d.* per yd.

24. $13\frac{1}{4}$ yds. of calico at 9*d.* per yd.; 8 pairs of socks at 1*s.* $6\frac{1}{2}$*d.* per pair; $4\frac{3}{4}$ yds. of ribbon at 9*d.* per yd.; and $2\frac{1}{2}$ dozen collars at $6\frac{1}{2}$*d.* each.

25. $16\frac{1}{2}$ lbs. of cheese at 10*d.* per lb.; $8\frac{1}{4}$ lbs. of butter at 1*s.* 5*d.* per lb.; $24\frac{1}{2}$ lbs. of bacon at $9\frac{1}{2}$*d.* per lb.; and 3 shillings worth of eggs at 14 for 1*s.*

26. $7\frac{1}{4}$ lbs. Stilton cheese at 1*s.* 2*d.* per lb.; 3 lbs. of butter at 1*s.* 5*d.* per lb.; $13\frac{3}{4}$ lbs. of bacon at 9*d.* per lb.; and 10 lbs. of lard at $9\frac{3}{4}$*d.* per lb.

27. 7 pairs of blankets at 18*s.* 6*d.* per pair; 47 yards of calico at $9\frac{1}{4}$*d.* per yard; $\frac{1}{2}$ dozen pairs of stockings at 2*s.* 11*d.* per pair; and 3 pairs of gloves at 1*s.* $10\frac{1}{2}$*d.* per pair.

28. 5 pairs of gloves at 2*s.* $11\frac{1}{2}$*d.* per pair; 47 yds. of chintz at $9\frac{3}{4}$*d.* per yd.; 1 gross of buttons at $3\frac{1}{4}$*d.* per dozen; and 60 yards of fringe at 1*s.* $3\frac{3}{4}$*d.* per doz. yds.

29. 2 stones of sugar at $2\frac{1}{2}$*d.* a lb.; 10 lbs. of tea at 1*s.* 10*d.* a lb.; 8 lbs. of coffee at 1*s.* 4*d.* a lb.; 12 lbs. of currants at $5\frac{3}{4}$*d.* a lb.; 20 lbs. of rice at $2\frac{1}{4}$*d.* a lb.; and 9 lbs. of candles at 11*d.* a lb.

30. 7 lbs. of raisins at $6\frac{1}{2}$*d.* per lb.; 4 lbs. of figs at $8\frac{1}{2}$*d.* per lb.; 6 lbs. of treacle at $3\frac{1}{2}$*d.* per lb.; $\frac{1}{4}$ lb. of pepper at 1*s.* 7*d.* per lb.; 6 lbs. of tea at 1*s.* $10\frac{1}{2}$*d.* per lb.; and 3 doz. bars of soap at $8\frac{3}{4}$*d.* per bar.

In the following bills, when the exact value of any item cannot be expressed in current coin, charge the farthing next above it:—

31. $\frac{1}{4}$ lb. of butter at $14\frac{1}{2}$*d.*; $\frac{3}{4}$ lb. of cheese at $6\frac{1}{2}$*d.*; per lb.

32. 3 ozs. of butter at 15*d.* per lb.; 2 eggs at 14 for 1*s.*

33. $\frac{1}{4}$ lb. of tea at 2*s.* $1\frac{1}{2}$*d.*; $\frac{1}{2}$ lb. of treacle at $2\frac{1}{4}$*d.*; per lb.

34. 2 ozs. of coffee at 1*s.* 7*d.*; 1 oz. of pepper at 7*d.*; per lb.

35. 1 lb. 2 oz. of beef at $7\frac{1}{2}$*d.*; $3\frac{1}{4}$ lbs. of mutton at $8\frac{1}{2}$*d.*; per lb.

36. 3 lbs. 6 ozs. of pork at $6\frac{1}{2}$*d.*; 5 ozs. of suet at 8*d.*; per lb.

37. $4\frac{5}{8}$ yds. of calico at 5*d.* a yd.; $\frac{1}{2}$ doz. buttons at 2*s.* 6*d.* per gross.

38. $\frac{5}{8}$ yd. of velvet at 6*s.* 9*d.* per yd.; 6 yds. of braid at $9\frac{1}{2}$*d.* per 3 doz. yds.

39. $3\frac{1}{4}$ lbs. of beef at $8\frac{1}{2}d.$ per lb.; $5\frac{1}{4}$ lbs. of mutton at $9\frac{1}{2}d.$ per lb.; and $7\frac{3}{4}$ lbs. of pork at $7\frac{1}{2}d.$ per lb.

40. $\frac{1}{4}$ lb. of tea at $1s.$ $10\frac{1}{2}d.$ per lb.; $1\frac{3}{4}$ lbs. of bacon at $7\frac{1}{2}d.$ per lb.; and $\frac{3}{4}$ lb. of cheese at $8\frac{1}{2}d.$ per lb.

41. $\frac{5}{8}$ yd. of serge at $1s.$ $10d.$ per yd.; $2\frac{1}{2}$ dozen buttons at $2\frac{3}{4}d.$ per dozen; and $3\frac{3}{4}$ yards of fringe at $6\frac{1}{4}d.$ per yard.

42. $5\frac{3}{8}$ yards of silk at $4s.$ $11d.$ per yard; 2 dozen and 8 buttons at $7\frac{1}{2}d.$ per dozen; and $7\frac{1}{2}$ yards of lining at $11\frac{3}{4}d.$ per yard.

43. 7 lbs. 5 ozs. of mutton at $9d.$ per lb.; $11\frac{3}{4}$ lbs. of beef at $8\frac{1}{2}d.$ per lb.; and 11 ozs. of suet at $8d.$ per lb.

44. 6 lbs. 1 oz. of veal at $9d.$ per lb.; $7\frac{1}{4}$ lbs. of lamb at $11\frac{1}{2}d.$ per lb.; and $1\frac{1}{4}$ lbs. of suet at $8\frac{1}{2}d.$ per lb.

45. Beef, 11 lbs. 6 ozs., at $8d.$ per lb.; mutton, 7 lbs. 10 ozs., at $9\frac{1}{2}d.$ per lb.; and veal, 10 lbs. 2 ozs., at $8\frac{1}{2}d.$ per lb.

46. 13 lbs. 5 ozs. of bacon at $6\frac{1}{2}d.$ per lb.; 5 lbs. 11 ozs. of cheese at $9\frac{1}{2}d.$ per lb.; and 8 lbs. 7 ozs. of lard at $10d.$ per lb.

Make out and receipt detailed accounts for—

47. 6 lbs. of tea at $2s.$ $1\frac{1}{2}d.$ per lb., 3 lbs. of coffee at $1s.$ $8d.$ per lb., and 3 ozs. of cloves at $1s.$ $4d.$ per lb., purchased on Jan. 7; $\frac{1}{4}$ lb. of pepper at $1s.$ $1\frac{1}{2}d.$ per lb., and 7 lbs. of rice at $3s.$ $10d.$ per stone, purchased on Feb. 15; and 2 doz. tablets of soap at $3\frac{1}{4}d.$ per tablet, 2 ozs. of nutmegs at $3s.$ $3d.$ per lb., and 3 tins of sardines at $11\frac{1}{2}d.$ per tin, purchased on Mar. 4; deducting a discount of $6d.$ in the pound from the total amount.

48. $2\frac{3}{8}$ yds. of ribbon at $1s.$ $9d.$ per yd., and 29 yds. of calico at $6\frac{1}{2}d.$ per yd., purchased on May 7; $\frac{1}{2}$ doz. collars at $8\frac{1}{2}d.$ each, 3 pairs of gloves at $1s.$ $11\frac{1}{4}d.$ per pair, and $\frac{7}{8}$ yd. of velvet at $9s.$ $11\frac{3}{4}d.$ per yd., purchased on June 6; and 3 doz. yds. of linen at $1s.$ $6\frac{1}{2}d.$ per yd., 20 yds. of huckaback at $11\frac{1}{2}d.$ per yd., and $2\frac{3}{4}$ yds. of oilcloth at $1s.$ $9\frac{1}{2}d.$ per yd., purchased on June 17; deducting a discount of $6d.$ in the pound from the total.

49. 3 packets of note-paper at $1s.$ $1\frac{1}{2}d.$ per packet, 1000 envelopes at $1\frac{1}{4}d.$ per packet of 25, and 3 gross of pens at $1s.$ $3\frac{1}{2}d.$ per gross, purchased on Aug. 3; 30 books at $1s.$ $6d.$ each, and 21 books at $1s.$ $4d.$ each, purchased on Sept. 12; and 2 quires of blotting-paper at $1s.$ $6\frac{1}{2}d.$ per quire, and 14 books at $3s.$ $6d.$ each, purchased on Sept. 27; allowing a discount of $2d.$ in the shilling on books only.

50. 12 books at $2s.$ $6d.$ each, 6 books at $4s.$ $6d.$ each, and 3 reams of foolscap paper at $4s.$ $3d.$ per ream, purchased on Oct. 3; 2 gross of pencils at $8\frac{1}{2}d.$ per dozen, 20 books at $9d.$ each, and 4 books at $3s.$ $6d.$ each, purchased on Oct. 23; and 500 envelopes at $1d.$ per packet of 25, 24 books at $2s.$ each, and 3 gross of penholders at $2s.$ $10d.$ per gross, purchased on Nov. 12; allowing a discount of $3d.$ in the shilling on books only.

XXVIII. DECIMALS.

NOTATION.

Express, in words, the meaning of—

1. ·7; ·07; ·007; ·0007; ·00007; ·000007; and ·0000007.
2. ·08; ·0008; ·8; ·00008; ·008; ·0000008; and ·000008.
3. ·23; ·45; ·486; ·271; ·083; ·803; ·0041; and ·0632.
4. 2·3; ·601; 5·03; 24·05; 17·17; ·089; 66·$\dot{6}$6; and ·6666.

Express in figures, with the decimal notation—

5. Three *tenths*; Seven *tenths*; Eight *hundredths*; Four *thousandths*.
6. One *tenth*; Six *hundredths*; Nine *hundredths*; Five *thousandths*.
7. Two *thousandths*; Seven *ten-thousandths*; Six *millionths*.
8. Eight *ten-thousandths*; Four *millionths*; Nine *thousandths*.
9. Two, and three-*tenths*; Twelve, and seven-*hundredths*.
10. Seventy, and seven-*thousandths*; Ten, and one-*ten-millionth*.
11. Twenty-one *hundredths*; Seventeen *hundredths*; Eleven *tenths*.
12. Twelve *hundredths*; Ninety-nine *hundredths*; Thirty-three *tenths*.
13. Two-hundred-and-two *thousandths*; Fourteen *millionths*.
14. One-hundred-and-nine *hundredths*; Eighty-seven *ten-thousandths*.

Write down, at sight, as a decimal, the value of—

15. ·23 × 10.	20. ·006 × 10.	25. 1·234 × 100.	30. ·2057 × 1000.
16. ·57 × 10.	21. ·701 × 10.	26. 5·003 × 100.	31. 2·315 × 1000.
17. 4·5 × 10.	22. 9·21 × 10.	27. 4·21 × 100.	32. 41·03 × 1000.
18. 7·2 × 10.	23. 5·34 × 10.	28. 5·03 × 100.	33. 5·2 × 1000.
19. ·08 × 10.	24. 72·45 × 10.	29. 12·1 × 100.	34. ·008 × 1000.

35. 2·3 ÷ 10.	40. 61 ÷ 10.	45. 153·1 ÷ 100.	50. 236·1 ÷ 1000.
36. 5·1 ÷ 10.	41. 29 ÷ 10.	46. 407·6 ÷ 100.	51. 400·9 ÷ 1000.
37. ·5 ÷ 10.	42. 60·3 ÷ 10.	47. 43·7 ÷ 100.	52. 21·3 ÷ 1000.
38. ·7 ÷ 10.	43. 8·02 ÷ 10.	48. 80·2 ÷ 100.	53. 80·5 ÷ 1000.
39. ·63 ÷ 10.	44. ·001 ÷ 10.	49. 2·3 ÷ 100.	54. 1·2 ÷ 1000.

Multiply 4·728, ·3045, ·00847, 23·067 and ·0000703, each

55. by 100. 56. by 1000. 57. by 10000.

Divide 2340·7, 70300·57, 400·1, ·03 and 4·367, each

58. by 10. 59. by 100. 60. by 10000.

XXIX. DECIMALS.

Addition and Subtraction.

Add together—

1. 3·25, 4·32, 7·04, 5·29 and 1·02.
2. 1·035, 2·503, 1·506, ·324 and ·017.
3. 7·5, 3·42, 1·354, 4·706 and ·83.
4. 1·073, ·457, 8·213, 11·007 and ·05.
5. 17·2, 1·35, ·043, ·245 and 20·8.
6. 4·07, ·435, ·009, 23·07 and 41·1.
7. 71, ·0341, 810·2, 53·245 and ·0003.
8. ·047, 2·7, 43·07154, 1·37036, 420.
9. 172·1, 3·40532, ·00878, 30·005, 2.
10. 3501, 3·501, 35·01, ·3501 and 350·1.

Subtract—

11. 2·75 from 7·28.
12. 5·247 from 8·319.
13. 43·57 from 75·37.
14. 38·72 from 81·42.
15. 3·427 from 14·0239.
16. 7·59 from 11·102.
17. ·635 from 2·18.
18. ·598 from 1·23.
19. 1·6875 from 17·04.
20. 3·3078 from 25·61.

Find the sum of—

21. 4031·06, 108·304, 9·001345, 76·739 and 250·0007.
22. ·608242, ·0315044, 1·8034, ·086, ·9106 and 20.

Find the difference between—

23. ·5 and ·05.
24. ·007 and ·07.
25. ·3456 and ·35.
26. ·86 and ·859.
27. ·9 and ·90.
28. ·09 and ·1.
29. ·0473 and ·437.
30. 43·87 and 4·387.
31. 2·6478 and 2·60578.
32. 6·561 and 65·6132.
33. ·34572 and 1·23.
34. 7·4 and 6·98075.
35. 2 and ·9871.
36. 7 and 3·4956.
37. 10 and ·01.
38. 170 and ·017.
39. 5·05 and 50·5.
40. 78·9 and ·978.

Find the value of—

41. 2·7 + ·031 − 1·5638 + 40·25 − 23·709.
42. 16·8 − 3·47513 − 7·261 + ·0083 − 3·4541.
43. 24·68 − 13·579 + 2·468 − 1·3579 − 10.
44. 900·87 − 80·076 − 7·0065 + ·60054 − 500·43.
45. 20 − ·02 + 2 − ·2 + 200 − ·002 + 22·2 − ·00222.
46. 333 − ·0333 + 33·3 − 3·33 − ·333 + ·00333.
47. Take the sum of 2·345, 23·45, 20·3045 and 30·0045 from 80·1.
48. Subtract 2·29997 from the sum of 1·8, ·2304, 11·50803, and ·03417.
49. By how many does the sum of ·1, ·2, ·3, ·4, ·5, ·6 and ·7 exceed the sum of ·1, ·02, ·003, ·004, ·005 and ·006?
50. By how many does the sum of 5·555 and ·5555 exceed their difference?

XXX. MULTIPLICATION OF DECIMALS.

Multiply—

1. 2·4 by 7.
2. ·47 by 12.
3. 1·43 by 17.
4. ·035 by 23.
5. 5·32 by 107.
6. 14·09 by 201.
7. ·087 by 40.
8. ·0053 by 70.
9. 4·17 by 230.
10. 15·6 by 3100.
11. ·043 by 7100.
12. ·0018 by 18000.
13. 7·3 by ·5.
14. 2·8 by ·6.
15. ·073 by ·9.
16. ·027 by 1·03.
17. ·043 by 2·05.
18. 1·65 by ·67.
19. ·305 by ·42.
20. 213·4 by ·0085.
21. 3·25 by 1·24.
22. 7·62 by 2·35.
23. ·0318 by 4·05.
24. ·6072 by ·0015.
25. 2·145 × ·014.
26. 315·2 × ·00107.
27. 3·402 × ·0203.
28. 30·635 × 1·206.
29. 3020 × ·015.
30. 2461 × ·207.
31. ·007853 × ·035.
32. ·3045 × ·00061.
33. ·03 × ·05 × ·07.
34. ·02 × ·004 × ·6.
35. 17 × ·07 × 1·7.
36. 31 × 3·1 × ·031.

Find the continued product of—

37. 2, ·3, ·04, ·005 and ·006.
38. ·7, ·07, ·007, ·0007 and 700.
39. 21, 2·1, ·21 and ·021.
40. ·23, ·023, 2·3 and 2300.

XXXI. DECIMALS.

Division by an Integer.

Divide, using *short* divisions—

1. 24·82 by 2.
2. 69·042 by 3.
3. 2·7545 by 5.
4. ·01768 by 8.
5. 1246·5 by 9.
6. ·1452 by 12.
7. 1·063 by 5.
8. 231·3 by 4.
9. 26·007 by 5.
10. 17·1 by 4.
11. ·34 by 8.
12. ·9 by 8.
13. 371·488 by 16.
14. 3·8475 by 25.
15. 1·7955 by 35.
16. 2·4304 by 28.
17. 14·1 by 16.
18. ·137 by 25.
19. 3·7 by 32.
20. ·0145 by 64.
21. 3·5 by 160.
22. 1·4 by 320.
23. 29·5 by 2500.
24. 831·2 by 6400.

Find, as a decimal, the complete quotient of—

25. 26·64 ÷ 37.
26. 4·983 ÷ 151.
27. 1·7388 ÷ 207.
28. ·01136 ÷ 71.
29. 1670·4 ÷ 174.
30. 177·1 ÷ 1012.
31. ·7 ÷ 175.
32. 18·8 ÷ 625.
33. 26·1 ÷ 2175.
34. 4006·7 ÷ 389.
35. 7·1 ÷ 3125.
36. 543·2 ÷ 4096.
37. 22708·8 by 3800.
38. 516·1 by 39700.
39. 21475·05 by 7041000.
40. 13723·226 by 8063000.

XXXII. DIVISION OF DECIMALS.

Divide, using *short* divisions—

1. 35·082 by ·3.
2. ·67144 by ·8.
3. ·61705 by ·07.
4. 2·15736 by ·04.
5. ·01331 by 1·1.
6. 34·56 by 1·2.
7. 1·73871 by ·009.
8. 4·00004 by ·0011.
9. 682·4 by ·05.
10. 35·16 by ·006.
11. 12·6 by ·0012.
12. ·5121 by ·08.
13. ·01 by ·004.
14. 1 by ·05.
15. 30 by ·008.
16. 7000 by ·0004.
17. 24·112 by 1·6.
18. ·27648 by ·16.
19. ·2317 by ·25.
20. 605·39 by 2·5.
21. 4·431 by ·32.
22. ·5709 by ·032.
23. ·03 by 6·4.
24. ·9763 by ·0064.
25. 2·1 by ·24.
26. 4·5 by ·036.
27. 6·1 by ·0025.
28. 49·7 by ·025.
29. 1·5 by ·0064.
30. 7 by ·32.

Divide—

31. ·0506 ÷ 2·3.
32. ·0507 ÷ ·039.
33. ·002232 ÷ ·031.
34. ·000153 ÷ ·17.
35. 7·7811 ÷ ·037.
36. 5·5811 ÷ ·067.
37. 30·5118 ÷ 50·6.
38. ·358307 ÷ ·059.
39. 4·96 ÷ 15·5.
40. 4·83 ÷ 9·2.
41. ·0341 ÷ ·124.
42. 1·71 ÷ ·76.
43. ·060248 ÷ 27·2.
44. 72·5026 ÷ 7·85.
45. 12·19192 ÷ 30·4.
46. ·65341 ÷ ·0475.
47. 1028·5 ÷ ·0017.
48. 17·25 ÷ ·0023.
49. 41 ÷ ·082.
50. 800 ÷ ·00125.
51. 35 ÷ ·0175.
52. 84·375 ÷ ·00375.
53. 1215013·8 ÷ 2·023.
54. 343·9836 ÷ ·01605.

Divide, to *four* places of decimals—

55. 1·3 by 7.
56. ·41 by 110.
57. ·43 by 8·1.
58. 5 by 8·4.
59. 2·4 by 19.
60. ·38 by 41.
61. ·6 by ·053.
62. 10 by ·029.
63. 48·1 by 9700.

64. Divide 23·065 by 3·5. Hence, without further work, write down the quotient of 23·065 ÷ ·0035.
65. Divide 210720·6 by ·4206. Hence, by inspection, find ·2107206 ÷ 42·06.
66. Divide 85·9625 by 26·45. Hence, find 85·9625 ÷ ·002645.
67. Divide 261 by 217·5. Hence, by inspection, find 26·1 ÷ 2·175; 2·61 ÷ ·0002175; and ·00261 ÷ ·2175.
68. Divide 51·61 by 39·7. Hence, by inspection, find ·5161 ÷ 3·97; 51610 ÷ ·397; ·5161 ÷ ·0397; and ·05161 ÷ 397.
69. Find the integral part of the quotient of 7·513 ÷ ·23 and the decimal remainder.
70. What integral number of times is ·27 contained in 140, and what decimal remains over?

XXXIII. DECIMALS.

CONVERSION INTO VULGAR FRACTIONS, AND VICE VERSÁ.

Write down in the form of *vulgar fractions*—

1. ·3; ·7; ·9; ·01.
2. ·1; ·07; ·01; ·009.
3. ·03; ·0001; ·00009.
4. ·007; ·00003; ·0000007.
5. ·13; ·23; ·017.
6. ·59; ·509; ·5009.
7. ·71; ·029; ·413.
8. ·017; ·113; ·1003.
9. ·871; ·0701; ·4359.
10. ·6789; ·00813; ·001793.
11. 3·1; 21·7; 1·009.
12. 5·3; 50·03; 5·0003.
13. 22·007; 220·07; 2020·7.
14. 170·3; 17·03; 1·00007.
15. 1·23; 12·57; 103·031.
16. 8·47; 19·351; 104·401.
17. 11·11; 1·111; 11·01011.
18. 70·77; 77·07; 7·07007.
19. 50·571; 17·3001; 1·7593.
20. 41·0209; 8·30459; 1·405039.

Reduce to vulgar fractions in their lowest terms—

21. ·2.	27. ·08.	33. 1·35.	39. ·0048.	45. 1·975.
22. ·5.	28. ·004.	34. 11·45.	40. ·0375.	46. 1·2625.
23. ·6.	29. ·15.	35. 2·125.	41. 1·725.	47. ·00075.
24. ·8.	30. ·25.	36. 10·045.	42. 3·0875.	48. ·000064.
25. ·02.	31. ·016.	37. 6·325.	43. 17·234.	49. ·15625.
26. ·05.	32. ·028.	38. 71·625.	44. 50·072.	50. 1·5625.

Write down, at sight, as *decimals*—

51. $\frac{1}{10}$; $\frac{7}{10}$; $\frac{1}{100}$.
52. $\frac{3}{10}$; $\frac{7}{100}$; $\frac{9}{100}$.
53. $\frac{8}{100}$; $\frac{2}{1000}$; $\frac{7}{10000}$.
54. $\frac{9}{1000}$; $\frac{5}{10000}$; $\frac{1}{1000000}$.
55. $\frac{17}{100}$; $\frac{27}{1000}$; $\frac{127}{1000}$.
56. $\frac{43}{100}$; $\frac{174}{10000}$; $\frac{2715}{100000}$.
57. $\frac{11}{10}$; $\frac{203}{10}$; $\frac{516}{100}$.
58. $\frac{203}{100}$; $\frac{9601}{100}$; $\frac{3254}{1000}$.
59. $\frac{571}{10}$; $\frac{571}{1000}$; $\frac{571}{100000}$.
60. $\frac{99}{10}$; $\frac{99}{1000}$; $\frac{99}{1000000}$.
61. $4\frac{3}{100}$; $7\frac{5}{10}$; $21\frac{23}{1000}$.
62. $100\frac{1}{100}$; $7\frac{9}{1000}$; $12\frac{14}{10000}$.
63. $1\frac{1}{10000}$; $231\frac{9}{10}$; $4\frac{73}{1000}$.
64. $51\frac{3}{100}$; $8\frac{8}{10000}$; $1000\frac{19}{1000}$.
65. $\frac{389}{10}$; $\frac{47}{10000}$; $80\frac{3}{100}$.
66. $\frac{23}{1000}$; $5\frac{715}{10000}$; $\frac{73561}{100}$.
67. $\frac{43256}{1000}$; $10\frac{7}{1000}$; $\frac{153}{100000}$.
68. $\frac{1753}{10000}$; $\frac{1001}{1000}$; $1457\frac{3}{100}$.
69. $\frac{101}{10}$; $10\frac{1}{1000}$; $\frac{101}{1000000}$.
70. $7\frac{1}{10000}$; $\frac{71}{10000}$; $\frac{700001}{10000}$.

Reduce to decimals—

71. $\frac{1}{5}$.	76. $\frac{7}{50}$.	81. $\frac{14}{25}$.	86. $1\frac{7}{125}$.	91. $\frac{43}{800}$.	96. $\frac{3}{32}$.
72. $\frac{4}{5}$.	77. $\frac{13}{50}$.	82. $\frac{1}{8}$.	87. $2\frac{3}{40}$.	92. $\frac{191}{250}$.	97. $\frac{27}{320}$.
73. $\frac{1}{2}$.	78. $\frac{3}{25}$.	83. $\frac{5}{8}$.	88. $7\frac{231}{500}$.	93. $\frac{1}{625}$.	98. $\frac{1}{64}$.
74. $\frac{1}{4}$.	79. $\frac{29}{20}$.	84. $\frac{7}{16}$.	89. $10\frac{3}{16}$.	94. $\frac{3}{6250}$.	99. $\frac{75}{128}$.
75. $\frac{3}{4}$.	80. $\frac{17}{20}$.	85. $\frac{103}{125}$.	90. $\frac{171}{80}$.	95. $1\frac{403}{1250}$.	100. $\frac{543}{3125}$.

XXXIV. RECURRING DECIMALS.

Find, by *short* divisions, the recurring quotient of—

1. $5{\cdot}2 \div 3$.
2. $25{\cdot}1 \div 11$.
3. $3{\cdot}412 \div 9$.
4. $32{\cdot}71 \div 6$.
5. $\cdot 5 \div 12$.
6. $1{\cdot}67 \div 60$.
7. $20{\cdot}9 \div 900$.
8. $4{\cdot}3 \div 1200$.
9. $\cdot 3 \div 7$.
10. $2{\cdot}5 \div 70$.
11. $\cdot 5 \div \cdot 9$.
12. $2{\cdot}8 \div \cdot 03$.
13. $4 \div \cdot 11$.
14. $31 \div \cdot 012$.
15. $11 \div \cdot 007$.
16. $4 \div \cdot 0007$.
17. $\cdot 5 \div 18$.
18. $8{\cdot}3 \div 44$.
19. $\cdot 23 \div 24$.
20. $17{\cdot}3 \div 36$.
21. $\cdot 5 \div 14$.
22. $\cdot 41 \div 81$.
23. $31 \div 5500$.
24. $7 \div 1440$.
25. $2 \div \cdot 27$.
26. $400 \div 6{\cdot}6$.
27. $\cdot 1 \div 7{\cdot}2$.
28. $\cdot 05 \div 5{\cdot}4$.
29. $31 \div \cdot 0022$.
30. $1 \div \cdot 048$.
31. $5 \div \cdot 0028$.
32. $\cdot 8 \div \cdot 081$.

Divide—

33. 7 by 101.
34. 5·5 by 37.
35. ·1 by 41.
36. ·01 by ·13.
37. ·3 by 7·4.
38. ·5 by 82.
39. 1·1 by 20·2.
40. ·89 by 3·9.
41. ·037 by 22·5.
42. ·1 by 9·25.
43. 40 by 16·5.
44. 7·05 by ·148.
45. 10·81 by 29·6.
46. ·45 by ·328.
47. 1·9 by 14·3.
48. 5·3 by 308.
49. 10 by 31.
50. ·4 by 17.

XXXV. RECURRING DECIMALS.

Express each of the following fractions as a recurring decimal:—

1. $\frac{1}{3}$.
2. $\frac{4}{9}$.
3. $\frac{3}{11}$.
4. $\frac{7}{11}$.
5. $\frac{1}{7}$.
6. $\frac{5}{7}$.
7. $\frac{7}{90}$.
8. $\frac{23}{300}$.
9. $\frac{47}{110}$.
10. $\frac{563}{1100}$.
11. $\frac{3}{70}$.
12. $\frac{31}{700}$.
13. $\frac{5}{6}$.
14. $\frac{11}{12}$.
15. $\frac{2}{15}$.
16. $\frac{19}{24}$.
17. $\frac{6}{35}$.
18. $\frac{5}{28}$.
19. $1\frac{1}{48}$.
20. $3\frac{1}{275}$.
21. $4\frac{7}{375}$.
22. $11\frac{7}{960}$.
23. $\frac{29}{37}$.
24. $\frac{8}{41}$.
25. $\frac{2}{13}$.
26. $\frac{1}{81}$.
27. $1\frac{1}{82}$.
28. $3\frac{5}{74}$.
29. $\frac{1}{63}$.
30. $\frac{19}{65}$.
31. $4\frac{1}{404}$.
32. $4\frac{1}{444}$.
33. $\frac{7}{192}$.
34. $\frac{9}{352}$.
35. $\frac{183}{6875}$.
36. $\frac{2201}{56250}$.
37. $5\frac{31}{156}$.
38. $2\frac{151}{189}$.
39. $\frac{6}{17}$.
40. $\frac{1}{19}$.

State, *from inspection*, which fractions in each of the following sets would yield *terminating* decimals:—

41. $\frac{3}{8}, \frac{5}{12}, \frac{4}{15}, \frac{7}{20}, \frac{11}{30}, \frac{1}{40}, \frac{19}{50}, \frac{13}{60}$.
42. $\frac{5}{9}, \frac{1}{14}, \frac{3}{16}, \frac{8}{25}, \frac{9}{32}, \frac{17}{48}, \frac{1}{55}, \frac{7}{64}$.
43. $\frac{1}{70}, \frac{1}{80}, \frac{1}{90}, \frac{1}{100}, \frac{1}{110}, \frac{1}{120}, \frac{1}{125}$.
44. $\frac{1}{140}, \frac{1}{144}, \frac{1}{160}, \frac{1}{180}, \frac{1}{200}, \frac{1}{250}$.
45. $\frac{7}{320}, \frac{11}{360}, \frac{13}{400}, \frac{17}{500}, \frac{1}{600}, \frac{3}{700}$.
46. $\frac{1}{620}, \frac{1}{625}, \frac{1}{630}, \frac{1}{635}, \frac{1}{640}, \frac{1}{645}$.

Without actually performing the division, find the *number of places of non-recurring decimal figures* in the decimal equivalent to—

47. $\frac{19}{740}$.
48. $\frac{7}{328}$.
49. $\frac{47}{1664}$.
50. $\frac{1}{3840}$.

XXXVI. RECURRING DECIMALS.

Express as vulgar fractions in their lowest terms:—

1. $\cdot\dot{2}$.
2. $\cdot\dot{5}$.
3. $\cdot\dot{6}$.
4. $\cdot\dot{8}$.
5. $4\cdot\dot{7}$.
6. $11\cdot\dot{1}$.
7. $\cdot\dot{9}$.
8. $7\cdot\dot{9}$.
9. $\cdot\dot{2}\dot{4}$.
10. $\cdot\dot{4}\dot{5}$.
11. $1\cdot\dot{7}\dot{2}$.
12. $4\cdot\dot{9}\dot{6}$.
13. $\cdot\dot{2}3\dot{4}$.
14. $\cdot\dot{2}7\dot{0}$.
15. $10\cdot\dot{3}1\dot{5}$.
16. $6\cdot\dot{8}3\dot{7}$.
17. $\cdot\dot{2}07\dot{9}$.
18. $\cdot\dot{4}06\dot{5}$.
19. $\cdot\dot{1}192\dot{4}$.
20. $\cdot\dot{0}569\dot{1}$.
21. $\cdot\dot{1}4285\dot{7}$.
22. $\cdot\dot{4}2857\dot{1}$.
23. $1\cdot\dot{5}7142\dot{8}$.
24. $3\cdot\dot{8}5714\dot{2}$.
25. $\cdot\dot{8}2417\dot{5}$.
26. $\cdot\dot{7}5824\dot{1}$.
27. $\cdot\dot{4}7619\dot{0}$.
28. $\cdot\dot{9}2307\dot{6}$.
29. $\cdot\dot{0}8641975\dot{3}$.
30. $\cdot\dot{0}11086474\dot{5}$.
31. $\cdot 8\dot{3}$.
32. $\cdot 3\dot{6}$.
33. $2\cdot 5\dot{7}$.
34. $1\cdot 0\dot{1}$.
35. $\cdot 2\dot{3}\dot{4}$.
36. $\cdot 3\dot{4}\dot{8}$.
37. $\cdot 0\dot{1}\dot{5}$.
38. $\cdot 97\dot{2}$.
39. $\cdot 01\dot{0}\dot{9}$.
40. $\cdot 02\dot{2}\dot{7}$.
41. $1\cdot 48\dot{3}$.
42. $2\cdot 708\dot{3}$.
43. $\cdot 2\dot{3}8\dot{7}$.
44. $\cdot 3\dot{8}3\dot{7}$.
45. $1\cdot 236\dot{1}$.
46. $3\cdot 002\dot{6}$.
47. $\cdot 10\dot{8}4\dot{6}$.
48. $\cdot 34\dot{4}5\dot{9}$.
49. $\cdot 8\dot{1}951\dot{2}$.
50. $\cdot 5\dot{0}731\dot{7}$.
51. $\cdot 00010\dot{6}$.
52. $\cdot 11458\dot{3}$.
53. $\cdot 0648\dot{1}\dot{4}$.
54. $\cdot 1003\dot{3}7\dot{8}$.
55. $\cdot 9\dot{2}8571\dot{4}$.
56. $\cdot 6\dot{5}7142\dot{8}$.
57. $\cdot 03\dot{4}2857\dot{1}$.
58. $\cdot 129\dot{1}4285\dot{7}$.
59. $\cdot 839\dot{2}8571\dot{4}$.
60. $\cdot 05\dot{7}6923\dot{0}$.

XXXVII. RECURRING DECIMALS.

ADDITION AND SUBTRACTION.

Find, without reducing to vulgar fractions, the value of—

1. $\cdot\dot{2}\dot{7}+\cdot\dot{3}45\dot{6}$.
2. $\cdot\dot{3}\dot{4}+\cdot\dot{5}7\dot{1}$.
3. $\cdot\dot{1}4285\dot{7}+\cdot\dot{0}7\dot{6}$.
4. $\cdot\dot{1}7690\dot{4}+\cdot\dot{3}8\dot{4}$.
5. $\cdot 2\dot{3}+\cdot 7\dot{5}\dot{1}$.
6. $\cdot 40\dot{1}\dot{5}+\cdot\dot{3}\dot{2}$.
7. $1\cdot 8\dot{5}\dot{4}+\cdot 5\dot{6}$.
8. $5\cdot 1\dot{4}\dot{7}+1\cdot 00\dot{8}$.
9. $\cdot 0\dot{1}+\cdot 00\dot{2}+\cdot 5$.
10. $\cdot 4\dot{1}\dot{3}+\cdot 8+\cdot 0\dot{4}$.
11. $\cdot 37\dot{4}\dot{6}-\cdot 08\dot{2}\dot{3}$.
12. $1\cdot 48\dot{6}\dot{3}-\cdot 73\dot{8}\dot{1}$.
13. $9\cdot 78\dot{6}\dot{3}-5\cdot 7\dot{8}\dot{3}$.
14. $\cdot 247\dot{6}-\cdot 10\dot{3}\dot{8}$.
15. $\cdot 4\dot{2}7\dot{5}-\cdot 30\dot{2}6\dot{7}$.
16. $3\cdot 4\dot{1}2\dot{7}-1\cdot\dot{0}8\dot{7}$.
17. $3\cdot\dot{2}47\dot{1}-2\cdot 2471$.
18. $1\cdot\dot{5}7\dot{6}-1\cdot 0576$.
19. $17-\cdot\dot{1}4285\dot{7}$.
20. $5-\cdot\dot{0}7692\dot{3}$.
21. $4\cdot 76\dot{3}\dot{0}-1\cdot 0\dot{2}8\dot{5}$.
22. $\cdot 20\dot{3}5\dot{1}-\cdot 000176\dot{4}$.
23. $\cdot\dot{5}134\dot{2}-\cdot\dot{3}\dot{6}$.
24. $\cdot\dot{7}4\dot{2}-\cdot\dot{3}16\dot{5}$.
25. $\cdot\dot{6}3\dot{9}+\cdot\dot{0}2\dot{7}+\cdot\dot{3}$.
26. $\cdot\dot{8}\dot{7}+\cdot\dot{6}+\cdot\dot{4}\dot{5}$.
27. $\cdot 2+\cdot\dot{4}\dot{5}+\cdot\dot{5}\dot{4}$.
28. $\cdot 4\dot{0}\dot{9}+\cdot 6\dot{1}+\cdot 4\dot{7}\dot{9}$.
29. $\cdot 6\dot{1}42\dot{3}+\cdot 2\dot{8}57\dot{6}$.
30. $\cdot\dot{4}45\dot{5}+\cdot\dot{0}09\dot{0}-\cdot\dot{4}\dot{5}$.

Simplify—

31. $123\cdot 546\dot{2}+34\cdot\dot{0}36\dot{7}+70\cdot\dot{4}\dot{5}$.
32. $4\cdot 3\dot{5}+765+7\cdot 2\dot{6}1\dot{3}+1\cdot 5\dot{4}\dot{7}$.
33. $3\cdot 7\dot{1}\dot{3}+2\cdot\dot{5}671\dot{2}+5\cdot 6+\cdot 12\dot{3}\dot{7}$.
34. $\cdot\dot{2}0879\dot{1}+\cdot\dot{0}7692\dot{3}+\cdot\dot{7}1428\dot{5}$.
35. $\cdot 1\dot{8}5714\dot{2}+\cdot\dot{2}8571\dot{4}+\cdot 0\dot{2}8571\dot{4}$.
36. $5\cdot 3\dot{8}5714\dot{2}+\cdot 960\dot{5}7142\dot{8}$.
37. $3\cdot 70\dot{4}\dot{3}+1\cdot\dot{0}56\dot{7}-2\cdot 513\dot{6}$.
38. $\cdot\dot{4}71\dot{6}-\cdot\dot{2}59\dot{0}+2\cdot 1305\dot{8}$.
39. $\cdot\dot{6}\dot{3}+\cdot\dot{5}4\dot{0}-\cdot\dot{1}7690\dot{4}+999$.
40. $\cdot\dot{9}756\dot{0}+\cdot 73\dot{4}145\dot{3}-\cdot\dot{6}097\dot{5}$.

XXXVIII. RECURRING DECIMALS.

MULTIPLICATION AND DIVISION.

Find, without reducing to vulgar fractions, the value of—

1. $\cdot 3\dot{2} \times 4$.
2. $3\cdot\dot{4}\dot{5} \times 3$.
3. $7\cdot\dot{2}\dot{4} \times 5$.
4. $\cdot\dot{4}5\dot{6} \times 12$.
5. $\cdot 7\dot{2}\dot{6} \times 45$.
6. $\cdot 3\dot{1}\dot{5} \times 23$.
7. $3\cdot 2\dot{4}3\dot{7} \times \cdot 5$.
8. $\cdot 07\dot{1}\dot{3} \times \cdot 08$.
9. $\cdot 4\dot{7}\dot{5} \times \cdot 015$.
10. $\cdot 3\dot{8}4\dot{7} \div 5$.
11. $7\cdot 3\dot{4}\dot{5} \div 4$.
12. $\cdot 372\dot{1}\dot{4} \div 16$.
13. $\cdot 70\dot{5}6\dot{1} \div 24$.
14. $1\cdot\dot{3} \div 37$.
15. $14\cdot\dot{5} \div 41$.
16. $23\cdot 465\dot{3} \div \cdot 02$.
17. $15\cdot 4\dot{6} \div \cdot 21$.
18. $4\cdot 8\dot{3} \div \cdot 035$.

19. Multiply $\cdot 36458\dot{3}$ by 47000.
20. Divide $\cdot 0062\dot{5}\dot{4}$ by $\cdot 00027$

Express as a decimal the product of—

21. $\cdot\dot{6}\dot{3} \times \cdot\dot{5}3\dot{9}$.
22. $\cdot 0\dot{6} \times \cdot 1\dot{6}\dot{3}$.
23. $\cdot\dot{2}\dot{7} \times \cdot 91\dot{6}$.
24. $\cdot\dot{9}\dot{0} \times 7\cdot 8\dot{3}$.
25. $\cdot\dot{8}\dot{1} \times \cdot 8\dot{1}$.
26. $6\cdot 7\dot{6} \times \cdot 0\dot{5}$.
27. $\cdot 049\dot{5} \times \cdot 30\dot{2}\dot{7}$.
28. $\cdot 56\dot{7} \times \cdot 48\dot{9}6\dot{3}$.
29. $1\cdot 1\dot{6} \times \cdot\dot{4}2857\dot{1}$.

Express as a decimal the quotient of—

30. $\cdot 1575 \div 5\dot{0}\dot{9}$.
31. $\cdot\dot{2}\dot{1} \div \cdot 0\dot{1}$.
32. $\cdot\dot{3}7\dot{0} \div \cdot\dot{5}$.
33. $1\cdot 6\dot{4} \div 10\cdot\dot{0}\dot{9}$.
34. $1\cdot\dot{2}3\dot{1} \div 3\cdot\dot{6}\dot{3}$.
35. $1\cdot 89 \div \cdot\dot{7}1428\dot{5}$.
36. $23\cdot\dot{6}4\dot{8} \div \cdot\dot{9}4\dot{5}$.
37. $17\cdot 61 \div \cdot\dot{5}28\dot{3}$.
38. $\cdot 8547 \div 1\cdot 3\dot{6}76\dot{5}$.

39. Multiply $6\cdot\dot{3}\dot{6}$ by $\cdot\dot{5}7142\dot{8}$.
40. Divide $\cdot 2\dot{1}4285\dot{7}$ by $\cdot\dot{5}8730\dot{1}$.

Simplify, giving results as decimals :—

41. $\cdot 065 + \cdot 75 \times \cdot 8\dot{3}$.
42. $2\cdot 4 \times 7\cdot\dot{6} - 1\cdot 84$.
43. $(12\cdot\dot{6} - 7\cdot 41\dot{6}) \times 2\cdot\dot{1}$.
44. $(8\cdot 41\dot{6} + 5\cdot 58\dot{3}) \div 2\cdot 1$.
45. $1\cdot 71 \div (\cdot\dot{1}4285\dot{7} + \cdot\dot{2}8571\dot{4})$.
46. $\cdot\dot{4}2857\dot{1} \times (4\cdot 3 + 3\cdot 4)$.
47. $\cdot\dot{4}2857\dot{1} \times 4\cdot 3 + 3\cdot 4$.
48. $1\cdot 71 \div \cdot\dot{1}4285\dot{7} - \cdot\dot{2}8571\dot{4}$.
49. $1\cdot 84 \times \cdot 8\dot{3} \div 6\cdot\dot{2}\dot{7}$.
50. $1\cdot 01\dot{9} \times \cdot 1\dot{3} \div \cdot 034$.
51. $\dfrac{53\cdot\dot{8}\dot{1}}{14\cdot 8}$.
52. $\dfrac{\cdot 019\dot{4}}{1\cdot 1\dot{6}}$.
53. $\dfrac{3\cdot 1\dot{2} - 2\cdot\dot{3}\dot{1}}{\cdot\dot{8}\dot{1}}$.
54. $\dfrac{1\cdot 8\dot{3} \times \cdot 43\dot{1}\dot{8}}{1\cdot 1875}$.
55. $\dfrac{2\cdot\dot{4}\dot{5} \times 3\cdot 1\dot{6} \div \cdot 01\dot{2}\dot{6}}{615\cdot 6}$.
56. $1\cdot 6 \times \dfrac{\cdot 5\dot{3}}{\cdot 6\dot{1}} \times \dfrac{\cdot 825}{\cdot 41\dot{6}}$.
57. $\dfrac{\cdot 875 \times \cdot\dot{2}7\dot{0}}{\cdot 125 + \cdot 125\dot{6}7\dot{5}} + \dfrac{3}{53}$.
58. $\dfrac{2\cdot 8 \times 11\cdot\dot{3}\dot{6}}{5\cdot 6\dot{8}\dot{1}} - \dfrac{1\cdot\dot{6}\dot{3}}{\cdot\dot{4}\dot{5}}$.
59. $\dfrac{2\cdot 791\dot{6} \times 3\cdot 2\dot{3}\dot{7}}{1\cdot 86\dot{1} \times \cdot 809\dot{3}\dot{4}} \times \cdot 08\dot{3}$.
60. $\dfrac{\cdot 041\dot{6} \div \cdot\dot{6}\dot{0}}{\cdot 02\dot{2}\dot{7} \div \cdot 00\dot{3}} \div \dfrac{\cdot 7\dot{3}}{\cdot 8}$.

XXXIX. DECIMALS.

Value of a Decimal of a Concrete Quantity.

Find the value of—

1. ·375*s*.
2. 4·625*s*.
3. £·85.
4. £2·175.
5. £·1375.
6. £·9375.
7. ·375 of 6*d*.
8. 3·75 of 3*d*.
9. ·7375 of 10*s*.
10. 1·0625 of 5*s*.
11. 1·5625 of 2*s*.
12. ·65625 of 4*s*.
13. 1·6875 of £2.
14. 3·0325 of £5.
15. ·596875 of £1.
16. ·778125 of £1.
17. ·4790625 of £10.
18. ·0053125 of £20.
19. ·6875 of 3*s*. 4*d*.
20. ·53125 of 6*s*. 8*d*.
21. 1·125 of 2*s*. 6*d*.
22. 2·825 of 5*s*. 10*d*.
23. 1·0625 of a guinea.
24. 1·0375 of £3, 10*s*.
25. ·375 of £3, 17*s*. 6*d*.
26. ·1875 of £4, 2*s*. 8*d*.
27. ·0136 of £14, 6*s*. $5\frac{1}{2}$*d*.
28. ·001024 of £16, 5*s*. $6\frac{1}{4}$*d*.
29. ·8125 of 1 cwt.
30. ·0625 of 3 cwts.
31. ·07890625 of a ton.
32. ·66796875 of a ton.
33. 2·34375 miles.
34. ·90625 of a cub. yd.
35. ·8125 of 2 tons 4 cwt.
36. 4·2128 of 5 cwt. 2 qr. 9 lb.
37. ·028125 of 1 mi. 6 fur.
38. ·390625 of 6 A. 1 R. 24 P.
39. Find the value of £·525 + 5·25 of 10*s*. + 52·5*s*.
40. Find the value of £1·285 − 5·115*s*. + 6·92 farthings.
41. Add together 2·5 of 1 ton 6 cwts., 3·125 of 2 qrs. 16 lbs., and 3·75 of 1 qr.
42. Add together ·9 of a pole, ·241 of a yd., ·97 of a foot, and ·684 of an inch.
43. From ·6409 of 1 ac. 2 ro. take the sum of ·1892 of 4 ac. and ·1894 of 3 ro.
44. Subtract the sum of ·3125 of 6 cwts. and ·032 of 3 cwts. 2 qrs. 14 lbs. 4 ozs. from 2 cwts.

Find the value of—

45. $\cdot 8\dot{3}$ of 2*s*. 6*d*.
46. $\cdot 02\dot{7}$ of 10*s*. 6*d*.
47. $\cdot 791\dot{6}$ of a guinea.
48. $\cdot 04\dot{8}$ of £1, 12*s*. $9\frac{3}{4}$*d*.
49. $\cdot 6\dot{1}$ of 1 yd.
50. $\cdot 0\dot{1}\dot{5}$ of 1 po.
51. $\cdot 01\dot{2}\dot{5}$ of 5 mi.
52. $2\cdot\dot{3}\dot{5}$ miles.
53. $\cdot\dot{4}2857\dot{1}$ of 1*s*. 2*d*.
54. $\cdot 3\dot{5}7142\dot{8}$ of £1, 1*s*.
55. $\cdot 017708\dot{3}$ of £1.
56. $\cdot 01420\dot{4}\dot{5}$ of £1, 2*s*.
57. Find the difference between £$\cdot 641\dot{6}$ and $3\cdot\dot{0}69\dot{3}$ of 8*s*. 5*d*.
58. Subtract $\cdot\dot{7}6923\dot{0}$ of 5*s*. 5*d*. from $\cdot 5\dot{0}\dot{9}$ of 9*s*. 2*d*.
59. From ·856 of 2 cwts. 26 lbs. take $3\cdot 2\dot{2}\dot{7}$ of 2 qrs. 10 lbs.
60. Take $\cdot 3\dot{5}$ of 5·625 tons from $\cdot 7\dot{0}\dot{9}$ of 5 tons 3 cwts. 14 lbs.

XL. DECIMALS.

Reduction of one Quantity to the Decimal of another.

Express—

1. 8*s.* as the decimal of £1.
2. 12*s.* as the decimal of £1.
3. 3*s.* .... £1.
4. 7*s.* .... £1.
5. 5*s.* .... £1.
6. 15*s.* .... £1.
7. 2*s.* 6*d.* .... £1.
8. 4*s.* 6*d.* .... £1.
9. 2*s.* 3*d.* .... £1.
10. 1*s.* 9*d.* .... £1.
11. 9*d.* .... 1*s.*
12. $1\frac{1}{2}$*d.* .... 1*s.*
13. $7\frac{1}{2}$*d.* .... 1*s.*
14. $5\frac{1}{4}$*d.* .... 1*s.*
15. $\frac{3}{4}$*d.* .... 1*s.*
16. $11\frac{1}{4}$*d.* .... 1*s.*
17. 15*s.* 6*d.* .... £5.
18. 10*s.* 6*d.* .... £3.
19. 4*s.* $4\frac{1}{2}$*d.* .... £10.
20. 14*s.* $7\frac{1}{2}$*d.* .... £5.
21. 1 qr. 7 lbs. .... 10 cwts.
22. 2 qrs. 21 lbs............. 1 ton.
23. 2 cwts. 1 qr. 7 lbs.... 1 ton.
24. 1 ton 3 cwts. 14 lbs.... 5 tons.
25. 1 lb. 5 ozs. .... 1 cwt.
26. 7 ozs. .... 2 cwts.
27. 1 chain .... 1 mile.
28. 2 ft. $7\frac{1}{2}$ in................ 100 yds.
29. 1 ac. 1 ro. 10 po. .... 12 acres.
30. 3 ac. 1 ro. 2 po. .... 50 acres.

31. Reduce 2*s.* $7\frac{1}{2}$*d.* to the decimal of 8 guineas.
32. Reduce 6*s.* $6\frac{3}{4}$*d.* to the decimal of 4 guineas.
33. What decimal of £100 is £5, 17*s.* $7\frac{1}{2}$*d.*?
34. What decimal of £20 is £7, 13*s.* $6\frac{3}{4}$*d.*?
35. Express £5, 3*s.* $4\frac{1}{2}$*d.* as the decimal of £1.
36. Express £14, 10*s.* $10\frac{1}{2}$*d.* as the decimal of £1.
37. What decimal of £1000 is $\frac{3}{5}$ of a guinea?
38. What decimal of £100000 is $2\frac{2}{5}$ of £4?
39. What decimal of a ton is 2 cwts. 3 qrs. 3 lbs. 8 ozs.?
40. What decimal of a mile is 11 po. 4 yds. $4\frac{1}{2}$ in.?
41. Express $1\frac{2}{7}$ of $\frac{3}{11}$ of £1, 12*s.* 1*d.* as the decimal of £100.
42. Express $\frac{4}{19}$ of ·153 of 12*s.* 8*d.* as the decimal of £10.
43. Reduce 2·307 of £1, 3*s.* $2\frac{1}{2}$*d.* to pence and the decimal of a penny.
44. Reduce ·1703 of £2, 13*s.* $3\frac{1}{4}$*d.* to pence and the decimal of a penny.
45. Reduce ·00125 of 2 qrs. $3\frac{1}{2}$ lbs. to ounces and the decimal of an ounce.
46. Reduce ·315 of 9 lbs. 7 ozs. 6 grs. to dwts. and the decimal of a dwt.

47. Reduce ·027 of 1 qr. 3 bush. 3 pks. 1 gal. to pints and the decimal of a pint.

48. Reduce 1·0374 of $3\frac{3}{7}$ acres to sq. yds. and the decimal of a sq. yd.

49. Subtract ·0015 of a week from ·375 of 1 hr. 20 min., and express the result in seconds and the decimal of a second.

50. Subtract ·0023 of 1 week 1 day from ·63 of 5 hours, and express the result in minutes and the decimal of a minute.

Express—

51. 8*d.* as a recurring dec. of 1*s.*

52. 5*d.* as a recurring dec. of 1*s.*

53. 1*d.* .... 1*s.*

54. $1\frac{3}{4}$*d.* .... 1*s.*

55. $10\frac{3}{4}$*d.* .... 1*s.*

56. $11\frac{1}{2}$*d.* .... 1*s.*

57. 13*s.* 4*d.* .... £1.

58. 16*s.* 8*d.* .... £1.

59. 7*s.* 2*d.* .... £1.

60. 5*s.* 1*d.* .... £1.

61. Reduce 10*s.* 4*d.* to the recurring decimal of £1, 7*s.* 6*d.*

62. .... 19*s.* .... £4, 8*s.*

63. .... 8*s.* 5*d.* .... £1, 4*s.*

64. .... 8*s.* 7*d.* .... £3.

65. .... 5*s.* 11*d.* .... 13*s.* 8*d.*

66. .... 3*s.* $8\frac{1}{2}$*d.* .... 12*s.* 4*d.*

67. .... 3*s.* 9*d.* .... 4*s.* 8*d.*

68. .... 4*s.* 6*d.* .... 5*s.* 5*d.*

69. .... 7 cwts. 21 lbs. .... 2 tons 15 cwt.

70. .... 2 qrs. 7 lbs. .... 8 cwts. 1 qr.

71. .... 1 inch .... 1 pole.

72. .... 1 ft. 6 in. .... $\frac{1}{4}$ mile.

73. .... $\frac{4}{5}$ of 1 yard .... $\frac{2}{5}$ of 3 fur.

74. .... $\frac{1}{8}$ of 4 ozs. 19 dwt. .... 16 dwts. 21 grs.

75. .... 11 sq. po. 11 sq. yds. .... 1 acre.

76. .... 1 day 17 hrs. .... 7 days 7 hrs.

77. What decimal of 2 guineas is £1, 14*s.* $9\frac{3}{4}$*d.*?

78. What decimal of 11 cwts. is 11 lbs. 11 ozs.?

79. Express the sum of $\cdot 00892708\dot{3}$ of £100 and $\cdot\dot{6}85714\dot{2}$ of £2, 2*s.* 7*d.* as the decimal of £41, 13*s.* 4*d.*

80. Reduce the difference between $\cdot 10\dot{1}5384\dot{6}$ of 1 qr. 12 lbs. 10 ozs., and $\cdot 0000651041\dot{6}$ of 3 tons to the decimal of 9 lbs.

XLI. MISCELLANEOUS EXERCISES.

1. Which is greater: Three-thousandths, or Three-hundredths?
2. Express in figures, as a decimal: Three and three-thousandths.
3. Express $\cdot 000007$ in words.
4. Write $\frac{123}{10000}$ as a decimal.
5. Add together $3\cdot 9064$, $\cdot 08456$, $327\cdot 265$, and $14\cdot 00983$.
6. Subtract $53\cdot 94306$ from $60\cdot 1531$.
7. Multiply $82\cdot 5604$ by $\cdot 08425$.
8. Divide $157\cdot 311$ by 3405.
9. Simplify $8\cdot 8 - 2\cdot 034 \times \cdot 35$.
10. How many times is $\cdot 0005$ of a penny contained in a shilling?

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11. Which is greater: Thirty-thousandths, or Three-hundredths?
12. Express Thirty-seven millionths in figures.
13. Write $\cdot 0047$ as a vulgar fraction.
14. Find the sum of $85\cdot 0609$, $4\cdot 68403$, $\cdot 00689$, and $529\cdot 873$.
15. Find the difference between $59\cdot 98063$ and $72\cdot 480527$.
16. Find the product of $\cdot 98236$ and $370\cdot 95$.
17. Divide, using short division, $271\cdot 403$ by 64.
18. Divide $2\cdot 3517$ by $\cdot 005$.
19. Simplify $(8\cdot 8 - 2\cdot 034) \times \cdot 35$.
20. A mowed $\cdot 175$ of a field, and B mowed $\cdot 0175$ of it: what decimal part of the field then remained unmown?

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21. Write down, at sight, the value of $4\cdot 7031 \times 100$.
22. State fully in words the meaning of $2\cdot 106$ inches.
23. Express $\frac{7}{10} + \frac{3}{1000}$ as a decimal.
24. Express $\cdot 0112$ as a vulgar fraction in its lowest terms.
25. What must be added to $91\cdot 006705$ to make $100\cdot 3$?
26. Find the square of $\cdot 0217$.
27. Divide, using short division, $\cdot 157035$ by $\cdot 0032$.
28. Simplify $2\cdot 34 \times 1\cdot 025 \div \cdot 0009$.
29. The product of two decimals is $\cdot 135$; one of them is $11\cdot 25$: find the other.
30. Reduce £$1\cdot 0375$ to pence.

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31. Write down, at sight, the value of $29 \cdot 35 \div 1000$.
32. Express $3 \cdot 00671$ as an improper vulgar fraction.
33. Find the value of $116 \cdot 001 - 47 \cdot 91308$.
34. Find the value of $1593 \div \cdot 0472$.
35. Find the value of $2 \cdot 405 \times \cdot 01308$.
36. Find the value of $27 \cdot 9875 + \cdot 0059625 + 381 \cdot 4 + 97 + \cdot 59623$.
37. Change $\frac{711}{128}$ into a decimal.
38. Simplify $\dfrac{6 \cdot 3 - 4 \cdot 26}{\cdot 0017}$.
39. Reduce £5·1875 to pence.
40. Express 2 qrs. 14 lbs. as the decimal of half a ton.

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41. Add Eleven *ten-thousandths* to Ten and eleven *thousandths*.
42. Write down, at sight, the quotient of $47 \cdot 83 \div 100$.
43. Find the value of $14 \cdot 1 - 7 \cdot 568 + 1 \cdot 42325 - 5 \cdot 008 + \cdot 2504 - 3 \cdot 197$.
44. Find the cube of $\cdot 023$.
45. Divide $\cdot 00567525$ by $\cdot 0805$.
46. Express $3\frac{19}{125}$ as a decimal.
47. Simplify $\dfrac{1 \cdot 44 \times \cdot 0032}{7 \cdot 2}$.
48. Express ·7875 of a mile in furlongs and poles.
49. How many times is £·125 contained in 17·5*s.*?
50. Reduce £3, 1*s.* $1\frac{1}{2}$*d.* to the decimal of £5.

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51. From One hundred, and seven-*tenths* take One-hundred-and-seven-*thousandths*.
52. Write down at sight the value of $57 \cdot 83 \times 1000$.
53. Find the continued product of $3 \cdot 4$, $\cdot 34$, $\cdot 034$ and 3400.
54. Find the value of $24 \cdot 048 - 3 \cdot 2735 - \cdot 08 - 12 \cdot 63575 - 6 \cdot 7 - 1 \cdot 311$.
55. Divide $204 \cdot 079$ by $\cdot 0437$.
56. Express $\cdot 71875$ as a vulgar fraction in its lowest terms.
57. Simplify $\dfrac{\cdot 00062}{\cdot 64}$.
58. Find the value of 5·25625 of 1 oz. 13 dwts. 8 grs.
59. Express ·42 of 4*s.* $10\frac{1}{2}$*d.* as the decimal of £1, 11*s.* 6*d.*
60. A book is $1\frac{3}{4}$ inches in thickness; the covers are each $\frac{3}{16}$ in. in thickness, and there are 500 pages; what decimal of an inch is the paper in thickness?

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61. Which is greater, $\cdot 3456$ or $\cdot 35$?

62. Write down at sight as a decimal the value of $\dfrac{41\cdot 23}{1000}$.

63. Without dividing, express $\frac{3}{16}$ as a decimal.

64. What must be added to $87\cdot 6993$ to make $392\cdot 0062436$?

65. Divide the product of $\cdot 004$ and $32\cdot 4$ by $6\cdot 4$.

66. Simplify $2\cdot 345 \times \cdot 062 + \cdot 85461$.

67. Simplify $\dfrac{22\cdot 4}{\cdot 25} + \dfrac{250}{\cdot 8} + \dfrac{1\cdot 2}{\cdot 0075}$.

68. Find the smallest integer which contains $7\cdot 4$ and $1\cdot 11$ each an integral number of times.

69. How many times is $\cdot 01$ of an inch contained in a yard?

70. Express the sum of £$0\cdot 1037$ and £$1\cdot 0713$ in £ *s. d.*

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71. What is the difference between $\cdot 700$ and $\cdot 7$?

72. Write down at sight as a decimal the value of $\dfrac{721 \div 100}{100}$.

73. Without dividing, express $\frac{407}{625}$ as a decimal.

74. Find $\cdot 479 + \cdot 479 + \cdot 479 + \cdot 479 + \cdot 479 + \cdot 479 + \cdot 479 + \cdot 479 + \cdot 479 + \cdot 479$.

75. Divide the difference between $\cdot 7$ and $\cdot 12$ by $\cdot 4$.

76. Simplify $6240\cdot 1 - 52\cdot 104 \div \cdot 0835$.

77. Simplify $\dfrac{\cdot 23 - \cdot 023}{\cdot 0023 \div 23000}$.

78. Find the smallest decimal which contains $\cdot 0289$ and $1\cdot 87$, each an integral number of times.

79. How many times is $\cdot 003$ of a shilling contained in a guinea?

80. Express the sum of $\cdot 625$ of £3 and $\cdot 175$ of £3, 2*s.* 6*d.* as the decimal of £50.

---

81. Find the value of $\cdot 91 \times 19 \times 1\cdot 01 \times 110$.

82. Without dividing, convert $\frac{2}{500} + \frac{5}{2000}$ into a decimal.

83. Reduce $\frac{1}{5} + \frac{1}{25} + \frac{1}{125} + \frac{1}{625} + \frac{1}{3125}$ to a decimal.

84. Find the sum of the squares of $\cdot 17$, $\cdot 017$ and $\cdot 0017$.

85. Simplify $\dfrac{\cdot 0038425 - \cdot 00183}{\cdot 013 + \cdot 022}$.

86. Simplify $(\cdot 03 + \cdot 016) \times \cdot 1 - \cdot 03611 \div 7\cdot 85$.

87. Find the value of $4\cdot 2128$ of 5 cwts. 2 qrs. 9 lbs.

88. Reduce $\cdot 0125$ of $\frac{4}{11}$ of 1 mile 23 poles to inches and the decimal of an inch.

89. $\cdot 278$ of a certain sum of money is £1, 3*s.* 2*d.* Find the sum.

90. Divide the sum of £10, 2*s.* 1*d.* between A, B and C, so that A may have $\cdot 6$ of the whole sum, B $\cdot 06$ of it, and C the rest.

---

91. Express $2\cdot 4 \times 3\cdot 15$ as a vulgar fraction in its simplest form.

92. Show that $\dfrac{1}{1 \times 2} + \dfrac{1}{2 \times 3} + \dfrac{1}{3 \times 4} + \dfrac{1}{4 \times 5} = \cdot 8.$

93. Find the difference of the squares of $5\cdot 4$ and $4\cdot 5$.

94. Simplify $22\cdot 891 + \cdot 23 \times (100\cdot 1 - 90\cdot 09) \div \cdot 013$.

95. Simplify $\dfrac{\cdot 00281 \times \cdot 0625}{13\cdot 39 - 11\cdot 985}$.

96. Find the value of $3\cdot 725$ of 25 miles 4 furlongs.

97. Reduce $\frac{2}{3}$ of $\cdot 69$ of 1 day 3 hours to minutes and the decimal of a minute.

98. What decimal of £5, 4*s.* 6*d.* is $\cdot 25$ of £1, 6*s.* $1\frac{1}{2}$*d.*?

99. Find the value of $\cdot 2$ of 6*s.* 8*d.* + $\cdot 4375$ of £1 + $\cdot 25$ of £1, 12*s.* 4*d.* + $\cdot 1$ of a crown.

100. A man after paying away $\cdot 75$ of his money to one person, and then $\cdot 75$ of the residue to another, had £6, 13*s.* left. How much had he at first?

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101. Which is greater: $\cdot 44$ or $\cdot\dot{4}$?

102. Find the complete quotient of $9\cdot 4 \div 11$.

103. Reduce $\frac{7}{27}$ to a decimal.

104. Convert $\cdot\dot{7}2\dot{9}$ into a vulgar fraction in its lowest terms.

105. Add together $2\cdot 13\dot{7}$, $3\cdot 8\dot{2}\dot{3}$ and $1\cdot 29$.

106. Subtract $3\cdot\dot{1}6\dot{2}$ from $5\cdot 714\dot{2}4\dot{6}$.

107. Multiply $\cdot\dot{5}\dot{4}$ by $3\cdot\dot{6}$.

108. Divide $\cdot\dot{1}2\dot{3}$ by $\cdot\dot{1}$, and express the result as a decimal.

109. Find the value of $\cdot\dot{0}\dot{9}$ of 2*s.* 9*d.*

110. What decimal of 1 cwt. is 16 lbs.?

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111. Express in figures, as a decimal, Seventeen ninety-ninths.

112. Find the complete quotient of $9\cdot 1 \div 41$.

113. Reduce $\frac{7}{24}$ to a decimal.

114. Convert $\cdot 97\dot{3}$ into a vulgar fraction in its lowest terms.

115. Find the sum of $\cdot 0\dot{2}\dot{7}$, $1\cdot 3\dot{4}\dot{5}$, and $2\cdot 08\dot{3}$.

116. Find the difference between $2\cdot\dot{5}3\dot{5}$ and $2\cdot 535$.

117. Express the product of $\cdot\dot{7}5\dot{9}$ and $\cdot\dot{4}\dot{5}$ as a decimal.

118. Divide $3\cdot5$ by $7\cdot\dot{0}\dot{7}$, and express the result as a decimal.

119. Find the number of square yards in $\cdot02\dot{2}\dot{7}$ of an acre.

120. What decimal of a guinea is equal to $\cdot375$ of £1?

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121. From sixty-three *hundred* take sixty-three *hundredths*.

122. Multiply $3\cdot7532$ by 1000, and divide $4\cdot27$ by 100.

123. Change $4\cdot2\dot{3}\dot{6}$ into a vulgar fraction in its lowest terms.

124. Reduce $\frac{7}{480}$ to a decimal.

125. Divide $\cdot00858\dot{3}$ by $\cdot024$, without changing the decimals into vulgar fractions.

126. Simplify $\dfrac{\cdot203 \times \cdot003 \times \cdot16}{\cdot008 \times \cdot0029}$.

127. Simplify $\dfrac{2\cdot8 \times 11\cdot\dot{3}\dot{6}}{5\cdot6\dot{8}\dot{1}}$.

128. Find the difference between $\cdot6$ of 6*s.* and $\cdot1\dot{6}$ of 16*s.*

129. Reduce $1\cdot047$ of 14 days 5 hrs. to mins. and the decimal of a minute.

130. How many pieces each $\cdot0014$ inches long can be cut from a line whose length is $2\cdot5$ inches? How long is the piece which is left over?

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131. Write down as a decimal, Eighty-three *nine-hundred-and-ninety-ninths.*

132. Reduce $\cdot016$ and $\cdot01\dot{6}$ to vulgar fractions in their lowest terms.

133. Convert $\frac{7}{160}$ and $\frac{160}{7}$ into decimals.

134. By what decimal must $\cdot000471$ be divided to give the quotient 200?

135. Simplify $\dfrac{\cdot5\dot{3}}{\cdot6\dot{1}}$ of $\dfrac{\cdot825}{\cdot41\dot{6}}$ of $1\cdot\dot{6}$.

136. Simplify $\dfrac{12\cdot4 + \cdot064 - \cdot066}{\cdot002.}$.

137. Multiply $2\cdot\dot{4}0\dot{7}$ by 83 without converting the decimal to a vulgar fraction.

138. Find the value of £$15\cdot125 + 17\cdot3025$*s.* $+ 9\cdot62$*d.*

139. Subtract $\cdot406$ of 2 acres 1 rood from $5\cdot25$ of 1 rood 13 perches, and express the answer in perches and the decimal of a perch.

140. How many bits, each $\cdot13$ of a foot long, can be cut from a length of 1 yard, and what decimal of an inch will be left over?

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141. Write down the product of $\cdot\dot{6}7\dot{0} \times 100$.

142. Which is greater: $\frac{3}{17}$ or $\cdot1\dot{7}$?

143. Express $\dfrac{\cdot\dot{2}1\dot{1}}{\cdot\dot{2}\dot{1}}$ as a vulgar fraction in its simplest form.

144. Simplify $\dfrac{(1\cdot005 + \cdot201)(1\cdot005 - \cdot201)}{1\cdot005 \times \cdot201}$.

145. Find the sum of $\cdot\dot{2}$, $\cdot\dot{0}\dot{2}$, $\cdot\dot{0}0\dot{2}$ and $\cdot\dot{0}00\dot{2}$.

146. Multiply $3\cdot2\dot{1}\dot{4}$ by $\cdot058$, without converting the decimals into vulgar fractions.

147. A book of 384 pages is exactly 1 inch thick exclusive of the cover. What decimal of an inch is the paper in thickness?

148. Express the difference between £137·75 and £13·775 in £ *s.* *d.*

149. Find the value of $\cdot41\dot{6}$ of $\dfrac{\cdot015 \text{ of } 2\cdot1}{\cdot035}$ of $\cdot0\dot{2}8571\dot{4}$ of 11*s.* 8*d.*

150. Sixty-six shillings are coined from a pound Troy of a metal containing 37 parts pure silver and three parts alloy. Express as the decimal of an ounce Troy the quantity of pure silver in one shilling.

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151. Which is greatest, and which least, of the decimals $\cdot\dot{0}3\dot{7}$, $\cdot0\dot{3}\dot{7}$, $\cdot03\dot{7}$?

152. Express $\dfrac{3}{10^3} + \dfrac{5}{10^5} + \dfrac{7}{10^7}$ as a decimal.

153. Simplify $\dfrac{3\cdot9 \times \cdot95 \times \cdot051}{\cdot085 \times \cdot0065 \times 3\cdot8}$.

154. Reduce $\cdot003\dot{5}1\dot{8}$ to a vulgar fraction in its lowest terms.

155. Multiply $\cdot\dot{0}2\dot{7}$ by $\cdot\dot{2}\dot{1}$, and express the result as a decimal.

156. Simplify $12\cdot8\dot{3} \div \dfrac{1\cdot\dot{2} \div 1\cdot\dot{3}}{1\cdot\dot{4} \div 1\cdot\dot{5}}$.

157. How many times does £387·5 contain ·001024 of 1*s.* 11¼*d.*?

158. Add together ·0075 of a week, ·463 of 3 days 4 hours, and 5·643 of 1 hour 6 min., and express the answer in minutes and the decimal of a minute.

159. Find the value of $\dfrac{\cdot43 \times \cdot43 - \cdot34 \times \cdot34}{\cdot43 - \cdot34}$ of 4*s.* 2*d.*

160. If $\cdot\dot{7}1428\dot{5}$ of an estate is pasture, $\cdot\dot{1}4285\dot{7}$ of it is wood, and the remaining 71 acres is arable land; of how many acres does the whole estate consist?

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161. Express $\cdot 00345\dot{1}8\dot{}$ as a vulgar fraction in its lowest terms.

162. Multiply $6\cdot 4\dot{7}$ by 1000, and $\cdot\dot{1}4285\dot{7}$ by 100.

163. Add together $2\cdot\dot{5}\dot{4}$, $\cdot 375$, $\cdot 125$ and $\cdot\dot{3}$.

164. Simplify $\dfrac{9\cdot 1 \times 12\cdot 1 \times 17\cdot 1}{1\cdot 33 \times 1\cdot 43 \times 1\cdot 65}$.

165. Divide 1 by $\cdot 0125$, and by $\cdot 0\dot{1}2\dot{5}$.

166. Multiply $\cdot 2\dot{4}\dot{7}$ by $2\cdot\dot{5}7142\dot{8}$, and give the result as a decimal.

167. Find the value of $\cdot 81380208\dot{3}$ of 6 cwts. without converting the decimal into a vulgar fraction.

168. Find the difference between $\cdot 76$ of 3*s.* $1\frac{1}{2}$*d.* and $\cdot 7\dot{6}$ of the same sum.

169. Express the sum of $\frac{1}{16}$ ton, $\frac{1}{16}$ cwt., $\frac{1}{16}$ qr., $\frac{1}{16}$ lb. and $\frac{1}{16}$ oz. in ounces and the decimal of an ounce.

170. A can do $\cdot 1\dot{6}$ of a piece of work in $\cdot\dot{0}7\dot{2}$ of a day, and B can do $\cdot 008\dot{3}$ of it in $\cdot 125$ of an hour; which is the faster workman?

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171. Express $\frac{13}{37}$ as a decimal without dividing.

172. Convert $\frac{6075}{8505}$ into a decimal.

173. Divide $1\cdot 9$ by $\cdot 0025$, and by $\cdot 1\dot{0}2\dot{7}$.

174. Divide $\cdot 674387$ by $402\cdot 35$ as far as five places of decimals.

175. Subtract $2\cdot\dot{4}2857\dot{1}$ from $3\cdot\dot{1}4285\dot{7}$, expressing the result as a vulgar fraction.

176. Simplify $\dfrac{5\cdot 6\dot{3}}{18\cdot\dot{7}} + \dfrac{5\cdot 61}{18\cdot 7}$, giving the result as a decimal.

177. Simplify $\dfrac{\cdot 0013133}{2\cdot 3 \times 57\cdot 1}$; also $\dfrac{\cdot 00\dot{1}\dot{3}}{\cdot 074}$, without converting the decimals into vulgar fractions.

178. Show that $\frac{3}{5}$ of $\frac{7}{90}$ of $\frac{1}{6}$ of 3*s.* 4*d.* is greater than $\cdot 6$ of $\cdot 0\dot{7}$ of $\cdot 1\dot{3}$ of 3*s.* 9*d.*

179. Express the sum of $\cdot 8\dot{3}$ of 13*s.* 4*d.* and $\cdot 13\dot{8}$ of £1, 4*s.* as the decimal of £5.

180. A and B go from England to America in different ships. A's ship travels without stopping at the rate of $42\cdot 5$ miles in every $\cdot 1\dot{6}$ of a day; and B's travels, also without stopping, at the rate of 44 yards every $\cdot 0025$ of an hour. Which makes the quicker voyage?

---

181. Without dividing, find the number of places of non-recurring decimal figures in the decimal which is equivalent to $\frac{31}{16400}$.

182. Find the sum of 3, $\cdot 3$, $\cdot 03$, $\cdot\dot{3}$, $\cdot 0\dot{3}$ and $\cdot\dot{0}\dot{3}$.

183. Multiply $3\cdot1\dot{2}8571\dot{4}$ by $\cdot00\dot{6}\dot{3}$, giving the result as a vulgar fraction.

184. Divide $47\cdot8\dot{1}\dot{3}$ by $\cdot3125$ without converting the decimals into vulgar fractions.

185. Show that $\frac{1}{29} = \cdot03448\frac{8}{29}$, and that $\cdot\dot{0}3\dot{7} = (\cdot\dot{3})^3$.

186. Simplify $\left(\dfrac{\cdot63 \times 1\cdot25}{1\cdot83 \times 3\cdot25} \div \dfrac{1\cdot6 \times 2\cdot25}{6\cdot1 \times 2\frac{1}{7}}\right) \div \cdot58\dot{3}$.

187. Find, without converting the decimals into vulgar fractions, the value of $£\cdot473614\dot{2} + £\cdot506594\dot{1}$.

188. What decimal of £1 must be added to $\cdot\dot{0}\dot{9}$ of £6, 13*s.* $4\frac{1}{2}$*d.* that the result may be 15*s.*?

189. How many times could $\cdot0007$ of a shilling be taken from £41 and what decimal of a penny would be left over?

190. What time is it when the part of the day that has gone is $\cdot\dot{7}$ of the part that remains?

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191. Without dividing, find the number of non-recurring decimal places in the decimal equal to $\frac{1}{192}$.

192. Express $\dfrac{35(\frac{3}{7} + \cdot8) + 99(\cdot\dot{5} + \frac{6}{11})}{63(\frac{3}{7} + \cdot\dot{5}) + 55(\cdot8 + \frac{6}{11})}$ as a vulgar fraction in its simplest form.

193. Reduce $\dfrac{1}{2 \times 3 \times 4} + \dfrac{1}{3 \times 4 \times 5} + \dfrac{1}{4 \times 5 \times 6}$ to a decimal.

194. Prove that $\frac{1}{17} = \cdot05882\frac{6}{17}$, and hence without further division obtain the value of $\frac{1}{17}$ to ten places of decimals.

195. Divide the square of $\cdot017$ by the square of $\cdot68$.

196. The quotient of $66420666 \div 7358$ is 9027: hence write down at sight the quotients of $6642\cdot0666 \div 7358000$ and $\cdot066420666 \div \cdot007358$.

197. Find the sum of $1020\cdot\dot{3}$, $102\cdot0\dot{3}$, $10\cdot\dot{2}0\dot{3}$, 10203, $\cdot10203$ and $1\cdot0\dot{2}0\dot{3}$.

198. Simplify $\frac{7}{12}$ of $5\cdot8 + 2\frac{4}{5}$ of $\cdot\dot{9}0476\dot{1} - 4\frac{1}{3}$ of $\cdot5\dot{6}9230\dot{7}$.

199. Express the difference between $\cdot\dot{3}7\dot{8}$ of 13*s.* $10\frac{1}{2}$*d.* and $\cdot\dot{3}7\dot{8}$ of 16*s.* 6*d.* as the decimal of $\cdot42\dot{6}$ of £1, 17*s.* 6*d.*

200. What day of the week is it when at noon on that day $\cdot3\dot{5}7142\dot{8}$ of the week is gone?

XLII. THE UNITARY METHOD,

OR SINGLE RULE OF THREE.

1. If 8 lbs. cost 10*s.*, what will 12 lbs. cost at the same rate?
2. If 15 lbs. cost 20*s.*, what will 21 lbs. cost at the same rate?
3. If 14 tons cost £8, what will 35 tons cost at the same rate?
4. If 10 tons cost £6, what will 45 tons cost at the same rate?
5. If 6 yds. cost 15*s.*, what will 22 yds. cost at the same rate?
6. If 18 yds. cost 21*s.*, what will 81 yds. cost at the same rate?
7. If 3 lbs. cost 5*s.*, what will 2 lbs. cost at the same rate?
8. If 4 lbs. cost 7*s.*, what will 3 lbs. cost at the same rate?
9. If 8 tons cost £6, what will 5 tons cost at the same rate?
10. If 10 tons cost £8, what will 7 tons cost at the same rate?
11. If 9 yds. cost 12*s.*, what will 2 yds. cost at the same rate?
12. If 18 yds. cost 27*s.*, what will 8 yds. cost at the same rate?
13. If 8 lbs. cost 12*s.*, how many lbs. cost 15*s.* at the same rate?
14. If 6 lbs. cost 10*s.*, how many lbs. cost 35*s.* at the same rate?
15. If 9 tons cost £6, how many tons cost £14 at the same rate?
16. If 12 tons cost £8, how many tons cost £18 at the same rate?
17. If 18 lbs. cost 15*s.*, how many lbs. cost 10*s.* at the same rate?
18. If 12 lbs. cost 39*s.*, how many lbs. cost 26*s.* at the same rate?
19. If 28 yds. cost 21*s.*, how many yards cost 15*s.* at the same rate?
20. If 9 yds. cost 57*s.*, how many yards cost 38*s.* at the same rate?
21. If a train travel 8 miles in 15 minutes, how many miles will it travel in 10 minutes at the same rate?
22. If a train travel 20 miles in 24 minutes, how many miles will it travel in 16 minutes at the same rate?
23. If a watch gain 25 seconds in 10 days, how many seconds will it gain in 18 days at the same rate?
24. If a watch lose 15 secs. in 6 days, how many seconds will it lose in 7 days at the same rate?
25. If a journey of 78 miles take 13 hours, how many hours would a journey of 60 miles take at the same rate?
26. At the rate of 42 miles in 4 hours, how many hours would it take to travel 91 miles?
27. If in 4 hours a man travel 26 miles, how far would he travel in 6 hours at the same rate?
28. If a steamer travel 161 miles in 14 hours, how far would it go in 8 hours at the same rate?

29. If in 15 weeks a labourer earn £12, in how many weeks would he earn £28?

30. If in 51 weeks a labourer earn £34, in how many weeks would he earn £10?

31. If 2 men hoe a field in 6 days, how long would 3 men take?

32. If 7 men hoe a field in 3 days, how long would 21 men take?

33. If 5 men can do a piece of work in 4 days, how long would 4 men take to do as much?

34. If 8 men can do a piece of work in 27 days, how long would 12 men take to do it?

35. If 4 men can mow a field in 5 days, how many days would 10 men take to mow it?

36. If 8 men can mow a field in 3 days, how long would 12 men take to mow it?

37. If 3 men can do a piece of work in 6 days, how long would 2 men take to do it?

38. If 49 men can do a piece of work in 32 days, how long would 28 men take to do as much?

39. If 18 men in 10 days do a certain amount of work, how long would 45 men take to do it?

40. If 24 men can finish a piece of work in 45 days, in how many days would 60 men finish it?

41. If 4 men can do a piece of work in 90 days, how many men could do the same in 36 days?

42. If 123 men can finish a certain work in 18 days, how many men would be required to do as much work in 2 days?

43. If 30 men took 14 days for a certain task, how many men could have done it in 20 days?

44. If 171 men take 12 days to perform a certain task, how many men would be able to complete the same in 19 days?

45. How long would food, sufficient to feed 35 men for 20 days, supply 50 men at the same rate?

46. How long would a supply of food, sufficient to feed 16 men for 27 days, supply 36 men with the same rations?

47. If a stack of hay will feed 8 horses for 18 weeks, how many horses would it feed for 16 weeks?

48. If a quantity of corn will feed 20 horses for 45 days, how many horses will it feed equally well for 36 days?

49. If I lend a man £40 for 3 months, for how many months should he lend me £60 in return?

50. If I lend a man £100 for 39 weeks, for how long ought he to lend me £75 in return?

51. If 4 lbs. of sugar cost 10*d.*, what will 7 lbs. of the same kind of sugar cost?

52. If 3 lbs. of tea cost 5*s.*, what will 8 lbs. of the same kind of tea cost?

53. If 8 lbs. of sirloin cost 6*s.* 4*d.*, what will 14 lbs. cost?

54. If 7 yards of calico cost 4*s.* 8*d.*, what will 13 yards of the same kind cost?

55. If 12 yards of ribbon cost 7*s.* 6*d.*, what will 40 yards of the same ribbon cost?

56. If 6 lbs. of lard cost 4*s.* 3*d.*, what is the cost of 10 lbs.?

57. If 28 cows cost £308, what, at the same rate, would 13 cows cost?

58. If 54 horses cost £3600, what, at the same rate, would 42 horses cost?

59. When 4 lbs. of fresh butter cost 5*s.*, what is the cost of 3 lbs.?

60. When a 9-lb. leg of mutton costs 6*s.* $4\frac{1}{2}$*d.*, what should be the price of a 7-lb. leg?

61. If 40 articles are worth £1, 16*s.*, what is the value of 35 of them?

62. Find the value of 56 articles, any 16 of which are worth £2, 10*s.*

63. If 26 gross of pens cost £1, 4*s.*, what would 65 gross of the same kind of pens cost?

64. When 51 score of cabbages cost 8*s.* 6*d.*, what should be given for 85 score?

65. If 17 sheep are worth £48, 9*s.*, what is the value of 11 similar sheep?

66. If 15 lambs are worth £32, 5*s.*, what is the value of 52 equally fine lambs?

67. If 5 copies of a book cost 5*s.* $7\frac{1}{2}$*d.*, what is the cost of 23 copies?

68. If 41 copies of a book cost 15*s.* $4\frac{1}{2}$*d.*, what is the price of 3 copies?

69. When 3800 fleeces are worth £950, what is the value of 8300 similar fleeces?

70. If 2640 fleeces sell for £660, what would 6240 fleeces sell for at the same rate?

71. What is the cost of 5 lbs. of tea, if 3 lbs. of the same tea cost 5*s.* $7\frac{1}{2}$*d.*?

72. What is the cost of 9 lbs. of coffee at the rate of 7*s.* $8\frac{1}{2}$*d.* for 5 lbs.?

73. Find the cost of 91 oranges, at 7 for sixpence.

74. Find the cost of 156 oranges, at 13 for a shilling.

75. Find the value of a dozen articles, any five of which are worth £1, 18*s.* 9*d.*

76. Find the value of 3 dozen articles when any seven of them are worth £1, 1*s.* 10½*d.*

77. Find the cost of 39 lambs at the rate of £5, 18*s.* for four.

78. Find the cost of 85 sheep at the rate of £16, 15*s.* for five.

79. What should be the rent of 7 acres, if the rent of 145 acres of land of the same kind is £228, 7*s.* 6*d.*?

80. Find the rent of 111 acres, if the rent of 10 acres of the same kind of land is £18, 6*s.* 8*d.*

81. If a labourer earns £5, 5*s.* in 7 weeks, in how many weeks does he earn £8, 5*s.*?

82. If a labourer earns £7, 4*s.* in 9 weeks, in how many weeks does he earn £4?

83. If 6 sheep cost £13, 10*s.*, how many, at the same rate, can be bought for £33, 15*s.*?

84. If 7 sheep cost £22, 8*s.*, how many, at the same rate, can be bought for £60, 16*s.*?

85. How many eggs, at 14 for a shilling, can be bought for £1, 2*s.* 6*d.*?

86. When eggs are sold at 24 for 1*s.*, how many can be bought for £1, 11*s.* 5½*d.*?

87. If a pigeon fly 17½ miles in 15 minutes, how long will it take to go 14 miles?

88. If an express train travel 4½ miles in 5 minutes, how long will it take to travel 36 miles at the same rate?

89. How far will a train go in 35 minutes at the rate of 6 miles in a quarter of an hour?

90. How far will a train go in 21 minutes at the rate of 50 miles per hour?

91. How long would 40 men take to do the work which 25 men can do in 24 days?

92. How long would 27 men take to do the work which 36 men can do in 12 days?

93. How many miles ought 22 tons to be carried for the money which pays the carriage of 10 tons for 33 miles?

94. How far should 14 cwts. be carried for the money which is paid for the carriage of 3 tons for 21 miles?

95. How long can 13 horses be kept on the food which lasts 7 horses for 52 days?

96. If a chest of tea last a family of 10 persons for 22 weeks, how long should an equally large chest last a family of 8 persons?

97. How far should 48 tons be carried for the same sum as will pay for the carriage of 36 tons for 144 miles?

98. If the carriage of $5\frac{1}{4}$ cwts. for 114 miles cost 4*s.* 8*d.*, how far ought $3\frac{1}{2}$ cwts. to be carried for the same money?

99. If the first-class railway fare for a journey of 57 miles be 9*s.* 6*d.*, what should be the first-class fare for a journey of 122 miles?

100. If the second-class fare for 102 miles is 10*s.* $7\frac{1}{2}$*d.*, what should be the second-class fare for 272 miles?

101. If 17 yards of silk cost £4, 8*s.* $6\frac{1}{2}$*d.*, what would 120 yards cost?

102. If 13 yards of cloth cost £7, 11*s.* 8*d.*, what would 73 yards cost?

103. If 18 gallons of beer cost £1, 1*s.*, what is the cost of 3 pints?

104. If 18 gallons of beer cost £1, 1*s.*, what is the cost of 11 pints?

105. If 3 cwts. cost 7*s.* 9*d.*, what will 4 tons 4 cwts. cost?

106. If 3 lbs. 3 ozs. cost 2*s.* 10*d.*, what will 17 lbs. 7 ozs. cost?

107. If 1 cwt. 2 qrs. 14 lbs. of sugar cost £3, 0*s.* 8*d.*, what is the cost of 7 lbs.?

108. If 1 cwt. 3 qrs. 4 lbs. of sugar cost £1, 17*s.* 6*d.*, what is the cost of 11 lbs.?

109. If 14 cwts. 8 lbs. of steel cost £54, 3*s.* 6*d.*, what is the cost of 17 cwts. 16 lbs.?

110. If 3 cwts. 12 lbs. cost £13, 13*s.* 6*d.*, what is the cost of 5 cwts. 20 lbs.?

111. If 9 gallons of beer cost 10*s.*, what is the value of a pint and a half?

112. If 3 lbs. of tea be worth 10*s.*, what is the value of an ounce and a half?

113. If 17 yards of calico cost 12*s.* 9*d.*, what is the cost of $12\frac{1}{4}$ yards?

114. If 15 yards of flannel cost £1, 13*s.* 9*d.*, how much will $20\frac{1}{3}$ yards cost?

115. If $4\frac{5}{6}$ yds. of cloth cost £4, 14*s.* 3*d.*, what is the cost of $5\frac{1}{12}$ yards?

116. If $6\frac{1}{4}$ tons of coal cost £6, 15*s.* 5*d.*, what is the price of 5 tons 3 cwts.?

117. If 2 tons 3 cwts. 3 qrs. of sugar cost £61, 5*s.* 0*d.*, what will 5 cwts. 1 qr. 16 lbs. cost?

118. If 4 cwts. 2 qrs. 7 lbs. of coffee cost £36, 3*s.* 11*d.*, what is the cost of 3 qrs. 14 lbs.?

119. If 2 roods 15 poles of land cost £59, 7*s.* 6*d.*, what would 8 ac. 0 ro. 17 poles of the same land cost?

120. When the value of 47 lbs. 1 oz. Troy of silver was £184, 16*s.* $0\frac{1}{2}$*d.*, what was the value of 84 lbs. 9 ozs.?

121. If $1\frac{1}{4}$ lbs. of butter cost 1*s.* $5\frac{1}{2}$*d.*, how much can be had for $5\frac{1}{4}$*d.*?

122. If $2\frac{1}{2}$ lbs. of butter cost 2*s.* $8\frac{1}{2}$*d.*, how much can be bought for $3\frac{1}{4}$*d.*?

123. If 12 bushels of wheat cost £2, 0*s.* 9*d.*, how many bushels can be bought for £55, 0*s.* 3*d.*?

124. How many bottles of wine can be bought for £26, 3*s.* 3*d.* if 15 dozen cost £34, 2*s.* 6*d.*?

125. If £3, 10*s.* is the wages of a servant for 18 weeks, for how many weeks would her wages be £8, 15*s.*?

126. If 2 qrs. 5 bush. of wheat be worth £3, 11*s.* 9*d.*, how much can be bought for £85, 8*s.* 4*d.*?

127. If 45 yards of silk cost £29, 18*s.* 3*d.*, how many yards can be bought for £9, 19*s.* 5*d.*?

128. If 288 yards of calico cost £4, 14*s.* 10½*d.*, how many yards can be bought for £1, 11*s.* 7½*d.*?

129. If the scale of a map is 25 miles to an inch, what length on the map represents 415 miles?

130. A map is made on the scale of 6 miles to an inch; what distance on the map represents 375 miles?

131. How many sheep worth £2, 17*s.* 6*d.* each should be given in exchange for 69 pigs worth £2, 12*s.* 6*d.* each?

132. How many horses worth £21, 2*s.* 6*d.* each should be given in exchange for 52 cows worth £11, 7*s.* 6*d.* each?

133. If 7 francs are worth 5*s.* 8¼*d.*, what is the value of 20570 francs?

134. If 3 dollars are worth 12*s.* 6*d.*, what is the value of 73025 dollars?

135. If 6 ac. 3 ro. 18 po. of land be let for £21, 9*s.* 9*d.*, what, at the same rate, is the rent of 34 ac. 1 ro. 10 po.?

136. If 3 ac. 2 ro. 12 po. of land grow 15 qrs. 3 bush. 2 pks. of wheat, what should be the produce of 7 ac. 3 ro. 36 po. of the same land?

137. If 44 yds. 1 ft. 9 in. of cloth cost £42, 7*s.* 9*d.*, what is the cost of 74 yds. 11 in.?

138. If 7 cwts. 2 qrs. 11 lbs. of flour cost £7, 1*s.* 10*d.*, what would 5 tons 17 cwts. of the same kind of flour cost?

139. If 13 cwts. 2 qrs. 16 lbs. of rice cost £15, 17*s.* 6½*d.*, what will 3 tons 1 cwt. 1 qr. 16 lbs. cost?

140. If 1 ton 13 cwts. 1 qr. 9 lbs. 5 ozs. cost £186, 13*s.* 3¾*d.*, what will 17 cwts. 7 ozs. cost?

141. If 1000 square yards of meadow produce a load of hay, how many loads should 25 acres yield?

142. How many miles will a coach, travelling at the rate of 7½ miles an hour, go between 10.15 A.M. and 5.45 P.M.?

143. If 4 tons are carried 193 miles for 17*s.* 4*d.*, what will it cost to carry 6 tons 3 cwts. 3 qrs. the same distance?

144. If, travelling at the rate of 12½ miles an hour, I require 15½ hours to complete a journey, in how many hours shall I complete it if I increase my rate of travelling by 7½ miles an hour?

145. A map is made on the scale of 12 miles to an inch; how far apart are two towns the names of which on the map are $7\frac{1}{2}$ inches apart?

146. Edinburgh is 330 miles from London; how far apart should these names be placed on a map the scale of which is 30 miles to an inch?

147. What should be the price of 4 lbs. 7 ozs. 18 dwts. of gold when 2 lbs. 11 ozs. 15 dwts. is worth £116, 17*s.* 6*d.*?

148. If gold is worth £4, 3*s.* 4*d.* per ounce, what is the value of a bar weighing 4 lbs. 5 oz. 5 dwts. 21 grs.?

149. The shadow of an upright staff 6 ft. high measures 7 ft. 6 in.; how high is a tower whose shadow, at the same time, measures 40 yards?

150. What is the height of a tower whose shadow measures 25 yards 10 in., when an upright stick 2 ft. 8 in. long has a shadow 2 ft. 2 in. long?

151. Silver is at 3*s.* 6*d.* per oz., gold at £3, 17*s.* $10\frac{1}{2}$*d.* per oz.; how much gold is equivalent to 7 lbs. 5 ozs. of silver?

152. If gold is £3, 17*s.* 6*d.* per ounce, and silver is £2, 14*s.* per lb., what is the value of a piece of silver of equal weight with a piece of gold worth £1085?

153. If 68 chests of tea, each weighing 2 qrs. 17 lbs., cost £625, 13*s.* 5*d.*, how much did 34 lbs. of the tea cost?

154. If a field of $3\frac{3}{4}$ acres is let for £13, 15*s.* a year, what should be the rent of land of the same quality measuring 15 acres 2 roods 4 poles 11 sq. yards?

155. If the rent of 810 A. 0 R. $33\frac{1}{2}$ P. be £1620, 8*s.* $4\frac{1}{2}$*d.*, how much land, at the same rent per acre, would be rented for £1389, 15*s.* 9*d.*?

156. If 26 acres 2 roods 25 poles of land be sold for £4691, 10*s.*, what, at the same rate, will 415 ac. 30 po. $2\frac{3}{4}$ sq. yds. sell for?

157. A clock which gains uniformly 10 seconds per hour is set right at 6 P.M. on April 22; when will it again denote correct time?

158. A clock which gains at a uniform rate shows the correct time at noon on May 1st, and again at noon on August 9th. How much has it gained at 3 P.M. on June 7th?

159. A watch loses 23 secs. every 13 days: it is 10 minutes fast at noon on May 18th; how much fast or slow will it be at noon on Nov. 3rd?

160. A clock gains uniformly 55 seconds per day. It is 3 minutes too fast at noon on Monday; what time will it indicate at 12 minutes past 7 on the next Saturday morning?

XLIII. THE UNITARY METHOD (*continued*),

OR DOUBLE RULE OF THREE.

1. If 4 men in 6 days earn £5, what will 15 men earn in 8 days?*
2. If 10 men in 12 days earn £18, what will 28 men earn in 15 days?
3. If £7 is earned by 6 men in 5 days, what will 14 men earn in 12 days?
4. If £20 is earned by 8 men in 12 days, what will 9 men earn in 22 days?
5. If 8 men earn £9 in 10 days, what will 20 men earn in 12 days?
6. If 4 men earn £16 in 15 days, what will 7 men earn in 24 days?
7. If 6 men in 3 days mow 12 acres, how many acres will 7 men mow in 9 days?
8. If 8 men in 4 days mow 28 acres, how many acres will 12 men mow in 6 days?
9. If 10 men mow 15 acres in 2 days, how many acres will 18 men mow in 6 days?
10. If 6 men mow 21 acres in 5 days, how many acres will 11 men mow in 9 days?
11. If 4 men in 15 days earn £16, what will 3 men earn in 10 days?
12. If 6 men in 5 days earn £7, 10*s*., what will 4 men earn in 3 days?
13. If 10 men in 3 days mow 20 acres, how many acres will 9 men mow in 2 days?
14. If 6 men in 5 days mow 24 acres, how many acres will 5 men mow in 4 days?
15. If 4 men in 6 days earn £5, 10*s*., what will 30 men earn in 2 days?
16. If 10 men in 12 days earn £18, what will 25 men earn in 7 days?
17. If £20 is earned by 6 men in 5 days, what will 2 men earn in 9 days?
18. If 8 men earn £9 in 10 days, what will 3 men earn in 22 days?
19. If 10 men mow 15 acres in 2 days, how many acres will 6 men mow in 5 days?

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* Here, and in all similar questions, the words "at the same rate" are implied.

20. If 3 men mow 20 acres in 8 days, how many acres will 11 men mow in 6 days?

21. If £8 is earned in 5 days by 6 men, how many men would earn £4 in 3 days?

22. If £20 is earned in 12 days by 8 men, how many men would earn £15 in 9 days?

23. If 4 men earn £6 in 12 days, how many men will earn £15 in 10 days?

24. If 10 men in 12 days earn £15, how many men will earn £36 in 8 days?

25. If 8 men earn £14 in 10 days, how many men will earn £63 in 15 days?

26. If the wages of 29 men for 12 days are £87, how many men should receive £405 for 54 days?

27. If 8 men in 4 days mow 28 acres, how many men will mow 21 acres in 6 days?

28. If 6 men mow 20 acres in 5 days, how many men will mow 70 acres in 3 days?

29. If 16 men reap 76 acres in 6 days, how many men will reap 114 acres in 9 days?

30. If 7 men reap 57 acres in 9 days, how many men will reap 38 acres in 2 days?

31. If 10 men earn £18 in 12 days, in how many days will 35 men earn £21?

32. If 6 men earn £9 in 5 days, in what time will 15 men earn £4, 10*s.*?

33. If 10 men mow 15 acres in 2 days, in how many days will 3 men mow 18 acres?

34. If 8 men in 4 days mow 28 acres, in how many days will 6 men mow 21 acres?

35. If 48*s.* is charged for the carriage of 8 cwts. for 128 miles, how far would 4 cwts. be carried for 6*s.*?

36. What is the cost of the carriage of 15 cwts. for 12 miles, if 6 cwts. is carried 20 miles for 4*s.*?

37. If $10\frac{1}{2}$ acres are mown by 3 men in 3 days, in what time could 112 acres be mown by 96 men?

38. If $11\frac{2}{3}$ acres are mown in 2 days by 6 men, how long will 3 men take to mow $17\frac{1}{2}$ acres?

39. If 14 men earn £9, 16*s.* in 7 days, what will 20 men earn in 4 days?

40. If 4 men earn £3, 18*s.* in $6\frac{1}{2}$ days, what will 17 men earn in $21\frac{1}{2}$ days?

41. If 17 men earn £6, 7*s.* 6*d.* in 2 days, how long will it take one man to earn £11, 5*s.*?

42. If 15 labourers earn £68, 12*s.* 6*d.* in 24 days, how long will 20 labourers take to earn £137, 5*s.*?

43. If the rent of 8 acres for half a year be £4, 16*s.*, what is the rent of 23 acres for 3 months at the same rate?

44. If the rent of 9 acres be £11, 5*s.* per year, find one quarter's rent of 350 acres at the same rate.

45. If 4 chests, each containing 20 lbs. of tea, cost £6, 6*s.* 8*d.*, what is the cost of 15 chests each containing 90 lbs. of the same kind of tea?

46. If the carriage of 2 tons for 60 miles be £1, 12*s.* 6*d.*, what, at the same rate, is the charge for the carriage of 17 tons for 14 miles?

47. If 16 horses eat 12 pecks of oats in 10 days, how long will 63 bushels last 56 horses?

48. If 7 bushels 2 pecks of corn feed 10 horses for 7 days, how many horses will 3 quarters 6 bushels feed for 10 days?

49. If 8 men dig a field of $9\frac{1}{2}$ acres in 19 days, how long will 5 men take to dig a field of 5 ac. 1 ro. 10 po.?

50. If the carriage of 13 cwts. 2 qrs. for 124 miles be £6, 9*s.* 4*d.*, how far can 1 ton 11 cwts. be carried for £2, 8*s.* 6*d.*?

51. If 4 lbs. of bread cost $5\frac{1}{2}$*d.* when wheat is 44*s.* per quarter, what should be the cost of 6 lbs. of bread when wheat is 48*s.* per quarter?

52. If a five-penny loaf weighs 4 lbs. when wheat is 35*s.* per qr., what should be the weight of a three-penny loaf when wheat is 32*s.* per qr.?

53. If 6 men do a piece of work in 10 days, working 8 hours a day, how many hours a day must 5 men work to do as much in 9 days?

54. How many men, in 14 days of 9 hours each, can do as much work as 21 men in 18 days of 8 hours each?

55. In how many days of 12 hours each would 100 men do as much work as 45 men can do in 16 days of 10 hours?

56. How many hours per day must 24 men work in order to do in 5 days twice as much work as 25 men, working 6 hours a day, can do in 4 days?

57. How many days would 36 men, working 7 hours a day, require to do a piece of work which 24 men, working 9 hours 20 minutes a day, can do in 9 days?

58. How many men, working 11 hours per day, can do in 91 days as much work as 143 men, working 9 hours per day, can accomplish in 133 days?

59. If 6 men do a piece of work in 30 days of 9 hours each, how many men could do ten times as much work in 25 days of 8 hours?

60. If 25 men do a certain amount of work in 14 days of 8 hours each, in how many days of 10 hours each would 7 men do $\frac{2}{5}$ of the same amount of work?

61. If 3 men reap 8 acres in 5 days, working 8 hours a day, in how many days will 8 men, working 12 hours a day, reap 192 acres?

62. If 12 men, working 8 hours a day, could do $\frac{4}{5}$ of a piece of work in 20 days, in how many days could 15 men, working 10 hours a day, do $\frac{7}{8}$ of it?

63. If 26 men, working 8 hours a day, earn £41, 12*s.* in 6 days, how much will 17 men, working 9 hours a day, earn in 39 days, if they all are paid at the same uniform rate per hour?

64. If 12 men, working 9 hours a day, earn £37, 16*s.* in 12 days, in how many days of 8 hours each would 21 men earn £102, 18*s.*, if paid at the same rate per hour?

65. If 3 pumps in 4 hours, working 45 minutes per hour, can discharge 16200 gallons of water from the hold of a ship; how many gallons can 5 pumps, working 40 minutes per hour for 72 hours, discharge?

66. If 5 men dig a trench in $1\frac{1}{3}$ days, working 9 hours a day; how long would one man take to dig a trench half as long again, of the same depth and half the width, working 10 hours a day?

67. If 14 men can build a wall 19 feet long and 4 feet high in 6 days of 10 hours each; how many men would be required to build a wall of the same kind, 38 feet long and 3 feet high, in 9 days of 7 hours each?

68. How many men, working 8 hours a day, could dig a trench 8 yards long, 8 feet wide, and 8 inches deep, in the same number of days that 9 men, working 12 hours a day, could dig a trench 12 yards long, 12 feet wide, and 12 inches deep?

69. If 16 men build a wall 25 yards long and 6 feet high in 14 days of 9 hours each, in how many days of 8 hours each would 7 men build a wall 30 yards long and 5 feet high?

70. If 20 men, working 7 hours per day, build a wall 800 feet long and 10 feet high in 14 days, how many hours a day must 15 men work to build a wall 900 feet long and 15 feet high in 21 days?

71. If 6 men dig a trench 15 yards long and 2 yards wide in 3 days of 12 hours each, how many days of 8 hours each will 8 men take to dig a trench of the same depth 10 yards long and 8 yards wide?

72. If 18 men dig a trench 630 yards long, 3 yards wide and 2 ft. deep in 12 days of 10 hours each, how many men would be required to dig a trench 330 yards long, 7 yards wide and 3 feet deep in 22 days of 9 hours each?

73. If 3 compositors, in 15 days of 10 hours each, can set in type 300 pages averaging 55 lines per page and 40 letters per line; in how many days of 8 hours each will 5 compositors set up 400 pages, averaging 60 lines per page and 44 letters per line?

74. If 6 compositors, in 16 days of $10\frac{1}{2}$ hours each, can set in type 720 pages, each page containing on the average 60 lines, and each line 40 letters; in how many days of 7 hours each will 9 compositors set in type 960 pages, averaging 45 lines per page and 50 letters per line?

75. If the carriage of 9 tons 11 cwts. 1 qr. for 114 miles cost £3, 11*s.* 3*d.*, how much should be charged for carrying 5 tons 4 cwts. 2 qrs. 39 miles further?

76. If, when meat is 9*d.* per lb., it costs £11, 16*s.* 3*d.* to supply a family of 12 persons for 24 weeks, what should be the cost of meat for a family of 18 persons for 14 weeks when the price has risen 3*d.* per lb.?

77. If 3 men working 8 hours a day can build a wall 80 feet long, 6 feet high and 2 feet thick, in 5 days; how many men working 7 hours a day will be required to build a wall of similar materials 100 feet long, 7 feet high, and twice as thick, in 25 days?

78. If 124 men in $5\frac{1}{2}$ days of 11 hours each can dig a trench $232\frac{1}{2}$ yards long, 3 ft. 8 in. wide, and 2 ft. 4 in. deep; in how many days, of 9 hours each, will 36 other men dig a trench $337\frac{1}{2}$ yards long, 5 ft. $7\frac{1}{5}$ in. wide, and 3 ft. 6 in. deep; supposing that each man of the second set is capable of doing as much work in 6 hours as each man of the first set can do in 7 hours?

79. A garrison of 3000 men is supplied with provisions for 15 weeks at the rate of 13 ounces per man per day. How many men must leave in order that the provisions may last those who remain for 26 weeks, allowing each man 10 ounces per day?

80. In 11 days 250 men working 9 hours a day complete 528 yds. of an embankment which is to be $1\frac{1}{2}$ miles long. How many additional men must be employed in order that, if all work 10 hours a day, the embankment may be finished in 33 days more?

Money.

4 farthings (*f.*) = 1 penny (*d.*).
12 pence = 1 shilling (*s.*).
20 shillings, or 240 pence = 1 pound (£).

Note.—A *florin* = 2*s.*; a *crown* = 5*s.*; a *guinea* = 21*s.*
A *sovereign* = 20*s.*, or 8 half-crowns, or 10 florins, or 40 sixpences, or 80 threepences, or 480 half-pence, or 960 farthings.

Time.

60 seconds (*sec.*) = 1 minute (*min.*).
60 minutes = 1 hour (*hr.*).
24 hours = 1 day.
7 days = 1 week (*wk.*).
365 days = 1 (common) year (*yr.*).
366 days = 1 leap year.

Note.—A *common year* = 52 weeks + 1 day; a *century* contains 100 years; a *lunar* month contains about 4 weeks.

The year is divided into 12 *calendar* months, of which February, in common years, contains 28 days; and, in leap years, 29 days;
"Thirty days hath September, April, June, and November;"
and the remaining seven calendar months each contain 31 days.

Leap year occurs once in four years (except at the end of a century). In order to discover whether any year (not the last in a century) is a leap year, *divide the number of the year by* 4, *and if there is* no *remainder it is leap year.* But if the year ends a century it is *not* leap year *unless the first two figures divide by* 4 *without remainder.*

Avoirdupois Weight.

16 drams (*dr.*) = 1 ounce (*oz.*).
16 ounces, or 7000 grains = 1 pound (*lb.*).
14 pounds = 1 stone (*st.*).
28 pounds, or 2 stones = 1 quarter (*qr.*).
4 quarters, or 8 stones, or 112 pounds } = 1 hundredweight (*cwt.*).
20 hundredweights = 1 ton.

Note.—This weight is used for all common substances; *e.g.* coal, meat, &c.

Troy Weight.

24 grains (*gr.*) = 1 pennyweight (*dwt.*).
20 pennyweights, or 480 grains = 1 ounce Troy (*oz. Tr.*).
12 ounces Troy = 1 pound Troy (*lb. Tr.*).

Note.—Troy weight is only used for gold, silver and jewellery.
The *grain* alone is the same in both Avoirdupois and Troy weights.

Long Measure.

12 inches (*in.*) = 1 foot (*ft.*).
3 feet, or 36 inches = 1 yard (*yd.*)
1760 yards = 1 mile (*mi.*).
$5\frac{1}{2}$ yards, or 11 half-yards = 1 pole (*po.*), rod, or perch (*per.*).
40 poles, or 220 yards = 1 furlong (*fur.*).
8 furlongs = 1 mile.

Note.—A *chain* = 22 yards, or 4 poles; 100 links = 1 chain.
A *fathom* = 6 feet; a *cable-length* = 120 fathoms; a *knot* = 6080 feet.
A *hand* = 4 inches. A *league* = 3 miles.
$2\frac{1}{4}$ inches = 1 *nail*, 4 nails = 1 quarter, 4 quarters = 1 yard.

Square Measure.

144 square inches (*sq. in.*) = 1 square foot (*sq. ft.*).
9 square feet = 1 square yard (*sq. yd.*).
$30\frac{1}{4}$ (*i.e.* $5\frac{1}{2} \times 5\frac{1}{2}$) square yards, or 121 square quarter-yards = 1 square pole (*sq. po.*, or P.), or square perch.
40 square poles = 1 rood (*ro.* or R.).
4 roods, or 4840 square yards = 1 acre (*ac.*, or A.).
640 acres or 1760×1760 square yards = 1 square mile (*sq. mi.*).

Note.—A *square chain* = 100×100 square links, or 22×22 square yards; 10 square chains = 1 acre.

Cubic Measure.

1728 (*i.e.* $12 \times 12 \times 12$) cubic inches (*cub. in.*) = 1 cubic foot (*cub. ft.*).
27 (*i.e.* $3 \times 3 \times 3$) cubic feet = 1 cubic yard (*cub. yd.*).

Capacity.

Liquids:
2 (imperial) pints (*pt.*) = 1 (imperial) quart (*qt.*)
4 quarts, or 8 pints = 1 gallon (*gal.*)

Dry goods; *e.g.* corn:
2 gallons = 1 peck (*pk.*)
4 pecks, or 8 gallons = 1 bushel (*bush.*)
8 bushels = 1 quarter (*qr.*)

Note.—A *hogshead of beer* = 54 gallons; a *hogshead of wine* = 63 gallons; a *pipe* of wine = 2 hogsheads. 4 *gills* = 1 imperial pint.
6 "reputed" quarts (*i.e.* common wine bottles) contain a gallon.
"A pint of pure water weighs a pound and a quarter."
A cubic foot of water weighs 1000 ounces.

Number.—12 units = 1 dozen; 12 dozen = 1 gross. 20 units = 1 score.

Paper.—24 sheets = 1 quire; 20 quires, or 480 sheets = 1 ream.

EXERCISES.—PART II.

XLIV. VARIOUS APPLICATIONS OF THE FOREGOING METHODS.

A. AGENTS OF DIFFERENT POWERS, &c.

1. If the work of 12 boys is equivalent to that of 7 men, how long will it take 15 boys to do as much work as 21 men can do in 5 days?

2. If 40 women can do a piece of work in 20 days, in how many days could 15 men do it, the work of 5 women being equal to that of 3 men?

3. If two men can do as much as 3 boys, and if 6 men can complete a certain task in 6 days, how long would 5 men and 6 boys together take to do it?

4. How long would it take 6 men and 16 women to do the work which 12 men can accomplish in 3 days; the work of 3 men being supposed to be equal to that of 4 women?

5. If 3 boys eat as much as 4 men, and it cost £1, 14*s.* 6*d.* to feed 3 men for a week, how much would it cost to feed, in a similar way, 7 boys for a week?

6. If 5 men and 9 boys could do a piece of work in 17 days, in how many days would 9 men and 12 boys do it; the work of 2 men being equal to that of 3 boys?

7. If 7 oxen eat 3 tons of hay in a month, how long would 35 tons feed 49 sheep, an ox consuming three times as much as a sheep?

8. If 8 men with 5 boys can do as much as 5 men with 18 boys, how many boys can do as much as 6 men?

9. If either 4 men, or 7 boys, could perform a piece of work in 5 hours, how long would 4 men and 21 boys take?

10. If 12 sheep, or 18 lambs, eat 96 bushels of turnips in a fortnight, how long will 14 sheep and 21 lambs take to eat 64 bushels?

11. If 7 men, or 12 boys, can hoe a field of 12 acres in 6 days working 10 hours a day, in how many days of 8 hours each can 14 men and 6 boys hoe a field of 60 acres?

12. If 5 men can mow 33 acres in 5 days working 11 hours a day, how many days would 4 men take to reap 32 acres working 10 hours a day, when it takes as long to reap 5 acres as to mow 6?

13. If 6 men earn as much as 9 women in a day, and 9 women as much as 12 boys, how long will it take 6 men, 9 women and 12 boys, working together, to earn £19, 16*s*., a boy's earnings being a 1*s*. a day?

14. If 14 labourers could dig a field of potatoes in 5 days, how many additional men would be required to finish the work 3 days sooner?

15. Fifty navvies are engaged to make an embankment 3 miles long. After working 6 days, and completing 6 furlongs, 30 of them strike work. How long will the rest take to finish it?

16. A contractor undertook to finish a certain work in 60 days. After 30 men had been employed for 20 days he found that only $\frac{1}{4}$ of the work was done. How many extra men must he employ in order to complete the work in the specified time?

17. A garrison of 5400 men was provisioned for 13 weeks, when a reinforcement arrived and the provisions only lasted 9 weeks; what was the strength of the reinforcement?

18. A garrison of 5985 men is provisioned for 180 days, but after 72 days it is reinforced by 171 men. How long do the provisions last?

19. A number of workmen perform a certain task in 60 days of 11 hours each. How many extra hours a day would they have to work to do the same amount in the same time, taking a half holiday on the last day in each week?

20. Three men can do as much work as 5 boys: the wages of 3 boys are equal to those of 2 men. A work on which 40 boys and 15 men are employed takes 8 weeks and costs £350: how long would it take, and what would it cost, if 20 men and 20 boys were employed?

B. Assets, Dividends, &c.

21. A bankrupt's debts are £6228; his available assets £188, 2*s*. 9*d*. How much can he pay in the pound?

22. A bankrupt's assets are £10,252, 4*s*. 8*d*. and his liabilities £13,901, 6*s*. 8*d*. How much can he pay in the pound?

23. A bankrupt's debts are £3427, 6*s*. 8*d*. and his assets are only £2184, 18*s*. 6*d*. How much will a creditor receive to whom he owes £375, 10*s*.?

24. A bankrupt pays a dividend of 5*s*. 6*d*. in the pound. One of his creditors received £27, 18*s*. 3*d*.; what was the amount of the debt due to this creditor?

25. A creditor received 16*s*. 3*d*. in the pound, and thereby lost £135, 10*s*.: how much was due to him?

26. A bankrupt paid a dividend of 8*s*. 2*d*. in the pound, and one of his creditors received £376, 5*s*. 7*d*.: what did this creditor lose?

27. On a debt of £230, the creditor lost £142, 11*s.* 0½*d.*: what dividend did the bankrupt pay?

28. A bankrupt pays 9*s.* 10*d.* in the pound; his assets, after deducting the "costs" of his bankruptcy, are £1888: find his debts?

29. A bankrupt's debts amount to £800, 10*s.*; his assets to £223, 15*s.* 6*d.* Of his debts, £48, 5*s.* have a prior claim and are paid in full: what dividend can he pay on the rest?

30. A bankrupt's liabilities are £2672, 17*s.* 6*d.*; his assets £1640. Debts amounting to £325, 12*s.* 6*d.* are fully secured, and the legal expenses connected with his bankruptcy amount to £140, 15*s.*: what sum will an unsecured creditor receive whose claim is for £47, 13*s.* 4*d.*?

C. Rates, Taxes, &c.

31. Find the income-tax on £653 at 7*d.* in the pound.

32. How much will the income-tax, at 8*d.* in the pound, amount to on an income of £523, 17*s.* 6*d.*?

33. A man's income-tax at 5*d.* in the pound amounts to £13, 17*s.* 6*d.*; find his income.

34. What is the income of a man who pays £60, 13*s.* 8*d.* income-tax at the rate of 10*d.* in the pound?

35. If the income-tax on £500, 10*s.* is £16, 13*s.* 8*d.*, what does it amount to on £510, 15*s.* at the same rate?

36. If £49, 4*s.* 8*d.* is the income-tax on £1477, what is it on £1020, 10*s.* at the same rate?

37. If a parish is rated at 2*s.* 1*d.* in the pound, what do the rates amount to on a house valued at £85, 10*s.* per annum?

38. A rate of 2*s.* 7½*d.* in the pound on a parish produces £745, 1*s.* 3*d.*; what is the rateable value of the parish?

39. If the rates on a house rented for £26 amount to £2, 14*s.* 2*d.*, what is the rental value of another house in the same parish on which the rates amount to £6, 15*s.* 5*d.*?

40. By the reduction of the income-tax from 7*d.* to 5*d.* in the pound a man saved £7, 15*s.* 9*d.*; what was his income?

41. Find the net income which remains after deducting income-tax at 8*d.* in the pound from £513, 12*s.* 6*d.*

42. Find the balance which results, after deducting income-tax at the rate of 9*d.* in the pound, from £5173, 6*s.* 8*d.*

43. Find the gross income which is reduced to £812 net by the payment of income-tax at 8*d.* in the pound.

44. A man's gross income is £666, 16*s.* 8*d.*; find his net income after an income-tax of 6*d.* in the pound has been paid.

45. A man's net income after payment of income-tax at 6*d.* in the pound is £1755; what would it be if the tax were 10*d.*?

D. FRACTIONS.

46. Three-eighths of a number is 27; find the number.
47. Five-sevenths of a number is 65; what is the number?
48. What number exceeds its fifth part by 44?
49. The sum of a number and its fifth part is 42; find the number.
50. What is the number whose fifth part exceeds its seventh part by 6?
51. Find the number the sum of whose fifth and seventh parts is 108.
52. What number is that the sum of the third, fourth and fifth parts of which is 47?
53. What number exceeds the sum of its fourth, sixth and eighth parts by 231?
54. Find the sum of money $\frac{19}{20}$ of which amounts to £22, 8*s.* $10\frac{1}{2}$*d.*
55. Five-ninths of a sum of money is 11*s.* $1\frac{3}{4}$*d.*; find the sum.
56. Two-thirds of a certain sum of money exceeds $\frac{3}{5}$ of it by £1, 1*s.* $1\frac{1}{4}$*d.*; find the sum.
57. A lost $\frac{2}{3}$ of all the marbles he had to B, won 17 from C, and then had 30; how many had he at first?
58. A paid $\frac{3}{5}$ of his money to B, and then had 8*s.* 2*d.* left. What did he pay to B?
59. A man went $\frac{16}{19}$ of his journey by train, $\frac{5}{57}$ of it by tram, and walked the remaining 8 miles. What was the length of his journey?
60. A pole stands with $\frac{1}{4}$ of its length in mud, $\frac{1}{3}$ in water, and 10 feet in air. How long is it?
61. A boy spent $\frac{1}{6}$ of his money at one shop, $\frac{7}{8}$ of the remainder at another, and then had $2\frac{1}{2}$*d.* left. What had he at first?
62. After paying $\frac{5}{12}$ of the contents of my purse to one person, $\frac{5}{7}$ of the remainder to another, and $\frac{5}{8}$ of what then remained to a third, there still remained 1*s.* 3*d.* How much was there in the purse at first?
63. A boy spent $\frac{3}{4}$ of his pocket-money in oranges, $\frac{1}{4}$ of what he had left in apples, and $\frac{3}{4}$ of what then remained in sweets, and had $1\frac{1}{2}$*d.* left. What had he at first?
64. The liquid in a cask fills $\frac{7}{9}$ of the cask; after 13 gallons have been drawn off it fills $\frac{4}{7}$ of the cask. How many gallons will the cask hold?
65. A cask is $\frac{7}{8}$ full; after 10 gallons have been drawn from it, it is one gallon short of half full. How many gallons will it hold?
66. If a man can do $\frac{3}{8}$ of a piece of work in $\frac{9}{16}$ of a day, how long will it take him to do the whole?
67. If in $\frac{3}{8}$ of a day a man does $\frac{15}{28}$ of his work, how long does it take him to do it all?

E. Time and Work.

68. A alone could mow a field in 10 days. B alone could mow it in 15 days. How long would it take them working together?

69. A can mow a field in 12 days; B in half that time: how long would they take together?

70. A can finish a piece of work in 21 hours; B could do the same in 28 hours: how long would they take working together?

71. A alone can fill a cart in 24 minutes; B alone in half an hour: how long would it take them together?

72. If the cold-water tap alone is opened a bath is filled in 9 minutes; if the hot-water tap alone is opened, in 12 minutes: how long would the bath take to fill if both taps were opened together?

73. A alone can mow a field in 6 days, B alone in 8 days, C alone in 12 days; how long would they take working together?

74. Three taps can separately fill a cistern in 10, 12 and 15 minutes respectively; how long would the cistern take to fill if all three were opened together?

75. A can do a piece of work in $2\frac{1}{4}$ days, B in $4\frac{1}{2}$ days; how long would they take together?

76. A can do a piece of work in $2\frac{1}{2}$ days, B in $3\frac{1}{3}$ days, C in $3\frac{3}{4}$ days; how long would they take to do it working together?

77. A can mow 4 acres in 3 days; B can mow 3 acres in 2 days: how long will they take together to mow 34 acres?

78. If a man eats 3 loaves in 2 days, and his wife eats 2 loaves in 3 days, how long would 26 loaves last the man and his wife?

79. If a man eats four threepenny loaves in 5 days, and his wife eats three twopenny loaves in 4 days, what does their bread bill amount to for the month of April?

80. A cistern is filled from a tap in 15 minutes; it can be emptied by a waste pipe in 25 minutes. If the cistern is empty and both are opened, how long will it take to fill?

81. One tap fills a cistern in 8 minutes; another empties it in 15 minutes. If the cistern be empty and then both taps are opened, how long will it take to fill?

82. A and B together mow a field in 8 days; A alone could do it in 18 days: how long would it take B alone?

83. A and B together dig a garden in 7 days; with C to help them they could do it in 5 days: how long would C take alone?

84. A, B and C together can finish a piece of work in 8 days; A alone could finish it in 20 days, and B alone in 24 days: how long would C alone take?

85. A alone can build a wall in 30 days, which B alone could build in 25 days. After A has worked alone for 10 days, B comes to help him. How many days after this will the work be done?

86. A and B together can do a piece of work in $7\frac{1}{2}$ hours, B and C in $10\frac{1}{2}$ hours, A and C in $8\frac{3}{4}$ hours. How many hours would A, B and C together take to do the work?

87. A and B together reap 5 acres in 3 days; B and C reap 3 acres in 2 days; A and C reap 7 acres in 5 days. How long would A alone take to reap $23\frac{1}{2}$ acres?

88. A is twice as good a workman as B: how long would each alone take to do what they can do together in 4 days?

89. A can do a piece of work in 30 days, and B can do the same in 20 days. After A has worked alone for 13 days, B helps him for 2 more days and then C joins them, and the work is completed 3 days later. How long would C alone take to do the whole work?

90. A, B and C together could do a piece of work in 60 days. They work at it together for 10 days, and then A leaves off, and B and C work together for 20 days more, when B leaves off, and C, working $\frac{1}{3}$ longer each day, then finishes the work alone in 96 more days. C, working at his former rate, could have done the whole in 222 days. How long would B take alone?

F. Time and Distance.

Express in feet per second the rates—

91. 60 miles per hour. **92.** 45 miles per hour.

93. $10\frac{1}{2}$ miles per hour. **94.** $3\frac{1}{2}$ miles per hour.

95. How many yards does a horse, trotting at the rate of 8 miles an hour, go in half a minute?

96. How many yards does a steamer, which travels at the rate of 18 knots an hour, go in $\frac{1}{4}$ of a minute?

Express the following rates in miles per hour—

97. 11 feet per second. **98.** $27\frac{1}{2}$ feet per second.

99. 32 feet per second. **100.** 625 feet per second.

101. A body moves over 100 yards in 9 seconds; express its rate in miles per hour.

102. How many miles would a man go in an hour who walks uniformly at the rate of 11 yards in 5 seconds?

103. If a train 88 yards long occupies 5 seconds in passing a signal-post, at the rate of how many miles per hour is it moving?

104. If telegraph posts are 55 yards apart, and a passenger finds that he passes 12 every minute, how many miles an hour is the train travelling?

105. How many seconds will a train 120 yards long, travelling at the rate of 30 miles an hour, take to pass completely over a bridge 122 yards long?

106. A train, 88 yards long, takes 12 seconds to pass completely over a bridge 66 yards long: how many miles an hour is the train travelling?

107. If a train, 80 yards long, going 42 miles an hour, takes a quarter of a minute to pass completely through a station, how long is the station?

108. If a train leaves London at 5 A.M., travelling at the rate of 40 miles an hour, at what time should it reach a town 136 miles distant, 25 minutes being spent in stoppages?

109. Two men start from the same place at the same time, one towards the north at $3\frac{1}{2}$ miles an hour, the other towards the south at $8\frac{1}{2}$ miles an hour: how far apart are they at the end of $7\frac{1}{2}$ hours?

110. A, who travels $8\frac{1}{2}$ miles per hour, and B, who travels $7\frac{1}{2}$ miles per hour, start at the same time from places 56 miles apart to meet each other: how many hours after the start will they meet?

111. Two boys start at the same time on bicycles from places 80 miles apart and ride to meet each other, travelling at 11 and $9\frac{1}{2}$ miles an hour respectively: how far apart will they be in $3\frac{1}{2}$ hours?

112. My friend, who walks 4 miles an hour, starts from a place 15 miles off at the same time that I start to meet him. When we meet I have walked 6 miles; at what rate do I walk?

113. Two persons, A and B, start at the same time from places $2\frac{1}{4}$ miles apart, A walking away from B at the rate of $3\frac{1}{4}$ miles per hour, and B following him at the rate of 4 miles an hour. How long will it take B to catch A?

114. A starts from a certain place and travels uniformly at the rate of $7\frac{1}{2}$ miles per hour. Two hours later B follows, travelling at the uniform rate of 12 miles per hour. How long will it take B to catch A?

115. An express train leaves London for York at 10 A.M. travelling at the rate of 42 miles an hour. A slow train leaves York at 8.30 for London and travels at the average rate of 24 miles an hour. How far from London will they be when they pass each other, the distance between London and York being 200 miles?

116. A train 88 yards long travelling at the rate of 45 miles per hour overtook a man running by the side of the line and passed him in 5 seconds. At what rate was this man running? The same train met another man walking at the rate of 3 miles per hour; how long did it take to pass him?

117. How long will it take a train 76 yards long travelling at the rate of 45 miles an hour to completely pass another train 78 yards long, travelling on a parallel line, at the rate of 35 miles per hour, when they move (i) in opposite directions, (ii) in the same direction?

118. An express train travelling at the rate of 52 miles an hour overtakes a coal train half as long again as itself, travelling on a parallel line at the rate of 16 miles an hour, and passes it completely in $12\frac{1}{2}$ seconds; find the length of each train.

119. In a game at hare and hounds the hare had a start of 10 minutes and ran at the uniform rate of $5\frac{3}{4}$ miles per hour; the hounds followed at the rate of $6\frac{1}{2}$ miles an hour. The length of the course was 8 miles. Was the hare caught?

120. A starts 3 minutes after B for a place $4\frac{1}{2}$ miles distant. B, on reaching his destination, immediately returns, and after walking a mile meets A. If A's speed be $3\frac{1}{3}$ miles per hour, what is B's speed?

121. A thin candle 8 inches long which burns at the rate of 3 inches in 2 hours, and a thick candle $5\frac{1}{2}$ inches long which burns at the rate of 2 inches in 3 hours, are lighted at the same time. After what interval will (i) both be the same length; (ii) the thick candle one inch longer than the thin one?

122. If the current of a river flow at the rate of 2 miles per hour and it take a man 3 hours to row 9 miles up the stream, how long will it take him to return?

Clocks.

At what time do the hands of a clock point in the same direction—

123. Between 3 and 4?

124. Between 7 and 8?

125. Between 5 and 6?

126. Between 4 and 5?

At what time are the hands in opposite directions—

127. Between 2 and 3?

128. Between 10 and 11?

129. Between 8 and 9?

130. Between 3 and 4?

At what times are the hands at right angles—

131. Between 6 and 7?

132. Between 1 and 2?

133. Between 12 and 1?

134. Between 9 and 10?

135. How soon after 4 o'clock will the hands of a clock be separated by an interval of 6 minute-spaces on the face?

136. At what time between 9 and 10 will the minute hand be exactly 8 minute-spaces in advance of the hour hand?

137. It is between 4 and 5 o'clock, and the minute hand is 10 minute-spaces in advance of the hour hand. What is the exact time?

138. It is between 11 and 12 o'clock, and the minute hand is 5 minute-spaces behind the hour hand. What is the exact time?

G. Races.

139. A can give B 5 yards' start in a race of 100 yards, and B can give C 4 yards in the same distance. How many yards can A give C?

140. A can give B 12 yards in 100, and B can give C 35 yards in $\frac{1}{4}$ mile. How many yards can A give C in 200?

141. A can score 100 points in a game while B scores 80, and B can score 100 while C scores 75: how many can C score while A scores 50?

142. A can give B 1 point in 10 in a certain game; B can give C 1 in 10; and C can give D 1 in 10: how many points can A give D in 1000?

143. A can give B 20 yards and C 41 yards start in a quarter of a mile: B can give C a start of 3 seconds over the same distance. How long does C take to run a quarter of a mile?

144. A can give B 10 yards and C 19 yards in 100 yards: B can give C $1\frac{1}{4}$ seconds in the same distance. How long does A take to run 100 yards?

H. Chains.

145. If 3 geese are worth 10 ducks, and 3 ducks are worth 4 chickens, and a couple of chickens cost 4*s*. 6*d*., what is the value of a goose?

146. If £1 is worth $25\frac{1}{2}$ francs, and 20 francs are worth $9\frac{1}{3}$ Dutch florins, how many Dutch florins are equivalent to £50?

147. If 35 napoleons are worth 192 thalers, 10 thalers worth 7 dollars, and 2 dollars worth 5 florins, how many napoleons are worth 144 florins?

148. If 6 pears are worth as much as 9 apples, 5 apples as much as 12 plums, 15 plums as much as 2 apricots, 4 apricots as much as 3 peaches, and 1 peach as much as 25 strawberries, how many strawberries should be given in exchange for 11 pears?

149. A clerk A can copy 5 lines while B copies 4, B can copy 2 while C copies 3, C copies 7 while D copies 8, and D copies 14 while E copies 17: how many lines can E copy while A copies 490?

150. If $\frac{1}{2}$ lb. of tea costs as much as $\frac{3}{4}$ lb. of coffee, $\frac{1}{2}$ lb. of coffee as much as $\frac{3}{4}$ lb. of chocolate, $\frac{1}{2}$ lb. of chocolate as much as $\frac{3}{4}$ lb. of biscuits, $\frac{1}{2}$ lb. of biscuits as much as $\frac{3}{4}$ lb. of jam, and $\frac{1}{2}$ lb. of jam costs 4*d*., what is the cost of a pound of tea?

XLV. RATIO AND PROPORTION.

RATIO.

Simplify the ratios—

1. $58 : 87$.
2. $803 : 73$.
3. $11\frac{1}{11} : 8\frac{5}{7}$.
4. $\cdot\dot{7}1428\dot{5} : 7\cdot5$.
5. $15\cdot51 : 18\cdot81$.
6. £5, 6*s.* 8*d.* : £8, 13*s.* 4*d.*
7. 3 tons 12 cwts. : 7 tons 16 cwts.
8. 7 hrs. 30 min. : 1 day.
9. $10\frac{1}{2}$ chains : 3 poles.
10. 5 fur. 121 yds. : 1 mi. 121 yds.
11. Which is greater $11 : 17$, or $1\frac{3}{4} : 2\frac{5}{8}$?
12. Which is greater $1\cdot3 : 2\cdot\dot{3}$, or 5*s.* 9*d.* : 11*s.* 8*d.*?
13. Prove that the ratio of £3, 8*s.* to £7 is equal to the ratio of $3\frac{1}{11}$ to $6\frac{4}{11}$.
14. Show that the ratio 2 lbs. 3 ozs. : 1 qr. 3 lbs. 8 ozs. is equal to the ratio 5 ins. : 2 yds.
15. What sum of money has to £1, 15*s.* 9*d.* the ratio $3\frac{1}{3} : 5\frac{1}{5}$?
16. What number has to $\cdot75$ the ratio of 3 tons to 9 lbs.?
17. Is the fraction $\frac{13}{15}$ increased, unaltered, or decreased, by subtracting 3 from both numerator and denominator?
18. Is the fraction $\frac{17}{18}$ increased, unaltered, or decreased, by adding 7 to both numerator and denominator?
19. If 2 be added to the numerator of the fraction $\frac{5}{11}$, what number must be added to the denominator in order that the value of the fraction may be unchanged?
20. If 3 be taken from the numerator of the fraction $\frac{9}{7}$, what number must be taken from the denominator that the value of the fraction may be unaltered?
21. If 7 be taken from the denominator of the fraction $\frac{27}{35}$, what number must be taken from the numerator that the value of the fraction may remain unchanged?
22. If 12 be added to the denominator of the fraction $\frac{56}{65}$, what number must be added to the numerator that the value of the fraction may remain unchanged?

Show by taking a numerical example that—

23. A ratio of greater inequality is diminished by adding the same number to both its terms.
24. A ratio of greater inequality is increased by subtracting the same number from both terms.
25. A ratio of less inequality is decreased by subtracting the same number from both terms.

PROPORTION.

Find a fourth proportional to—

26. 12, 21 and 56.
27. $7\frac{1}{2}$, $5\frac{1}{4}$ and $9\frac{1}{3}$.
28. ·3, 5·6 and £2, 10*s.*
29. 3 yds. 1 ft., 2 ft. 3 ins. and 5 cwts.

Find the missing term in each of the following proportions—

30. $1\frac{1}{4}$: $1\frac{1}{3}$ :: $1\frac{1}{2}$: ——.

31. 3 lbs. : 5 ozs. :: 8*s.* : ——.

32. 21 : 24 :: —— : 5*s.* 4*d.*

33. 2 ft. : 3 yds. :: —— : 15.

34. $\cdot\dot{3}$: —— $= \cdot 1\dot{6}$: $\cdot\dot{2}\dot{7}$.

35. 2 gals. 1 qt. : —— = 13 acs. 2 ro. : 3 acs. 2 ro.

36. —— : $4\frac{2}{5}$ = 25 : 33.

37. —— : 10 tons 10 cwts. = £6 : 5 guineas.

Find a mean proportional between—

38. 16 and 49. 39. 32 and 200. 40. 68 and $1\frac{8}{9}$.

41. Are 51, $42\frac{1}{2}$ and $35\frac{5}{12}$ in *continued* proportion?

42. Find a *third* proportional to 3·4 and 5·1.

43. Prove that—

·15 : 4·5 :: 8*d.* : £1 :: 2 lbs. 3 ozs. : 2 qrs. 9 lbs. 10 ozs.

44. Are £100, £700, £103 and £721 proportionals?

45. Are 17, 51, 17—2 and 51—2 proportionals?

46. The shadow of a tower is $16\frac{1}{2}$ yards long when the shadow of an upright stick 4 feet long is 3 feet 8 inches; what is the height of the tower?

47. On a ground-plan made on the scale of half an inch to a yard the length of a house is represented by a line $7\frac{1}{2}$ inches long; how long is the house?

48. If the circumference of a circle of radius 7 inches is 3 ft. 8 ins., what is the circumference of a circle of radius 4 yds. 2 ft.?

49. The longest side of a triangular field measures $16\frac{1}{2}$ chains. On a plan of the field the sides are represented by lines $1\frac{7}{8}$, $2\frac{1}{4}$ and $2\frac{3}{4}$ inches long respectively. Find the lengths of the other sides of the field.

50. On a plan of a field the sides measure 5, 3, $4\frac{1}{2}$ and 7 inches. The shortest side of the field itself measures 17 chains 10 links; find the length of each of the other three sides.

XLVI. PROPORTIONAL PARTS, AND PARTNERSHIP.

1. Divide 308 into two parts which shall be in the ratio $9 : 19$.
2. Divide 1602 into two parts in the ratio $35 : 143$.
3. Divide 483 into parts proportional to 3, 7, 11.
4. Divide 46875 into four parts proportional to 1, 4, 4, 6.
5. Divide 22 into parts in the ratio of 86 to 68.
6. Divide 28 into parts proportional to 29, 37 and 46.
7. Divide 133 into two parts which shall be in the ratio $\frac{2}{3} : \frac{3}{5}$.
8. Divide 716 in the ratio $1\frac{3}{7} : 2\frac{5}{6}$.
9. Divide 46875 into parts proportional to $\frac{1}{16}$, $\frac{1}{8}$, $\frac{1}{4}$, $\frac{1}{2}$.
10. Divide 150 into parts proportional to 2, 3·2, and 4·8.
11. Divide 2601 into parts proportional to 1·6, ·16, and 1·13.
12. Divide 171 in the ratio of 2·7 to ·72.
13. Divide 36·5 into two parts, which shall be in the ratio of $5 \cdot \dot{6}$ to 6·5.
14. Divide £28900 into four parts proportional to 8, 5, 3, 1.
15. Divide a mile into six lengths proportional to 3, 5, 13, 17, 21 and 29.
16. Divide £4, 12*s.* 6*d.* in the ratio $3 : 7$.
17. Divide £405, 2*s.* 6*d.* among three persons in the ratios $6 : 7 : 8$.
18. Divide £97, 18*s.* $8\frac{1}{2}$*d.* into parts proportional to the numbers 7, 9 and 13.
19. Divide £32, 10*s.* between A and B, so that A's share may have to B's share the ratio of 17 to 83.
20. Divide £4, 8*s.* in the ratio $53 : 43$.
21. Divide £30 into parts proportional to 230, 250 and 270.
22. Divide £154 between 4 persons, so that their shares may be proportional to $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$.
23. Divide £1330 between 3 persons, in the ratios $\frac{2}{3} : \frac{3}{4} : \frac{4}{5}$.
24. The sides of a triangle are proportional to $2\frac{1}{5}$, 3, $3\frac{1}{4}$. The sum of their lengths is 338 feet. Find the length of each side.

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25. A, B and C are partners in a firm. A's capital is £1000, B's £2000 and C's £1500. What should each receive out of profits which amount to £3000?
26. A puts into a business £2500, B £1500 and C £1000; the profits of the business amount to £625; how should this sum be divided amongst the partners?

27. A man divided £504 amongst his three sons, whose ages were 19, 21 and 23, in sums proportional to their ages. Three years afterwards he similarly divided an equal sum. How much did each receive in all?

28. 684 policemen are to be distributed among three towns in proportion to the population—viz., 10,944; 12,312; 25,992. How many policemen will be sent to each town?

29. A bankrupt, whose estate is worth £698, has four creditors A, B, C, D; he owes A £270, B £300, C £150, D £100. The legal expenses of his bankruptcy amount to £83. How much will each creditor receive?

30. The proceeds of a business amounting to £2689, 8*s.* are divided among the four partners whose contributions to the capital were severally £543, 10*s.*, £274, 10*s.*, £685, 13*s.* 4*d.* and £417, 6*s.* 8*d.*; how much does each partner receive?

31. On the dissolution of a partnership, £16,690, 11*s.* 3*d.* was divided between the two partners in the ratio $47\frac{1}{4} : 33\frac{3}{4}$. How much did each receive?

32. Distribute 500 policemen between three towns, of population 19,000, 8500, 23,600 respectively, as nearly as possible in proportion to population.

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33. Divide £150 between A, B and C, so that A may receive £7 as often as B receives £8, and B £4 as often as C receives £5.

34. Divide £60, 2*s.* $1\frac{3}{4}$*d.* between A, B and C, so that A may have half as much again as B, and B half as much again as C.

35. Divide £150 among A, B and C, so that B may have twice as much as A, and C twice as much as A and B together.

36. Divide £3, 18*s.* 4*d.* among three persons, A, B, C, so that B may receive seven-eighths as much as A, and C five shillings more than B.

37. Divide £520 among four persons so that the first may have twice as much as the second, the second twice as much as the third, and the first and third together as much as the second and fourth together.

38. Divide £954, 9*s.* between A, B and C, so that A's share may be to B's as 3 is to 5 and B's to C's as 10 is to 11.

39. Divide £2100 among A, B, C and D, so that A's share may be to B's as 7 : 6, B's to C's as 5 : 4, and C's to D's as 3 : 2.

40. Divide 114 gallons into 4 measures, so that the first shall be to the second as 3 is to 5, the second to the third as 7 is to 9 and the third to the fourth as 15 is to 17.

41. If 7 men do as much work as 12 women, and 5 women as much as 8 children, divide £69 among 4 men, 6 women and 9 children in proportion to the work they do.

42. If 12 men can do the work of 19 women, and 8 women that of 11 boys, divide £584, 15*s.* among 14 men, 6 women and 10 boys in proportion to their work.

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43. Three men hire a pasture for £42, 12*s.* One puts in 10 oxen for 9 months, another 12 oxen for 8 months, the third 14 oxen for 7 months. What part of the rent should each pay?

44. A, B and C are partners in a business. A puts in £900 for 6 months, B £1200 for 4 months and C £1800 for a year. Divide profits amounting to £742 fairly among them.

45. Three persons A, B, C, rent the grazing of a park for £570. A puts 126 oxen in the park for 3 months; B puts in 162 oxen for 5 months; and C puts in 216 oxen for 4 months. What part of the rent should each person pay?

46. A and B enter into partnership each with a capital of £1000. At the end of the first year the profits are £500, and A then invests his share in the business, but B withdraws £200 of his capital. How should the profits, amounting to £492, be divided at the end of the second year?

47. A begins to trade with a capital of £1500. He is joined 5 months later by B with £1200. At the end of the year the profits amount to £453, 7*s.* 6*d.* Find, to a penny, the share of each partner.

48. A begins business with a capital of £850. Three years later he takes B, with a capital of £1100, into partnership, and two years later still they are joined by C with £1400. At the end of 8 years from the commencement the business is sold for £5500; what part of this sum is due to A?

49. Two merchants A and B trade together for a year. A invests £1200 in the business for the whole time. B puts in £300 at first, £300 more at the end of 3 months, and again £600 at the end of another 3 months. The total profits amount to £270; how should this sum be divided between them?

50. A and B enter into partnership; A with £2000, and B with £1800, of capital. Nine months later they are joined by C with £2400. At the end of the year the profits are £656, 5*s.*; find, within a penny, the share of this which belongs to C.

51. A began business with a capital of £550, and was joined during the year by B with a capital of £600. At the end of the year B received £75 out of profits amounting to £240. When did B enter the business?

52. Two traders, A and B, enter into partnership with capitals of £800 and £960 respectively. Two months later A increases his capital by £300, and three months later still B increases his by £560. One month after this A increases his capital by £500. Ten months from the commencement of their partnership the profits are divided. How must this be done?

After a second interval of 10 months profits are again divided and each partner receives the same amount, one of them having during the interval contributed an additional £400 of capital. Which did this, and when?

53. Two farmers A and B rent a field for 10 weeks and pay £5 for it. A puts in 3 horses for 5 weeks, 4 cows for 8 weeks, and 12 sheep for the whole time; whereas B has 1 horse and 30 sheep in the whole time. If the cost of the keep of a horse, a cow, and a sheep are in the ratio 5 : 3 : 1, find, to the nearest penny, what share of the rent each should pay.

54. A and B enter into partnership; A puts in £1800, but at the end of 4 months withdraws £450 of this, and again at the end of 9 months £200. If at the end of the year their just shares of the profits are as 5 is to 3, how much capital did B put into the business?

55. Three partners A, B and C had shares in a business proportional to the numbers 4, 5 and 6 respectively. C retired, and then A and B had equal shares. If, for his share, C received altogether £15,000 from A and B, what sum should each have contributed?

56. A and B enter into partnership, their capitals being as 5 : 6; at the end of two months they withdraw respectively one-third and one-fourth of their capital, and four months afterwards B increases his capital by the addition of one-third of his original capital. How are they to divide the profits, amounting at the end of the year to £379, 2*s.* 6*d.*?

57. A, B and C enter into partnership, their capitals being as 1 : 2 : 3. After 6 months A withdraws half his capital, but 3 months later restores it. Four months from the commencement B withdraws one-third of his capital, and 2 months later C adds twice as much as B withdrew. How must profits, amounting to £658, 10*s.*, be divided at the end of the year?

58. A, B and C are partners in a bank in which they invest £12,800, £8000 and £5000 respectively. C, as manager, is to be paid $\frac{3}{20}$ of the gross yearly profits, while A and B are each to be allowed 2 guineas a day for attendance. At the end of the year the gross profits amount to £7650, and A has attended 6 days, and B 4 days. What is the total sum due to B?

59. The values of equal weights of three metals are as 2 : 5 : 19; volumes proportional to 3, 4 and 6 are mixed: what value of the first metal is contained in a lump of the mixture worth £7?

60. A piece of metal weighing 11 cwts. 3 qrs. has been formed by compounding three metals in quantities which, by measure, are as 5 : 3 : 2; but the weights of equal volumes of them would be as 7 : 11 : 13. What weight of each of the component metals has been used?

XLVII. AREAS.

Rectangular Surfaces.

Find the area of a rectangle—

1. 8 feet long and 7 feet wide.
2. 13 feet long and 11 feet wide.
3. 3 ft. 6 ins. long and 2 ft. 3 ins. wide.
4. 5 ft. 4 ins. long and 4 ft. 8 ins. wide.
5. 10 ft. 7 ins. long and 6 ft. 2 ins. wide.
6. 3 yds. 2 ft. long and 2 yds. 1 ft. 6 ins. wide.
7. $4\frac{3}{5}$ ins. long and $2\frac{5}{6}$ ins. wide.
8. $1\frac{7}{8}$ ins. long and $1\frac{5}{6}$ ins. wide.
9. 8 ft. $4\frac{1}{2}$ ins. long and 5 ft. $7\frac{1}{2}$ ins. wide.
10. 3 ft. $7\frac{1}{4}$ ins. long and 1 ft. $11\frac{1}{2}$ ins. wide.

Find in acres, &c., the area of a rectangular field—

11. 176 yards long and 110 yards wide.
12. 480 yards long and 242 yards wide.
13. 550 yards long and 33 yards wide.
14. 27 poles long and 19 poles wide.
15. 14 chains long and 8 chains wide.
16. 17 chains long and 12 chains wide.

Find the area of a square, the length of a side of which is—

17. 1 ft. 1 in.	20. 220 yards.	23. 9 chains.
18. $7\frac{3}{8}$ inches.	21. 77 yards.	24. 7 chains 25 links.
19. 2 yds. 2 ft. 7 ins.	22. 360 yards.	25. 5 chains 75 links.

Find the area of—

26. A rectangular floor 18 ft. 3 ins. long and 16 ft. 6 ins. wide.
27. A page of a book $7\frac{1}{2}$ inches long and $5\frac{1}{4}$ inches wide.
28. A tablecloth $3\frac{1}{4}$ yds. long, $1\frac{5}{8}$ yds. wide.
29. A piece of tape $\frac{3}{4}$ in. wide, 12 yds. long.
30. A piece of wall-paper 12 yards long and 21 inches wide.
31. A straight road 187 miles long, 10 yards wide.
32. A straight path $3\frac{1}{4}$ miles long and 2 ft. 9 ins. wide.
33. A straight road 5 furlongs in length and of the average width of 1 chain 5 links.
34. An oblong field the dimensions of which are 55·5 chains and 10·125 chains.

Find the length of a rectangle*—

35. 119 sq. ft. in area and 7 ft. in width.
36. 195 sq. yds. in area and 13 yds. in width.
37. 35 square yards in area and 5 feet in width.
38. $6\frac{3}{4}$ sq. ins. in area and $1\frac{7}{8}$ ins. in width.
39. 7 sq. ft. in area and 8 ins. in width.
40. $5\frac{5}{6}$ sq. ft. in area and $2\frac{2}{3}$ ins. in width.
41. 2 acres in area and 88 yards in width.
42. 3 ac. 1 ro. in area and $27\frac{1}{2}$ yds. in width.

Find the breadth of a rectangle—

43. 8 sq. ft. 36 sq. ins. in area and $5\frac{1}{2}$ feet in length.
44. 5 sq. yds. in area and 3 yds. 2 ft. 3 ins. in length.
45. $2\frac{1}{2}$ sq. ft. in area and 2 ft. $9\frac{3}{5}$ ins. in length.
46. 1 sq. ft. in area and 13 yds. 1 ft. in length.
47. 4 ac. 3 ro. in area and $9\frac{1}{2}$ chains in length.
48. 1 acre in area and 1 chain 25 links in length.
49. 3 ac. 3 ro. in area and 33 poles in length.
50. 5 ac. 1 ro. 10 po. in area and $514\frac{1}{4}$ yds. in length.

Find the length of—

51. A rectangular ceiling the breadth of which is 14 feet and its area 259 square feet.
52. A strip of paper $1\frac{1}{2}$ inches wide, the area of which is 1 square foot.
53. A roll of matting 18 inches wide which would cover a floor 27 feet long and 12 ft. 8 ins. wide.
54. A nine-inch board the area of which is 3 square feet.

Find the width of—

55. A window-blind 7 feet long and $2\frac{5}{8}$ sq. yds. in area.
56. A straight canal a mile long which covers two acres of ground.
57. A strip of paper 12 yards long and $2\frac{1}{2}$ sq. yds. in area.
58. A straight road $16\frac{1}{2}$ miles long which occupies 84 acres of ground.
59. A "piece" of wall-paper 12 yards long which covered 63 sq. ft. of wall-space.
60. 17 equal planks each 18 feet long, which formed a floor whose area was $184\frac{7}{8}$ sq. ft.

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* Exercises in finding the length of the side of a *square* when the area is given will be found under Square Root.

How many—

61. Times is the area of a rectangle 3 ins. long and 2 ins. wide contained in that of a rectangle 4 ft. 6 ins. long and 1 ft. 10 ins. wide?

62. Cards each $3\frac{1}{2}$ ins. long by $2\frac{3}{4}$ ins. wide would just cover an oblong table 5 ft. 6 ins. long by 3 ft. 6 ins. wide?

63. Mats each 1 yd. long and 15 ins. wide would cover a floor 15 ft. long and 14 ft. wide?

64. Bricks each 9 ins. by 4 ins. would pave a kitchen 24 ft. by 16 ft. 6 ins.?

65. Postage-stamps $\frac{15}{16}$ in. long by $\frac{5}{8}$ in. wide would cover an envelope $7\frac{1}{2}$ ins. long by $3\frac{3}{4}$ ins. wide?

66. Tiles six inches square would pave a passage six feet wide and six yards long?

67. Tiles $2\frac{1}{2}$ ins. square would pave a hall 10 ft. square?

68. Paving-stones each having a face 6 sq. ins. in area would pave a straight road 5 chains long and 5 yards wide?

69. Slabs each 2 sq. ft. in area would pave a floor 16 yards square?

70. Slabs each 2 feet square would pave a courtyard 16 square yards in area?

71. Pieces of turf 1 ft. 3 ins. square would make a grass-plot 40 feet square?

72. Square yards of floor-cloth would cover a floor 29 feet long and 23 feet wide?

How many yards of patternless carpet would just cover a floor—

73. 21 feet long, 14 ft. wide; carpet 3 ft. wide?

74. 28 feet 6 ins. long, 19 ft. 6 ins. wide; carpet a yard wide?

75. 27 ft. long, 18 ft. wide; carpet 2 ft. wide?

76. 15 yds. long, 4 yds. 2 ft. wide; carpet 4 ft. wide?

77. 27 ft. long, 16 ft. wide; carpet $\frac{3}{4}$ yd. wide?

78. 56 ft. long, 18 ft. 8 ins. wide; carpet 28 ins. wide?

79. 6 yards square; carpet $\frac{3}{4}$ yd. wide?

80. 14 feet square; carpet $31\frac{1}{2}$ ins. wide?

81. 21 ft. long, 15 ft. wide; carpet $\frac{3}{4}$ yd. wide, leaving an uncarpeted margin 18 ins. wide all round the floor?

82. 20 ft. 10 ins. long, 16 ft. wide; carpet 2 ft. wide, leaving a margin 1 foot wide uncarpeted all round the room?

83. 7 yds. long, 6 yds. wide; carpet $\frac{3}{4}$ yd. wide, leaving a margin $\frac{3}{4}$ yd. wide uncarpeted all round the floor?

84. 20 ft. 6 ins. square; carpet 18 ins. wide, leaving an uncarpeted margin 9 ins. wide all round?

Find the cost of plain carpet sufficient to just cover a floor—

85. 18 ft. 6 ins. long and 12 ft. 3 ins. wide at 3*s.* per sq. yard.

86. 18 feet long and 15 feet broad, with carpet 30 inches wide, at 5*s.* per yard.

87. 20 ft. 3 ins. long by 13 ft. 4 ins. wide, with carpet 2 feet 3 inches wide, at 5*s.* a yard.

88. 10 yds. 2 ft. long and 7 yds. 1 ft. broad, with carpet $\frac{3}{4}$ yd. wide, at 4*s.* 6*d.* a yard.

89. 12 ft. 9 ins. by 16 ft. 6 ins., with carpet 33 ins. wide, at 3*s.* 8*d.* a yard.

90. 7 yds. 8 ins. long and $5\frac{1}{4}$ yds. wide, with carpet 27 ins. wide, at 3*s.* $4\frac{1}{2}$*d.* a yard.

91. 40 feet long by 30 feet broad, with carpet at 5*s.* 9*d.* per square yard, leaving uncovered a space of 3 feet wide all round the room.

92. 100 ft. long by 25 ft. broad, with carpet at 4*s.* 6*d.* per square yard, leaving uncarpeted a space 4 feet wide all round.

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93. Find the cost of staining a border 15 inches wide all round a floor 20 ft. long and 17 ft. broad at 1*s.* per square yard.

94. Find, to the nearest penny, the cost of staining a border 16 ins. wide all round the floor of a room 16 ft. 8 ins. square, at 9*d.* per square yard.

95. A room is 25 feet long by 17 feet broad. What will it cost to carpet it at 6*s.* 9*d.* a square yard, leaving uncovered a margin one yard wide? Find the extra cost of covering this space with India matting at 1*s.* 3*d.* a square yard.

96. All round the floor of a room, which is 28 feet long and 22 feet wide, there is a border 2 feet wide which is left uncarpeted. Find the cost of staining the border at 1*s.* $1\frac{1}{2}$*d.* a square yard. Find also the number of yards of carpet, 27 inches wide, required for covering the rest of the floor, and the cost of this carpet at 3*s.* 9*d.* a yard.

97. A room, 20 feet long and 16 feet wide, has a stained border 2 feet wide; the rest of the floor is covered with carpet at 27*s.* per square yard; the staining costs 9*d.* per square yard. Find the whole expense.

98. A carpet 19 feet 6 inches by 15 feet 9 inches, which costs 16*s.* per square yard, is laid down in a room measuring 23 feet by 17 feet, and the rest of the floor is covered with floor-cloth at 8*d.* per square foot. Find the total cost.

99. A room 22 ft. by 19 ft. has in it a Turkey carpet 19 ft. 6 ins. by 15 ft. 9 ins., costing 12*s.* per square yard; the rest of the floor is covered with felt at 4*d.* per square foot; find the cost of the carpet and of the felt.

100. What will it cost to make a gravel walk 7 feet wide along the inner edge of each side of a square field whose side is 110 yards long, at 1*s.* 6*d.* a square yard?

101. In a courtyard 67 ft. 6 ins. long and 42 ft. 9 ins. wide there is a footway, 5 ft. 6 ins. wide, the whole length of the yard. What is the cost of paving the whole, the price per square yard for the footway being 3*s.* 6*d.* and for the remainder 3*s.*?

102. A room is 24 ft. long, $19\frac{1}{2}$ ft. wide. What will be the total cost of painting, at 1*s.* 3*d.* per square yard, a border a yard wide, and a skirting-board 18 in. high, all round the floor?

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103. Carpet 2 ft. wide, at 4*s.* 9*d.* per yd., sufficient to just cover a rectangular floor would cost £6, 10*s.* $7\frac{1}{2}$*d.*; find the area of the floor.

104. The cost of matting, $\frac{3}{4}$ yd. wide at 1*s.* 10*d.* per yd. which just covers a floor 21 feet long, is £5, 2*s.* 8*d.*; how broad is the floor?

105. Drugget at 3*s.* 9*d.* per yd., which just covers a floor 16 ft. square, costs £4, 5*s.* 4*d.*; how wide is the drugget?

106. The cost of carpet 21 ins. wide which just covers a floor whose dimensions are 16 ft. 6 ins. and 13 ft. 3 ins. is £7, 5*s.* 9*d.*; find the price of the carpet per yard.

107. A carpet for a room cost £12, 5*s.* The room was $25\frac{1}{2}$ feet long and $19\frac{1}{2}$ feet wide, and between the edge of the carpet and the walls was a margin $2\frac{1}{4}$ feet wide. Find the cost of the carpet per square yard.

108. The cost of patternless carpet 2 ft. wide at 7*s.* per yd. for the middle of a floor 18 feet wide is £19, 15*s.* 6*d.* A margin 1 ft. wide is left uncarpeted all round the floor. How long is the floor, if the value of the quantity of carpet turned under for the hem is 3*s.* 6*d.*?

109. The cost of staining a border 2 feet wide all round a room 30 feet long by 27 feet wide, is 17*s.* 8*d.*; find the price per square yard.

110. In making a pavement 7 feet wide all round the inside of a square, 648 slabs, each 2 ft. long and 1 ft. 9 ins. wide, were used. Find the length of the side of the square.

111. A pavement 6 feet wide all round the outside of a square grass-plot is formed of 552 slabs each 2 ft. long by 1 ft. 6 ins. wide. What is the length of a side of the grass-plot?

112. Each of two rooms is 18 feet wide, but one is twice as long as the other; the floor of the shorter room is covered with carpet at 9*s.* per yard, of the longer at 6*s.* per yard, the width of the carpet in each case being $\frac{3}{4}$ of a yard. Find the length of each room in feet, when the difference in the cost of the two carpets is £9.

Walls of a Room, &c.

Find in feet (i) the perimeter; (ii) the area of the walls; of a rectangular room—

113. 18 ft. long, 14 ft. wide, 11 ft. high.
114. 10 ft. high and the floor 15 ft. square.
115. 20 ft. long, 17 ft. wide, 9 ft. high.
116. 21 ft. long, 16 ft. broad, 10 ft. 6 ins. high.
117. 17 ft. 6 ins. in length, in breadth, and in height.
118. 24 ft. long, $16\frac{1}{2}$ ft. wide, 12 ft. 3 ins. high.
119. 23 ft. 8 ins. long, 17 ft. 4 ins. broad, 11 ft. 6 ins. high.
120. 41 ft. 2 ins. long, 32 ft. 4 ins. broad, 18 ft. 6 ins. high.
121. 23 ft. 8 ins. long, 15 ft. 10 ins. wide, 11 ft. 11 ins. high.
122. 6 yds. 2 ft. 3 ins. long, $5\frac{1}{4}$ yds. broad, 9 ft. $10\frac{1}{2}$ ins. high.

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123. Find the area of the sides and ends of a rectangular box 15 ins. long, 10 ins. wide and 7 ins. high.

124. Find the area of the whole external surface of a rectangular box 14 ins. long, 9 ins. wide and $4\frac{1}{2}$ ins. high.

125. Find the total area of the walls, ceiling and floor of a rectangular room 12 ft. 6 ins. long, 11 ft. 6 ins. wide and 9 ft. 3 ins. high.

126. How many square feet of sheet-lead would line an open rectangular tank which measures inside 4 ft. in length, 2 ft. in depth and 2 ft. 9 ins. in breadth?

127. How many square yards of paper would just completely cover the outside of a rectangular box 6 ft. long, 3 ft. wide and 2 ft. high?

128. A rectangular box, without lid, is made of wood 1 inch thick; it measures outside 3 ft. 8 ins. in length, 2 ft. 2 ins. in breadth and 1 ft. 7 ins. in height; find the total area of its inside surface.

129. A box, with lid, made of wood $\frac{3}{4}$ inch thick, measures outside 4 ft. $4\frac{1}{2}$ ins. in length, 2 ft. $7\frac{1}{2}$ ins. in width and 1 ft. 9 ins. in height; find the total area of its inside surface.

130. How many sheets of paper each 3 feet long and 2 feet wide would just completely cover the entire surface of the walls of a rectangular room 22 ft. long, 18 ft. wide and 12 ft. high?

Making no allowance for doors, &c., how many—

131. Square yards of paper would cover the walls of a room 18 ft. long, 15 ft. wide and 12 ft. high?

132. Yards of paper 2 ft. wide would cover the walls of a room 20 ft. long, 16 ft. wide and 11 ft. 6 ins. high?

Making no allowance for doors, &c., how many—

133. Yards of paper 2 ft. 6 ins. wide would cover the walls of a room 20 ft. 6 ins. long, 9 ft. 6 ins. wide and 13 ft. 6 ins. high?

134. "Pieces" of paper 12 yards long and 21 inches wide would cover the walls of a room 24 ft. 3 ins. long, 14 ft. 3 ins. broad and 9 ft. high?

135. Sheets of paper 9 ins. square would cover the walls of a room 16 ft. 7 ins. long, 14 ft. 2 ins. wide and 8 ft. 3 ins. high?

136. Feet of 7-inch board would line to a height of 3 ft. 6 ins. the walls of a room 21 ft. long and 14 ft. wide?

Find the cost of paper (making *no* allowance for doors, windows, &c., or for matching the pattern), sufficient to cover the walls of a room—

137. 21 ft. long, $16\frac{1}{2}$ ft. wide and 10 ft. high at $7\frac{1}{2}d.$ per sq. yd.

138. 25 ft. long, 18 ft. 6 ins. wide and 10 ft. high; paper 2 ft. wide at $3d.$ a yd.

139. 21 ft. 6 ins. long, 15 ft. 3 ins. broad and 12 ft. high; paper 21 inches wide at $7s.\ 6d.$ per piece of 12 yards.

140. 27·7 ft. long, 19·55 ft. wide and 14·4 ft. high; paper 2·7 ft. wide at $1s.\ 3d.$ per piece of 12 yards.

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141. What is the cost of paper half a yard wide, at $1s.\ 6d.$ per piece of 12 yards, which would just cover the side-walls and ceiling of a passage 12 yards long, 6 ft. wide and $10\frac{1}{2}$ ft. high?

142. The length, breadth and height of a covered tank are 24 ft., 17 ft. and $14\frac{1}{2}$ ft. respectively. Find the cost of painting it all over outside at $1d.$ per sq. ft.

143. An open rectangular cistern, made of sheet-iron, is 7 ft. long, 4 ft. wide and 2 ft. 6 ins. deep. Find the cost of painting it all over, inside and outside, at $1\frac{1}{2}d.$ per sq. ft.

How many sq. ft. of plaster are there on the walls and ceiling of a room—

144. 10 feet high, 20 ft. long and 16 ft. wide, having a door 8 ft. by 4 ft., and two windows each 5 ft. by 3 ft.?

145. 28 ft. long, 20 ft. wide, 10 ft. high, having a door 7 ft. by 4 ft., a fireplace 5 ft. by 4 ft., and two windows each 5 ft. by 3 ft.?

146. 24 ft. long, 18 ft. broad, 12 ft. high, having two doors each 6 ft. 6 ins. by 3 ft. 4 ins.; two windows, opening down to the floor, each 10 ft. high by 5 ft. wide; a fireplace 5 ft. high by 4 ft. 6 ins. wide; and a skirting-board a foot high?

Find the cost of paper sufficient for the following rooms, allowing for waste 2 complete pieces over and above what is saved from windows, &c., the paper being bought *by the piece* 12 yards long and 21 ins. wide—

147. 28 ft. 6 ins. long, 18 ft. 9 ins. wide, 12 ft. high; price of paper 2*s.* 6*d.* per piece.

148. 31 ft. 4 ins. long, 17 ft. 3 ins. wide, 13 ft. 6 ins. high; price of paper 1*s.* $10\frac{1}{2}$*d.* per piece.

149. 20 ft. 8 ins. long, 18 ft. 6 ins. broad, 12 ft. high; price of paper 2*s.* 9*d.* per piece.

150. 30 ft. 7 ins. long, 17 ft. 8 ins. wide, 14 ft. 2 ins. high; price of paper 3*s.* 3*d.* per piece.

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151. The area of the walls of a room 13 ft. $7\frac{1}{2}$ ins. long and 13 ft. 3 ins. broad is 645 sq. ft.; how high is the room?

152. The area of the walls of a room 18 ft. 3 ins. long and 12 ft. high is 764 sq. feet; how broad is the room?

153. The area of the whole outside surface of a covered box 14 ins. long and 8 ins. wide is 3 sq. ft. 78 sq. ins.; how high is the box?

154. The area of the whole inside surface of an open tank 2 ft. 3 ins. wide and 1 ft. 9 ins. deep is $36\frac{5}{8}$ sq. ft.; how long is the tank?

155. The area of the sides, ends and bottom of a tank 17 ft. long and 5 feet deep is $345\frac{1}{2}$ sq. ft.; how wide is the tank?

156. The cost of painting at 5*d.* per sq. yd. the two sides and the ceiling of a dark passage 30 yds. long and 10 ft. high is £5, 14*s.* 7*d.*; how wide is the passage?

157. If paper 30 inches wide at $7\frac{1}{2}$*d.* per yard sufficient to just cover the entire walls of a room 31 ft. long and 23 ft. wide would cost £5, 3*s.* 6*d.*, how high is the room?

158. If paper at 2*d.* per yard sufficient to just cover the entire walls of a room 21 ft. long, 16 ft. wide, and 10 ft. high would cost 13*s.* 4*d.*, how wide is the paper?

159. The cost of painting the walls and ceiling of a room 17 ft. long, 14 ft. wide, and 10 ft. 6 ins. high, having a door 7 ft. by 3 ft. 9 ins., a window 6 ft. by 4 ft. 6 ins. and a fireplace 3 ft. by 3 ft. 6 ins., is £3, 8*s.* $9\frac{1}{4}$*d.*; what is the price per square foot?

160. A room 20 ft. long has two windows each 7 ft. by 4 ft., two doors each 6 ft. 6 ins. by 3 ft. 4 ins. and a fireplace 4 ft. by 3 ft. 3 ins. The cost of painting the walls at $1\frac{1}{2}$*d.* per sq. ft. is £4, 2*s.* $2\frac{1}{2}$*d.*, and the cost of polishing the floor at 1*s.* per sq. yd. is £1, 13*s.* 4*d.* How wide and how high is the room?

XLVIII. VOLUMES;

RECTANGULAR SOLIDS.

Find the volume of a rectangular solid of—

	Length.	Breadth.	Thickness.
1.	9 ft.;	4 ft.;	3 ft.
2.	17 ft.;	7 ft.;	5 ft.
3.	3 yds. 2 ft.;	2 yds. 1 ft.;	2 ft.
4.	5 yds.;	7 feet;	1 yd. 2 ft.
5.	3 ft. 9 in.;	1 ft. 8 in.;	1 ft. 2 in.
6.	6 ft. 6 in.;	4 ft. 4 in.;	3 ft. 3 in.
7.	3 yds.;	2 ft.;	11 in.
8.	17 yds.;	17 ft.;	17 in.

Find the volume of a cube the length of an edge of which is—

9. $1\frac{3}{4}$ in. **10.** 2 ft. 4 in. **11.** 1 ft. 1 in. **12.** 3·1 in.

Find the third dimension of a rectangular solid of volume—

13. 364 cub. ft.; breadth 7 ft.; thickness 4 ft.

14. 105 cub. ft.; breadth 3 ft. 6 in.; thickness 2 ft. 3 in.

15. 1 cub. ft. 876 in.; length 4 ft. 8 in.; thickness 2 ft. 7 in.

16. 8 cub. yds. 4 ft.; length 5 yds. 1 ft.; width 15 in.

17. 57 cub. ft. 828 in.; length 7 ft. 5 in.; breadth 2 ft. 7 in.

18. 4 cub. yds. 1 ft. 1528 in.; length 16 ft. 10 in.; breadth 7 ft. 10 in.

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19. How many cubic feet of air are contained in a room 17 ft. long, 15 ft. 3 in. wide and 12 ft. high?

20. How many cubic feet of timber are there in a beam 24 ft. long, 8 inches wide and 6 inches thick?

21. How many cubic feet of wood are there in the boards of a floor 18 ft. long and 16 ft. wide the boards being $1\frac{1}{4}$ inches thick?

22. What is the height of a room 18 ft. 6 in. long and 14 feet broad, which contains 2849 cub. ft. of air?

23. A room contains 2684 cub. ft. of space; its length is 16 ft. 6 in., and its height is 10 ft. 8 in.; what is its breadth?

24. What length must be cut off a beam $7\frac{1}{2}$ in. wide and 4 in. thick, that the volume of the piece cut off may be one cubic foot?

25. What is the length of a uniform beam 36 sq. inches in section, if its volume is 1 cub. yd.?

26. What is the thickness of a board 9 inches wide, when the volume of a piece 21 ft. 4 ins. long is one cubic foot?

27. How many children will a school-room 60 feet long, 28 feet wide and $13\frac{1}{2}$ feet high accommodate, allowing 140 cubic feet of space for each child?

28. A school-room 40 feet long and 15 feet high accommodates 120 children, allowing 120 cubic feet of air for each child: how wide is the room?

29. A school-room 72 feet long accommodates 180 persons allowing 12 sq. ft. of floor and 200 cub. ft. of space per head: how wide and how high is the room?

30. If a cubic foot of lead be made into a sheet 32 feet long and 6 feet wide, what decimal of an inch is the sheet in thickness?

31. Find the value of a uniform beam 23 feet long, and 64 sq. inches in section, at 2*s.* 3*d.* per cubic foot.

32. Find the cost, at 4*s.* 3*d.* per cubic yard, of a coat of gravel 2 inches thick for a court-yard 9 yards long and 7 yards wide.

33. A pond of 3 acres is covered with ice 6 inches thick; if a cubic foot of ice weigh 896 ozs., find in tons the weight of the ice on the pond.

34. A level field of 4 acres is flooded to a depth of 4 inches; if a cubic foot of water weigh 63 lbs., find in tons the weight of water on the field.

35. Find in tons the weight of a block of stone containing $670\frac{5}{16}$ cub. ft., if a block of the same kind of stone which is 5 ft. long, 3 ft. 9 in. broad and 2 ft. 6 in. thick weighs 7500 lbs.

36. If a cubic foot of iron weighs 480 lbs., what is the weight in tons, &c., of 14,000 iron bars each 10 feet long, $1\frac{1}{2}$ in. wide and $\frac{3}{8}$ in. thick?

37. An inlaid chess-board is composed of $1\frac{3}{4}$-inch squares, $\frac{1}{16}$ in. thick, of ebony and boxwood arranged alternately. How many cubic inches of ebony are there in it?

38. A gravel walk 6 feet wide runs round a grass-plot 60 feet long and 40 feet wide. Find the cost of the gravel, 3 inches deep, at 3*s.* per cubic yard.

39. A lawn 23 yards long and 17 yards wide is surrounded by a path 5 feet wide. Find the cost of a coat of gravel, 3 inches thick, for the path at 4*s.* 6*d.* per load (*i.e.* per cubic yard).

40. If the cost of digging a trench 2 feet wide and 18 inches deep at 9*d.* per cubic yard be £3, 19*s.* 3*d.*, how long is it?

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41. How many wooden blocks, each 7 in. by 5 in. by 3 in., could be packed in a cellar 14 feet long, 10 feet wide and 8 feet high?

42. How many bars of soap, 18 in. long, 3 in. wide and 3 in. thick, could be packed in a box the internal dimensions of which are 5 ft., 3 ft. and 2 ft. 9 in.?

43. How many cubes of $\frac{3}{4}$ in. edge could be packed in a box 1 ft. 3 in. long, 9 in. wide and $3\frac{1}{2}$ in. deep, inside?

44. How many gross of exercise-books, each book being 8 in. long, $6\frac{1}{2}$ in. wide and $\frac{1}{8}$ in. thick, could be packed in a box 2 ft. 8 in. long, 2 ft. 2 in. wide, and 1 ft. 6 in. deep, inside?

45. How many bricks would be required for a wall 80 yards long, 10 feet high and 18 inches thick, if a brick, with its share of mortar, measures 9 in. by $4\frac{1}{2}$ in. by 3 in.?

46. How many tiles six inches square and 1 in. thick could be packed in a cellar 12 ft. long, 11 ft. wide and 7 ft. high?

47. In a builder's yard there are two stacks of bricks; the first stack is $18\frac{3}{4}$ ft. long, $3\frac{3}{4}$ ft. wide and $8\frac{1}{4}$ ft. high; the second is $11\frac{1}{4}$ ft. long, $9\frac{3}{4}$ ft. wide and $5\frac{1}{4}$ ft. high. How many more bricks are there in one stack than in the other, the dimensions of a brick being 9 in., $4\frac{1}{2}$ in. and 3 in.?

48. Two boxes, whose inside dimensions are 2 ft. 6 in., 2 ft. $2\frac{1}{2}$ in., 2 ft. and 2 ft. $11\frac{1}{2}$ in., 2 ft., 1 ft. $10\frac{1}{2}$ in. respectively, are filled with copies of a book which is $7\frac{1}{2}$ in. long, 6 in. wide and $\frac{1}{2}$ in. thick. How many more books are there in one box than in the other?

49. If a brick with its share of mortar measures 9 by $4\frac{1}{2}$ by 3 inches, how many bricks would be required for a wall 100 yds. long and 8 ft. high, the lowest course of bricks being 18 inches wide, the second course $13\frac{1}{2}$ inches wide, and the remaining courses each 9 inches wide?

50. Supposing that a brick with its share of mortar occupies $9 \times 4\frac{1}{2} \times 3$ cubic inches of space; how many bricks are there in a wall 5 ft. high and 9 inches thick which surrounds a square the side of which measures, on the inner side of the wall, 18 yards?

If a pint of water weighs a pound and a quarter, and a cubic foot of water weighs 1000 ozs., how many—

51. Gallons would a tank 6 ft. long, 4 ft. wide and 2 ft. 4 in. deep, hold?

52. Gallons would a cistern 6 ft. 8 in. long, 5 ft. 4 in. wide and 4 ft. 6 in. deep, hold?

53. Imperial quart bottles could be filled with the contents of a full tank 3 ft. 6 in. long, 2 ft. 4 in. wide and 2 ft. 3 in. deep, and what fraction of a pint would be left over?

54. Feet wide is a tank 24 ft. long and 8 ft. deep which will just hold 7200 gallons?

55. Feet long is a tank 4 ft. 3 in. wide and 2 ft. 6 in. high which holds $531\frac{1}{4}$ gallons?

56. Acres are there in a field upon which 45,375 tons of rain-water fall in the course of a year, the average rain-fall being $2\frac{1}{3}$ per calendar month?

How many cubic inches of wood are there in a box, with lid—

57. 2 ft. long, 1 ft. 10 in. wide and 1 ft. 5 in. high, the boards of which it is made being 1 in. thick?

58. 3 ft. long, 2 ft. wide, 1 ft. $5\frac{1}{2}$ in. high, of wood $\frac{3}{4}$ in. thick?

59. 5 ft. 7 in. by 4 ft. 7 in. by 3 ft. 1 in., of wood 1 inch thick?

60. 4 ft. by 3 ft. by 2 ft., the boards being $\frac{3}{4}$ in. thick?

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61. How many cubic inches of iron are there in a safe which measures outside 2 ft. 6 in. in length, 2 ft. in width and 2 ft. 9 in. in height, the iron being 2 in. thick throughout?

62. How many cubic inches of wood are there in a box, without lid, made of board half an inch thick, the length being 3 ft. 2 in., width 1 ft. 10 in. and height 1 ft. 8 in.?

63. What length of 7-inch board, $\frac{3}{4}$ in. thick, would make a packing-case 2 ft. 3 in. long, 1 ft. 9 in. wide and 1 ft. $10\frac{1}{2}$ in. high, allowing a length of 3 in. of board for waste in sawdust?

64. How many cubic feet of stone are there in a rectangular trough, of internal length, 4 ft. 6 in.; width, 1 ft. 9 in.; depth, 1 foot, the stone being everywhere 3 in. thick?

65. A closed tank made of wood $1\frac{7}{8}$ in. thick, and lined throughout with sheet-lead $\frac{1}{8}$ in. thick, measures outside 5 ft. in length, 2 ft. 4 in. in breadth and 2 ft. 8 in. in depth; how many gallons of water will it hold?

66. A stone trough is 4 ft. long, 1 ft. 6 in. wide, 1 ft. 3 in. deep, inside, and the stone is everywhere 3 in. thick. If the weight of a cub. ft. of the stone be 175 lbs., and of water be 62 lbs., what would the trough weigh when full of water?

67. A hollow rectangular vessel, without lid, formed of material 1 in. thick, whose external dimensions are 13 ft. 6 in., 6 ft. 8 in. and 6 ft. 11 in. respectively, weighs 3 qrs. 25 lbs. Find the weight of a solid mass of the same material and the same dimensions.

68. Gold can be beaten out so thin that 275,625 gold leaves placed one upon another are but an inch in thickness. If a cubic foot of gold weigh 10 cwt. 95 lbs., what is the weight in grains of a piece of gold leaf 8 inches long and 7 inches wide?

69. Find the cost of bricks, at £1, 17*s.* 6*d.* per 1000, for a wall 5 ft. high and 9 inches thick surrounding (with the exception of a doorway 6 ft. wide) a rectangular garden which measures, outside the wall, 40 yards in length and 24 yards in breadth. The dimensions of a brick to be taken as 9 in. by $4\frac{1}{2}$ in. by 3 in., and no allowance to be made for mortar.

70. A safe 2 ft. 6 in. long, 2 ft. wide and 2 ft. 9 in. high, measured externally, is made of iron 2 in. thick, and weighs $23\frac{1}{4}$ cwt.; how high is a safe 2 ft. 10 in. long and 2 ft. 1 in. wide, made of iron $2\frac{1}{2}$ in. thick, and weighing $35\frac{1}{4}$ cwt.?

XLIX. DUODECIMALS.

Express, as duodecimals,

1. 5 ft. $10\frac{1}{2}$ in.
2. 6 ft. $7\frac{1}{2}$ in.
3. 4 ft. $3\frac{1}{4}$ in.
4. $11\frac{3}{4}$ in.
5. $8\frac{5}{8}$ in.
6. $2\frac{1}{8}$ in.
7. $6\frac{3}{8}$ ft.
8. 4 yds. $2\frac{3}{4}$ ft.
9. 5 yds. $1\frac{3}{16}$ ft.
10. 3 sq. ft. 72 in.
11. 2 sq. ft. 36 in.
12. 4 sq. ft. 88 in.
13. 136 sq. in.
14. 143 sq. in.
15. $89\frac{1}{2}$ sq. in.
16. $5\frac{3}{4}$ sq. ft.
17. $6\frac{5}{8}$ sq. ft.
18. 5 sq. yds. $1\frac{1}{16}$ sq. ft.
19. $10\frac{3}{4}$ cub. ft.
20. $17\frac{3}{8}$ cub. ft.
21. $3\frac{13}{16}$ cub. ft.
22. $3\frac{1}{2}$ cub. in.
23. $86\frac{3}{4}$ cub. in.
24. $117\frac{1}{8}$ cub. in.
25. 6 cub. ft. 436 in.
26. 11 cub. ft. 864 in.
27. 2 cub. ft. 1524 in.

Express in feet and inches,

28. 2 ft. $5'$.
29. 1 ft. $7'.6''$.
30. 5 ft. $4'.9''$.
31. 11 ft. $10'.8''$.
32. 7 ft. $0'.4''$.
33. 16 ft. $2'.7''.6'''$.
34. 5 sq. ft. $9'$.
35. 14 sq. ft. $3'.8''$.
36. 6 sq. ft. $11'.9''$.
37. 1 sq. ft. $8'.7''.6'''$.
38. $10'.11''.8'''$.
39. $9'.4''.5'''.6^{iv}$.
40. 12 cub. ft. $2'$.
41. 7 cub. ft. $0'.5''$.
42. 3 cub. ft. $5'.7''.3'''$.
43. $10'.8''.11'''$.
44. $8'.7''.10'''.6^{iv}$.
45. $5'.10''.0'''.8^{iv}.9^{v}$.
46. Add 18 ft. $9'.8''$; 2 ft. $5'.7''$; $11'.10''.8'''$.
47. Add 15 sq. ft. $3'.7''$; 1 sq. ft. $11'.8''.5'''$; and $6'.0''.8'''.6^{iv}$.
48. Add 12 sq. ft. $5'.8''$; 3 sq. ft. $10'.5''.9'''$; and $11'.8''.7'''$.
49. Subtract 2 cub. ft. $8'.7''.5'''.6^{iv}$ from 7 cub. ft. $2'.0''.3'''$.
50. Multiply 2 ft. $7'.8''$ (i) by 12; (ii) by 2; (iii) by 13.
51. Multiply 5 sq. ft. $6'.9''.4'''$ (i) by 12; (ii) by 72; (iii) by 47.
52. Divide 16 ft. $3'.5''$ (i) by 12; (ii) by 10; (iii) by 24.
53. Find, by Practice, the cost of 5 sq. ft. $8'.6''$ at 6*s*. 6*d*. per sq. ft.
54. Find, by Practice, the cost of 4 cub. ft. $6'.10''.8'''$ at £2, 12*s*. 6*d*. per cub. ft.

Find, by duodecimals, the area of a rectangle of dimensions—

55. 3 ft. $4'.6''$ and 3 ft.
56. 4 ft. $5'.9''$ and $3'$.
57. 1 ft. $6'.3''$ and $9''$.
58. $8'.4''$ and $8''$.
59. 3 ft. $5'$ and 2 ft. $3'$.
60. 4 ft. $9'$ and 2 ft. $6'$.
61. 7 ft. $10'$ and 5 ft. $8'$.
62. 12 ft. $8'.2''$ and 7 ft. $10'$.
63. 21 ft. $9'.7''$ and 12 ft. $9'$.
64. 4 ft. $3'.9''$ and 2 ft. $5'.6''$.
65. 11 ft. $10'.5''$ and 8 ft. $9'.7''$.
66. 23 ft. $8'.9''$ and 14 ft. $3'.6''$.

Find, by duodecimals, the area of a square whose side is—

67. 4 ft. $7'$.
68. $5'.5''$.
69. 1 ft. $6'.8''$.
70. 13 ft. $9'.6''$.

Find, by duodecimals, the volume of a rectangular solid of dimensions—

71. 2 ft. 9′; 2 ft. 3′; 2 ft.
72. 1 ft. 8′; 1 ft. 6′; 1 ft. 3′.
73. 4 ft. 7′; 3 ft. 8′; 2 ft. 6′.
74. 3 ft. 4′. 6″; 1 ft. 6′; 8′.
75. 16 ft. 8′; 13 ft. 9′; 3′
76. 32 ft. 9′; 1 ft. 3′; 2′.
77. 2 ft. 5′. 6″; 2 ft. 2′; and 1 ft. 6′.
78. 5 ft. 7′. 9″; 3 ft. 8′; and 2 ft. 9′.
79. 11 ft. 4′. 3″; 8 ft. 9′. 7″; and 3 ft. 10′.
80. 16 ft. 2′. 8″; 4 ft. 11′. 2″; and 2 ft. 8′. 6″.
81. 14 ft. 7 in.; 13 ft. $8\frac{1}{2}$ in.; and 3 yds. 2 ft. 7 in.
82. 7 yds. 2 ft. 9 in.; 16 ft. 3 in. 10 pts.; and 5 yds. $11\frac{1}{2}$ in.

Find, by duodecimals, the volume of a cube whose edge is—

83. 2 ft. 4′. 84. 3 ft. 8′. 85. 8′. 8″. 86. 7 ft. 7′. 7″.

Find by duodecimals—

87. The number of square feet of glass in 24 windows, each containing 4 panes, each pane 1 ft. 7′ long and 10′. 9″ wide.
88. The number of square feet of glass in 37 windows, each containing 12 panes, each pane 11′. 6″ long and 8′. 3″ wide.
89. The cost of glazing 60 windows, each containing 12 panes, each pane 1 ft. 2′. 6″ long and 10′. 6″ wide, at 1*s.* 6*d.* per sq. ft.
90. The cost of glazing 96 windows, each containing 9 panes, each pane 1 ft. 5′. 6″ long and 1 ft. 1′. 3″ wide, at 16*d.* per sq. ft.
91. The cost, to a penny, of digging out a cellar 18 ft. 4′ long, 12 ft. 9′ wide, and 14 ft. 6′ deep, at 1*s.* 6*d.* per cubic yard.
92. In sq. yds., ft., and ins. the area of the entire surface of a block 11 ft. 1′. 6″ long, 11 ft. 8′ wide, 6 ft. 6′. 8″ thick.
93. The area of the entire surface of a cube of edge 8 yds. 2 ft. $6\frac{2}{3}$ in.
94. The value of 3 dozen blocks of granite, each 15 ft. 8′ long, 10 ft. 4′ wide, and 6 ft. 9′ thick, at £1, 16*s.* per cub. yd.
95. How many more cubic inches of water a tank 12 ft. 6′ long, 5 ft. 3′ wide, and 3 ft. 9′ deep, will hold than a cubical tank whose edge is 6 ft. 3 in.
96. The value, to a penny, of a block 11 ft. $4\frac{1}{3}$ in. long, 3 ft. $3\frac{5}{12}$ in. wide, and 2 ft. $4\frac{1}{4}$ in. thick, at 8*s.* 9*d.* per cubic foot.
97. The quotient of 118 sq. ft. 10′. 6″. 6‴ $\div$ 21 ft. 3′. 6″.
98. The quotient of 390 sq. ft. 5′. 3″. 6‴. 6^{iv} $\div$ 23 ft. 4′. 5″.
99. The thickness of a rectangular solid of volume 17 cub. ft. 1′. 11″. 10‴. 6^{iv}, length 3 ft. 1′. 6″, and breadth 2 ft. 4 in.
100. The length of a rectangular solid of volume 303 cub. ft. 3′. 5″. 7‴. 4^{iv}, breadth 8 ft. 4 in., and thickness 2 ft. $7\frac{2}{3}$ in.

L. APPROXIMATION.

DECIMALS.

Write down the nearest approximation
(i) in *three* places, (ii) in *five* places, of decimals to—

1. 4·5632765.	**4.** ·2789325.	**7.** $2\cdot068\dot{3}$.
2. ·0326549.	**5.** 6·1499325.	**8.** $473\cdot\dot{6}$.
3. 15·2476256.	**6.** ·4233655.	**9.** $\cdot0\dot{5}\dot{8}$.

Find the nearest approximation, within the limit mentioned, to—

10. The sum of 24·175, 4·775625, 18·49636, ·1356 and ·085725, to *two* places of decimals.

11. 7·4632 + 12·05768 + ·0195 + 23·40585 + 301·62385 + 1·45367, to *one* place of decimals.

12. The sum of $\cdot5\dot{6}\dot{3}$, $18\cdot4078\dot{3}$, $6\cdot\dot{7}$, ·05975, $2\cdot48\dot{6}$ and 21·34625, to *three* places of decimals.

13. The difference between 4·452296 and 104·235 to *three* places of decimals.

14. $12\cdot1\dot{6} - \cdot857425$ to *four* places of decimals.

15. $8\cdot56340075 - 2\cdot08\dot{3}$ to *five* places of decimals.

16. Find the integer nearest to the sum of 87·563, 910·2675, 7·864, 204·4793, 8·5, 47·5, 60·075 and ·5675.

17. Find to the nearest million the sum of 4756320, 10478630, 1562500, 8479000, 2936860 and 12075680.

18. Find, within one-thousandth of the whole, the sum of 7·875, 1·20584, 12·67325, 148·8366 and ·27695.

19. Find, within one-hundredth of the whole, the sum of $4\cdot\dot{7}$, 12·6834, 3·14159, $\cdot\dot{0}7\dot{3}$, 42·587632 and 28·5.

20. Find, within one-millionth of the whole, the sum of 54763·25, 80·4725, 6·47965, 97318·476, 293·647875 and ·086795.

21. Find, within one-thousandth of the whole, the difference between 87·6187345 and 147·53225.

22. Find, within one-millionth of the whole, $438\cdot573 - 37\cdot8\dot{3}$.

Find the nearest approximation in *three* places of decimals to—

23. $14\cdot5637525 \times 8$.	**31.** $\cdot0814275 \div 7$.	**39.** $813\cdot576385 \times 2\frac{1}{2}$.
24. $2\cdot41573825 \times 12$.	**32.** $7\cdot375284 \div 12$.	**40.** $8\cdot241675 \times 3\frac{3}{4}$.
25. $\cdot13068875 \times 40$.	**33.** $32\cdot6410565 \div 60$.	**41.** $52\cdot50\dot{2}91\dot{6} \times 2\frac{3}{4}$.
26. $\cdot8302545 \times 15$.	**34.** $\cdot45690385 \div 32$.	**42.** $4\cdot638225 \times 3\frac{1}{3}$.
27. $6\cdot62078\dot{3} \times 800$.	**35.** $2\cdot5069578 \div \cdot5$.	**43.** $23\cdot7225 \times 4 \div 100$.
28. $26\cdot508472 \times \cdot4$.	**36.** $5\cdot2506725 \div \cdot56$.	**44.** $491\cdot75 \times 2\frac{1}{2} \div 100$.
29. $3\cdot55381\dot{6} \times \cdot05$.	**37.** $56\cdot48\dot{3} \div 73$.	**45.** $76\cdot48\dot{3} \times 3\frac{1}{4} \div 100$.
30. $8\cdot51\dot{6} \times 1\cdot6$.	**38.** $4\cdot3259 \div 1\cdot7$.	**46.** $506\cdot41\dot{6} \times 3\frac{3}{4} \div 100$.

Decimalize to *three* places—

47. £2, 16s. 5d.
48. £13, 18s. 2d.
49. £1, 16s. 7d.
50. £43, 13s. 1d.
51. £4, 15s. 7d.
52. £0, 9s. 9d.
53. £63, 11s. 1d.
54. £10, 3s. 8d.
55. £3, 3s. 6d.
56. £75, 19s. 10d.
57. £2, 1s. 11d.
58. 17s. $8\frac{1}{2}$d.

Decimalize to *five* places—

59. £1, 9s. 10d.
60. £16, 12s. 7d.
61. £0, 13s. 11d.
62. £0, 5s. 8d.
63. £453, 16s. 2d.
64. £197, 13s. 5d.
65. £4, 18s. $8\frac{1}{2}$d.
66. £87, 3s. $3\frac{3}{4}$d.
67. £142, 1s. $7\frac{1}{4}$d.

Find, to the nearest *penny*, the value of—

68. £4·816.
69. £1·584.
70. £17·908.
71. £13·7028.
72. £·6363.
73. £38·3565.
74. £123·88375.
75. £1·5951$\dot{6}$.
76. £11·0983.

Find, to the nearest *farthing*, the value of—

77. £47·5465.
78. £·685.
79. £1·519.
80. £11·046535.
81. £546·46.
82. £3·298156.

Find, to the nearest penny, the value of—

83. £4·5735 + £15·032 + £1·4723575 + £·01275 + £12·428766.
84. £13·469725 + £4·8$\dot{3}$ + £·16775 + £23·0251$\dot{6}$ + £305·970$\dot{5}\dot{7}$.
85. £52·470895 – £6·528775.
86. £17·18$\dot{3}$ – £9·241$\dot{6}$.

Find, to the nearest penny, *by Decimalized Practice*,* the—

87. Value of $1672\frac{1}{2}$ things at £3, 11s. $7\frac{1}{2}$d. each.
88. Cost of $893\frac{1}{4}$ things at £1, 17s. $5\frac{1}{4}$d. each.
89. Rent of 956 A. 3 R. 10 P. at £1, 6s. 3d. per acre.
90. Rent of 1857 A. 2 R. 24 P. at £1, 14s. $8\frac{1}{2}$d. per acre.
91. Value of 1473 tons 13 cwts. at £2, 11s. 5d. per ton.
92. Dividend on £1485, 17s. 6d. at 5s. $6\frac{1}{2}$d. in the pound.
93. Dividend on £884, 16s. 8d. at 11s. $8\frac{1}{2}$d. in the pound.
94. Value of 1 cwt. 2 qrs. 17 lbs. at £2, 13s. 8d. per cwt.
95. Cost of 3 cwts. 1 qr. 16 lbs. 9 ozs. at £14, 16s. 6d. per cwt.
96. Rent of 2 ac. 3 ro. 31 po. at £1, 11s. 8d. per acre.
97. Rent of 4 ac. 2 ro. $37\frac{3}{4}$ po. at £1, 8s. 6d. per acre.
98. Value of 10 Troy ozs. 397 grs. at £3, 17s. $10\frac{1}{2}$d. per oz.
99. 2 tons 11 cwts. 2 qrs. $17\frac{1}{2}$ lbs. at £2, 15s. $11\frac{1}{2}$d. per ton.
100. 39 cwts. 3 qrs. $24\frac{1}{4}$ lbs. at £5, 13s. 8d. per ton.

* Further exercises in this method may be obtained from pages 89 and 92, the results *to the nearest penny* being easily seen from the *exact* answers given.

Contracted Multiplication.

Obtain, in the number of places of decimals mentioned within brackets, the nearest approximation to—

101. $90 \cdot 760125 \times 8 \cdot 0127$ (*two*).
102. $12 \cdot 345675 \times 1 \cdot 23456$ (*two*).
103. $\cdot 9846725 \times 7 \cdot 90845$ (*two*).
104. $5436 \cdot 7928 \times 5 \cdot 36419$ (*two*).
105. $\cdot 0598765 \times 2 \cdot 107685$ (*three*).
106. $4 \cdot 7603561 \times 5 \cdot 7140232$ (*three*).
107. $\cdot 04327184 \times 80 \cdot 85642$ (*two*).
108. $2 \cdot 706815 \times 85 \cdot 07685$ (*two*).
109. $\cdot 91843275 \times 51 \cdot 0826$ (*three*).
110. $416 \cdot 087925 \times \cdot 0807685$ (*two*).
111. $34 \cdot 76912 \times 41 \cdot 2804$ (*three*).
112. $41 \cdot 69207 \times \cdot 004037$ (*three*).
113. $1705 \cdot 32 \times \cdot 014623$ (*two*).
114. $42 \cdot 506 \times \cdot 00840542$ (*four*).
115. $\cdot 0521685 \times \cdot 0084032$ (*four*).
116. $\cdot 810901 \times \cdot 809101$ (*three*).
117. $4 \cdot 132167 \times \cdot 01098$ (*two*).
118. $\cdot 234678 \times \cdot 378426$ (*six*).
119. $8 \cdot 2\dot{1}5\dot{7} \times \cdot \dot{8}23\dot{1}$ (*four*).
120. $456 \cdot 8\dot{3} \times \cdot \dot{1}4285\dot{7}$ (*six*).

121. $(4 \cdot 1506312)^2$ (*three*).
122. $(3 \cdot 14159)^2$ (*three*).
123. $(\cdot 57075)^2$ (*two*).
124. $(12 \cdot 0575)^2$ (*four*).
125. $(\cdot 02138607)^2$ (*six*).
126. $(2 \cdot 1703)^3$ (*two*).
127. $(3 \cdot 1416)^3$ (*two*).
128. $(\cdot 41506)^3$ (*two*).
129. $(\cdot 0286038)^3$ (*five*).
130. $(\cdot \dot{5}13\dot{2})^3$ (*three*).
131. $(1 \cdot 0325)^4$ (*three*).
132. $(1 \cdot 03)^8$ (*three*).
133. $(1 \cdot 025)^5$ (*five*).
134. $(1 \cdot 04)^9$ (*five*).
135. $(1 \cdot 05)^{11}$ (*three*).

136. Find $\cdot 015632 \times 14 \cdot 0514 \times \cdot 72315$ to *two* places of decimals.
137. Find $\cdot 0012345 \times 805 \cdot 6231 \times 12 \cdot 14285$... *three* ....
138. Find $\cdot 265137 \times 128 \cdot 23 \times 9 \cdot 2671$ .... *four*........................
139. Find $\cdot 3141\dot{6} \times 21 \cdot 5\dot{7}\dot{2} \times 41 \cdot 5\dot{3}\dot{8}$ .... *three* ....

140. Find the integer nearest to $72 \cdot 96415 \times 18 \cdot 74625$.
141. Find the integer nearest to the square of $41 \cdot 38716$.
142. Find the integral part of the product of $43 \cdot 126725$ and $39 \cdot 425683$.
143. Find, to the nearest *thousand*, $76045 \cdot 285 \times 91 \cdot 8375$.
144. Find, to the nearest *thousandth* of unity, $30 \cdot 7865 \times \cdot 4312$.
145. Find, within *one-thousandth of the whole*, $143 \cdot 325 \times 64 \cdot 495$.
146. Find, within *one-thousandth of the whole*, $7 \cdot 056275 \times 1 \cdot 175875$.
147. Find, within *one-hundredth of the whole*, $1 \cdot 92648 \times \cdot 182356$.
148. Find, within *one-millionth of the whole*, $45 \cdot 105275 \times 608 \cdot 05625$.
149. Find, within *one-thousandth of the whole*, $(15 \cdot 607)^3$.
150. Find, within *one-thousandth of the whole*, the continued product of $\cdot 000386$, $\cdot 010274$ and $65 \cdot 25$.

Find, to the nearest penny, the value of—

151. $£\cdot 57325 \times 2\cdot 54$.

152. $£75\cdot 41\dot{6} \times \cdot 1352$.

153. £14, 17*s.* 8*d.* $\times$ 203·75.

154. £138, 13*s.* 6*d.* $\times$ 315·425.

155. 426 tons 13 cwts. 3 qrs. 7 lbs. at £815, 12*s.* 6*d.* per ton.

156. 273 miles 6 fur. 120 yds. at £613, 12*s.* 8*d.* per mile.

157. $£8175\cdot 8\dot{3} \times (1\cdot 025)^5$.

158. $£506\cdot 75 \times (1\cdot 05)^7$.

159. £851, 10*s.* $\times (1\cdot 04)^{12}$.

160. $£3500 \times (1\cdot 05)^{20}$.

Contracted Division.

Find, in the number of places of decimals mentioned within brackets, the nearest approximation to—

161. $8109\cdot 75 \div 15623$ (*three*).

162. $314210\cdot 8 \div 18306$ (*two*).

163. $\cdot 0264508 \div 1\cdot 2054$ (*four*).

164. $8\cdot 14623 \div 12\cdot 954$ (*three*).

165. $8\cdot 567145 \div 47\cdot 632$ (*three*).

166. $142\cdot 875 \div 603\cdot 625$ (*three*).

167. $72\cdot 2416 \div \cdot 2365$ (*two*).

168. $32\cdot 5172 \div \cdot 046325$ (*two*).

169. $\cdot 046325 \div 32\cdot 5172$ (*four*).

170. $\cdot 03765 \div 13\cdot 2416$ (*four*).

171. $\cdot 3060875 \div 2\cdot 3641525$ (*five*).

172. $352\cdot 4258 \div 23\cdot 575$ (*four*).

173. $6\cdot 25040875 \div 124\cdot 528$ (*six*).

174. $324\cdot 816 \div 7\cdot 248057$ (*six*).

175. $1\cdot 425 \div \cdot 0\dot{5}6\dot{7}$ (*three*).

176. $8\cdot 2\dot{1}5\dot{7} \div \cdot\dot{8}23\dot{1}$ (*four*).

177. $\cdot 36\dot{2}\dot{5} \div 14\cdot\dot{6}\dot{9}$ (*five*).

178. $17\cdot\dot{3} \div 219\cdot 1\dot{5}7\dot{3}$ (*six*).

179. $1847\cdot 345 \times 1\cdot 72 \div 2185$ (*two*).

180. $241\cdot 345 \times \cdot 2075 \div 3\cdot 1416$ (*two*).

181. $\dfrac{516\cdot 5 \times 852}{36500}$ (*three*).

182. $\dfrac{\cdot 8856 \times 5\cdot 3718}{12\cdot 6784}$ (*three*).

183. $\dfrac{7\cdot 14765}{8\cdot 76 \times 4\cdot 25}$ (*three*).

184. $\dfrac{407\cdot 6275 \times \cdot 1568}{(3\cdot 1416)^2}$ (*two*).

185. $\dfrac{17\cdot 525 \times \cdot 015745}{3\cdot 13675 \times 21\cdot 24325}$ (*four*).

186. Find the integer nearest to $1\cdot 002591 \div \cdot 038764875$.

187. Find the integral part of the quotient of $4863\cdot 375 \div 1\cdot 9258725$.

188. Find, within *one-thousandth of the whole*, $73\cdot 15688 \div 158\cdot 45$.

189. Find, within *one-thousandth of the whole*, $3568\cdot 5 \div 16\cdot 517845$.

190. Find, within *one-millionth of the whole*, $14\cdot 68705 \div \cdot 0316283$.

Find, to the nearest penny, the value of—

191. $£863\cdot 48\dot{3} \div 617$.

192. $£103\cdot 725 \div 318\cdot 546$.

193. £21769, 11*s.* 10*d.* $\div$ 8576.

194. £1490, 13*s.* 8*d.* $\div$ 13·825.

195. $£31\cdot 1426 \times 12\cdot 18 \div 73$.

196. £1156, 17*s.* 6*d.* $\times$ 4·86 $\div$ 365.

Find, to *five* places of decimals, the sum of the series—

197. $\frac{1}{1\times2} + \frac{1}{1\times2\times3} + \frac{1}{1\times2\times3\times4} + \dots\dots\dots\dots$

198. $\frac{1}{1\times2} - \frac{1}{1\times2\times3} + \frac{1}{1\times2\times3\times4} - \dots\dots\dots\dots$

199. $\frac{1}{1\times3} + \frac{1}{1\times3\times5} + \frac{1}{1\times3\times5\times7} + \dots\dots\dots\dots$

200. $\frac{1}{1\times3} - \frac{1}{1\times3\times5} + \frac{1}{1\times3\times5\times7} - \dots\dots\dots\dots$

Find, to the nearest penny, by contracted work in Decimals, the

201. Value of 5 tons 11 cwts. 3 qrs. at the rate of £41, 13*s*. 7*d*. for 7 tons 9 cwts. 1 qr.

202. Value of 78 ozs. 210 grs. of gold, if 53 ozs. 300 grs. are worth £175, 6*s*. 3*d*.

203. Dividend paid by a bankrupt whose debts are £2957, 18*s*. 8*d*. and his available assets £1025, 13*s*. 6*d*.

204. Dividend to be paid by a bankrupt whose assets amount to £10,223, 14*s*. 8*d*.; his liabilities to £13,901, 16*s*. 6*d*., and the "costs" of his bankruptcy to £473, 18*s*.

205. Shares of profits amounting to £473, 18*s*. 3*d*. due to each of two partners whose capitals are £2756 and £1350 respectively.

206. Average cost per ton if two truck-loads of coal are bought, the first, containing 9 tons 9 cwts. 3 qrs., for £5, 8*s*. 6*d*.; and the second, containing 8 tons 18 cwts. 2 qrs., for £5, 3*s*. 6*d*.

207. Find, to the nearest *penny*,
parts of (i) £56, 16*s*. 4*d*. proportional to 67, 47 and 29;
(ii) £147, 13*s*. 9*d*. .... 13, 15·5 and 19;
(iii) £1413, 12*s*. .... 13, $19\frac{1}{4}$ and 22.

Also, to the nearest *quarter*,
parts of (iv) 57 tons 13 cwts. 3 qrs........ 7, 17 and 37.

and to the nearest *pole*,
parts of (v) 473 A. 2 R. 37 P. .... 9, 9·5 and 9·25.

208. Find, within a thousandth of the whole, the area of a rectangle 34 ft. 5 in. long and 26 ft. $7\frac{1}{2}$ in. wide.

209. If the length of a rectangle is known to be greater than 6·135 ft. and less than 6·136 ft., and its breadth greater than 4·027 ft. and less than 4·028 ft., within how many square inches can its area be determined with certainty?

210. If a bicycle-wheel has a circumference of 7·145 ft. and makes, on the average, 6500 revolutions per hour, find, to the nearest hundredth of a mile, the distance travelled in 5 hrs. 27 min.

LI. PERCENTAGES.

Find, *mentally*, the percentage equivalent to the fraction—

1. $\frac{1}{2}$.
2. $\frac{1}{4}$.
3. $\frac{1}{5}$.
4. $\frac{1}{20}$.
5. $\frac{1}{25}$.
6. $\frac{1}{50}$.
7. $\frac{3}{4}$.
8. $\frac{2}{5}$.
9. $\frac{7}{10}$.
10. $\frac{9}{20}$.
11. $\frac{11}{25}$.
12. $\frac{17}{50}$.
13. $\frac{1}{3}$.
14. $\frac{1}{8}$.
15. $\frac{1}{12}$.
16. $\frac{1}{40}$.
17. $\frac{3}{40}$.
18. $\frac{7}{200}$.
19. ·625.
20. ·1275.

Find, *mentally*, in lowest terms, the fraction equivalent to—

21. 8 per cent.
22. 24 per cent.
23. 15 per cent.
24. 35 per cent.
25. 78 per cent.
26. 6 per cent.
27. $4\frac{1}{2}$ per cent.
28. $3\frac{1}{3}$ per cent.
29. $6\frac{1}{4}$ per cent.
30. $66\frac{2}{3}$ per cent.
31. $8\frac{1}{3}$ per cent.
32. 12·5 per cent.

Find the percentage equivalent to the fraction—

33. $\frac{3}{16}$.
34. $\frac{5}{19}$.
35. $\frac{17}{120}$.
36. $\frac{7}{17}$.
37. $\frac{11}{305}$.
38. ·375.

Find, in lowest terms, the fraction equivalent to—

39. $3\frac{1}{8}$ per cent.
40. $4\frac{4}{9}$ per cent.
41. $6\frac{7}{8}$ per cent.
42. $2\frac{3}{16}$ per cent.
43. 42·5 per cent.
44. 18·4 per cent.

Find—

45. 72 per cent of 375.
46. 12 per cent of 3775.
47. $3\frac{3}{4}$ per cent of 4720.
48. $16\frac{2}{3}$ per cent of 492.
49. 6·25 per cent of 648.
50. 8·375 per cent of 42400.

How much per cent is—

51. 19 of 475?
52. 235 of 750?
53. 189 of 504?
54. 4455 of 5445?
55. 23·75 of 475?
56. 58·8 of 3920?

57. If $7\frac{1}{2}$ per cent of a certain number is 69, what is that number?
58. Find the number $8\frac{1}{3}$ per cent of which is 48.
59. Of what number is 109·8 twelve per cent?

Find, *mentally*, 5 per cent of—

60. £94.
61. £116.
62. £275.
63. £347.
64. £12, 10*s.*
65. £23, 5*s.*
66. £8, 15*s.*
67. £17, 10*s.*
68. £36, 12*s.* 6*d.*
69. £4, 6*s.* 8*d.*
70. £9, 13*s.* 4*d.*
71. £51, 4*s.* 2*d.*
72. £86, 8*s.* 4*d.*
73. £132, 16*s.* 8*d.*
74. £189, 11*s.* 8*d.*
75. £537, 18*s.* 4*d.*

Find, *mentally*, neglecting fractions of a penny, 5 per cent of—

76. £10, 2*s*.	79. £14, 14*s*. 8*d*.	82. £107, 7*s*. 2*d*.
77. £3, 12*s*.	80. £43, 15*s*. 9*d*.	83. £181, 9*s*. 8*d*.
78. £26, 19*s*.	81. £62, 11*s*. 10*d*.	84. £467, 13*s*. 10*d*.

Find, *mentally*, neglecting fractions of a penny, $2\frac{1}{2}$ per cent of—

85. £34.	88. £7, 10*s*.	91. £10, 16*s*. 8*d*.
86. £57.	89. £14, 11*s*.	92. £4, 7*s*. 6*d*.
87. £12, 10*s*.	90. £28, 6*s*. 8*d*.	93. £59, 14*s*. 3*d*.

Find—

94. 4 per cent of £7, 10*s*.	99. $3\frac{1}{2}$ per cent of £50, 16*s*. 8*d*.
95. 4 per cent of £71, 5*s*.	100. $2\frac{1}{4}$ per cent of £26, 13*s*. 4*d*.
96. 3 per cent of 6 tons 5 cwts.	101. $7\frac{1}{4}$ per cent of £125.
97. 85 per cent of 19 ozs. Troy.	102. $4\frac{3}{4}$ per cent of £202, 10*s*.
98. 15 per cent of 193·5 grs.	103. $5\frac{5}{8}$ per cent of £423, 13*s*. 4*d*.

How much per cent is—

104. 2*d*. of 1*s*.?	110. 1*s*. $7\frac{1}{2}$*d*. of £1, 12*s*. 6*d*.?
105. 2*s*. 6*d*. of £1?	111. 12*s*. 9*d*. of £1, 9*s*. 9*d*.?
106. $1\frac{1}{2}$*d*. of £1?	112. £38, 10*s*. of £924?
107. 19*s*. 6*d*. of £1?	113. 42 lbs. of 2 cwts.?
108. 6*s*. 8*d*. of £5?	114. 16 grs. of 1 oz. Troy?
109. 8*s*. 4*d*. of £2?	115. 2 hrs. 24 mins. of 5 days?

If a tradesman allow 5 per cent "discount for cash", find, neglecting fractions of a penny, how much ready money would pay for an article priced—

116. 14*s*. 10*d*.	119. £4, 3*s*. 9*d*.	122. £11, 7*s*. 9*d*.
117. £2, 12*s*.	120. £6, 16*s*. 8*d*.	123. £22, 14*s*. 6*d*.
118. £1, 8*s*. 6*d*.	121. £5, 5*s*.	124. £17, 13*s*. 3*d*.

If a tradesman allow $2\frac{1}{2}$ per cent discount for prompt payment of an account, find, neglecting fractions of a penny, the ready money which would pay for goods invoiced at—

125. £4, 17*s*. 7*d*.	128. £7, 8*s*. 5*d*.	131. £5, 13*s*. 10*d*.
126. £3, 5*s*. 10*d*.	129. £18, 10*s*. 9*d*.	132. £1, 19*s*. 1*d*.
127. £2, 11*s*. 6*d*.	130. £25, 12*s*. 3*d*.	133. £29, 1*s*. 6*d*.

Increase—

134. 450 by 2%.	136. £14, 10*s*. by 5%.	138. £43, 10*s*. 8*d*. by $12\frac{1}{2}$%.
135. 236 by 25%.	137. £33, 15*s*. by 4%.	139. £93, 12*s*. 3*d*. by $16\frac{2}{3}$%.

Increase, giving the result to the nearest penny—

140. £47, 10*s.* by 4%.
141. £51, 13*s.* 4*d.* by 3%.
142. £18, 15*s.* by $3\frac{1}{2}$%.
143. £91, 12*s.* 3*d.* by $2\frac{1}{4}$%.
144. £156, 18*s.* 8*d.* by $4\frac{1}{2}$%.
145. £284, 6*s.* 10*d.* by $3\frac{3}{4}$%.

Decrease—

146. 1890 by 10%.
147. 4620 by 15%.
148. £51, 10*s.* by $2\frac{1}{2}$%.
149. £137, 13*s.* 3*d.* by $33\frac{1}{3}$%.
150. 11 tons by $7\frac{1}{2}$%.
151. 9 tons 16 cwts. by $12\frac{1}{2}$%.

Decrease, giving the result to the nearest penny—

152. £36, 11*s.* 2*d.* by 2%.
153. £16, 7*s.* 11*d.* by 7%.
154. £73, 15*s.* 7*d.* by $3\frac{1}{4}$%.
155. £105, 5*s.* 9*d.* by $2\frac{1}{3}$%.
156. £272, 1*s.* 10*d.* by $4\frac{1}{4}$%.
157. £518, 16*s.* 4*d.* by $2\frac{3}{8}$%.

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158. If a man with an income of £570 a year spends 85 per cent of it, what does he save annually?

159. The population of a town was 143,500 in 1881, and it had decreased 3 per cent at the next census; what was its population in 1891?

160. The population of a town at one census was 172,800; at the next it was 181,440; find the rate per cent of increase.

161. Standard gold contains 22 parts of pure gold to 2 parts of alloy. Find the percentage of alloy in a sovereign.

162. If 17·5 per cent of a number be 1946, what is the number?

163. If $3\frac{1}{2}$ per cent of a sum of money is £22, 10*s.* 11*d.*, what is that sum?

164. The death-rate one week in a town of 82,000 inhabitants was 17·5 per thousand. How many persons died?

165. The population of a certain town increased during a certain interval from 24,500 to 25,250; find, to two places of decimals, the rate per cent of increase.

166. If a bankrupt pays a dividend of 12*s.* 6*d.* in the pound, how much per cent do his creditors lose?

167. If the price of coal is 16*s.* 8*d.* a ton, subject to discount of 5 per cent for cash, how much ready money must be paid for 6 tons 8 cwts.?

168. Between 1871 and 1881 the population of a certain town increased by $24\frac{3}{4}$ per cent, and in the latter year it was 206,087; what was it in the former?

169. If 8 per cent of a garrison of 4125 men desert to the enemy who number 6000, what is the gain per cent to the enemy?

170. In a concert hall there are 600 reserved seats and 1300 unreserved seats; when $14\frac{1}{2}$ per cent of the former and 10 per cent of the latter are unoccupied, of how many persons does the audience consist?

171. If 11 men working 10 hours a day earn 643·5 francs in 13 days, in how many days would 25 men, working 9 hours a day, earn 2673 francs when the pay for an hour's work has risen 20 per cent?

172. In an examination, A obtains 48 per cent and B 33 per cent of full marks. Their marks, if added together, amount to 567. What are full marks?

173. An arithmetic examination paper is worked by 2500 candidates, of whom one-fifth are girls and the rest boys. If 5 per cent of the boys, and 40 per cent of the girls, fail, what percentage of the total number of candidates pass?

174. The populations of the upper and lower parts of a town were equal. After the former had fallen 20 per cent, and the latter risen 15 per cent, the total number of inhabitants was 39,390. What was the population of each part at first?

175. At a census three towns had populations of 17,650, 19,600 and 18,760. At the next census the first had decreased 18 per cent, the second had increased 21 per cent, and the total population of the three had increased by 4691 persons: find the change per cent in the population of the third town.

176. A man whose income is £900 a year pays an income-tax of 8*d.* in the pound; what percentage of (i) his gross income, (ii) his net income, is his tax?

177. After paying 10 per cent of his income in rates and taxes, a man has £1350 left. Find his whole income.

178. In a battle 5 per cent of an army were killed, 18 per cent wounded, 8 per cent were taken prisoners and 243 men deserted. The effective force then remaining numbered 5691. Find the number of men at first.

179. If a man can travel 360 miles in 12 days of 8 hours each, how many hours a day less need he travel, if he increase his rate 20 per cent, in order to accomplish 450 miles in 20 days?

180. If the income-tax were $3\frac{1}{2}$ per cent on a man's income, instead of 8*d.* in the £, what difference would it make to a man whose annual income is £3125?

181. A bankrupt's debts amount to £21,140, and his assets to £4832. Allowing $12\frac{1}{2}$ per cent for expenses, what would the dividend be?

182. In a forest 5 per cent of the trees are blown down by a gale, and, after 3 per cent of those remaining have been cut down, there still stand 55,290 trees. How many trees were there in the forest before the gale occurred?

183. If A pays 8*d.* in the pound income-tax and other charges amounting to 5 per cent on the remainder of his income, what will he have left out of an income of £600 per annum?

184. The net rental of an estate after deducting 7*d.* in the pound for income-tax, and 5 per cent on the remainder for the expenses of collecting, is £479, 11*s.* 10*d.*; find the gross rental.

185. A man after paying 5 per cent on his gross rental for collecting, and income-tax at the rate of 8*d.* in the £1 on it, has left £672, 15*s.* 9*d.*; find his gross rental.

186. If the length and breadth of a rectangle were each increased 20 per cent, how much per cent would its area be increased?

187. If the length of a rectangle were increased 15 per cent and its breadth decreased 15 per cent, what would be the change per cent in its area?

188. If air is composed of 75·55 per cent of nitrogen, 23·22 per cent of oxygen and 1·23 per cent of carbonic acid, how much is there of each gas in a chamber containing 6548 cubic feet of space?

189. If air contains 23·01 per cent of its weight of oxygen, and if a cubic foot of air weigh 1·23 ounces avoirdupois; find, to the nearest ounce, the weight of the oxygen in a room 18 feet by 27 ft. 4 in. by 12 feet.

190. At an examination in which full marks were 1000, A got 20 per cent more than B, B 10 per cent more than C, C 20 per cent less than D. If A got 660, what percentage of full marks was obtained by D?

191. In an examination paper of 5 questions, 5 per cent of the candidates answer all; 5 per cent none; 25 per cent, and 20 per cent, of the rest answer respectively one and four; also 24·5 per cent of the total number of candidates answer two, and 200 candidates answer three, questions. How many candidates are there?

192. A rate of $4\frac{3}{4}$ per cent was to be paid on a certain sum, but income-tax at 6*d.* per £ of rate was to be deducted from the rate. The net rate paid was £342, 14*s.* 3*d.* Find the sum on which it was paid.

193. If I add 5 per cent to a certain number and deduct 5 per cent from the same number, I obtain two numbers which differ by 51. Find these numbers.

194. If an income-tax of 8 pence in the pound brings in a net revenue of 15 millions, and the cost of collection is 4 per cent of the sum collected, what is the total amount of the incomes taxed?

195. If, when the income-tax is raised from 6*d.* to 10*d.* in the pound, the resulting revenue is only increased 10 per cent, what is the decrease per cent in the amount of incomes taxed?

*196. At the census of 1891 the population of England was 27,501,362; of Wales, 1,501,163; of Scotland, 4,025,647; and of Ireland, 4,704,750. Find to the nearest *tenth* in decimals the percentage of the total population of the British Isles then inhabiting each country.

Find the numbers required to complete the following tables, giving the percentages of the numbers for 1893, correct to the nearest *tenth* in decimals:—

RAILWAY PASSENGER TRAFFIC RECEIPTS.

197.

L. & N.W.R.	1894.	1893.	Increase or decrease per cent.
	£	£	
First Class,.........	481,248	498,469	... ...dec.
Second ,, ...	240,762	257,782	... ...dec.
Third ,, ...	2,875,012	2,813,434	... ...inc.
Total, ...	3,597,022	3,569,685	... ...inc.

198.

G.N.R.	1894.	1893.	Increase or decrease per cent.
	£	£	
First Class,.........	159,837	164,349	... ...
Second ,, ...	20,824	23,779	... ...
Third ,, ...	1,137,357	1,115,204	... ...
Total, ...	1,318,018	1,303,332	... ...

199.

G.W.R.	1894.	1893.	Increase or decrease per cent.
	£	£	
First Class,.........	239,828	249,406	... ...
Second ,, ...	262,810	291,980	... ...
Third ,, ...	2,782,897	2,718,825	... ...
Total, ...	3,285,535	3,260,211	... ...

NUMBER OF PASSENGERS CARRIED ON ENGLISH RAILWAYS.

200.

	1894.	1893.	Increase or decrease per cent.
Class.	No.	No.	
Ordinary 1st,......	29,821,010	30,048,982	... ...
,, 2nd,......	60,161,714	59,989,640	... ...
,, 3rd, ...	821,430,202	783,138,430	... ...
Season Tickets,....	1,184,861	1,574,876	... ...
Total,..........	912,597,787	874,751,928	... ...

* Exs. 196 to 200 should be worked by the contracted methods of Chap. L.

LII. PROFIT AND LOSS.

Find the gain, or loss, per cent of cost price in the following cases:—

1. Cost £30; sold for £36.
2. ...... £4; .... £4, 10*s.*
3. ...... £36; .... £30.
4. ...... £8; .... £7, 5*s.*
5. ...... £3, 2*s.* 6*d.* ... £3, 7*s.* 6*d.*
6. Cost £10, 10*s.*; sold for £3, 10*s.*
7. ...... 3*s.* 9*d.*; .... 4*s.* $1\frac{1}{2}$*d.*
8. ...... 12*s.* 6*d.*; .... 14*s.* 9*d.*
9. ...... £5, 16*s.* 8*d.* ..... £5, 14*s.* 4*d.*
10. ...... £1, 3*s.* 9*d.*.......... £1, 8*s.* 6*d.*

Find the selling price in the following cases:—

11. Cost £8, 5*s.*, and sold at a profit of 20 per cent of the cost.
12. ...... 7*s.* 6*d.*, ... 10 ....
13. ...... £1, 15*s.*, ... 15 ....
14. ...... £5, 12*s.*, ... $12\frac{1}{2}$ ....
15. ...... 2*s.* 2*d.*, ... $37\frac{1}{2}$ ....
16. ...... 12*s.* 6*d.*, and sold at a loss of 12 ....
17. ...... £12, 10*s.*, ... 30 ....
18. ...... £1480, ... $7\frac{1}{2}$ ....
19. ...... £5, 12*s.* 6*d.*, ... 4 ....
20. ...... £1, 6*s.* 8*d.*, ... $6\frac{7}{8}$ ....

Find the cost price of an article—

21. Sold for 14*s.*, at a profit of 5 per cent of the cost.
22. .... £1330, ... 14 ....
23. .... 9*s.* 9*d.*, ... 17 ....
24. .... £1, 5*s.* $4\frac{1}{2}$*d.*,......... $8\frac{3}{4}$ ....
25. .... 88 guineas, ... 110 ....
26. .... 7*s.* 8*d.*, at a loss of 8 ....
27. .... 480 guineas......... 10 ....
28. .... $11\frac{3}{4}$*d.*, ... 6 ....
29. .... £4, 11*s.*, ... $13\frac{1}{3}$ ....
30. .... £16, 10*s.*, ... $72\frac{1}{2}$ ....

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31. For how much must goods, bought for £31, 10*s.*, be sold so as to gain $7\frac{1}{2}$ per cent of the cost?
32. What profit per cent is made if £7, 4*s.* 11*d.* be gained by an outlay of £123, 6*s.* 8*d.*?
33. If 123 yards of cloth are bought for £8, 4*s.*, and sold at 1*s.* 6*d.* a yard, how much profit per cent on the cost is made?
34. A man sold a horse for £36, losing 4 per cent of the cost price. How much did he pay for the horse?

35. What was the cost price of goods which were sold for £95, 3*s.* 3¼*d.* at a profit of 5¼ per cent of the cost?

36. If by selling an article at 19*s.* 6*d.* 17 per cent of the cost is gained, what did the article cost?

37. A tradesman gains 17 per cent of the cost by selling an article for £17, 11*s.*; what is the cost price?

38. If a horse be sold for £90 at a gain of 20 per cent of the cost, what was the cost price of the horse?

39. If goods are bought at £1, 3*s.* 4*d.* a cwt., at what price per lb. must they be retailed to bring in 10 per cent profit?

40. If sugar is bought at £20, 16*s.* 8*d.* a ton, at how much per lb. must it be sold to realize 12 per cent profit?

41. If 10 cwts. of tea are bought for £108, and sold again at 2*s.* 6*d.* per lb., what is the gain per cent on the capital employed?

42. If tea be bought at 3*s.* a lb., and sold at the rate of £19, 12*s.* 0*d.* per cwt., what is the gain per cent?

43. A grocer bought 150 lbs. of tea. Selling it at 3*s.* 7*d.* a lb. he gained 7½ per cent. Find what he gave for the whole.

44. Eggs bought at 15 for 1*s.* are sold at 10 for 9*d.* What is the gain per cent on the outlay?

45. If herrings are bought at 2*s.* 6*d.* per 100 and sold at two for 1½*d.*, what is the gain per cent?

46. If eggs be bought at 10*d.* per dozen and sold at 10 for a shilling, what is the gain per cent?

47. Find the gain per cent in buying oranges at seven for 6*d.* and selling them at six for 7*d.*

48. If eggs be bought at 9*d.* per dozen, how many must be sold for a shilling so as to gain 60 per cent?

49. If eggs are bought at the rate of 10 for a shilling, how many ought to be sold for 11s. 3*d.* so as to gain 12½ per cent?

50. By selling tea at 3*s.* a lb. a grocer gains 4 per cent; for what must he sell it that he may gain 17 per cent?

51. If by selling tea at 2*s.* 3*d.* per lb. I lose 10 per cent, at what price must I sell it to gain 15 per cent?

52. If by selling cloth at 9*s.* per yard there is a gain of 5 per cent, how much per cent will be gained by selling it at 11*s.* 3*d.* per yd.?

53. If sugar sold at 3*d.* a lb. gives a profit of 12 per cent, at what price per cwt. must it be sold to gain 15 per cent?

54. By selling goods for £120 a merchant gains 25 per cent; how much money would he have lost if he had sold them for £88?

55. By selling 8 tons of tea at 1*s.* 6½*d.* per lb. a merchant loses 7½ per cent. For what must he sell the whole so as to gain 10%?

56. If a grocer gains one-eighth of the cost price by selling tea at 3*s.* per lb., how much per cent would he gain if he sold it at 3*s.* 4*d.*?

57. By selling an article for £49, 17*s.* 6*d.* 5 per cent is lost; what should it be sold for in order to gain 14 per cent?

58. If I lose 5 per cent by selling a piece of land for £2337, what shall I gain or lose per cent by selling it for £2644, 10*s.*?

59. A sells his house for £1855, and gains thereby 6 per cent on the original cost. How much per cent would he have lost by selling it at £1715?

60. If I lose $6\frac{1}{4}$ per cent of the cost when I sell goods for £18, 15*s.*, at what price must I sell them to gain $6\frac{1}{4}$ per cent?

61. If 6 per cent be gained by selling 340 lbs. of tea for £59, 10*s.*, at what price per pound, to the nearest halfpenny, must it be sold in order to gain 10 per cent?

62. By selling 624 yds. of cloth at a loss of 4 per cent I lost altogether £15, 12*s.*; what were the buying and selling prices per yd.?

63. A man sells oranges at a penny a piece, and so doing makes a profit of 44 per cent. How many does he buy for £10?

64. If $5\frac{1}{2}$ per cent be lost by selling articles at 7*s.* a gross, find the gain or loss per cent by selling them at 5*s.* $1\frac{1}{3}$*d.* a hundred.

65. Seven per cent was lost by selling a carriage for £69, 15*s.*, and $4\frac{1}{2}$ per cent was gained by selling a horse for £94, 1*s.* Find the total gain or loss.

66. A man bought 9 sheep for £25, and 8 oxen for £95. He sold the sheep at a gain of 12 per cent, and the oxen at a gain of 10 per cent. What was his total profit?

67. A man buys two articles for £8 and £36 respectively, and sells them again, losing 40 per cent on the first and gaining 15 per cent on the second. How much does he gain or lose altogether?

68. How much is gained or lost per cent by buying a number of oranges at 5 for twopence, and selling half of them at two a penny and half at three a penny?

69. A person buys equal quantities of eggs at the rates of 2 a penny and 3 a penny. What does he gain or lose per cent by selling them all at the rate of sixteen for a shilling?

70. If a milk-seller makes a profit of £438 per annum by selling milk at 4*d.* per quart, his rate of profit being 60 per cent, how many gallons does he sell on the average daily?

71. Find the cost price of 300 engravings to a dealer, who sold them all at a profit of 2*s.* 6*d.* each, and of $16\frac{2}{3}$ per cent.

72. A man sells an article for £15, less 5 per cent discount for cash. If he has bought it for £12, 10*s.*, what is his gain per cent on his outlay?

73. A tradesman marks his goods at $22\frac{1}{2}$ per cent above cost price, but allows for ready money a discount of 5 per cent on the price marked. What profit does he make on a ready-money customer's payment of £279, 6*s.*?

74. A grocer mixes 28 lbs. of tea at $2s.\ 0\frac{1}{4}d.$ per lb., with 3 at $2s.\ 8d.$; at what price per lb. must he sell the mixture to make 20 per cent profit?

75. A tobacconist mixes together two kinds of tobacco, worth $5s.$ and $6s.$ per lb. respectively, in such a manner as to have 3 lbs. of the cheaper to 2 lbs. of the dearer. He sells the mixture at $6s.\ 3d.$ per lb.; what does he gain per cent?

76. If 300 lbs. of coffee worth $1s.\ 10d.$ per lb. are mixed with the same quantity worth only $1s.\ 6d.$ per lb., what should be the selling price per lb. to gain 10 per cent on the outlay?

77. A grocer buys coffee at £8, $10s.$ per cwt. and chicory at £2, $10s.$ per cwt., and mixes them in the proportion of 5 parts chicory to 7 of coffee. If he sells the mixture at £7 per cwt. what is his gain per cent?

78. A grocer buys 17 lbs. of tea at $2s.$, 20 at $1s.\ 9d.$ and 54 at $1s.\ 6d.$, and mixes them. He sells the mixture so as to make a profit of $21\frac{1}{3}$ per cent. What does he charge for it per lb.?

79. A grocer has two qualities of tea. By selling the first at $3s.\ 6d.$ per lb. he gains 25 per cent, and by selling the second at $3s.$ he gains 20 per cent. If he sell a mixture of one part of the first with two parts of the second at $3s.\ 2d.$ per lb., what is his gain per cent?

80. If 40 lbs. of tea at $2s.\ 3d.$ per lb. be mixed with 60 lbs. at $1s.\ 10d.$ per lb., how must the mixture be sold so as to make a profit of 25 per cent?

81. Spirits at $25s.\ 4d.$, $22s.\ 10d.$, and $21s.\ 8d.$ per gallon are mixed in the proportion of the numbers 1, 2, 3 respectively, and sold at $4s.\ 6d.$ per $1\frac{1}{4}$ pint bottle. Other expenses amounting to $2\frac{1}{2}d.$ per bottle, what is the gain per cent?

82. If the prime cost of 125 gallons of wine is 100 guineas, and 20 pints are lost in bottling, what percentage of profit will be made on the whole by selling the remainder at $48s.$ per dozen bottles, six bottles containing a gallon?

83. A wine merchant buys brandy at $16s.$ per gallon; dilutes it with water and then sells the mixture at $15s.$ per gallon. His profits by the sale are 20 per cent. How much water is there in each gallon of the mixture?

84. A merchant mixes 12 gallons of spirits at $10s.\ 6d.$ a gallon with 15 gallons at $12s.$ a gallon, and 7 gallons of water. What is the cost price per gallon of the mixture, and what will be gained per cent by selling the whole at $10s.\ 8d.$ per gallon?

85. If 48 gallons of spirit at $12s.$ per gallon, $2\frac{2}{3}$ gallons at $10s.\ 6d.$ and $19\frac{1}{2}$ gallons at $1s.\ 4d.$, be mixed with $19\frac{5}{6}$ gallons of water, and the mixture sold at $7s.\ 10\frac{1}{2}d.$ per gallon, what is the gain per cent?

86. A wine merchant buys 120 gallons of spirit for £96. How much

water does he mix with it if, when it is sold at 18*s.* a gallon, he makes a profit of 20 per cent?

87. A man buys 416 gallons of spirit at 10*s.* a gallon. How much water must he add to it that he may gain 10 per cent on his outlay and retail the mixture at 8*s.* a gallon?

88. A milkman buys daily 36 gallons of milk at 8*d.* per gallon. How much water does he mix with it if, by selling the mixture at 9*d.* per gallon, he clears 25 per cent profit?

89. If a person had sold his horse for £75 instead of for £72, he would have gained 5 per cent more than he did. Find the cost price of the horse.

90. If by selling cloth at 12*s.* per yd. 5 per cent more is gained than by selling it at 11*s.* 6*d.* per yd., what was the cost price per yd.?

91. If 7 per cent more be gained by selling a house for £146, 13*s.* 4*d.* than by selling it for £137, 6*s.* 8*d.*, find the original cost.

92. A tradesman marks an article 15 per cent above cost price, but he allows his customer 5 per cent discount for ready money. What rate per cent of profit does he really gain?

93. A manufacturer formerly sold an article at 11*s.*, gaining thereby 10 per cent. The cost of manufacturing has lately advanced 25 per cent. Find at what price it must now be sold so as still to gain 10 per cent.

94. A wine merchant mixes his brandy with one-sixth of its own volume of water, and then sells it at such a price that if it were pure he would gain 5 per cent. What per cent is his profit?

95. A sold a horse to B, gaining $7\frac{1}{2}$ per cent on what it cost him; B sold it to C for £70, 19*s.*, gaining 10 per cent on what it cost him. What did A pay for the horse?

96. If the manufacturer makes a profit of 20 per cent, the wholesale dealer a profit of 25 per cent, and the shopkeeper a profit of 40 per cent, what was the cost of the manufacture of an article bought at a shop for 17*s.* 6*d.*?

97. A makes an article and sells it to B, B sells it to C, then C sells it to D for 3 guineas; A makes 20 per cent, B $12\frac{1}{2}$ per cent, C 40 per cent profit. What did it cost A to make it?

98. An article of commerce passes successively through the hands of three dealers, each of whom adds to the price, as his profit, 10 per cent of the price he paid for it. What did the first dealer pay for goods which the third sold for £11, 1*s.* 10*d.*?

99. If sugar is sold at £1, 7*s.* $8\frac{3}{4}$*d.* per cwt., after passing through the hands of three dealers, each of whom makes 10 per cent profit on it, what did it cost the first dealer per stone?

100. A person buys 3 houses, for which he pays sums proportional to 4, 3 and 2. He sells them all at once, clearing 10 per cent by the first, 6 per cent by the second, but losing 4 per cent on the third. His entire gain is £101. Find what each cost.

LIII. PERCENTAGES.

COMMISSION, BROKERAGE, &C.

Find the commission on—

1. £1472, 15*s.* at 5%.
2. £853, 6*s.* 8*d.* at 3%.
3. £128, 3*s.* 4*d.* at $2\frac{1}{2}$%.
4. £4857, 1*s.* 8*d.* at 1%.
5. £2091, 19*s.* 7*d.* at 4%.

Find the brokerage on—

6. £550 at $\frac{1}{4}$%.
7. £3450 at $\frac{1}{8}$%.
8. £1936, 13*s.* 4*d.* at $\frac{1}{8}$%.
9. £759 at $1\frac{1}{4}$%.
10. £6786, 13*s.* 4*d.* at $\frac{3}{16}$%.

Find, to the nearest penny, the commission on—

11. £492, 12*s.* 9*d.* at 5%.
12. £2351, 18*s.* 6*d.* at 5%.
13. £885, 9*s.* 4*d.* at $2\frac{1}{2}$%.
14. £70,312, 8*s.* 3*d.* at $2\frac{1}{2}$%.
15. £421, 5*s.* 10*d.* at 3%.
16. £5791, 10*s.* 8*d.* at 1%.
17. £1080, 15*s.* at $\frac{1}{8}$%.
18. £349, 17*s.* 9*d.* at $\frac{3}{4}$%.
19. £981, 11*s.* 6*d.* at $\frac{5}{16}$%.
20. £3489, 6*s.* 9*d.* at $\frac{7}{8}$%.

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21. The rental of an estate is £2356, 10*s.* Find the agent's commission at 5 per cent.
22. Find the premium at $4\frac{1}{2}$ per cent on an insurance policy for £7250.
23. A man insures his life for £2500 at £2, 17*s.* per cent; find his annual premium.
24. An auctioneer's charge, at £2, 12*s.* 6*d.* per cent, for a sale was £19, 5*s.* What did the goods sell for?
25. Find the brokerage at 12*s.* 6*d.* per cent on £666, 13*s.* 4*d.*
26. A broker buys goods to the value of £970, and his charge is $\frac{5}{8}$ per cent. How much has his employer to pay altogether?
27. What is the value of a cargo if the insurance premium at 8 per cent amounts to £610?
28. After deducting 5 per cent as his agent's commission, a landlord received £8647, 17*s.* Find the agent's commission.
29. An agent collecting rents at a commission of $4\frac{3}{8}$ per cent deducted his commission and paid over £741, 1*s.* $10\frac{1}{2}$*d.* What amount of rents did he collect?
30. A cargo worth £855 was insured for that sum, the premium being $6\frac{3}{4}$ per cent. The cargo was lost, and the broker who obtained a settlement charged $\frac{1}{2}$ per cent. What was the owner's actual loss?

LIV. SIMPLE INTEREST.

Find the Interest on—

1. £721, 13*s.* 4*d.* for 1 year at 3 per cent per annum.
2. £313, 2*s.* 6*d.* ... 1 .... 4 ....
3. £191, 5*s.* ... 2 years at 5 ....
4. £1562, 10*s.* ... 2 .... 3 ....
5. £142, 17*s.* 6*d.* ... 4 .... 5 ....
6. £141, 13*s.* 4*d.* ... $\frac{1}{2}$ a year at 4 ....
7. £2408, 6*s.* 8*d.* ... $\frac{1}{2}$ .... 6 ....
8. £351, 7*s.* 6*d.* ... 4 years at $2\frac{1}{2}$ ....
9. £206, 16*s.* 6*d.* ... 5 .... $3\frac{1}{3}$ ....
10. £3304, 11*s.* 8*d.* ... $4\frac{1}{4}$ .... 4 ....
11. £512, ... $6\frac{2}{3}$ .... $5\frac{1}{4}$ ....
12. £960, ... $3\frac{1}{4}$ .... $2\frac{1}{2}$ ....
13. £6750, ... $3\frac{1}{2}$ .... $2\frac{3}{4}$ ....
14. £232, 13*s.* 4*d.* ... 5 .... $2\frac{1}{4}$ ....
15. £289, 17*s.* 6*d.* ... 4 .... $2\frac{1}{2}$ ....

Find the Amount of—

16. £117, 10*s.* in 1 year at 5 per cent per annum.
17. £362, 10*s.* ... $\frac{1}{2}$ a year... 4 ....
18. £88, 6*s.* 8*d.* ... 3 years... 3 ....
19. £991, 10*s.* ... $1\frac{1}{2}$ .... 5 ....
20. £3643, 6*s.* 8*d.* ... $3\frac{1}{2}$ .... $4\frac{3}{4}$ ....

Find the Interest on—

21. £175, for 7 months at 4 per cent per annum.
22. £1347, ... 15 .... 3 ....
23. £821, 5*s.* ... 8 .... $3\frac{1}{2}$ ....
24. £25, 6*s.* 8*d.* ... $7\frac{1}{2}$ .... 5 ....
25. £1226, 8*s.* ... 5 .... $2\frac{1}{2}$ ....
26. £37, 10*s.* ... 219 days .... 3 ....
27. £2160, 12*s.* 6*d.*... 73 .... 10 ....
28. £353, 2*s.* 6*d.* ... 146 .... 2 ....
29. £93, 15*s.* ... 2 years 292 days at $2\frac{1}{2}$ per cent.....
30. £7950, 18*s.* 4*d.*... 3 ...... 105 .... 3 ....
31. £737, 18*s.* 4*d.* from Mar. 3 to May 15, at 5 per cent....
32. £210, 16*s.* 8*d.* ...... Ap. 7 ... Nov. 12, ... $3\frac{1}{3}$ ....
33. £105, 8*s.* 4*d.* ...... June 1 ... Oct. 25, ... $2\frac{1}{2}$ ....

Find the Amount of—

34. £133, 6*s.* 8*d.* from Sept. 27, 1892, to Feb. 20, 1894, at 3%.

35. £215, 12*s.* 6*d.* ...... July 13, 1892, ... May 1, 1895, ... 4%.

Find, *neglecting any fraction of a penny*,
(i) the Interest on, (ii) the Amount of—

36.	£274, 11*s.* 9*d.*	for 1	year	at 3	per cent per annum.
37.	£89, 18*s.* 2*d.*	... $\frac{1}{2}$	a year	... 4	....
38.	£1703, 5*s.* 10*d.*	... $\frac{1}{4}$	....	$2\frac{1}{2}$	....
39.	£26, 13*s.* 7*d.*	... 2	years	... 3	....
40.	£716, 12*s.* 5*d.*	... 3	....	$4\frac{1}{2}$	....
41.	£69, 9*s.* 10*d.*	... 4	months	... 4	....
42.	£1025, 8*s.*	... 11	....	$2\frac{1}{2}$	....
43.	£3383, 19*s.*	... 292	days	... $3\frac{1}{3}$	....
44.	£10, 10*s.* 10*d.*	... 146	....	$4\frac{1}{2}$	....
45.	£107, 13*s.* 5*d.*	... 219	....	$3\frac{1}{4}$	....

Find, *mentally*, neglecting any fraction of a penny, the Interest

at 5 per cent on

46. £325, for a year.

47. £256, ... half a year.

48. £143, ... half a year.

49. £291, ... 3 months.

50. £238, 6*s.* ... a month.

51. £85, 11*s.* ... a month.

52. £63, 14*s.* ... 2 months.

at $2\frac{1}{2}$ per cent on

53. £267, for a year.

54. £1261, ... a year.

55. £362, ... half a year.

56. £142, ... a month.

57. £13, ... a month.

58. £491, 10*s.* ... a month.

59. £211, 5*s.* ... a month.

60. £188, 12*s.* ... 2 months.

*Find, *to the nearest penny*, the Interest on—

61.	£186, 11*s.*	for 143	days at	4	per cent per annum.
62.	£423, 7*s.*	... 229	....	3	....
63.	£4917, 14*s.* 6*d.*	... 317	....	5	....
64.	£2038, 5*s.* 10*d.*	... 87	....	3	....
65.	£53, 13*s.*	... 91	....	$2\frac{1}{2}$	....

66. £2553, 8*s.* from Jan. 17 to May 23 at 2 per cent......

67. £906, 3*s.* 8*d.* ...... Mar. 11 ... June 24 ... 5 ....

68. £26, 7*s.* 5*d.* ...... June 24 ... July 29 ... 3 ....

69. Eleven guineas ...... July 1 ... Nov. 30 ... $2\frac{1}{2}$ ....

70. £197, 17*s.* 4*d.* ...... Aug. 14 ... Dec. 31 ... $3\frac{1}{2}$ ....

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* Exs. 61 to 70 may conveniently be worked by the contracted methods of Chap. L.

At what Rate per cent per annum would the simple interest on

71.	£350,	be £21,	for 2	years?
72.	£325,	... £42, 5*s.*	... 4	....
73.	£1020,	... £290, 14*s.*	... 6	....
74.	£560,	... £41, 13*s.*	... $3\frac{1}{2}$	....
75.	£4373, 6*s.* 8*d.*	... £246,	... $1\frac{1}{4}$	....
76.	£21, 17*s.* 6*d.*	... £5, 2*s.* 1*d.*	... 7	....
77.	£737, 10*s.*	... £19, 13*s.* 4*d.*	... 8	months?
78.	£2500,	... £365, 12*s.* 6*d.*	... $3\frac{1}{4}$	years?
79.	£1350,	... £271, 13*s.* 9*d.*	... $5\frac{3}{4}$	......
80.	£466, 13*s.* 4*d.*	... £5, 16*s.* 8*d.*	... 5	months?

At what Rate per cent at simple interest would

81.	£560,	*amount* to	£679,	in 5	years?
82.	£2560,	....	£3328,	... 5	....
83.	£308, 6*s.* 8*d.*	....	£481,	... 14	....
84.	£450,	....	£470, 5*s.*	... $1\frac{1}{2}$	....
85.	£537, 16*s.* 8*d.*	....	£591, 12*s.* 4*d.*	... 4	....
86.	£657, 10*s.*	....	£664, 1*s.* 6*d.*	... 146	days?
87.	£285, 12*s.* 6*d.*	....	£291, 6*s.* 9*d.*	... 6	months?
88.	£422, 10*s.*	....	£435, 3*s.* 6*d.*	... 9	months?
89.	£183, 6*s.* 8*d.*	....	£230, 1*s.* 8*d.*	... $1\frac{5}{12}$	years?
90.	£6825, 10*s.*	....	£7061, 11*s.* 9*d.*	... 101	days?

In what Time would the simple interest on

91.	£650,	be £130,	at 5	per cent per annum?
92.	£2000,	... £315,	... $4\frac{1}{2}$	....
93.	£840,	... £1050,	... 5	....
94.	£80,	... £1,	... $2\frac{1}{2}$	....
95.	£550, 10*s.*	... £220, 4*s.*	... $2\frac{1}{2}$	....
96.	£112, 10*s.*	... £400, 10*s.*	... 4	....
97.	£783, 6*s.* 8*d.*	... £14, 13*s.* 9*d.*	... $4\frac{1}{2}$	....
98.	£953, 11*s.* 3*d.*	... £2, 12*s.* 3*d.*	... 4	....
99.	£484, 3*s.* 4*d.*	... £12, 2*s.* 1*d.*	... 3	....
100.	£5062, 10*s.*	... £75, 18*s.* 9*d.*	... $2\frac{1}{2}$	....

In what Time, at simple interest, would

101.	£175,	*amount* to	£196,	at 4	per cent per annum?
102.	£3200,	....	£3424,	... $3\frac{1}{2}$	....
103.	£950,	....	£1199, 7*s.* 6*d.*	... $5\frac{1}{4}$	....
104.	£2652,	....	£3779, 2*s.*	... 5	....

In what Time, at simple interest, would

105.	£1260,	*amount* to	£1307, 5*s.*	at $3\frac{3}{4}$	per cent per ann.?
106.	£183, 6*s.* 8*d.*	....	£230, 1*s.* 8*d.*	... $4\frac{1}{2}$	....
107.	£10055,	....	£11186, 3*s.* 9*d.*	... $2\frac{1}{2}$	....
108.	£221, 17*s.* 6*d.*	....	£226, 6*s.* 3*d.*	... 3	....
109.	£740,	....	£750, 15*s.* 10*d.*	... $3\frac{1}{2}$	....
110.	£133, 16*s.* 8*d.*	....	£134, 4*s.* 11*d.*	... $2\frac{1}{4}$	....

On what Principal would the simple interest be

111.	£17,	for	1 year	at 4	per cent per annum?
112.	£237, 10*s.*	...	2 years	... 5	....
113.	£111, 12*s.*	...	1 year	... 3	....
114.	£162, 15*s.*	...	2 years	... $3\frac{1}{2}$	....
115.	£514, 10*s.*	...	14	... 7	....
116.	£39, 7*s.* 6*d.*	...	$\frac{1}{2}$ a year	... 5	....
117.	£40, 17*s.* 3*d.*	...	$3\frac{1}{2}$ years	... 4	....
118.	£100,	...	73 days	... $2\frac{1}{2}$	....
119.	£125,	...	5 months	... $4\frac{1}{2}$	....
120.	4*s.* 6*d.*	...	10 days	... 5	....

What Principal would *amount*, at simple interest, to

121.	£795,	in	2 years	at 3	per cent per annum?
122.	£234,	...	1 year	... 4	....
123.	£539, 15*s.*	...	6 years	... $4\frac{1}{2}$	....
124.	£647, 10*s.*	...	4 yrs. 2 mo.	... 4	....
125.	£423, 2*s.* 6*d.*	...	3 yrs. 8 mo.	... $3\frac{1}{2}$	....
126.	£338, 1*s.* 9*d.*	...	$1\frac{1}{2}$ years	... 5	....
127.	£728, 12*s.* 9*d.*	...	7 months	... $4\frac{1}{2}$	....
128.	£366, 1*s.*	...	42 days	... $2\frac{1}{2}$	....
129.	£178, 1*s.* 6*d.*	...	3 yrs. 73 dys.	... $6\frac{1}{4}$	....
130.	£1058, 13*s.* 3*d.*	...	6 months	... $4\frac{3}{8}$	....

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131. Find the simple interest on £1375 for $4\frac{1}{2}$ years at £2, 16*s.* per cent.

132. Find the simple interest on £953, 11*s.* 3*d.* for 20 days at 4 per cent per annum.

133. Find, mentally, the interest on £315, 15*s.* for a month at 5%.

134. Find the simple interest on £2970, 16*s.* 8*d.* for 3 years 73 days at £3, 2*s.* 6*d.* per cent.

135. Calculate the simple interest on £725 from 1st January to 15th March, 1897, at $3\frac{3}{4}$ per cent.

136. Find to the nearest penny the interest on £536 for 124 days at 5 per cent per annum.

137. Find to the nearest penny the simple interest on £147, 11*s.* 3*d.* from April 11 to September 19, at $2\frac{1}{2}$%.

138. Find to a penny the interest on £425 from Jan. 1st to May 4th in Leap year, at $2\frac{1}{2}$ per cent.

139. A dealer buys a machine for £525 cash and sells it, after nine months, for £423, 10*s.* How much did he lose, considering money worth 6 per cent per annum?

140. If the simple interest on a certain sum of money amount to £119, 2*s.* 9*d.* in 2 years 3 months, what is the interest on six times the sum for one-fourth of that time at the same rate?

141. In what time will £200 amount to £232, 0*s.* 8*d.* at $6\frac{1}{6}$ per cent simple interest?

142. In what time will the simple interest on £2541, 17*s.* 6*d.* amount to £228, 15*s.* $4\frac{1}{2}$*d.* at 3 per cent per annum?

143. What sum of money will amount to £769, 5*s.* at the end of $8\frac{1}{3}$ years at 4 per cent simple interest?

144. At what rate per cent per annum will the simple interest on £1560 amount to £245, 14*s.* in $3\frac{1}{2}$ years?

145. On what sum will the simple interest amount to £82, 10*s.* in 4 years at $5\frac{1}{2}$ per cent per annum?

146. At what rate simple interest will £855 amount to £956, 10*s.* $7\frac{1}{2}$*d.* in 5 years?

147. In how many years at 4 per cent per annum simple interest will the interest be one-quarter of the principal?

148. In what time will money double itself at $3\frac{3}{4}$ per cent simple interest?

149. On what date will £650, lent on June 1st, 1896, amount to £708, 10*s.* at 5 per cent per annum simple interest?

150. The interest for one year on £2550, after deduction of income-tax at 6*d.* per £, is £82, 17*s.* 6*d.* Find the rate per cent.

151. What sum lent on a mortgage at $4\frac{1}{2}$ per cent would yield an annual income of £191, 5*s.*?

152. A man mortgages his estate for £3500, the interest at 5 per cent per annum being payable half-yearly. Find the amount of one half-yearly instalment of interest.

153. A money-lender charged £3 for the loan of £40 for a month; at what rate per cent per annum did he charge interest?

154. The rate of interest in a savings-bank is £2, 13*s.* 4*d.* per cent per annum. A man deposits a sum of money; his interest at the end of 11 months was 14*s.* 8*d.* What sum did he deposit?

155. What would be the interest on a loan of £10 for 3 weeks at the rate of $\frac{1}{8}$ per cent per *diem*?

156. On what sum is £1, 10*s*. 5 months' interest at the rate of 1*s*. 6*d*. per £1 per annum?

157. Compare the interest on £247, 10*s*. at 5 per cent for two years with that on £375 at $3\frac{3}{8}$ per cent for the same time.

158. The interest on £2000 for 3 years at $3\frac{1}{2}$ per cent is to be divided between A, B, and C; A's share being four times B's, and C's being equal to A's and B's together. Find the shares.

159. A question being proposed in an examination, to find the simple interest on a certain sum of money for $2\frac{1}{2}$ years at $3\frac{1}{4}$ per cent, a candidate by mistake reckoned it for $2\frac{1}{4}$ years at $3\frac{1}{2}$ per cent, and so obtained a result too little by £26, 4*s*. 8*d*. What ought the answer to have been?

160. A man lent £1800 on house property at a certain rate per cent, and £1200 on land at $\frac{1}{2}$ per cent less. His total income from both sources was £114. Find the rates of interest.

161. If £860 amounts to £963, 4*s*. in 3 years, in what time will £1060 amount to £1166 at the same rate, simple interest?

162. In what time will £1455 amount to £2328, if, in 3 years, £430 amount to £494, 10*s*. at simple interest at the same rate?

163. Divide £3300 into two parts such that the simple interest on the one at 3 per cent for $4\frac{1}{2}$ years would be equal to the simple interest on the other at $2\frac{1}{4}$ per cent for 5 years.

164. Divide £3783 into two parts such that the interest on the first for 3 months at $4\frac{1}{2}$ per cent shall be equal to that on the second for 7 months at $2\frac{1}{4}$ per cent.

165. A has £2000; he lends £800 to B, and receives as interest £18 half-yearly; he lends £650 to C, and receives as interest £4, 17*s*. 6*d*. quarterly; the rest of his money he lends to D at 5 per cent per annum. Find the average rate of interest A makes on his capital.

Find to a penny, by the "*Third, tenth, and tenth*" rule, the

166. Interest on £3259, 14*s*. for 283 days at $2\frac{1}{2}$%.

167. Amount of £8563, 17*s*. 9*d*. from April 13 to Dec. 31 at $2\frac{3}{4}$%.

168. Interest on £837, 15*s*. 7*d*., from July 7 to Aug. 23, at $2\frac{1}{4}$%.

169. A merchant supplies a tradesman with goods to the cash value of £650 on May 10. The tradesman pays, on account, £160 on June 24; £175 on Sep. 8; and £130 on Nov. 16. Find the balance owing on Dec. 31, allowing interest at 6 per cent.

170. A man opens an account at a bank, on July 10, by depositing £320. On July 30 he pays in £85; on Aug. 23 he draws £120; on Sep. 14 he draws £56, 10*s*.; and on Nov. 2 he pays in £27, 15*s*. Find the interest, at 2%, due to him on Dec. 31.

LV. COMPOUND INTEREST.

Find the Amount, at compound interest, of—

1. £2500 in 2 years at 5 per cent per annum.
2. £4250 ... 2 .... 4 ....
3. £600 ... 3 .... 5 ....
4. £320 ... 2 .... $2\frac{1}{2}$ ....
5. £750 ... 4 .... 10 ....

Find the Compound Interest on—

6. £625 for 2 years at 3 per cent per annum.
7. £3125 ... 3 .... 3 ....
8. £8000 ... 3 .... $2\frac{1}{2}$ ....
9. £1500 ... 2 .... $3\frac{1}{2}$ ....
10. £3150 ... $2\frac{1}{2}$ .... 5 ....

Find, neglecting any fraction of a penny,

the Amount of—

11. £540 .... in 2 yrs, at 3 %.
12. £1500 .... 3 ... 4 ...
13. £2450 .... 3 ... 3 ...
14. £173, 17*s.* 6*d.* ... 2 ... 4 ...
15. £99, 19*s.* 9*d.* ... 3 ... 2 ...
16. £2610 .... 3 ... 5 ...
17. £975, 15*s.* .... 3 ... 5 ...
18. £453, 10*s.* .... 2 ... $2\frac{1}{2}$ ...
19. £1607, 11*s.* 3*d.* ... 3 ... $3\frac{1}{3}$ ...
20. £2481, 6*s.* 8*d.* ... 3 ... $2\frac{1}{2}$ ...

the Compound Interest on—

21. £364, 5*s.* ...... for 2 yrs. at $2\frac{1}{4}$ %.
22. £7275 .... 2 ... $4\frac{1}{4}$ ...
23. £528, 15*s.* .... 2 ... $2\frac{3}{4}$ ...
24. £608, 15*s.* 4*d.* ... 3 ... $2\frac{3}{4}$ ...
25. £172 .... 3 ... $4\frac{3}{4}$ ...
26. £1400 .... $2\frac{1}{2}$ ... 4 ...
27. £623, 17*s.* .... $3\frac{1}{2}$ ... $2\frac{1}{2}$ ...
28. £482, 9*s.* 8*d.* .... $3\frac{1}{4}$ ... $3\frac{1}{3}$ ...
29. £340 .... $3\frac{1}{2}$ ... $3\frac{1}{2}$ ...
30. £3185, 3*s.* 4*d.* .... $2\frac{3}{4}$ ... $3\frac{3}{4}$ ...

Find, to the nearest penny, the Amount of—

31. £825, 10*s.* ...... in 2 years at 3 per cent per annum.
32. £428, 12*s.* 8*d.* ... 3 .... 3 ....
33. £666, 13*s.* 4*d.* ... 4 .... 5 ....
34. £4523, 17*s.* 6*d.* ... 4 .... $2\frac{1}{2}$ ....
35. £157, 16*s.* 7*d.* ... 2 .... $6\frac{2}{3}$ ....
36. £1000 .... 5 .... $3\frac{1}{2}$ ....
37. £4765 .... 3 .... $3\frac{3}{4}$ ....
38. £760 .... 3 .... $4\frac{7}{8}$ ....
39. £1234 .... $3\frac{1}{2}$ .... $2\frac{1}{2}$ ....
40. £243, 10*s.* .... $2\frac{1}{4}$ .... $2\frac{3}{4}$ ....

Find, to the nearest penny, the Amount, at compound interest, payable *half-yearly*, of—

41. £428 .... in 2 years at 4 per cent per annum.
42. £753, 10*s*. .... $1\frac{1}{2}$ .... 5 ....
43. £2670 .... 2 .... $2\frac{1}{2}$ ....
44. £115, 12*s*. 8*d*. ... $1\frac{1}{2}$ .... $3\frac{1}{2}$ ....
45. £3500 .... 3 .... $3\frac{1}{3}$ ....
46. £973, 18*s*. .... $1\frac{1}{2}$ .... $3\frac{3}{4}$ ....

Find, to the nearest penny, the Comp. Int., payable *quarterly*, on—

47. £680 for 1 year at 4 per cent per annum.
48. £2130 ... 1 .... 5 ....
49. £517, 15*s*. for $\frac{3}{4}$ of a year at 3 per cent per annum.
50. £850 .... $1\frac{1}{4}$ years ...... $2\frac{1}{2}$ ....

Find, to the nearest pound, the Amount, at compound interest, of—

51. £1250 in 8 years at 4%.
52. £800 ... 10 .... 5 ..
53. £851, 10*s*... 12 .... 4 ..
54. £1013, 10*s*. in 7 years at 5%.
55. £500 .... 16 .... 3 ..
56. £7000 .... 20 .... 5 ..
57. £675 for $6\frac{1}{2}$ years at 4% per annum, payable *half-yearly*.
58. £2350 for $7\frac{1}{2}$ years at 5% per ann. payable *half-yearly*.

Find the difference between the simple and compound interest on—

59. £425 .... for 3 years at 4 per cent per annum.
60. £1333, 6*s*. 8*d*. ... 4 .... 5 ....
61. £7650 .... 2 .... $3\frac{1}{2}$ ....
62. £615, 10*s*. .... 3 .... $2\frac{1}{2}$ ....
63. £2517, 11*s*. 8*d*. ... $2\frac{1}{2}$ .... $3\frac{3}{4}$ ....

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*In what Time, at compound interest, would—

64. £750 *amount* to £843·648 ...... at 4 per cent per annum?
65. £3000 .... £3472, 17*s*. 6*d*. ... 5 ....?
66. £1250 gain £197, 0*s*. $7\frac{1}{2}$*d*. *interest* at 5 per cent?
67. £480 ...... £43, 7*s*. 5*d*. (nearly)........... $2\frac{1}{2}$ ....?
68. £1800 ...... £170, 1*s*. .... $3\frac{1}{3}$ ....?
69. £25000 ...... £1787, 14*s*. 6*d*. .... 3 ....?
70. £520, 16*s*. 8*d*. amount to £552, 14*s*. 3*d*. at 4% payable hf.-yearly?

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* **For Exs. on finding the Rate in the case of Comp. Int. see under Cube Root.**

What Principal would, at compound interest—

71. *Amount* to £1352	in 2	years at 4	per cent?
72. .... £4862, 0*s.* 6*d.*	... 4	 5	.... ?
73. .... £6892, 2*s.*	... 3	.... $2\frac{1}{2}$	.... ?
74. Gain £164, 14*s.* *interest*	... 2	.... $3\frac{1}{3}$	.... ?
75. ...... £5751, 2*s.* 6*d.*............	$2\frac{1}{3}$	.... $3\frac{3}{4}$	.... ?

76. Find, neglecting any fraction of a shilling, the compound interest on £720 for $2\frac{1}{2}$ years at $3\frac{1}{2}$ per cent.

77. Find, to the nearest penny, the difference between the first and third year's interest on £4735 at $3\frac{1}{4}$ per cent.

78. Find the exact difference between the simple and compound interest on £2450 for 3 years at 3 per cent.

79. Find, to the nearest shilling, the difference between the simple and compound interest on £4560 for 2 years at 4 per cent per annum, the interest being payable half-yearly.

80. Find, to the nearest pound, the compound interest on £300 for 10 years at $2\frac{1}{2}$ per cent.

81. If the present population of a town is 35,200, and the births average annually 23 per 1000 and the deaths 18 per 1000 of the population at the beginning of the year, what will be the population of the town at the end of 4 years?

82. The difference between the simple and compound interest on a certain sum for 2 years at 4% is £1, 8*s.* Find the sum.

83. The compound interest on a certain sum for 2 years at $2\frac{1}{2}$ per cent exceeds the simple interest for the same time at the same rate by £4, 7*s.* 6*d.* Find the sum.

84. The difference between the second and third year's interest on a certain sum at 5 per cent is £5, 5*s.* Find the sum.

85. If at the beginning of each year a man invests £100 at compound int. at 4%, what will he be worth at the end of the fifth?

86. If a man borrowed £300 at 4 per cent, and repaid the principal and interest by equal annual instalments of £50, in how many years would he be out of debt?

87. £820 is borrowed for 2 years at 5 per cent, to be repaid, principal and interest, in two equal annual instalments: find them.

88. If £1700 be borrowed for 2 years, to be repaid, with interest at 4 per cent, in two equal annual instalments, what must be the amount of each instalment?

89. If £100 amounts to £127 in 5 years at compound interest, what would £2000 amount to in 10 years at the same rate?

90. If, in a certain time, at a certain rate per cent, £500 amounts, at compound interest, to £864, what would £3125 amount to in double that time at the same rate?

LVI. DISCOUNT AND PRESENT WORTH.

Practical, or Banker's, Discount.

Find, neglecting any fraction of a penny, the Banker's discount, at 5 per cent, on—

1. £325	due 3 mo. hence.		**5.** £400	due 71	days hence.
2. £214, 12*s.*	... 3	....	**6.** £215	... 123	....
3. £257, 10*s.*	... 6	....	**7.** £342, 7*s.* 6*d.*	... 64	....
4. £507, 15*s.*	... 6	....	**8.** £152, 18*s.* 9*d.*	... 101	....

Find, neglecting any fraction of a penny, the cash value of a "bill" for—

9. £280 if discounted, at 5 per cent, 3 months before it was due.

10. £417, 12*s.* 6*d.* .... 5 .... 6 ....

11. £325 .... 4 .... 57 days...........................

12. £253, 16*s.* 6*d.* .... 3 .... 163 ....

Find the Banker's charge for cashing a "bill" for—

13. £410	drawn June 19,	at 3 mo.,	discounted July 1,	at 5%.
14. £224	.... June 20	... 3	.... July 24	... 4%.
15. £2000	.... May 5	... 3	.... June 10	... $3\frac{1}{4}$%.
16. £550, 6*s.* 8*d.*	.... Jan. 3	... 5	.... Jan. 11	... 4%.
17. £847, 15*s.* 9*d.*	.... Feb. 5	... 6	.... Ap. 30	... 3%.

Find the Commercial present worth of a "bill" for—

18. £300	drawn Mar. 12,	at 3 mo.,	discounted Mar. 24,	at 5%.
19. £275	.... Sep. 24	... 3	.... Oct. 4	... 5%
20. £1400	.... Ap. 5	... 6	.... Ap. 26	... 4%.
21. £670, 10*s.*	.... June 17	... 3	.... July 2	... 4%.
22. £375, 12*s.* 6*d.*	.... May 8	... 4	.... July 12	... $3\frac{1}{2}$%.
23. £1123	.... Nov. 7	... 3	.... Nov. 28	... 5%.
24. £573, 14*s.*	.... Oct. 30	... 6	.... Nov. 14	... 5%.
25. £2476, 15*s.*	.... Feb. 20	... 9	.... Mar. 15	... $4\frac{1}{2}$%.

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26. How much will a broker, who charges 5 per cent discount, give for a bill for £600 due 7 months hence?

27. If I take to a bank a bill for £350 which is due in 63 days, what shall I receive for it, discount being charged at the rate of 5 per cent?

28. A bill for £1000 at 90 days sight has been presented for acceptance on February 10th, 1888, and on March 10th is discounted at 5 per cent. How much will the discounter pay for the bill?

29. "*London, June* 26, 1894.

"Three months after date I promise to pay to Mr. John Brown, or order, Three hundred pounds for value received.

Thomas Jones."

The above promissory note was cashed on July 7, at 5 per cent; how much was received for it?

30. The banker's discount on a bill due in 2 months at $5\frac{1}{2}$ per cent is £19, 5*s*. What is the amount of the bill?

31. The commercial present worth of a sum due 219 days hence, at 5 per cent, is £499, 11*s*. What is the sum?

32. A 4 months' bill drawn on 1st Feb. and discounted on 23rd Mar., at 5 per cent, realized £742, 10*s*. Find the amount of the bill.

33. If £217, 16*s*. was the cash value on April 7 of a bill for £220, drawn Mar. 16, at 3 months, at what rate per cent was it discounted?

34. If the banker's charge on Sep. 25 for discounting, at 5 per cent, a bill drawn Sep. 11, at 3 months, was £6, 3*s*., find the "face value" of the bill.

35. If the banker's charge for cashing, at 5 per cent, a bill for £520 drawn May 8, at 3 months, was £5, 4*s*., on what date was it cashed?

Theoretical, or "True", Discount.

Find the Theoretical (i) discount, (ii) present worth, of—

36. £780 due in 1 yr. at 4 %.
37. £4200 .... 1 ...... 5 ...
38. £664 .... 15 mo.. 3 ...
39. £407 .... 6 ......$3\frac{1}{2}$ ...
40. £1030 .... 8 ......$4\frac{1}{2}$ ...
41. £284, 18*s*. due in 6 mo. at $3\frac{1}{2}$ %.
42. £843, 15*s*. .... 3 .... 5 ...
43. £474, 14*s*. .... 73 days... 5 ...
44. £150, 10*s*. .... 70 .... 5 ...
45. £503, 10*s*. .... 84 ....$4\frac{1}{2}$ ...

46. Find the difference between the interest and the "true" discount on £694, 3*s*. 4*d*. for 11 months at $4\frac{1}{2}$ per cent.

47. Find the difference between the Commercial and the Theoretical discount, at 5 per cent, on £1680, due 3 months hence.

48. The "true" discount on £405 due 6 months hence is £5. Find the rate per cent.

49. The theoretically true present value of £627 due 146 days hence is £618, 15*s*. Find the rate per cent.

50. For how many months is £828 the "true" present worth of £848, 14*s*. at 6 per cent?

51. For what time does the mathematical discount on £1028, 6*s*. at 5 per cent amount to £6, 6*s*.?

52. The theoretical present value of a certain sum due 73 days hence at $4\frac{1}{2}$ per cent is £262, 10*s*. Find the sum.

53. On what sum due 1 yr. hence is £12 the "true" discount at 4% ?

54. The "true" discount at 4 per cent on a certain sum due 5 months hence is £13, 10*s.* 10*d.* Find the sum.

55. The difference between the interest and the theoretical discount on a certain sum for 1 year, at 4%, is £2. Find the sum.

56. If the difference between the "true" and the "banker's" present worth of a certain sum due in 3 months, at 4 per cent, is 3*s.* $4\frac{4}{5}$*d.*, what is the sum?

57. The interest on £110 is £11. Find the "true" discount on the same sum for the same time at the same rate.

58. The banker's discount at 6 per cent on a sum due in 2 months exceeds the "true" discount by 10*s.* Find the latter.

59. The "true" discount on a sum due 3 months hence is £7, 10*s.* The interest on the same sum for 3 months at the same rate is £7, 16*s.* Find the sum and the rate per cent.

60. Show, by an example, that the difference between the banker's and the "true" discount on any sum for any part of a year, at any rate per cent, is the interest on the "true" discount for that time at that rate.

Long Period Present Value.

Find, to the nearest penny, the present value of—

61.	£850	due 2	years hence at	5	per cent *compound* interest.
62.	£250	... 3	....	4	....
63.	£725, 10*s.*	... 4	....	5	....
64.	£4500	... 5	....	5	....
65.	£2000	... 10	....	4	....

66. How much ready money could be obtained in lieu of a legacy of £1000 due at the end of 3 years, allowing compound interest at 5 per cent?

67. Find the present value of an annuity of £100 for 4 years, the first instalment to be due a year hence, allowing compound interest at 5 per cent.

68. Find the present value of a fellowship of £200 a year for 6 years, reckoning 4 per cent compound interest, the first payment being made a year hence.

69. What annuity, to continue for 3 years, the first payment being made a year hence, can be purchased for £800, reckoning by compound interest at 5 per cent?

70. What annuity, to continue for 5 years, the first payment to be due 1 year hence, will £2000 cash purchase, supposing money to be worth 4 per cent?

LVII. STOCKS AND SHARES.

STOCKS.

Find the cost of—

1. £800 *stock* at 87.
2. £2500 .... 92.
3. £650 .... 78.
4. £525 *stock* at 108.
5. £1320 .... 105.
6. £1400 .... 91½.
7. £900 *stock* at 85½.
8. £1700 .... 132½.
9. £450 .... 96¼.
10. £8400 .... 87¼.

How much Stock—

11. At 80 can be bought for £1000?
12. ... 96 .... £840?
13. ... 112 .... £980?
14. ... 75 .... £624?
15. ... 102 .... £1360?
16. At 97½ would cost £1560?
17. ... 124¾ .... £2495?
18. ... 139½ .... £744?
19. ... 61¼ .... £490?
20. ... 105⅝ .... £3380?

What Annual Income is obtained from—

21. £3400 *stock* paying 3 p. cent?
22. £750 .... 4 ....
23. £1450 .... 5 ....
24. £850 .... 2½ ....
25. £725 .... 3 ....
26. £8700 *stock* paying 4½ p. cent?
27. £1730 .... 6½ ....
28. £640 .... 2¾ ....
29. £666, 13*s.* 4*d.* .... 7 ....
30. £1412, 10*s.* .... 4 ....

What Annual Income is obtained by *investing*—

31. £1500 *cash* in 4 p.c.'s at 128?
32. £1890 .... 5 .... 135?
33. £910 .... 2½ .... 104?
34. £840 .... 4½ .... 126?
35. £1225 .... 2¾ .... 98?
36. £486, 10*s.* *cash* in 3 p.c.'s at 84?
37. £958, 10*s.* .... 5½ ....142?
38. £655 .... 4 ....98¼?
39. £2300 .... 4½ ....par?
40. £435, 15*s.* .... 2¾ ......115½?

Find the Half-yearly Dividend obtained from—

41. £1670 *stock* paying 4 per cent per annum.
42. £233, 6*s.* 8*d.* .... 4½ ....
43. £730 .... 3 ....
44. *Investing* £780 in 5½ per cent stock at 143.
45. .... £760 ... 2 .... 71¼.
46. .... £220 ... 3½ .... par.

Find the Quarterly Dividend obtained from—

47. £780 *stock* in 2½ per cents.
48. £3000 *stock* in 2¾ per cents.
49. *Investing* £6500 in 2½ per cent consols at 104.
50. .... £1520 ... 2¾ .... 114.

Find the cost of—

51. £2600 *stock* at $91\frac{3}{4}$ (Brokerage $\frac{1}{4}$ p. c.).
52. £950 .... 61 ....
53. £2450 .... $106\frac{3}{8}$ (Brokerage $\frac{1}{8}$ p. c.).
54. £646, 13*s*. 4*d*... $104\frac{7}{8}$ ....
55. £500 .... 80 (Brokerage 2*s*. 6*d*. p. c.).

How much cash would be realized by the sale of—

56. £360 *stock* at $95\frac{1}{4}$? (Brokerage $\frac{1}{4}$ p. c.)
57. £700 .... $200\frac{1}{2}$? ....
58. £875 .... $112\frac{1}{8}$? (Brokerage $\frac{1}{8}$ p. c.)
59. £1760 .... $107\frac{1}{2}$? ....
60. £1530 .... 120? (Brokerage 2*s*. 6*d*. p. c.)

How much Stock—

61. At $74\frac{3}{4}$ could be *bought* for £420? (Brokerage $\frac{1}{4}$.)
62. ... $81\frac{1}{4}$ .... £815? ....
63. ... $111\frac{7}{8}$ .... £980? (Brokerage $\frac{1}{8}$.)
64. ... $166\frac{1}{4}$ would be *sold* for £2490? (Brokerage $\frac{1}{4}$.)
65. ... 65 .... £518? ....
66. ... $102\frac{3}{4}$ .... £3284? (Brokerage $\frac{1}{8}$.)

Find the Annual Income obtained by *investing*—

67. £2940 in 5 p. c. stock at $104\frac{3}{4}$ (Brokerage $\frac{1}{4}$).
68. £900 ... $2\frac{3}{4}$ .... $109\frac{7}{8}$ (Brokerage $\frac{1}{8}$).

Find the Half-yearly Dividend obtained by *investing*—

69. £342, 10*s*. in $4\frac{1}{2}$ p. c. stock at $68\frac{1}{4}$ (Brokerage $\frac{1}{4}$).
70. £2250 .... 7 .... $179\frac{3}{4}$ ....

Find the Quarterly Dividend obtained by *investing*—

71. £1242 in $2\frac{1}{2}$ p. c. consols at $107\frac{7}{8}$ (Brokerage $\frac{1}{8}$).
72. £672 ... $2\frac{3}{4}$ .... $111\frac{7}{8}$ ....

*Find the net Annual Income, after deducting income-tax at 6*d*. in the pound, obtained from—

73. £840 *stock* paying 5 per cent.
74. £1600 .... $2\frac{1}{2}$ ....
75. *Investing* £1200 in 4 p. c. stock at 96.
76. .... £528 ... $3\frac{1}{2}$ .... 88.
77. .... £1700 ... 5 .... $127\frac{1}{4}$ (Brokerage $\frac{1}{4}$).
78. .... £1000 ... $2\frac{1}{2}$ .... $103\frac{7}{8}$ (Brokerage $\frac{1}{8}$).

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* Brokerage is to be neglected whenever it is not mentioned in the question.

Find the net Half-yearly Dividend, after deducting income-tax at 8*d.* in the pound, obtained from—

79. £600 *stock*, paying $4\frac{1}{2}$ per cent.

80. £1600 .... $2\frac{3}{4}$ ....

81. *Investing* £540 in 5 p. c. stock at 108.

82. .... £1715 ... 4 .... 98.

83. .... £2460 ... $4\frac{1}{2}$ .... 92 (Brokerage $\frac{1}{4}$).

84. .... £714 ... $6\frac{1}{4}$ .... $169\frac{3}{4}$ ....

Find the net Quarterly Dividend, after deducting income-tax at 9*d.* in the pound, obtained from—

85. £3200 stock in $2\frac{1}{2}$ per cent consols.

86. £8000 .... $2\frac{3}{4}$ ....

87. *Investing* £720 in $2\frac{1}{2}$ p. c. consols at 105.

88. .... £480 ... $2\frac{3}{4}$ .... 110.

89. .... £1270 ... $2\frac{1}{2}$ .... 111 (Brokerage $\frac{1}{8}$).

90. .... £1850 ... $2\frac{3}{4}$ .... $115\frac{1}{2}$ ....

What rate per cent interest on capital is obtained by investing in—

91. $2\frac{1}{2}$ p. c. stock at 112?

92. 4 .... $97\frac{1}{2}$?

93. $7\frac{1}{2}$ .... $202\frac{1}{2}$?

94. 3 p. c. stock at $86\frac{1}{4}$?

95. $5\frac{1}{2}$ .... $137\frac{1}{2}$?

96. $3\frac{1}{2}$ .... $61\frac{1}{4}$?

Which investment pays better—

97. 4 per cents at 104, or $5\frac{1}{2}$ per cents at 144?

98. 3 .... 80, ... $4\frac{1}{2}$ .... 90?

99. $3\frac{1}{4}$ .... $97\frac{1}{4}$, ... $3\frac{1}{2}$ .... 105?

00. $3\frac{1}{2}$ .... $98\frac{1}{4}$, ... $3\frac{3}{4}$ .... 105?

101. 3 .... $97\frac{1}{2}$, ... 5 .... $162\frac{3}{8}$?

102. $4\frac{1}{2}$ .... $162\frac{3}{8}$, ... $2\frac{3}{4}$ .... par?

Find the change in annual income when—

103. £5000 stock paying $2\frac{1}{2}$ per cent is sold at 87 and the proceeds are invested in 5 per cents at 150.

104. £3000 4 p. c. stock is sold at 95 and $6\frac{1}{2}$ p. c. stock at 120 is bought with the proceeds.

105. £5000 stock in $3\frac{1}{2}$ p. c.'s is sold at 81, and with the proceeds is bought 4 p. c. stock at 108.

106. £4500 stock which pays 4 per cent is sold out at 114, and the proceeds are invested in $6\frac{1}{4}$ per cent stock at 150.

107. £7400 stock paying 3 per cent is sold out at $99\frac{1}{8}$ (Brokerage $\frac{1}{8}$), and the proceeds are invested in 6 per cent stock at $147\frac{3}{4}$ (Brokerage $\frac{1}{4}$).

Find the change in annual income when—

108. £10,000 stock in $3\frac{1}{2}$ p. c.'s is sold out at $94\frac{5}{8}$ (Brokerage $\frac{1}{8}$), and with the proceeds 6 per cent. debentures at $104\frac{3}{4}$ are bought (Brokerage $\frac{1}{4}$).

109. £5000 stock paying 3 per cent is sold at $84\frac{1}{4}$, and with the proceeds 4 per cent stock at $95\frac{3}{4}$ is bought (Brokerage $\frac{1}{4}$ in both transactions).

110. £2350 stock in the $2\frac{1}{2}$ per cent consols is sold at 110 (Brokerage $\frac{1}{8}$), and the proceeds are invested in railway stock at $146\frac{1}{4}$, paying $4\frac{1}{2}$ per cent (Brokerage $\frac{1}{4}$).

Find, *neglecting any fraction of a penny*, the—

111. Cost of £3453, 12*s.* 6*d.* stock at 97.

112. Annual income from £865, 10*s.* 6*d.* stock paying 4 per cent.

113. Amount of stock at 79 which can be bought for £1463, 10*s.*

114. Amount of stock at $94\frac{1}{2}$ which can be bought for £2000 (Brokerage $\frac{1}{4}$).

115. Cash realized by the sale of £776, 12*s.* 8*d.* stock at 89.

116. Cash realized by the sale of £2340 stock at $111\frac{1}{2}$ (Brokerage $\frac{1}{8}$).

117. Half-yearly dividend, less income-tax at 10*d.* in the pound, on £843, 6*s.* 8*d.* stock paying $4\frac{1}{2}$ per cent.

118. Half-yearly dividend obtained by investing £907, 10*s.* in 3 per cent stock at $84\frac{1}{2}$ (Brokerage $\frac{1}{4}$).

119. Quarterly dividend, less income-tax at 7*d.* in the pound, on £727, 10*s.* stock paying 4 per cent.

120. Quarterly dividend, less income-tax at 5*d.* in the pound, obtained by investing £935 in $2\frac{3}{4}$ per cent consols at $113\frac{5}{8}$ (Brokerage $\frac{1}{8}$ p. c.).

Shares.

Find the cost of—

121. Twelve £10 shares in a gas company, at 13.

122. Thirty-five £5 shares in a water company, at 7.

123. A hundred £1 shares in a mining company, at $\frac{5}{8}$.

124. Seven £100 shares in a brewery, at 136.

125. Forty-two £50 shares in a banking company, at $87\frac{1}{2}$.

126. 50 £1 shares in a company, at $\frac{1}{2}$ *premium.*

127. 75 £5 .... $2\frac{1}{4}$ ....

128. 24 £1 .... $\frac{1}{4}$ *discount.*

129. 60 £2 .... $1\frac{1}{4}$ ....

130. 85 £1 .... $1\frac{3}{16}$ *premium.*

Find the annual income from—

131. Eighty-two £5 shares in a company paying 7 p. c.
132. Thirty-six £10 .... 12 ...
133. 45 £1 .... $3\frac{1}{2}$ ...
134. 150 £50 .... 15 ...
135. 325 £5 .... $1\frac{1}{2}$ ...

How many—

136. £1 shares, at $2\frac{1}{2}$, can be bought for £175?
137. £1 .... $\frac{7}{8}$ .... £350?
138. £5 .... $4\frac{1}{2}$ .... £270?
139. £10 .... $17\frac{1}{2}$ .... £157, 10*s.*?
140. 10*s.* ....7*s.* 9*d.*...................... £27, 2*s.* 6*d.*?
141. £1 shares, at $\frac{1}{4}$ *discount*, can be bought for £105?
142. £1 .... $\frac{1}{2}$ *premium*, ... £81?
143. £5 .... $1\frac{3}{4}$ *premium*, ... £540?
144. £10 .... $4\frac{1}{2}$ *discount*, ... £27, 10*s.*?
145. £50 .... $7\frac{1}{2}$ *premium*, ... £402, 10*s.*?

Find the cost, allowing 1 per cent for brokerage and stamp, of—

146. 100 £1 shares at $1\frac{3}{8}$.
147. 200 £5 .... $4\frac{3}{16}$.
148. 25 £10 .... $12\frac{1}{2}$.
149. 60 £1 shares at $\frac{1}{4}$ *premium.*
150. 10 £10 .... $2\frac{3}{4}$ *discount.*

How much cash is realized by the sale of—

151. Forty £1 shares at $2\frac{3}{4}$? (Brokerage $\frac{1}{2}$ p. c.)
152. Sixty £2 .... $1\frac{1}{4}$? ....
153. 200 £1 .... $5\frac{5}{16}$ *premium*? (Brokerage $\frac{1}{2}$ p. c.)
154. 80 £10 .... $4\frac{3}{4}$ *discount*? ....
155. 25 £5 .... $1\frac{7}{8}$ *premium*? ....

What rate per cent interest on capital is obtained by investing in—

156. £5 shares, at $7\frac{1}{2}$, in a company paying $6\frac{1}{2}$ p. c.?
157. £1 .... $\frac{7}{8}$ .... $3\frac{1}{4}$ ...
158. £10 .... $22\frac{1}{2}$ .... 12 ...

Which is the more profitable investment—

159. £10 shares, at £$12\frac{1}{2}$ each, in a gas company which pays $5\frac{1}{2}$ p. c., or £5 shares at $6\frac{1}{4}$ in a water company paying 5 p. c.?
160. £1 shares, at $\frac{1}{8}$ premium, paying $4\frac{3}{4}$ p. c., or £1 shares at $\frac{1}{4}$ discount, paying $3\frac{1}{4}$ p. c.?

[*Note.*—Unless specially mentioned, Brokerage need not be considered.]

161. How much $3\frac{1}{2}$ per cent stock must one hold in order to obtain from it an annual income of £68, 1*s.* 6*d.*?

162. An income of £1000 is made up of £240 from 6 per cent stock, £340 from 8 per cent stock, and the remainder from $3\frac{1}{2}$ per cent stock. How much of each stock is held?

163. What sum must be invested in $2\frac{3}{4}$ per cent consols at $96\frac{1}{4}$ in order to obtain an income of £100 a year?

164. If $4\frac{1}{2}$ per cents are at 119, how much must I invest to obtain an income of £225?

165. A man obtains an income of £120 by investing in 4 per cent stock at 116. How much money does he invest?

166. What sum of money invested in the 3 per cents at $97\frac{1}{2}$ will give an annual income of £150?

167. If the $2\frac{3}{4}$ per cents stand at 99, what sum must be invested in the stock to secure a dividend of £25 per quarter?

168. The income derived from investing a certain sum in $3\frac{1}{2}$ per cent stock at $106\frac{3}{4}$ is £120, 12*s.* Find the sum invested.

169. What is the amount of money which, invested in the $2\frac{3}{4}$ per cents at $98\frac{3}{4}$, will produce an annual income of £12, 2*s.* 11*d.*?

170. What sum invested in $4\frac{1}{2}$ per cents at $89\frac{3}{4}$ will yield an income of £450 per annum? (Brokerage $\frac{1}{4}$ per cent.)

171. How much money would have to be invested in 3 per cents, at 87, in order to produce a net income of £295, after deducting income-tax at 4*d.* in the pound?

172. When certain 3 per cents are at $91\frac{7}{8}$, what sum must be invested in them in order to secure an income of £708, after paying income-tax at 4*d.* in the pound and brokerage $\frac{1}{8}$ per cent?

173. What sum must be invested in 3 per cents at $90\frac{7}{8}$ to yield a net yearly income of £702, after deducting income-tax at 6*d.* in the pound? (Brokerage $\frac{1}{8}$.)

174. If the 3 per cents are at $90\frac{5}{8}$, what sum must be invested in them in order to obtain an annual income of £470, after paying income-tax at 5*d.* in the pound? (Brokerage $\frac{1}{8}$ per cent.)

175. A man holds £4560 railway stock: the first half-year's dividend is at the rate of $3\frac{1}{2}$ per cent, and for the second half of the year the dividend is at the rate of 3 per cent. What is his income for the year?

176. What difference is there between the incomes arising from the investment of £4850 in $3\frac{1}{2}$ per cents at 97 or in the $3\frac{3}{4}$ per cents at par?

177. I have £1000 to invest. Shall I obtain more income by investing it in French 3 per cent Rentes at 78, or Swedish 4 per cents at 104, or Norwegian $3\frac{1}{2}$ per cents at 91?

178. How much $4\frac{1}{2}$ per cent stock must a person have who obtains from it a net yearly income of £337, 2*s.* 6*d.*, after deducting income-tax at 8*d.* in the pound?

179. A person buys £500 stock at 66, and afterwards £500 more of the same stock at 69. He sells out the whole at 89. Find the increase in his capital.

180. What income, to the nearest penny, will be derived from investing £5000 in the $2\frac{3}{4}$ per cent consols at $98\frac{1}{4}$?

181. A man obtains an income of £210 by investing £6160 in $2\frac{3}{4}$ per cent stock. At what price does he buy the stock?

182. What is the price of $2\frac{3}{4}$ per cent stock when an investment of £2109, 7*s.* 6*d.* produces an income of £61, 17*s.* 6*d.* per annum?

183. What is the price of 3 per cent stock, when a sum of £4353, 4*s.* 7*d.* when invested will produce an income of £132, 5*s.*?

184. What sum must be invested in the purchase of 5 per cent railway debentures at $130\frac{1}{2}$ so as to produce an income of £120 a year, after paying an income-tax of 8*d.* in the pound?

185. I sell £4000 of a 4 per cent stock at 144 and invest the proceeds in a $2\frac{1}{2}$ per cent stock. At what price must I buy the $2\frac{1}{2}$ per cent stock to obtain the same income as before?

186. A person who had 45 shares in a gas company sold them at 42 and invested his money in Russian 5 per cents at 105. What income did he then obtain?

187. What is the clear annual income derived from investing £6050 in the 3 per cents at $90\frac{3}{4}$, after deducting an income-tax of 4*d.* in the pound?

188. How much must be invested in the 3 per cents at 99 to produce the same income as would be obtained by investing £1508 in the $2\frac{1}{2}$ per cents at 87?

189. Find the change in income due to selling out at $222\frac{3}{4}$ £3600 stock paying $7\frac{1}{2}$ per cent, and investing the proceeds in $4\frac{1}{2}$ per cent stock at 111. (Brokerage $\frac{1}{4}$.)

190. Which yields the larger return in income for the same amount invested,—4 per cents at $151\frac{1}{2}$ (Brokerage $\frac{1}{4}$), or $2\frac{1}{2}$ per cents at par (Brokerage $\frac{1}{8}$)?

191. What rate per cent does a man get for his money by investing in £10 bank shares selling at 25, and paying 10 per cent?

192. Certain £10 shares in a company are at $23\frac{3}{4}$. What is the yield per cent per annum upon the capital invested for a year in which a dividend of 12 per cent is paid?

193. What rate per cent of interest does one get on money invested in a 4 per cent stock, the price of which (including brokerage) is $119\frac{5}{8}$?

194. A man invests £1980 in $3\frac{1}{2}$ p. c.'s at 99, and £3220 in $4\frac{1}{2}$ p. c.'s at 105. Find the average rate of interest he makes on his capital.

195. A company pays a dividend of 6 per cent, and its £5 shares sell for £7, 5*s.* What percentage does an investor obtain for his money?

196. What must be the price of 6 per cent stock in order that money invested in it may yield $4\frac{1}{2}$ per cent?

197. Find the price of the 3 per cents when a person receives $3\frac{1}{2}$ per cent for his money after paying 2*d.* in the pound income-tax.

198. What is the price of the 3 per cents when they give 3 per cent clear, after paying income-tax at 9*d.* in the pound?

199. What is the price of 3 per cent stock if, after an income-tax of 8*d.* in the pound has been deducted, an investor gets 4 per cent interest on his money?

200. What is the price of a 5 per cent stock when money invested in it yields 4 per cent interest, after paying income-tax at 9*d.* in the pound? (Brokerage $\frac{1}{8}$.)

201. If by selling out $2\frac{1}{2}$ per cent consols at 105, and investing in another stock at 126, a man could double his income, what dividend must the latter pay?

202. A man transfers his money from 4 per cents at 102 to 3 per cents at 95. Find the change per cent in his income.

203. A person invested £630 in stock at $94\frac{1}{2}$ and sold immediately, gaining £10. At what price did he sell?

204. How much stock must be bought in 3 per cents at $89\frac{1}{8}$ in order that by selling it at $91\frac{5}{8}$ a profit of £45 may be made?

205. If £3000 be invested in a certain 5 per cent stock, it produces annually £25 more than if it be invested in 3 per cent stock at 90. Find the price of the 5 per cent stock.

206. By the conversion of consols from 3 per cent to $2\frac{3}{4}$ per cent, a person's income is diminished by £27 per annum. What cash would he obtain by selling out at $97\frac{1}{8}$?

207. A person sells out of $3\frac{1}{2}$ per cents at $92\frac{3}{4}$ and realizes £18,550. If he invests $\frac{2}{5}$ of the proceeds in 4 per cents at 96, and the remainder in 3 per cents at 90, find the alteration in his income.

208. A man sells twenty £10 shares in a company which pays a dividend of $5\frac{1}{2}$ per cent, at £13, 10*s.* each, and invests the proceeds in £1 shares, at 15*s.* each, in another company which pays a dividend of 3 per cent. Find the change in his income.

209. A person would obtain £22, 10*s.* a year more if he invested his money in a $5\frac{1}{2}$ per cent stock at 132, than he would if he invested in $2\frac{1}{2}$ per cent consols at 105. What money has he?

210. A person bought consols at 94 and sold the same at $95\frac{5}{8}$, thereby gaining £55. What cash did he receive for the stock? (Brokerage $\frac{1}{8}$ per cent.)

211. Which is the better investment—5 per cents quoted at $142\frac{1}{2}$, $\frac{5}{8}$, or 4 per cents at $113\frac{3}{4}$, $\frac{7}{8}$?

212. At what price must a person buy 4 per cent stock with the proceeds of the sale of £9600 three per cent stock at 96 if no change results in income?

213. At what price must I buy a 4 per cent stock with the proceeds of the sale of £9600 3 per cent stock at $86\frac{3}{8}$, in order to make no change in my income, allowing $\frac{1}{8}$ per cent brokerage?

214. A person invests £13,650 in a 4 per cent stock at 91. On the stock falling to 75 he sells out, and investing the proceeds in an 8 per cent stock he finds that he thereby loses in interest £60. What is the price of the latter stock?

215. How much stock in $2\frac{1}{2}$ per cent consols must be sold at 108 in order with the proceeds to purchase £4315 railway stock at 132? (Brokerage on consols $\frac{1}{8}$, and on railway stock $\frac{1}{4}$ per cent.)

216. A man invests in consols at $106\frac{1}{2}$, and on their rising to £108 he sells out, thus increasing his capital by £20. How much stock did he buy, and what did he pay for it? (Brokerage $\frac{1}{8}$.)

217. What must be the price of 5 per cent stock in order that, after deducting income-tax at 8*d.* in the pound, it may yield $3\frac{5}{8}$ per cent interest on capital invested in it?

218. What is the market price of the $2\frac{1}{2}$ per cent consols when, after deducting income-tax at 6*d.* in the pound, 2 per cent interest on capital invested in them is made? (Brokerage $\frac{1}{8}$ per cent.)

219. A man invested £560 in 3 per cents at $99\frac{1}{2}$, and sold out when they rose to $102\frac{1}{4}$. How much (neglecting fractions of a penny) did he gain after paying $\frac{1}{8}$ per cent brokerage on each transaction?

220. A person has an income of £600 a year from Victoria four per cents: if he sells out at $101\frac{1}{4}$ and invests in three per cents at $91\frac{1}{8}$, what is his loss of income?

221. A man buys into a certain stock at $145\frac{3}{4}$ and at the end of six months sells out at $197\frac{1}{2}$. If, including the half-year's interest at $7\frac{1}{2}$ per cent per annum, he gains £5550, what amount of stock did he buy?

222. A man sells out £10,000 three per cents at 101, and £5000 four and a half per cents at 108, and invests the proceeds in five per cents at 124. Find his income before and after the transaction.

223. A person sells stock paying $6\frac{1}{2}$ per cent at $128\frac{1}{2}$ and invests in stock paying 3 per cent at $72\frac{1}{2}$. By how much per cent will the interest of his investment be altered?

224. A and B have each £7200 to invest. A buys Costa Rica stock, which pays no dividend, at 15; B buys Portuguese 3 per cents at 48. At the end of a year A sells at $15\frac{3}{4}$; B, having received the year's dividends, at $47\frac{1}{2}$. Which had made the better investment?

225. A speculator buys £1000 stock at 80 and pays for it with money which he borrows at 5 per cent. At the end of a year he sells out (the stock having paid a dividend of 4 per cent), and, after repaying his loan with interest, he has £40 left. At what price did he sell out?

226. A man owns £7500 of 3 per cent stock. He sells out and invests the proceeds in 5 per cent stock at 120, thereby increasing his income by £25. At what price did he sell out?

227. By the conversion of consols from 3 per cents to $2\frac{3}{4}$ per cents a person's income was reduced by £7 a year. What cash would he have obtained by selling out at $102\frac{1}{4}$? (Brokerage $\frac{1}{8}$.)

228. A man holding 4 per cent stock which brought him in £250 a year, sold at $120\frac{1}{4}$, and bought other stock, paying a dividend of $5\frac{1}{2}$ per cent, at 156. What gain in income did he effect? (Brokerage $\frac{1}{4}$.)

229. A person holding £2450 three per cent stock sells out at $93\frac{1}{2}$, and invests the proceeds in $4\frac{1}{2}$ per cents, thereby increasing his income by £31, 13*s.* 9*d.* At what price did he buy?

230. A person had a certain capital, half of which he invested in 3 per cents at 90, and the other half in 5 per cents at 110. If his total income was £6883, 10*s.*, what was his capital?

231. A man's income from three per cent stock is £750. He sells out half his stock at 96, and invests it in other securities at 120. If his income be increased by £50, what interest do these securities pay?

232. Starting with £2310 I buy into the 3 per cents at 99; and when they have risen to 101 I sell out and reinvest in foreign 4 per cents at par. What is the change of income?

233. A person invested £2353 in 6 per cent stock at 181: a year afterwards he sold out when the stock was at 178, and invested the proceeds in a $2\frac{1}{2}$ per cent stock at 26. What difference does the change of investment make in his income?

234. A person sells out of the three per cents and realizes £18,000. Would it be more to his advantage to reinvest this money in the three and a half per cents at $101\frac{7}{8}$ or in a mining company paying a dividend of $1\frac{3}{4}$ per cent per annum, of which the £10 shares were quoted at $5\frac{3}{8}$?

235. If an income-tax of 6*d.* in the pound be deducted before payment of the dividend on $2\frac{1}{2}$ per cent stock at 91, how much money must be invested in that stock to yield a net income of £208 per annum?

236. How much money must I invest in a $3\frac{1}{2}$ per cent stock at $135\frac{3}{4}$ so that, after paying $\frac{1}{4}$ per cent commission, a stamp of 10*s.* per £100 stock, and 6*d.* in the pound income-tax on the dividend, I may have a net income of £1000 a year?

237. The 3 per cents are at $96\frac{7}{8}$, and $3\frac{1}{4}$ per cents at 105. What amount invested in the former would produce an annual

income of £1 more than if invested in the latter, one-eighth per cent brokerage being charged on each investment?

238. A man sells out $2\frac{3}{4}$ per cent consols at $96\frac{1}{4}$, and, by investing the proceeds in shares which pay an annual dividend of £4 per share, raises his income 5 per cent. Find the price of a share.

239. What income would result from the investment of £990 in £50 shares in a company, if the shares, £20 being paid up, sell at £45, and the company pays a dividend of 12 per cent?

240. A person buys shares in a railway at $19\frac{1}{2}$ when £15 has been paid, and after paying a call of £10 per share, sells them again at £32, 9*s*. How much per cent does he gain?

241. A man buys 50 shares, £10 paid up, at £12, 5*s*., and, after paying a call of £5 per share, sells them at £16, 10*s*. What capital does he sacrifice?

242. If £2650 is invested partly in $4\frac{1}{2}$ per cent stock at 104, and partly in 4 per cent stock at 96, and the resulting incomes are equal, find how much is invested in each stock.

243. A man invests £9256, 10*s*., partly in 5 per cent stock at 150, and partly in $2\frac{3}{4}$ per cent stock at 99, and his income from each source is the same. What sums did he invest in each stock?

244. A man invested one-third of his capital in 5 per cents at $142\frac{1}{2}$ and two-thirds in 4 per cents at $113\frac{3}{4}$, and the total resulting income was £455, 13*s*. 4*d*. Find his capital.

245. A man invests £7000, part in 3 per cents at 97, and the remainder in 4 per cents at 104. Find how much he invests in each if the returns from both sources are equal.

246. If £7594 be invested partly in $2\frac{1}{2}$ per cents at $110\frac{1}{2}$ and partly in $2\frac{3}{4}$ per cents at $115\frac{1}{2}$, and the resulting incomes are equal, how much was invested in each stock? (Brokerage $\frac{1}{8}$.)

247. A man invests £16,213 partly in £100 shares at 105, bearing a dividend (free from income-tax) of 8 per cent, and partly in a mortgage at 5 per cent interest on which he pays income-tax at 8*d*. in the pound. His net income from each source is the same. What is his whole income?

248. A man invests half his capital in the $2\frac{1}{2}$ per cents at 88, and the other half in 4 per cents at 116, and his total income from both sources is £642. What was his capital?

249. A man calls in a sum of £10,000 lent on mortgage at 4 per cent and invests the money in £3000 L. & N.W. Ry. 4 p.c. deb. stock at 133, and £3000 G.W. Ry. 5 p.c. stock at 167. The balance of the £10,000 he places on deposit at $2\frac{1}{2}$ per cent. What change is made in his income?

250. Which is the better investment (the security being equal), 3 per cents perpetually at par, or 5 per cents at 124, the latter stock being redeemable in 20 years at par? (Simple interest is to be reckoned.)

251. A person is left a legacy on which he pays 10 per cent succession duty. He invests the rest of the money in a 3 per cent stock at 75, bringing him in £270 per annum. To how much did the legacy amount?

252. A man has £3000 of a 4 per cent stock, which he sells out at 120, and then invests £1900 in a $2\frac{3}{4}$ per cent stock at 95. What rate of interest does he get for the remainder, if his income is, on the whole, diminished by £14?

253. A person wishes to found a scholarship of £48 a year. How much money must he for this purpose invest in $2\frac{1}{2}$ per cent stock at 87, in order that the scholar may have his £48 clear after paying income-tax at 8*d.* in the pound?

254. A and B invested equal sums; A in $3\frac{1}{2}$ per cents at 120, B in $2\frac{3}{4}$ per cents at 96. After this A sold out at par and invested in the $2\frac{3}{4}$ per cents, which had fallen in price. A now found his income the same as B's. What was the price at which he bought the $2\frac{3}{4}$ per cents?

255. A person having bought a certain amount of $2\frac{3}{4}$ per cent stock at 95, afterwards sold it, and with the proceeds bought $3\frac{1}{2}$ per cent stock; he obtained £900 less stock than before, but his income was unchanged. How much money did he originally invest?

256. £4600 was invested in consols at 92. Part of the stock was sold out at 93 and part at 88, and the original capital was thus decreased by £5. How much stock was sold out at each price?

257. A man invests £5900 in $2\frac{1}{2}$ per cent consols at $110\frac{1}{2}$, and afterwards sells out part of the stock at $108\frac{1}{4}$ and the remainder at 113, thus increasing his capital by £25. How much stock did he sell out at each price? (Brokerage $\frac{1}{8}$ p. c.)

258. A man invests £2730 in 3 per cents at $90\frac{3}{4}$, and sells out part of the stock when they have risen to $93\frac{3}{4}$ and the remainder when they have fallen to $85\frac{1}{4}$. He lost £10 by the transaction. How much did he sell out at first? (Brokerage $\frac{1}{4}$ per cent.)

He invests the proceeds in $4\frac{1}{2}$ per cents so as to cause an increase in his income of £16, 13*s.* 4*d.* At what price did he buy the $4\frac{1}{2}$ per cents? (Brokerage $\frac{1}{4}$ per cent.)

259. Some 4 per cent stock at $115\frac{3}{8}$, and some 5 per cent stock at $134\frac{7}{8}$, are sold, and the proceeds are invested in $4\frac{1}{2}$ per cent stock at $123\frac{1}{2}$; if the change does not alter the income, what is the ratio of the quantities of stock sold?

260. The capital of a company consists of £1,100,000 in $4\frac{1}{2}$ per cent debenture stock, the same amount in 6 per cent preference shares, and the same amount in ordinary shares. If the net annual profits are £209,000, calculate the amount per cent available for dividend on the ordinary shares after payment of the debenture and preference claims.

261. A man has £9170 in 3 per cent consols. He sells out at $101\frac{3}{8}$ and buys Indian bonds at $98\frac{3}{4}$, and when these have risen to $106\frac{1}{16}$ he sells out and reinvests in consols at $98\frac{1}{8}$. If the interest on consols has meanwhile been reduced to $2\frac{3}{4}$ per cent, find the permanent change in his income, allowing $\frac{1}{8}$ per cent for brokerage on each transaction.

262. A person invests £4800 in 4 per cents at 80, and at the end of each year invests the dividend which becomes due, in the same stock. Supposing the stock to remain at 80 for 3 years, find his dividend at the end of the third year.

263. A person who has £10,257, 10*s.* three per cent stock calculates that by selling it and investing in $3\frac{1}{4}$ per cent stock at $93\frac{1}{4}$ he can increase his annual income by £10, 9*s.*; but, before he can effect the exchange, each stock rises $\frac{1}{4}$ per cent; by how much is his income really increased?

264. A man invests £1404, 16*s.* in $2\frac{3}{4}$ per cents at 99; how much must he also invest in $3\frac{1}{2}$ per cents at 105, so that the rate of interest on the whole may be 3 per cent?

265. A person invests £7000 in the 3 per cents at 99, and pays income-tax at 8*d.* in the pound. On the stock rising to 102 he sells out and invests the proceeds in railway stock at 163, paying 5 per cent free of income-tax. Calculate to the nearest penny the change in his net income.

266. A man purchases £1400 stock in three per cent consols at $94\frac{1}{2}$, and also invests £3150 in the purchase of Russian inscribed five per cent loan at $94\frac{1}{4}$. How much stock has he standing in his name? If he sells the consols at $95\frac{1}{8}$ and the Russians at $96\frac{1}{2}$, what does he gain or lose by the transaction? (Brokerage on consols $\frac{1}{8}$, on Russians $\frac{1}{4}$.)

267. Find, neglecting fractions of a penny, the change of income in transferring £2800 from Goschens ($2\frac{3}{4}$ per cents) to India $3\frac{1}{2}$ per cents, the former being quoted at $96\frac{5}{8}$, $\frac{7}{8}$, and the latter at 106, $\frac{1}{4}$. (Brokerage $\frac{1}{8}$.)

268. Show that ·0067 of the amount of $2\frac{3}{4}$ per cent consols will give the quarterly dividend, with sixpence in the pound income-tax deducted, with an error not greater than one penny for every £1000 stock.

269. Find, neglecting fractions of a penny, the net half-yearly dividend, after deducting income-tax at 8*d.* in the pound, resulting from the investment of £16,000 in railway stock at $179\frac{1}{2}$, paying a dividend at the rate of $5\frac{1}{2}$ per cent, the charge for brokerage being $\frac{1}{4}$ per cent.

270. A bill for £15,000 at 3 months was discounted at $4\frac{1}{2}$ per cent and the proceeds invested in $2\frac{3}{4}$ per cent consols at 95, brokerage being $\frac{1}{8}$ per cent, and income-tax 5*d.* in the pound. Find, to the nearest penny, the net quarterly income.

LVIII. FOREIGN MONEY.

DECIMAL COINAGES.

Express at sight—

1. 103·05 *fr.* in francs and cents.
2. 1587·5 *fr.* ....
3. 236·7 *M.* in marks and pf.
4. 76·055 *M.* ....
5. 846·75 *l.* in lire and cent.
6. 48·25 *fl.* in florins and kreuzer.
7. 93·45 *R.* in roubles and kopeks.
8. $7·45 in dollars and cents.
9. $154·7 ....
10. $6·0575 ....

Express at sight—

11. 74 *fr.* 25 *c.* in francs.
12. 8 *fr.* 5 *c.* ....
13. 105 *M.* 50 *pf.* in marks.
14. 16 *M.* $5\frac{1}{2}$ *pf.* ....
15. 24 *l.* 5 *c.* in lire.
16. 6 *fl.* 25 *kr.* in florins.
17. 62 *fl.* 5 *kr.* ....
18. 150 *R.* 19 *ko.* in roubles.
19. $17, 35 *c.* in dollars.
20. $175, 5 *c.* ....
21. Add 96 *fr.* 78 *c.*, 108 *fr.* 5 *c.*, 2086 *fr.* 25 *c.*, and 6 *fr.* 80 *c.*
22. Add 910 *M.* 5 *pf.*, 17 *M.* 50 *pf.*, 4 *M.* 20 *pf.*, and 1050 *M.* 85 *pf.*
23. Subtract $48, 5 *c.* from $203, 70 *c.*

Multiply—

24. 7 *fr.* 35 *c.* by 29.
25. 4·50 *fr.* by 234.

Multiply—

26. 6 *M.* 60 *pf.* by 63.
27. $4·75 by 147.

Divide—

28. 162 *fr.* 61 *c.* by 23.
29. $747·90 by 54.
30. How many times is 23 *M.* 50 *pf.* contained in 1574 *M.* 50 *pf.*?

If a franc $= 9\frac{1}{2}d.$, a mark $= 11\frac{3}{4}d.$, a lira $= 9\frac{1}{2}d.$, a florin $= 20d.$, a rouble $= 37\frac{1}{2}d.$, and a dollar $= 50d.$, express in £, *s. d.* to the nearest penny—

31. $81, 17 *c.*
32. $473·56.
33. 78 *fr.* 50 *c.*
34. 456·75 *fr.*
35. 4 *M.* 85 *pf.*
36. 63·50 *M.*
37. 27 *fl.* 15 *kr.*
38. 104 *fl.* 25 *c.*
39. 18 *l.* 50 *c.*
40. 350 *R.*
41. 47 *R.* 80 *ko.*
42. 17·25 *R.*
43. 18560 *M.*
44. 22550 *fr.*
45. 225·50 *fr.*
46. 185·60 *M.*

Express approximately—

47. £7, 12*s.* 6*d.* in francs and cts.
48. £37, 14*s.* 8*d.* ....
49. £21, 3*s.* 5*d.* ....
50. £405, 12*s.* ....
51. £15, 4*s.* 9*d.* in marks and pf.
52. £2, 5*s.* 2*d.* ....
53. £93, 15*s.* 6*d.* ....
54. £3, 13*s.* 4*d.* in dollars and cents.
55. £216, 10*s.* ....
56. £107, 12*s.* 9*d.* in florins and kr.
57. £31, 13*s.* 7*d.* ....
58. £2, 11*s.* 10*d.* in lire and cent.
59. £53, 4*s.* in roubles and kopeks.
60. £105, 14*s.* 9*d.* ....

LIX. THE METRIC SYSTEM.

Length.

Read, mentioning the special denomination of each separate figure—

Km.	Km.	m.	m.	cm.
1. 4·573.	**2.** 45·73.	**3.** 4·573.	**4.** 457·3.	**5.** 457·3.

Read as a length expressed in *kilometres and metres*—

Km.	Km.	Km.	Km.	Km.
6. 4·573.	**7.** 45·73.	**8.** 457·3.	**9.** 20·534.	**10.** 2·0534.

Read as a length expressed in *metres and centimetres*—

m.	m.	m.	cm.	mm.
11. 3·65.	**12.** 36·5.	**13.** 1·375.	**14.** 846.	**15.** 7305.

Express in *kilometres and decimals of a kilometre*—

16. 2 *Km.* 3 *Hm.* 4 *Dm.* 7 *m.*
17. 5 *Mm.* 1 *Km.* 7 *Dm.* 2 *dm.*
18. 90 *Km.* 125 *m.*
19. 13 *Km.* 25 *m.*
20. 7 *Km.* 500 *m.*
21. 4 *Km.* 5 *m.*

Express in the denomination *kilometres*—

22. 8750 *m.* **23.** 43·7 *m.* **24.** 4 *Dm.* **25.** 4 *dm.* **26.** 20 *cm.*

Express in the denomination *metres*—

27. 6 *Km.* **28.** 13·5 *Km.* **29.** 7 *Dm.* **30.** 7 *dm.* **31.** 45 *mm.*

Express in the denomination *centimetres*—

32. 3 *m.* **33.** 7·5 *m.* **34.** 6 *dm.* **35.** 3 *mm.* **36.** 3 *Km.*

37. Find (in kilometres) the sum of 47 *Km.*, 58 *Dm.*, 85 *m.*, 2·45 *Km.*, 1375 *m.* and 9 *Km.* 80 *m.*

38. Find (in metres) the sum of 3 *Dm.*, 17·5 *Dm.*, 6 *dm.*, 48 *cm.*, 152 *cm.*, 10·5 *m.* and 5 *Hm.*

39. Find (in centimetres) the sum of 1 *m.* 25 *cm.*, 10 *cm.* 5 *mm.*, 3 *dm.* 3·5 *mm.*, 2 *m.* 5 *cm.* and 7 *m.* 50 *cm.*

40. Find (in *Km.*) the difference between 8 *Km.* 250 *m.* and 18 *Km.* 25 *m.*

41. Find (in metres) the difference between 9 *Dm.* 7 *m.* and 9 *m.* 7 *dm.*

42. Find (in *cm.*) the difference between 1 *m.* 5 *cm.* and 86 *cm.* 5 *mm.*

Multiply *—

43. 6 *m.* 25 *cm.* by 8.
44. 2 *Km.* 45 *m.* by 12.
45. 1 *Km.* 725 *m.* by 34.
46. 13 *m.* 50 *cm.* by 600.
47. 3 *m.* 5 *cm.* by 308.
48. 1 *Km.* 5 *m.* by 760.

Divide, giving the remainder (if any) in centimetres—

49. 18 *Km.* 140 *m.* by 4.
50. 369 *Km.* by 180.
51. 43 *m.* 20 *cm.* by 73.
52. 7 *m.* 5 *cm.* by 11.
53. 2 *Km.* 860 *m.* by 571.
54. 128 *m.* by 42.

* These products should be given in the highest denomination mentioned in the question.

How many times is—

55. 3 *Km.* 50 *m.* contained in 143 *Km.* 350 *m.*?

56. 17 *m.* 50 *cm.* .... 315 *m.*?

57. 13 *Km.* 5 *m.* .... 9129 *Km.* 510 *m.*?

58. 7 *cm.* 4 *mm.* .... 22 *m.* 57 *cm.*?

59. How many times can 12 *m.* 35 *cm.* be taken from 3 kilometres, and what remains?

60. How many times can 7·5 millimetres be subtracted from 3·25 decimetres, and what is left over?

Find the cost of—

61. 24 metres 8 centimetres of silk at 4 francs 25 cents. per metre.

62. 18 metres 5 centimetres of cloth at 3 francs 80 cents. per metre.

63. 8 kilometres 25 metres of fencing at 5 francs 75 cents. per metre.

64. 6 kilometres 850 metres, at 28·60 francs per kilometre.

65. 43 metres 25 centimetres, at 3 marks 8 pfennigs per metre.

66. 3 kilometres 640 metres, at 4 marks 50 pfennigs per metre.

67. 6 centimetres 5 millimetres, at 7·75 francs per centimetre.

68. 14 metres 5 decimetres, at 2 florins 25 cents. per metre.

69. How many revolutions will a wheel, whose circumference is 2 metres 15 centimetres, make in travelling a distance of 4 kilometres 42 metres?

70. How many metres of calico, at 1 franc 5 centimes per metre, can be bought for 175 francs 35 centimes?

71. If 7 metres 5 centimetres of cloth cost 22 francs 56 centimes, what will 5 metres 70 centimetres of the same cloth cost?

72. If 24 metres 50 centimetres of linen cost 19 florins 60 cents., how much can be bought for 49 florins 20 cents.?

73. If a bicyclist travel at the average rate of 13 *Km.* 750 *m.* per hour, how far will he go in 3 hours 45 minutes?

74. How many times can a piece 3 *cm.* 4 *mm.* long be cut from a rod 9 *dm.* long, and what is the length in millimetres of the remnant?

Find to the nearest five cents. (or five pf.) the value of—

75. 13 *m.* 45 *cm.* at 74 *fr.* 85 *c.* per metre.

76. 7 *m.* 4·25 *cm.* at 2 *fr.* 35 *c.* per centimetre.

77. 8 *Km.* 17 *m.* at 6 *fr.* 85 *c.* per kilometre.

78. 2 *Km.* 326 *m.* at 7 *M.* 68 *pf.* per kilometre.

79. 16 *m.* 32·5 *cm.* at 1 *M.* 45 *pf.* per metre.

80. 23 *m.* 88 *cm.* at 9 *fl.* 95 *c.* per metre.

WEIGHT.*

Find the cost of—

81. 13·5 kilogrammes of sugar at 60 centimes per kilogramme.
82. $28\frac{1}{2}$ kilogrammes of butter at 1 franc 20 cents. per kilogramme.
83. 23·8 kilogrammes of copper at 2 francs 5 cents. per kilogramme.
84. 14 kilogrammes 750 grammes of coffee at 3 francs 80 centimes per kilogramme.
85. 82 kilogrammes 125 grammes, at 3 marks 44 pfennigs per kilogramme.
86. 17 kilogrammes 250 grammes, at 1 mark 24 pfennigs per kilogramme.

87. If a kilogramme of copper is worth 1 franc 29 centimes, of what weight is 723 francs 69 centimes the value?
88. Find to the nearest centigramme the quotient of 8·4 *g.* ÷ 53.
89. How many times is 3 *Kg.* 500 *g.* contained in 845 *Kg.* 25 *g.*, and how many grammes are left over?
90. If 74·5 *Kg.* of sugar cost 56·2 francs, what is the cost of 96·85 *Kg.*?
91. If 56 *Kg.* 500 *g.* of lead are worth 64 marks 41 pfennigs, find the value of 18 *Kg.*

Find, to the nearest five cents. (or five pf.), the value of—

92. 7 *Kg.* 385 *g.* at 1 *fr.* 30 *c.* per *Kg.*
93. 3 *Kg.* 465 *g.* at 1 *M.* 75 *pf.* per *Kg.*
94. 7·25 quintal at 25 *fr.* 50 *c.* per tonneau.
95. 4·75 tonneaux at 3 *fr.* 75 *c.* per quintal.

CAPACITY.*

Find the cost of—

96. 3·25 litres of milk at 20 centimes per litre.
97. 7 litres 5 decilitres of vin ordinaire at 70 *c.* per litre.
98. 3·25 hectolitres of wine at 54 francs per hectolitre.

99. If 12 hectolitres of wine cost 576 francs, what is the cost per litre?
100. Find the cost of 85·25 hectolitres at 24 francs 50 centimes per decalitre.

* Exs. 1–60 may be made to apply to Weight or Capacity, by merely substituting the word "gramme", or "litre", for "metre".

Area.

Express in square metres—

101. 3 *sq. Dm.*	**103.** 3 *sq. Km.*	**105.** 23 hectares.
102. 3 *sq. dm.*	**104.** 23 ares.	**106.** 23 centiares.

Express in square centimetres—

107. 7 *sq. dm.* **108.** 7 *sq. m.* **109.** 485 *sq. mm.* **110.** 3·4 ares.

Express in ares—

111. 580 *sq. m.* **112.** 43 *sq. Dm.* **113.** 198 *ca.* **114.** 6·3 *Ha.*

115. Express in square centimetres the sum of 53 *sq. dm.*, 63·5 *sq. dm.*, 8 *sq. m.*, 7·4 *sq. m.* and 654 *sq. cm.*

116. Express in hectares the sum of 6 *Da.* 3 *a.*, 5 *Ha.* 17 *a.*, 13·25 *Ha.*, 157 *a.*, 80·5 *a.* and 246 *ca.*

117. A field is divided into 17 allotments, each containing 3·75 ares; find the size of the field in decares.

118. How many times is 13 *sq. cm.* 50 *sq. mm.* contained in 3 *sq. dm.* 10·5 *sq. cm.*?

119. Find the rent of a farm of 89 *Ha.* 25 *a.* at 57 *fr.* 50 *c.* per hectare.

120. Find the cost of 2 *Ha.* 72 *a.* of land at 2 *fr.* 25 *c.* per sq. metre.

121. Find the area of a floor 5 *m.* 75 *cm.* long and 4 *m.* 20 *cm.* wide.

122. Find the area of a rectangular field 126 *m.* long and 43 *m.* 50 *cm.* wide.

123. Find the cost of paving a floor 7 *m.* 5 *dm.* long and 4 *m.* 8 *dm.* wide, at 3 *M.* 80 *pf.* per square metre.

124. Find the cost of matting, 75 *cm.* wide, to cover a floor 5 *m.* 5 *dm.* long and 4 *m.* 4 *dm.* wide at 4 *fr.* 50 *c.* per metre.

Find, to the nearest five cents. (or five pf.), the cost of—

125. Paving a passage 25 *m.* long and 2 *m.* 40 *cm.* wide, with tiles 15 *cm.* square, at 1 *fr.* 50 *c.* per dozen.

126. Electroplating a metal box 21 *cm.* long, 12 *cm.* wide, and 8 *cm.* high, all over, at 2 *fr.* 75 *c.* per square decimetre.

127. Lining an open tank 4 *m.* 60 *cm.* long, 2 *m.* 25 *cm.* wide, and 1 *m.* 50 *cm.* deep, at 4 *M.* 75 *pf.* per square metre.

Volume.

128. Express 3 *cub. m.* in cubic centimetres.

129. Express 4850 *cub. dm.* in cubic metres.

130. Express 15 decasteres in cubic metres.

131. Express 15 *cub. dm.* in steres.

VOLUME (*continued*).

132. Find, in cubic metres, the volume of a rectangular block 3 *Dm.* long, 8 *m.* wide, and 7 *dm.* thick.

133. How many cubic decimetres of air are there in a room 4 *m.* 20 *cm.* long, 3 *m.* 75 *cm.* wide, and 3 *m.* 5 *cm.* high?

134. Find the cost of gravel ·75 of a decimetre thick for a path 86·5 metres long and 2·2 metres wide, at 4 francs per cubic metre.

135. Given that a cubic centimetre of water weighs a gramme, how many kilogrammes of water would cover the floor of a skating rink, 20 metres long and 15 metres wide, to a depth of one decimetre?

136. A cask containing a hectolitre of wine bursts in a cellar 4 metres square: to what depth is the floor flooded?

137. How many hectolitres of water will a tank, 3 metres long, 2·8 metres wide, and 1·25 metres deep, hold?
Find also, in kilogrammes, the weight of the water.

138. Find, in tonneaux, the weight of water in a reservoir having an area of 2·35 hectares, the average depth of the water being 2 metres 35 centimetres.

CONVERSION OF METRIC TO ENGLISH MEASURES, AND VICE VERSA.*

Given that 1 metre = 39·37079 inches, and 1 gallon = 277·274 cubic inches, convert, correct to *five* significant figures—

139. 3 *cm.* into *inches.*
140. 1 *Km.* ...... *yards.*
141. 8 *Km.* ...... *miles.*
142. 41 *m.* ...... *feet.*
143. 1 *sq. m.* ...... *sq. in.*
144. 36 *sq. cm.* ...... *sq. in.*
145. 1 *Ha.* ...... *acres.*
146. 1 *cub. m.* ...... *cub. in.*
147. 1 *cub. m.* ...... *cub. yds.*
148. 1 *litre* ...... *pints.*
149. 1 *gramme* ...... *grains.*
150. 1 *Kg.* ...... *lbs. Av.*
151. 1 *inch* into *cm.*
152. 5 *feet* ...... *dm.*
153. 1 *yard* ...... *m.*
154. 3 *miles* ...... *Km.*
155. 1 *sq. in.* ...... *sq. cm.*
156. 5 *acres* ...... *Ha.*
157. 3 *R.* 24 *P.* ...... *ares.*
158. 1 *cub. in.* ...... *cub. cm.*
159. 7 *cub. yds.* ...... *cub. m.*
60. 1 *gallon* ...... *litres.*
161. 1 *oz. Troy* ...... *grammes.*
162. 25 *lbs. Av.* ...... *Kg.*

* These Exercises should be worked by the contracted methods of Chap. L.

163. Taking 8000 metres to be equal to 5 miles, how many square metres are there in an acre?

164. Find the difference in yards between 5 miles and 8 kilometres. (A kilometre = 39370·8 inches.)

165. A cubic centimetre of water weighs one gramme. Find the volume of 350 kilogrammes of a liquid which is twice as heavy as water.

166. A metre contains 39·3708 inches. Find, to the nearest integer, the number of metres in a mile.

167. A kilometre being 1093·638 yards, find to four places of decimals how many kilometres there are in 100 English miles.

168. Given that a metre is 3·3708 inches longer than a yard, find which is greater: 10 square metres, or 12 square yards.

169. Express 39 chains in metres, correct to the nearest centimetre.

170. Calculate whether posting at a shilling per mile is less or more expensive than posting at 65 centimes per kilometre, having given that 1 metre = 39·37 inches, and 25 francs = £1.

171. Find the number of square centimetres in the area of a rectangle 5 ft. 3 in. long by 2 ft. 6 in. wide.

172. If £1 = 25·2 francs and 1 acre = ·40467 hectare, find to the nearest franc the rent per hectare equivalent to 25*s.* per acre.

173. A metre being equal to 39·371 inches, and a franc being equal to 9·38 pence, what is the value in English money of a yard of silk worth $7\frac{1}{2}$ francs a metre?

174. If the railway fare in France for a distance of 384 *Km.* be 25·28 francs, find, to the nearest farthing, the rate per mile. (1 metre = 39·3708 inches; £1 = 25·2 francs.)

175. Given that 1 oz. Troy = 31·1035 grammes, express 25 tons 2 cwt. 27 lbs. in the metric system.

176. Given that 1 gramme = 15·4323 grains, express 2 cwt. 3 qrs. 11 lbs. in the metric system.

177. Determine the value of $2\frac{1}{4}$ cwt. in marks, if a ton be equivalent to 1015 kilos, and 50·75 kilos cost 95·8 marks.

178. I bought 40 metres of velvet at 12 francs 60 centimes a metre. I sold 25 yards of it at 9*s.* $11\frac{3}{4}$*d.* a yard, and the rest at 12*s.* a yard. What was my gain per cent? (1 metre = 39·37 inches; £1 = 25·22 francs.)

179. If the cost of the carriage of goods in England be 1*d.* per ton per mile, find the equivalent cost of carriage in France in centimes per 1000 kilogrammes per kilometre, assuming that a kilometre is equal to 5 furlongs, a kilogramme to 2·2 lbs., and 100 centimes to one-twenty-fifth of a pound sterling.

180. A metre being equal to 39·37 inches, and a gramme being equal to 15·43 grains, find the weight in grammes of a cubic metre of air, when 100 cubic inches of the air weigh 31 grains.

LX. FOREIGN EXCHANGES.*

Exchange—

1. £530 into francs, at 25·20 *fr.* for £1.
2. £285 ..... marks, ... 20·35 *M.* ... £1.
3. £364 ..... lire, ... 27·15 *l.* ... £1.
4. £1080 ..... kroner, ... 18·31 *kron.* .. £1.
5. £635 ..... gulden, ... 12·12 *gul.* ... £1.
6. £186 ..... Dutch florins, ... 12 *fl.* 2 *st.*† ... £1.
7. £242 ..... American dollars, ... 49*d.* ... \$1.
8. £1230 ..... roubles, ... $25\frac{1}{4}d.$ ... 1 *R.*
9. £470 ..... Spanish dollars, ... $37\frac{1}{2}d.$ ... 1 *dol.*
10. £345 ..... milreis, ... $39\frac{1}{4}d.$ ... 1 *mlr.*

Exchange—

11. 8570 *francs* into sterling, at 25·16 *fr.* for £1.
12. 3250 *marks* .... 20·36 *M.* ... £1.
13. 1875 *pesetas* .... 30·50 *pes.* ... £1.
14. 656 *florins* .... 12·09 *fl.* ... £1.
15. 4870 *lire* .... 26·84 *l.* ... £1.
16. 2895 *roubles* .... 93·70 *R.* ... £10.
17. 1680 *dollars* .... \$4·86 ... £1.
18. 64850 *rupees* .... 1*s.* $2\frac{1}{8}d.$ ... 1 rupee.
19. 498 *tael* .... 2*s.* 11*d.* ... 1 tael.
20. 564 *yen* .... 2*s.* $1\frac{1}{2}d.$ ... 1 yen.

21. Exchange £483, 16*s.* 8*d.* into francs at 25·21.
22. .... £176, 12*s.* 6*d.* ..... marks ... 20·54.
23. .... £1053, 10*s.* ..... kroner ... 18·32.
24. .... £251, 7*s.* 9*d.* ..... Dutch florins ... 12 $2\frac{3}{4}$.
25. .... £873, 15*s.* ..... rupees ... 1*s.* $2\frac{7}{8}d.$
26. .... £531, 6*s.* ..... milreis ... $39\frac{3}{8}d.$

27. Exchange 15000 kroner into sterling at 18·19.
28. .... 3875 marks .... 20·38.
29. .... 14850 francs .... $25·16\frac{1}{2}$.
30. .... 973 florins .... 12·09.
31. .... 1584 roubles .... 93·72.
32. .... 10860 rupees .... 1*s.* $2\frac{5}{32}d.$

* *Approximate* results required. † 20 stivers (*st.*)=1 *fl.*

33. Find the cost in London of a bill on Paris for 5000 francs, when the short exchange, London on Paris, is $25 \cdot 24$.

34. A merchant in London purchases goods in New York, the price of which is 3475 dollars, and remits bills in payment. Find the cost; exchange in London on New York being $4 \cdot 83$.

Find the "sight" quotations corresponding to the following "long" rates, supposing that the rate of discount is 4 p. c.:—

35. London on Paris, 3 mo., 25 32.

36. London on Brussels, 3 mo., 25 $37\frac{1}{2}$.

37. London on New York, 60 days, $48\frac{3}{4}$.

38. Berlin on London, 3 mo., 20 26.

39. St. Petersburg on London, 3 mo., 93 70.

40. Calcutta on London, 4 mo., 1*s.* $2\frac{5}{16}$*d.*

Find, when the rate of discount is 3 p. c., the quotation for 3 months' bills corresponding to the following "short" rates:—

41. London on Paris, cheques, $25 \cdot 21\frac{1}{2}$.

42. London on Amsterdam, sight, 12 $1\frac{7}{8}$.

43. Amsterdam on London, sight, $12 \cdot 09\frac{1}{2}$.

44. Lisbon on London, sight, $40\frac{1}{4}$*d.*

Find the rate of discount when the quotations are—

45. London on Paris, cheques, $25 \cdot 15\frac{1}{2}$; 3 mo., $25 \cdot 31\frac{1}{4}$.

46. Berlin on London, 8 days, $20 \cdot 35\frac{1}{4}$; sight, $20 \cdot 37$.

Find the cost in London of the bills described below, the rate of discount being 4 p. c., allowing for brokerage $\frac{1}{10}$ p. c. and stamp 1*s.* p. c.—

	Bill.	Course of Exchange.
47.	1890 florins, sight......	London on Vienna, 3 mo., $12 \cdot 13$.
48.	4896 marks, sight......	London on Hamburg, 3 mo., $20 \cdot 35$.
49.	1693 florins, sight......	London on Rotterdam, 3 mo., 12 $3\frac{3}{4}$.
50.	\$8550, sight	London on New York, sight, $48\frac{1}{2}$.

Find the cost of the following bills on London, payable at sight, supposing the rate of discount to be 3 p. c.:—

51. £423, 12*s.* 8*d.*...Paris on London, $25 \cdot 18$, cheques.

52. £857, 10*s.*........St. Petersburg on London, $93 \cdot 72$, 3 mo.

53. £238, 15*s.* 6*d.*...Rome on London, $26 \cdot 84$, 3 mo.

54. £192, 13*s.* 9*d.*...Calcutta on London, 1*s.* $2\frac{1}{8}$*d.* T.T.

55. If the cheque exchange in London on Paris is $25 \cdot 30$, and the rate of discount for 3 months' bills in London is at 5 per cent per annum, what debt in Paris can be discharged by a person in London who holds a 3 months' bill on London for £1000?

56. A merchant wishes to pay a debt of 5000 roubles in St. Petersburg, when the course of exchange is—London on St. Petersburg, 3 mo., $25\frac{1}{8}$; St. Petersburg on London, 3 mo., 93·70. Will it be better for him to remit bills to St. Petersburg, or for his agent there to draw upon him, supposing the rates of discount to be 3 p. c. in London and 4 p. c. in St. Petersburg?

57. A London merchant having to pay a Berlin merchant for goods received, finds the rates of exchange as follows:—London on Berlin, at 3 months, 20·53 marks for £1; Berlin on London, at 3 months, 20·27 marks for £1. Will it be better for him to remit direct to Berlin, or for his correspondent in Berlin to draw upon him in London? (Disc. 4 p. c. in either case.)

Find the arbitrated rate of exchange at—

58. London on Vienna, through Berlin, when the direct rates are—London on Berlin, £1 = 20·54 marks; Berlin on Vienna, 170·30 marks = 100 florins.

59. Paris on Berlin, through London, when the direct rates are—Paris on London, £1=25·18 fr.; London on Berlin, £1=20·58 marks.

60. New York on London, through Hamburg and Paris, when the direct rates are—New York on Hamburg, \$95 = 400 marks; Hamburg on Paris, 80·50 marks = 100 francs; Paris on London, £1 = 25·18 francs.

61. A New York merchant owes 20,000 marks in Berlin, the exchange at New York on Berlin being 4·85 dollars per 20 marks, and on London 4·875 dollars per pound sterling. If the exchange at London on Berlin is 20·75 marks per £1, will the merchant find it better to remit direct from New York, or through London, and what difference will it make?

62. A merchant in America wishes to send to London £7000, when exchange on London is at a premium of 5 per cent. He may send it through France or through Hamburg. (1 dollar = 5·20 francs = 2·85 marks; £1 = 25·60 francs = 13·5 marks.) Which course will be most to his advantage?

63. How much English money will it cost a London merchant to remit \$2545·20 to the United States by way of Paris and Madrid, when the course of exchange between London and Paris is 25 fr. 60 c. for £1; Paris and Madrid, 1 fr. for 1 peseta; Madrid and New York, 5 pes. 60 c. for \$1?

64. If £1 contains 113 grains pure gold, 15·432 grains = 1 gramme, and 20 francs contains 5·8 grammes pure gold; calculate the par of exchange between English and French currency.

65. Find the par of exchange between the pound sterling and the rupee, having given that the price of silver is 40*d.* the ounce of standard fineness (37 parts pure in 40 standard), and that a rupee contains $\frac{3}{8}$ oz. of standard silver (11 parts pure in 12).

LXI. THE METHOD OF NINE MULTIPLES.

Form a table of the products of 32·185 by each of the numbers from 1 to 9, and use it in obtaining the product of 32·185 and—

1. 34. **2.** 92. **3.** 187. **4.** 556. **5.** 7·2. **6.** 46·35.

Make a similar table of the products of ·38265, and use it in obtaining, correct to four places of decimals, the product of ·38265 and—

7. 48·25. **8.** 2·065. **9.** ·3975. **10.** ·38265.

Given that the price of one article is £·1375, construct a table from which the price of any number of these articles may be found by addition, and use it in obtaining, in £ *s. d.*, the price of—

11. 47. **12.** 183. **13.** 258. **14.** 462. **15.** 2967.

Given that the price of 1 ton is 13*s.* 8*d.*, make a table of the decimalized prices of 1 to 9 cwts., and use it in finding, correct to the nearest penny, the price of—

16. 14 cwts.	**19.** 36 cwts.	**22.** 2 tons 14 cwts. 2 qrs.
17. 17 cwts.	**20.** 2 tons 13 cwts.	**23.** 7 tons 9 cwts. 3 qrs.
18. 12 cwts. 2 qrs.	**21.** 5 tons 8 cwts.	**24.** 28 tons 12 cwts. 1 qr.

Given that 1 metre = 1·093633 yards, continue the table up to 9 metres, and use it in converting into yards, correct to the nearest tenth—

25. 85 *m.*	**27.** 672 *m.*	**29.** 94·8 *m.*	**31.** 87·5 *m.*
26. 246 *m.*	**28.** 1703 *m.*	**30.** 306·5 *m.*	**32.** 593·64 *m.*

Make, also, a similar table for converting yards into metres, and then express in metres, correct to the nearest centimetre—

33. 208 *yds.* **34.** 497 *yds.* **35.** $75\frac{1}{2}$ *yds.* **36.** $1643\frac{3}{4}$ *yds.*

Given that 1 *Kg.* = 2·204621 *lbs. Av.*, continue the table up to 9 *Kg.*, and use it in converting into English weight, correct to the nearest ounce—

37. 174 *Kg.* **38.** 3468 *Kg.* **39.** 392·8 *Kg.* **40.** 16·075 *Kg.*

Make a table of nine multiples for use in changing English into French money when the exchange is $20{\cdot}17\frac{1}{2}$ francs for £1, and use it in finding the value in French money of—

41. £228.	**43.** £2095, 10*s.*	**45.** £38, 8*s.* 9*d.*
42. £1376.	**44.** £436, 15*s.*	**46.** £653, 11*s.* 4*d.*

Make, also, a similar table for use in changing francs into £ *s. d.* at the same rate, and use it in the case of—

47. 8079 *fr.* **48.** 42650 *fr.* **49.** 354·5 *fr.* **50.** 1583·65 *fr.*

LXII. SQUARE ROOT.

Find, *by inspection*—

1. $\sqrt{900}$.	4. $\sqrt{10000}$.	7. $\sqrt{25 \times 9}$.	10. $\sqrt{144 \times 36}$.
2. $\sqrt{2500}$.	5. $\sqrt{360000}$.	8. $\sqrt{49 \times 16}$.	11. $\sqrt{4 \times 8 \times 8}$.
3. $\sqrt{6400}$.	6. $\sqrt{1210000}$.	9. $\sqrt{81 \times 121}$.	12. $\sqrt{7 \times 9 \times 9 \times 7}$.

13. $\sqrt{7^2}$.	17. $\sqrt{3^2 \times 2^4 \times 11^2}$.	21. $\sqrt{5 \times 29 \times 4 \times 29 \times 5}$.
14. $\sqrt{3^4}$.	18. $\sqrt{23^2 \times 5^2 \times 2^2}$.	22. $\sqrt{3 \times 4 \times 11 \times 6 \times 11 \times 2}$.
15. $\sqrt{5^2 \times 7^2}$.	19. $\sqrt{13 \times 49 \times 13}$.	23. $\sqrt{8 \times 16 \times 6 \times 3}$.
16. $\sqrt{2^6 \times 12^2}$.	20. $\sqrt{4 \times 17 \times 25 \times 17}$.	24. $\sqrt{57 \times 5 \times 19 \times 2 \times 30}$.

Find, by factors, the square root of—

25. 576.	29. 4356.	33. 16384.	37. 540225.
26. 1024.	30. 6561.	34. 59049.	38. 40960000.
27. 1296.	31. 9216.	35. 105625.	39. 1234321.
28. 3969.	32. 108900.	36. 390625.	40. 81162081.

Extract the square root of—

41. 1849.	51. 167281.	61. 10975969.	71. 404130609.
42. 3364.	52. 173889.	62. 12432676.	72. 845355625.
43. 7396.	53. 277729.	63. 19175641.	73. 1095874816.
44. 11449.	54. 552049.	64. 22297284.	74. 1156136004.
45. 43681.	55. 603729.	65. 27489049.	75. 1383542416.
46. 56169.	56. 1164241.	66. 36180225.	76. 8617223241.
47. 66049.	57. 1461681.	67. 81144064.	77. 8674873321.
48. 84681.	58. 3108169.	68. 94264681.	78. 10856806416.
49. 103041.	59. 3452164.	69. 189475225.	79. 9000426005041.
50. 106929.	60. 5322249.	70. 231496225.	80. 50085018863929.

Find, as a vulgar fraction, the square root of—

81. $\frac{1}{10000}$.	87. $\frac{36}{361}$.	93. $1\frac{9}{16}$.	99. $1\frac{165}{196}$.	105. $8\frac{289}{784}$.
82. $\frac{16}{25}$.	88. $\frac{324}{961}$.	94. $1\frac{25}{144}$.	100. $1\frac{72}{289}$.	106. $19740\frac{1}{4}$.
83. $\frac{49}{400}$.	89. $\frac{225}{729}$.	95. $1\frac{64}{225}$.	101. $2\frac{167}{529}$.	107. $72002\frac{7}{9}$.
84. $\frac{64}{121}$.	90. $\frac{1225}{4356}$.	96. $2\frac{46}{49}$.	102. $8\frac{17}{169}$.	108. $301675\frac{9}{16}$.
85. $\frac{81}{625}$.	91. $\frac{1024}{6561}$.	97. $32\frac{1}{9}$.	103. $33\frac{16}{25}$.	109. $1\frac{223}{12321}$.
86. $\frac{4}{441}$.	92. $\frac{9}{14641}$.	98. $1\frac{29}{196}$.	104. $65\frac{64}{81}$.	110. $134\frac{155}{169}$.

Find as a vulgar fraction—

111. $\sqrt{22\frac{11}{49}}$.	114. $\frac{7}{8}\sqrt{441}$.	117. $\sqrt{\frac{4}{25}}$ of $\frac{25}{49}$.	120. $\sqrt{\frac{12}{27}}$.
112. $\sqrt{371\frac{46}{49}}$.	115. $28\sqrt{3\frac{1}{16}}$.	118. $\sqrt{2\frac{1}{4}} \div \sqrt{1\frac{9}{16}}$.	121. $\sqrt{\frac{125}{320}}$.
113. $\sqrt{1\frac{1261}{1764}}$.	116. $\sqrt{\frac{2}{5} \text{ of } \frac{10}{49}}$.	119. $\sqrt{2\frac{1}{4}} \div 1\frac{9}{16}$.	122. $\sqrt{2\frac{84}{150}}$.

Find, by inspection, the square root of—

123. 1·21.
124. 1·44.
125. ·04.
126. ·09.
127. ·64.
128. ·81.
129. ·0001.
130. ·0036.
131. ·0049.
132. ·000004.
133. ·000025.
134. ·000144.

Extract the square root of—

135. 4·41.
136. ·0289.
137. ·5329.
138. ·0000000484.
139. 213·16.
140. 9·7969.
141. ·091809.
142. 1274·49.
143. 20·8849.
144. 25·5025.
145. 1·002001.
146. 121·2201.
147. 19740·25.
148. 2·832489.
149. 901·8009.
150. 70702·81.
151. ·01595169.
152. ·00819025.
153. 1227·8016.
154. 1280·9241.
155. 18·593344.
156. 315956·41.
157. 4281·0849.
158. 0·08450649.
159. 6248·9025.
160. 64·128064.
161. 125·686521.
162. 14636·1604.
163. 1830879·61.
164. ·03598609.
165. 3·54455929.
166. 58095·4609.
167. 30·86358025.
168. 429496·7296.
169. 60691890·25.
170. 63·84169801.
171. 240398·012416.
172. 4281·35971041.
173. 3601207301·2036.

Find the first *six* significant figures of the square root of—

174. 2.
175. 3.
176. 5.
177. 70.
178. 2·5.
179. 48·4.
180. 0·51.
181. ·051.
182. ·3.
183. ·4.
184. 3·1416.
185. ·00056.
186. ·00001.
187. 4032·24.
188. 66·13531715.

Find, correct to three places of decimals, the square root of—

189. $1\cdot\dot{6}$.
190. $\cdot\dot{3}$.
191. $7\cdot08\dot{3}$.
192. $\cdot\dot{7}\dot{2}$.
193. $\cdot00\dot{4}2\dot{3}$.
194. $20\cdot41\dot{6}$.
195. $\cdot0\dot{1}4285\dot{7}$.
196. $\cdot\dot{6}\dot{4}$.

Find, *as a vulgar fraction*, the square root of—

197. $\cdot\dot{4}$.
198. $\cdot00\dot{1}$.
199. $\cdot02\dot{7}$.
200. $\cdot13\dot{4}$.
201. $3\cdot36\dot{1}$.
202. $4\cdot98\dot{7}$.
203. $8\cdot02\dot{7}$.
204. $\cdot0802\dot{7}$.

Find, correct to three places of decimals—

205. $\sqrt{\frac{7}{9}}$.
206. $\sqrt{\frac{5}{16}}$.
207. $\sqrt{1\frac{3}{25}}$.
208. $\sqrt{4\frac{9}{64}}$.
209. $\sqrt{\frac{3}{5}}$.
210. $\sqrt{\frac{1}{11}}$.
211. $\sqrt{\frac{5}{12}}$.
212. $\sqrt{1\frac{1}{8}}$.
213. $\sqrt{2\frac{1}{6}}$.
214. $\dfrac{\sqrt{29}}{\sqrt{24}}$.
215. $\sqrt{\frac{22}{7}}$.
216. $\sqrt{\frac{23}{32}}$.
217. $\sqrt{\frac{53}{150}}$.
218. $\sqrt{\frac{465}{37}}$.
219. $\sqrt{\dfrac{\cdot128}{12\cdot5}}$.
220. $\sqrt{\dfrac{\cdot00125}{\cdot18}}$.

Having given that $\sqrt{2} = 1{\cdot}414213...$, $\sqrt{3} = 1{\cdot}732050...$, $\sqrt{5} = 2{\cdot}236067...$, $\sqrt{6} = 2{\cdot}449489...$, $\sqrt{7} = 2{\cdot}645751...$;
find, correct to four places of decimals—

221. $7\sqrt{2}$.
222. $40\sqrt{3}$.
223. $13 - 4\sqrt{5}$.
224. $4\sqrt{2} - 2\sqrt{3}$.
225. $4\sqrt{3} \times \sqrt{2}$.
226. $\sqrt{2} \times \sqrt{10}$.
227. $\sqrt{2 + \sqrt{3}}$.
228. $3\sqrt{4 - \sqrt{7}}$.
229. $\dfrac{5}{\sqrt{5}}$.
230. $\dfrac{3}{\sqrt{3}}$.
231. $\dfrac{12}{\sqrt{6}}$.
232. $\dfrac{2 + \sqrt{3}}{\sqrt{2}}$.
233. $\dfrac{11 - \sqrt{5}}{\sqrt{3}}$.
234. $\dfrac{1 + \sqrt{7}}{2\sqrt{2}}$.
235. $\dfrac{1}{\sqrt{2} + 1}$.
236. $\dfrac{1}{\sqrt{5} - 1}$.
237. $\dfrac{1}{\sqrt{6} - 1}$.
238. $\dfrac{1}{\sqrt{7} + 1}$.
239. $\dfrac{1}{8 - 3\sqrt{7}}$.
240. $\dfrac{3}{3\sqrt{3} - 5}$.
241. $\dfrac{\sqrt{2}}{2 - \sqrt{3}}$.
242. $\dfrac{4\sqrt{3}}{10 + 7\sqrt{2}}$.
243. $\dfrac{\sqrt{3} - 1}{\sqrt{3} + 1}$.
244. $\dfrac{3 + 2\sqrt{2}}{3 - 2\sqrt{2}}$.
245. $\dfrac{11\sqrt{3} - 7\sqrt{7}}{11\sqrt{3} + 7\sqrt{7}}$.
246. $\dfrac{6\sqrt{5} + 5\sqrt{6}}{6\sqrt{5} - 5\sqrt{6}}$.

Calculate to five places of decimals the value of—

247. $\sqrt{\left(\dfrac{\sqrt{7} - \sqrt{3}}{\sqrt{7} + \sqrt{3}}\right)}$.
248. $\sqrt{\left(\dfrac{\sqrt{13} + 3}{\sqrt{13} - 3}\right)}$.
249. $\dfrac{1}{\sqrt{7} + \dfrac{1}{\sqrt{7} + \dfrac{1}{\sqrt{7} + \dfrac{1}{\sqrt{7}}}}}$

250. Find the value of $\dfrac{7 + \sqrt{5}}{6 - \sqrt{5}} - \dfrac{7 - \sqrt{5}}{6 + \sqrt{5}}$ to three places of decimals.

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251. The area of a square floor is 53 sq. yds. 7 sq. ft.; find the length of a side.
252. If the area of a square floor is 30 sq. yds. 5 sq. ft. 1 sq. in., what is the length of a side?
253. The length of a rectangular floor is double its breadth, and its area is 59 sq. yds. 8 sq. ft. 2 sq. in.; find its length.
254. Find the perimeter of a square floor the area of which is 51 sq. yds. 3 sq. ft. 36 sq. in.
255. A square field contains $2\frac{1}{2}$ acres; find the length of its side.
256. The area of a square falls short of 10 acres by 439 square yards; find the length of each side.
257. A square lawn contains 1 acre 6 perches $19\frac{1}{2}$ sq. yds.; find the length of its side.

258. Find the perimeter of a square field of 10 acres.

259. What is the breadth of a rectangular field three times as long as it is broad, if its area is 18 A. 0 R. 3 P.?

260. Find the length of the side of a square equal in area to the sum of two squares whose sides are 28 yards, and 11 yds. 2 feet.

261. How long will it take a man to walk round a square field, whose area is 40 acres, at the rate of 4 miles an hour?

262. A square field contains 21 ac. 3 ro. 1 po.: how long will it take a man to run round it at the rate of $7\frac{1}{2}$ miles an hour?

263. Find the cost of fencing a square field containing 10 acres at 4*s.* 6*d.* per yard.

264. How much will it cost to surround a field, whose area is 3 acres 3 roods 619 sq. yards, with netting at $1\frac{1}{2}d.$ a foot?

265. If the cost of making a square lawn be £351, 18*s.* $4\frac{1}{2}d.$, at the rate of 3*s.* $4\frac{1}{2}d.$ per square yard, how many feet long is it?

266. A square field contains $2\frac{1}{2}$ acres. What is the cost of making a path three yards wide inside the field round the boundary at 1*s.* 6*d.* per sq. yard?

267. If it cost £643, 10*s.* 9*d.* to level a square cricket ground at 9*d.* per square yard, what will it cost to enclose it with a fence at 7*s.* 6*d.* per yard?

268. A sum of £252, 1*s.* was divided among a number of persons, and each person received as many shillings as there were persons. How much did each person receive?

269. If a piece of silk costs £15, 15*s.* $2\frac{1}{4}d.$, and the number of yards in its length is the same as the number of pence in the cost of one yard, what is the length of the silk, and its price per yard?

270. A tourist found that he had spent on the average daily half as many shillings as there were days in his holiday. He spent altogether £57, 12*s.*; how many days did his tour last?

271. When a regiment of 962 men is drawn up in a solid square, one man is left out; find the number of men in the front rank.

272. A company of men can be formed into a hollow square 9 deep, having 970 men in the front rank of each side. How many men would there be in the front rank of each side if the company were formed into a solid square?

273. A beam, the section of which is square, is 24 feet long and its volume is $8\frac{1}{6}$ cubic feet; find its width.

274. A square chess-board contains 64 equal squares, and the area of each square is 1·3225 sq. in. The rim round the board is two-fifths of an inch wide. Find the length of the board.

275. The sum of the areas of three squares is 1 sq. yd. 6 sq. ft. 94 sq. in. The first is four times as large as the second, and the second is nine times as large as the third. Find the lengths of the sides of the squares.

276. Find the square root of the sum of the squares of 3·9, 5·2, and 15·6.

277. Subtract the number whose square root is ·01 from the number whose square is ·01871424.

278. Find, within one millionth of the complete result, the square root of 37·7.

279. Find the first six significant figures in the square root of ·005.

280. Show that $\sqrt{7}$ is intermediate in value between $\frac{66}{25}$ and $\frac{53}{20}$.

281. Find, within an inch, the length of the side of a square floor the area of which is 195 square feet.

282. Find, within an inch, the length of one of the sides of a square field whose area is 3 acres.

283. Find, approximately, the length of the side of a square which is equal in area to a rectangle 660 yds. long and 376 yds. broad.

284. Find the least integer by which 290304 must be either multiplied or divided, so that the result may be a perfect square.

285. Find the least integer which (i) added to, (ii) subtracted from, 47962 makes the resulting number a perfect square.

286. Find the least integer by which 2008008 must be multiplied so as to make the product a perfect square.

287. Evaluate $\sqrt[4]{81 \times 256}$ and $\sqrt[4]{23\frac{266}{625}}$.

288. Find the fourth root of 112550881.

289. Find the fourth root of 53·1441.

290. Find the fourth root of 17 to four places of decimals.

291. Find a mean proportional between 651 and 2604.

292. Find correct to four places of decimals a mean proportional between 21·437 and 437·21.

293. A rectangular schoolroom is 21 yds. 1 ft. long and 16 yds. wide; find the distance between opposite corners.

294. The side of a square is 17 feet; find, to the nearest inch, the distance between opposite corners.

295. A rectangular field is 6 chains 80 links in length and 5 chains 10 links in breadth; find the distance between two opposite corners of the field.

296. The diagonal of a square is 7 inches long; find, within a thousandth of an inch, the length of a side of the square.

297. A ladder 41 feet long, placed with its foot 9 feet from a wall, just reaches to the top of the wall. How high is the wall?

At what rate per cent compound interest will—

298. £625 amount to £676 in 2 years?

299. £1500 gain £101, 13*s.* 4*d.* in 2 years?

300. £1024 amount to £1305, 0*s.* $3\frac{3}{4}$*d.* in 4 years?

LXIII. CUBE ROOT.

Find, *by inspection*, the cube root of—

1. 8000.
2. 27000.
3. 64000.
4. 125000.
5. 8×27.
6. 1728.
7. $4^3 \times 6^3$.
8. $7^3 \times 3^6$.

Find, by inspection—

9. $\sqrt[3]{11 \times 11 \times 11 \times 7 \times 7 \times 7}$.
10. $\sqrt[3]{13 \times 13 \times 13 \times 8}$.
11. $\sqrt[3]{700 \times 2 \times 49 \times 5}$.
12. $\sqrt[3]{121 \times 64 \times 2 \times 44}$.

Find, by factors, the cube root of—

13. 729.
14. 3375.
15. 13824.
16. 21952.
17. 35937.
18. 884736.
19. 1953125.
20. 182284263.

Extract the cube root of—

21. 4913.
22. 12167.
23. 39304.
24. 50653.
25. 110592.
26. 274625.
27. 493039.
28. 571787.
29. 2248091.
30. 8365427.
31. 857375000.
32. 1191016000.
33. 12812904.
34. 26463592.
35. 284890312.
36. 397065375.
37. 1334633301.
38. 24414238701.
39. 83568086848.
40. 411166897856.
41. 1027243729000.

Find, as a vulgar fraction, the cube root of—

42. $\frac{27}{8000}$.
43. $\frac{125}{1728}$.
44. $\frac{64}{343}$.
45. $1\frac{91}{125}$.
46. $1\frac{217}{512}$.
47. $20\frac{51}{64}$.
48. $41\frac{301}{1331}$.
49. $2345\frac{22}{343}$.
50. $423987\frac{457}{512}$.
51. $2\cdot\dot{3}7\dot{0}$.
52. $\cdot 000\dot{2}9\dot{6}$.
53. $\cdot 004\dot{6}2\dot{9}$.

Find, by inspection—

54. $\sqrt[3]{\cdot 008}$.
55. $\sqrt[3]{\cdot 027}$.
56. $\sqrt[3]{1\cdot 728}$.
57. $\sqrt[3]{\cdot 000001}$.

Extract the cube root of—

58. 2·197.
59. 29·791.
60. ·438976.
61. 6434·856.
62. 34·965783.
63. 768575·296.
64. 29993·266043.
65. 135400835·375.
66. 2079995·797125.

Find, correct to two places of decimals, the cube root of—

67. 2.
68. 3.
69. 7.
70. $\frac{5}{8}$.
71. $\frac{4}{27}$.
72. $1\frac{9}{64}$.
73. 3·5.
74. ·6315.
75. ·08.
76. $\cdot 1\dot{6}$.
77. $\frac{1}{3}$.
78. $\frac{5}{11}$.
79. $3\frac{1}{7}$.
80. $\frac{17}{24}$.
81. $5\frac{104}{105}$.

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82. Find the integral part of the cube root of 7777777.
83. The volume of a cubical block is 2248091 cubic inches; find the length of its edge.

84. The volume of a cubical block is 2248·091 cubic feet; find the area of one of its faces.

85. Find the area of the total surface of a cube whose volume is 91 cub. ft. 216 cub. in.

86. Prove that the length of a side of a square, the area of which is 15876 square yards, is to that of an edge of a cube the contents of which are 94818816000 cubic inches as 189 : 190.

87. The sides of a rectangular vessel are 10 ft. 1 in., 11 ft. 11 in., and 14 ft. 1 in. long respectively. Find the length of a side of a cubical vessel of equal volume.

88. Given that a cubic metre is equal to 35·316581 cubic feet, find the length in feet of a linear metre, correct to four places of decimals.

89. The volume of wood in a cubical box of uniform thickness is $732\frac{3}{8}$ cubic inches. If the capacity of the box is a cubic foot, what is the thickness of the wood?

90. Two cubical tanks contain together 6,133,248 cubic feet, and the edge of one is twice as long as the edge of the other; find the edge of each.

91. A beam, the section of which is square, is 27 times as long as it is wide and its volume is 8 cubic feet; find its length.

92. Three numbers are to one another as 2 : 3 : 4. The sum of their cubes is 33957. Find the numbers.

93. A room contains 4394 cubic feet of air; its length is double its width and its width is equal to its height. Find the cost of carpeting it at 4*s.* 6*d.* per square yard.

94. Find the smallest integral multiplier of 16435188 which will convert it into a perfect cube.

95. Find the least integer which added to 33333 makes the result a perfect cube.

Find, *by inspection*, the cube root of each of the following *perfect cubes*:—

96. 1331.	98. 5832.	100. 85184.	102. 571787.
97. 4096.	99. 15625.	101. 205379.	103. 912673.

104. Find, by factors, $\sqrt[5]{164916224}$.

105. Find the sixth root of $1838\frac{17}{64}$.

106. Find the sixth root of 2 to three places of decimals.

107. Find the ninth root of 2357947691.

108. Find, approximately, the length of the side of a cubical cistern which holds 1000 gallons.

At what rate per cent, compound interest, will—

109. £3125 amount to £3515, 4*s.* in 3 years?

110. £1562, 10*s.* gain £95, 12*s.* 9*d.* in 3 years?

LXIV. SCALES OF NOTATION.

Add together, in

1. *Scale Five*, 4321; 203; 1304 and 32.
2. *Scale Eight*, 14735; 20664; 176 and 5640.
3. *Scale Seven*, 6354; 142503; 6554; 253 and 465.
4. *Scale Four*, 3120; 223; 1032; 2303 and 321.
5. *Scale Eleven*, 12879; 46t3; t52; 48t and 1t369.
6. *Scale Twelve*, 4578; 3607t; 14e75; te7 and 864.

Subtract, in

7. *Scale* 6, 35042 from 51200.
8. *Scale* 3, 10212 from 11020.
9. *Scale* 7, 40546 from 152364.
10. *Scale* 9, 785614 from 1230203.
11. *Scale* 11, 5t34t2 from 942t30.
12. *Scale* 12, t98te from 17e0t2.

Multiply, in

13. *Scale* 5, 20431 by 34.
14. *Scale* 8, 5746 by 75.
15. *Scale* 6, 3452 by 243.
16. *Scale* 7, 16054 by 1056.
17. *Scale* 11, 2t385 by 72t.
18. *Scale* 12, 64et9e by t8e.

Divide, in

19. *Scale* 9, 7134504 by 8.
20. *Scale* 11, 362t076 by 11.
21. *Scale* 4, 30321 by 23.
22. *Scale* 5, 4203312 by 34.
23. *Scale* 8, 7642435 by 507.
24. *Scale* 12, 5e78783 by 2t7.

25. Change 5784 from the common scale to scale 7.
26. .... 12957 .... 3.
27. .... 64021 from scale 10 to scale 11.
28. .... 79463 .... 12.
29. .... 3441 from scale 5 to the common scale.
30. .... 2463 .... 8 ....
31. .... 5342 .... 6 to scale 4.
32. .... 25361 .... 7 .... 9.
33. .... 30t5e .... 12 .... 3.
34. .... 32103 .... 4 .... 11.
35. Express 142857 in the Octenary scale.
36. Transform 31415 from the Senary to the Nonary scale.
37. Transform 687 from the Denary to the Binary scale.
38. Change 33201 from the Quaternary to the Undenary scale.
39. Change 4t3e0 from the Duodecimal to the Quinary scale.
40. Change 66666 from the Septenary to the Decimal scale.

41. Change 82·125 from the common scale to scale 6.
42. .... 417·3125 .... 12.
43. .... 1201·12 from scale 3 to the common scale.
44. .... 55·6 .... 7 ....
45. .... 675·24 .... 8 .... 11.
46. .... $3130 \cdot 0\dot{2}$ .... 4 .... 2.

Extract the square root of—

47. 21652 in the scale of 7.
48. 373444 .... 9.
49. 148115 in the scale of 11.
50. 3106571 .... 8.

51. Find the square root of t2t1 in the duodenary scale.
52. Express in the common scale the greatest and least numbers which can be expressed with four digits in scale 6.
53. How many times is the greatest number of three figures in scale 4 contained in the greatest number of four figures in scale 8?
54. The numbers 345 and 303 are in scale 7; find the cube root of their product in that scale.
55. In what scale of notation is 357234 expressed by 3015333?
56. Express the decimal ·5625 as a duodecimal.
57. Transform 275·9375 from the decimal to the duodecimal scale.
58. Transfer 355·41 from scale 6 to scale 10.
59. 234·521 is in scale 6; express this number in scale 12.
60. In what scale does the duodecimal number 3tte become 15143?
61. 50010 is in scale 7; find its prime factors in that scale.
62. Express $\frac{1}{3}$ as a radix fraction in the scale of 5.
63. Express the decimal $51 \cdot 2\dot{7}$ in the duodecimal scale.
64. Change $37 \cdot 2291\dot{6}$ from scale 10 to scale 12.
65. When can a vulgar fraction in the senary scale be converted into a terminating radix fraction? Will the denary fraction $\frac{19}{432}$ produce a terminating or a recurring radix fraction in the senary scale?
66. $\cdot 14\dot{6}$ is in scale 9; convert it into a vulgar fraction in its lowest terms in that scale.
67. $\cdot \dot{2}5\dot{3}$ is in scale 6; express it as a vulgar fraction in that scale.
68. $\cdot 5\dot{2}\dot{4}$ is in scale 12; express it as a vulgar fraction in that scale.
69. Find which of the series of weights 1 lb., 2 lbs., 4 lbs., 8 lbs., 16 lbs. &c., must be taken in order to weigh 233 lbs.
70. How can weights of 1, 3, 3^2, 3^3, 3^4, &c. lbs. be used in a balance so as to weigh (i) 334 lbs.; (ii) 574 lbs.?

LXV. MISCELLANEOUS EXERCISES.

1. Shakespeare was born in the year MDLXIV and died in MDCXVI. At what age did he die?

2. If a man spends as much in four months as he earns in three, how much does he save in a year out of an income of £150?

3. Find, without waste of labour, the total cost of 112 lbs. of meat at $11\frac{1}{2}d.$ per lb., 112 lbs. at $7\frac{3}{4}d.$, 112 lbs. at $10\frac{1}{4}d.$, and 112 lbs. at $9\frac{1}{2}d.$

4. Resolve 6006 and 7854 into their prime factors; and deduce their greatest common measure.

5. Express $\frac{13}{17}$ with numerator 117, and $\frac{560}{840}$ with denominator 6.

6. Simplify $\frac{117}{285}$ of $1\frac{61}{91} - \dfrac{5\frac{6}{7}}{13\frac{2}{3}} + 2\frac{1}{7}$ of $\left(\dfrac{11\frac{5}{7}}{5\frac{6}{7}} - 1\frac{1}{13} \div \frac{25}{26}\right)$.

7. Divide ·005868 by ·036, and arrange the divisor, dividend, and quotient in descending order of magnitude.

8. The weights of the four oarsmen in a boat are respectively 10 st. 6 lbs., 10 st. 9 lbs., 11 st. 1 lb., and 11 st. 3 lbs., and the average weight of the whole crew, including the coxswain, is 10 st. Find the weight of the coxswain.

9. If a swimmer takes 25 strokes a minute, and goes $2\frac{1}{4}$ yards each stroke, how long does it take him to swim 300 yards?

10. How many men, engaged at $5\frac{1}{2}d.$ per hour for 9 hours on each of 23 days, earn the same wages as 22 men at $5\frac{3}{4}d.$ per hour for $9\frac{1}{2}$ hours on each of 18 days?

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11. Multiply 3478192 by 509070, and test the result by casting out nines.

12. An equal number of men and women earned £54, 12*s.* in 4 weeks, the men receiving 15*s.* and the women 7*s.* 9*d.* a week; how many men were there?

13. Find the sum of all the numbers between 200 and 300 which are exactly divisible by 9.

14. Show that $\dfrac{3255}{4557}$ is equal to $\dfrac{2805}{3927}$.

15. Simplify $\dfrac{1}{2 + \dfrac{2}{3 + \dfrac{3}{4 + \frac{4}{5}}}}$ and $\dfrac{1}{2 - \dfrac{2}{3 - \dfrac{3}{4 - \frac{4}{5}}}}$.

16. Find to four significant figures the quotient of $1·376 \div 14·72$.

17. Make out a bill for 2 doz. pairs of gloves at 2*s.* $11\frac{1}{2}d.$ a pair, 37 yds. of chintz at $9\frac{3}{4}d.$ a yard, 1 gross of buttons at $3\frac{1}{4}d.$ a dozen, 164 yds. of fringe at 1*s.* $3\frac{3}{4}d.$ a doz. yards.

18. Find by Practice the value of 157 miles 3 fur. 24 poles of telegraph-wire at £11, 10*s.* per mile.

19. How many tacks if put 6 inches apart would be required for a carpet 16 feet long and 12 feet wide?

20. If a certain amount of food last 165 men 40 days, how long will three times the amount last 100 men, who eat on an average one-fifth as much again?

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21. Find the number nearest to 2146537 which is exactly divisible by 4275.

22. Two equal sums were divided, the one amongst 141 men, and the other amongst a certain number of women; each man received twenty-five shillings, and each woman eighteenpence less; how many women were there?

23. If $\frac{3}{7}$ of $4\frac{1}{4}$ be subtracted from a certain fraction, and the difference be multiplied by 70, the product obtained is $110\frac{1}{2}$; find the fraction.

24. Divide $990\cdot6$ by $\cdot3048$, and then write down the quotient of $99\cdot06$ divided by $3\cdot048$.

25. Find the value in cubic feet and inches of $\cdot226\dot{8}5\dot{1}$ of a cubic yard.

26. If a bankrupt pay 11*s.* 4*d.* in the pound, how much will a creditor receive to whom he owes £51, 8*s.* 9*d.*?

27. If 3 men, or 5 women, can do a piece of work in 10 days, in what time could it be done by 4 men and 10 women working together?

28. Divide 801 into two parts which shall be to one another as 2 is to 7.

29. Two pipes together fill a cistern in 1 hour: one of them alone fills it in 3 hours; how long does the other alone take to fill it?

30. Two trains start at 9 A.M. from stations 200 miles apart, and approach each other, their rates being 25 and 35 miles an hour respectively; when will they meet?

If their lengths are 60 and 72 yds., how long will they take to pass each other completely?

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31. Find both the greatest and least numbers each of four digits which are exactly divisible by 17.

32. Find the value of $\dfrac{£4651,\ 9s.\ 4\frac{1}{2}d.}{365}$, and of $\dfrac{£12,\ 18s.\ 6\frac{1}{2}d.}{15s.\ 2\frac{1}{2}d.}$.

33. Subtract $(3\frac{5}{7} - 2\frac{26}{35}) \times \dfrac{\frac{5}{17}}{\frac{3}{14}}$ from $78\frac{1}{7} + 23\frac{4}{21}$.

34. Simplify (i) $19\cdot425 - 26\cdot05 + 57\cdot0785 - 30\cdot515$; (ii) $\cdot\dot{9}2307\dot{6} \times 1\cdot04$.

35. Divide £701, 12*s.* 9*d.* among A, B, and C, so that A may have £3, 7*s.* 1*d.* more than B, and B £5, 11*s.* 10*d.* more than C.

36. A garrison of 2000 men has provisions for 54 days; at the end of 15 days a reinforcement arrives, and it is found that there are now provisions for only 20 days. What was the number of the reinforcement?

37. A man's gross income is £863; find his net income after deducting 4*d.* in the pound for income-tax.

38. Find a number which bears the same ratio to 100 that 44 does to 1760.

39. A can do a piece of work in 6 days, B in 8 days, and C in 12 days; B and C work together for 2 days and then A takes C's place; how soon will the work be finished?

40. A starts at 9 A.M. at the rate of 9 miles per hour; at 9.30 B follows him at the rate of $10\frac{1}{2}$ miles per hour; at what time will B overtake A, and how far will they have travelled?

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41. Reduce 175 ozs. Troy to ozs. Avoirdupois.

42. If eggs are bought at three for twopence and sold at four for threepence, how many must be sold to realize a profit of two shillings?

43. The least common multiple of two numbers is 2145, their greatest common measure is 13, and one of the numbers is 143. What is the other?

44. Which is the greater $\dfrac{2\frac{3}{8}+3\frac{1}{4}}{7\frac{2}{3}-1\frac{5}{6}}$ or $\dfrac{3\frac{2}{7}+4\frac{2}{5} \text{ of } \frac{20}{33}}{6\frac{2}{3}-1\frac{5}{6} \text{ of } \frac{20}{33}}$?

45. Subtract $\cdot\dot{1}9047\dot{6}$ of a guinea from $\cdot 208\dot{3}$ of a pound.

46. What would a rate of 1*s.* 11*d.* in the pound amount to on property valued at £530 a year?

47. How many blocks of wood, 8 inches by 4 inches, will pave a floor 53 feet by 18 feet?

48. If a train goes 15 miles an hour, what is its speed in feet per second? If the train is 110 yards long, how long will it take, going at this speed, to cross a bridge 165 yards long?

49. If either 14 men, or 23 boys, could do a certain piece of work in 25 days, how long would 23 men and 14 boys take to do it?

50. Divide a legacy of £345, 12*s.* among three persons so that their respective shares shall be as the numbers 7, 6, 5.

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51. A certain number is divided by 63 by short divisions; the first divisor is 7 and the first remainder 1; the second remainder is 8 and the quotient is 66; find the number.

52. Taking the prices per quarter of wheat for the 5 years commencing 1837 to be £2, 15*s.* 10*d.*, £3, 4*s.* 6*d.*, £3, 10*s.* 8*d.*, £3, 6*s.* 4*d.*, and £3, 4*s.* 4*d.*; also for the 5 years ending 1889 to be £1, 12*s.* 10*d.*, £1, 11*s.*, £1, 12*s.* 6*d.*, £1, 11*s.* 10*d.*, and £1, 9*s.* 9*d.*, determine how much less the average price per quarter for the latter 5 years is than the average price per quarter for the former 5 years.

53. How many pieces, each $\cdot 0015$ inches long can be cut from a line whose length is $2\cdot 41$ inches? How long is the piece which is left over?

54. Divide $\dfrac{2\frac{1}{7}}{11} \times \dfrac{2\frac{1}{3}+6\frac{2}{9}}{\frac{3}{70} \div \left(\frac{4}{7}-\frac{1}{2}\right)}$ by $\dfrac{1\cdot 8\dot{3} \times \cdot 43\dot{1}\dot{8}}{1\cdot 1875}$.

55. Find, by Practice, the rent of a farm of 899 acres 2 roods 30 poles at £1, 16*s.* 4*d.* an acre.

56. A bankrupt's assets are £1472, 7*s.* 6*d.* and his debts amount to £7634, 5*s.*; if the expenses of the bankruptcy amount to £200, what dividend will the creditors receive?

57. If 4 men and 8 boys working together can do 5 times as much work per hour as a man and a boy together, compare the working power of a boy with that of a man.

58. A train 160 ft. long is travelling at the rate of 25 miles an hour. Find (i) how long it will be in passing completely over a bridge 240 yds. long; (ii) how long it will be in passing completely a train 170 ft. long travelling in the opposite direction at the rate of 20 miles an hour.

59. Find the cost of staining, at $10\frac{1}{2}d.$ per sq. yd., a border 2 feet wide all round the floor of a room which is 30 feet long and 19 feet wide.

60. If 3 pears are worth 7 apples, 19 apples worth 4 apricots, and 5 apricots worth 22 plums, how many pears should be given in exchange for 616 plums?

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61. A number is divided by 5, 7, 11 in succession; the remainders are 4, 3, 9 respectively, and the final quotient is 170: find the number.

62. Find the least number that must be added to 222222 so that the sum may be divisible by 47, 53, and 59.

63. Show that $\dfrac{2\frac{1}{4}s.}{£2\frac{1}{8}} + \dfrac{7\frac{1}{2} \text{ lbs.}}{1\frac{1}{4} \text{ cwt.}} = \dfrac{1 \text{ ac. } 3 \text{ ro. } 32 \text{ po.}}{20 \text{ ac. } 3 \text{ ro. } 8 \text{ po.}}$.

64. How many times can a jug holding ·078125 of a gallon be filled from a vessel containing 786·375 gallons, and what part of a pint would then be left in the vessel?

65. After paying *5d.* in the pound income tax, a man has a net income of £1457. Find his gross income.

66. An estate consists of plots of land containing, respectively, 8 ac. 2 ro. 22 per., 1 ro. 29 per., 1 ro. 35 per., 7 ac. 37 per., 6 ac. 3 ro. 27 per., 2 ac. 2 ro. 8 per., 8 ac. 2 ro. 16 per., 15 ac., 9 ac., 11 per., and 2 ac. 18 per. What sum would be realized by selling the whole at £25, 10*s.* an acre?

67. The hands of a watch indicate half-past three; in how many minutes will the hands be together?

68. A number of tiles six inches square and half an inch thick are piled up into a stack 8 feet long, 2 ft. 6 in. wide and 4 ft. 5 in. high: find the number.

69. After spending $\frac{5}{9}$ of the money in my purse, then $\frac{3}{4}$ of the remainder, and then $\frac{4}{13}$ of what was still left, I found I had $3s.\ 9\frac{1}{2}d.$ over. How much had I at first?

70. A contractor engages to finish 3 miles of road in 200 days, and employs for that purpose 90 workmen. After they have been working 65 days, he finds that they have completed only 6 furlongs of it. How many additional men must he employ to complete the work in the specified time?

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71. Simplify $21 + 36 \div 4 - 4 \times 2 - 22$, and $13 - 24 + 87 - 93 + 66 - 49$.

72. The eldest of three sisters is 5 years older than the second, and the second is 3 years older than the youngest; their united ages amount to 44 years; find the age of each.

73. If a boy weighed half as much again as he does he would weigh 11 st. 11 lbs.; what is his weight?

74. If sugar is bought wholesale at £16 per ton, what is the lowest price per lb. in English money at which it can be sold retail without loss, and what is then the gain on each ton?

75. What number is the same multiple of 6289 that 7701 is of 453?

76. Find as shortly as you can the sum of $\frac{224}{225}$ and $\frac{399}{400}$. Express in its simplest form $\cdot 002 \times 36\cdot 25 \div \cdot 029 - \dfrac{102\cdot 85 \times \cdot 04}{1\cdot 7}$.

77. Find the cost of 85 kilog. 25 gram. of butter at 2 francs 75 centimes per kilogramme.

78. Find the simple interest on £369, 3*s.* 4*d.* for 3 years 73 days at $9\frac{3}{8}$ per cent per annum?

79. If a woman's work is equal to that of two boys, or to half that of a man; how long will it take 5 women and 6 boys to do what 3 men can do in 24 days?

80. A and B row a mile race. A takes 38 strokes per minute, and B 73 strokes in 2 minutes; but 25 of B's strokes are as effective as 26 of A's. Which wins?

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81. Simplify $(3^2 - 5^2 + 7^2)(5^2 - 3^2 + 7^2)(7^2 - 3^2 - 5^2)$.

82. How many lbs. of tea at 2*s.* 2*d.* per lb., with double that quantity of coffee at 1*s.* 7*d.* per lb., can be bought for £4, 10*s.* 8*d.*?

83. Find the least number which when divided by 39 leaves remainder 38, and when divided by 65 leaves remainder 64.

84. Add $1\frac{3}{845}$, $1\frac{11}{676}$, and $1\frac{22}{169}$; and find the value of $\dfrac{35 - 8\frac{1}{3} \times 4\frac{1}{5}}{35 \div 8\frac{1}{3} + 4\frac{1}{5}}$.

85. Divide $\dfrac{1\cdot 08}{\cdot 18}$ of $\cdot 36$ by $\dfrac{3\cdot 6}{\cdot 05}$ of $\cdot 6$; and multiply $1\cdot 2\dot{7}2\dot{9}$ by $\cdot 012\dot{3}$, giving the results in decimals.

86. Find by Practice the yield of corn from 41 ac. 3 ro. 20 poles of land at 3 qrs. 3 bush. 1 peck per acre.

87. Find the cost of 14 litres 2 decilitres of liqueur at 72 francs 25 centimes per decalitre.

88. At what time between 5 and 6 o'clock is the minute hand of a watch 19 minute spaces in advance of the hour hand?

89. How much money put out to interest at 3 per cent per annum will yield an income of £238, 15*s.* 3*d.*?

90. The outer circumference of an iron roller 4 ft. 7 in. broad is 9 ft. 2 in. How many revolutions must it make, and how far must it travel along a road, to cover an acre of ground?

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91. By what number (greater than 100) has 463759 been multiplied if the last three figures on the right of the product are 757?

92. Find, to the nearest farthing, the cost of 17 lbs. 6 ozs. of beef at $8\frac{1}{2}$*d.* per lb.

93. Reduce $\frac{5}{24}$ and $\frac{7}{11}$ to decimals, and find their product correct to three decimal places.

94. Find by Practice the cost of 888800 bricks at £1, 17*s.* 6*d.* per thousand.

95. Find the cost of 12 tons 2 cwt. 2 qrs. if 10 tons 9 cwt. 56 lb. cost £97, 15*s.* 4*d.*

96. For what time is £12, 17*s.* 2*d.* the simple interest on £128, 11*s.* 8*d.* at 4 per cent. per annum?

97. If 6*s.* 6*d.* be gained by selling 26 yards at 3*s.* $4\frac{1}{2}$*d.* per yard, what is the gain per cent?

98. A and B ride a race of 50 miles on bicycles. The wheel of A's bicycle makes three revolutions in 2 seconds, and that of B 960 revolutions in 10 minutes 15 seconds. A accomplishes the distance in 3 hours 20 minutes, and wins by 5 minutes. Find the circumference of the wheel of each bicycle.

99. How many postage stamps, each $\frac{15}{16}$ of an inch long and $\frac{3}{4}$ of an inch wide, would make a frieze a foot wide all round a room 16 feet 6 in. long and 14 feet 9 in. wide?

100. The wages of A and B, working together, for 8 days would pay B alone for 18 days; for how many days' work would they pay A?

101. Find the smallest number by which 174636 must be divided in order that the quotient may be a perfect square.

102. Find, to the nearest farthing, the price of goods per lb. which cost £3, 2*s.* 6*d.* per cwt.

103. Find the value of $\frac{3}{8}$ of $\dfrac{\frac{7}{13}-\frac{1}{7}}{\frac{21}{65} \text{ of } 1\frac{6}{7}+\frac{17}{26}}$ of ·0125 of £76, 1*s.* 4*d.*

104. Find by Practice the cost of 227 tons 15 cwts. 2 qrs. of iron at £4, 13*s.* 4*d.* per ton.

105. If the rent of 547 acres for half a year be £211, 19*s.* 3*d.*, what will be the rent of 8 acres for three quarters of a year.

106. Find the simple interest on £821, 5*s.* 0*d.* for 8 months at $3\frac{1}{4}$ per cent per annum.

107. A can run 10 yards to B's 9: how many yards start must A give B in a mile to make an even race?

108. By selling goods for £240 a tradesman gains 25 per cent; how much would he have gained per cent by selling them for £204?

109. What will it cost to cover with zinc the sides and bottom of a cistern whose length, breadth, and depth are respectively 7 ft. 10 in., 5 ft. 4 in., and 1 ft. 9 in. at 6*s.* 9*d.* per square yard?

110. A, B, and C together can do a piece of work in 4 days; B alone can do it in 10 days, and C alone in $12\frac{1}{2}$ days; in what time can A do it alone?

111. Find the size of the smallest field which could be divided into an exact number of allotments of either 1 ro. 2 po., 1 ro. 14 po., or 2 ro. 16 po.

112. Simplify $\dfrac{\frac{2}{3}+\frac{4}{5} \text{ of } \frac{6}{7}-\frac{8}{9} \text{ of } \frac{10}{11}}{\frac{1}{2} \text{ of } \frac{3}{5}+\frac{3}{4} \text{ of } \frac{6}{7}} \times \dfrac{1\frac{3}{8} \div 5\frac{1}{8}}{\frac{4}{17} \text{ of } 1\frac{8}{9}-\frac{7}{9} \text{ of } \frac{3}{11}}.$

113. Find a decimal of a pound which exceeds a penny by a millionth of a shilling.

114. Find the cost of 3 decametres 5 decimetres of cloth at 7 francs 5 centimes per metre.

115. Find the simple interest on £47, 12*s.* 6*d.* (i) for $10\frac{2}{3}$ years at $2\frac{1}{2}$ per cent; (ii) for 15 months at 4 per cent; and (iii) to the nearest penny for 3 days at 3 per cent.

116. Twelve boys do a third of a piece of work in six days, when two are taken ill; how long will it take the remainder to finish it?

117. A clock set right at 9 A.M. is $7\frac{1}{2}$ min. fast at noon; what is the right time when the clock indicates 9 P.M.?

118. What length of plank 7 inches wide and $2\frac{1}{2}$ inches thick contains a cubic foot of timber?

119. At a game of skill A can give B 20 points in 100, B can give C 15 in 100. How many can A give C?

120. The train for B leaves A, travelling at the rate of 40 miles per hour, at the same time that the train for A, travelling 32 miles per hour, leaves B. When they pass each other one has gone 20 miles further than the other; find the distance between A and B.

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121. Find the sum of all the common measures of 315 and 420.

122. Simplify $\dfrac{111}{1881}$ of $\dfrac{141}{1771}$ of $\dfrac{161}{1551}$ of $\dfrac{171}{1221}$.

123. If ninety-one million locusts weigh a ton, how many weigh $1\frac{3}{13}$ ounces?

124. Divide 18·56 by 4·33 to (i) one place; (ii) two places; (iii) three places of decimals, giving the remainder in each case.

125. Find $(\cdot 41\dot{6} + \cdot 2\dot{2}3\dot{4} + \cdot 00\dot{4}\dot{5}) \times 37$.

126. On a map made on the scale of 2 miles to an inch, what area represents 2560 acres?

127. If 14 men in 12 days earn £28, how much will 10 men earn in 7 days when there has been a rise of 10 per cent in wages?

128. At what rate per cent simple interest will a sum of money double itself in 15 years?

129. Find, to the nearest penny, the compound interest on £28, 12*s.* 9*d.* for 2 years at $2\frac{3}{4}$ per cent.

130. A slow train, going 20 miles an hour, starts from A at 9 A.M.; an express follows, going 45 miles an hour, at 9.50. If the slow train arrives at B 5 minutes before the express, find the distance from A to B.

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131. Show, without dividing, that 477576 is exactly divisible by 396.

132. Simplify $12\frac{89}{131} \times 13\frac{28}{151} \times 14\frac{27}{101} \times 18\frac{75}{181}$.

133. Find as shortly as possible (i) $864\cdot 75 \times \cdot 125$; (ii) $578\cdot 64 \times 9\cdot 8$;

$$\text{(iii)}\quad \frac{5\cdot 7 \times 5\cdot 7 - 4\cdot 3 \times 4\cdot 3}{5\cdot 7 - 4\cdot 3}.$$

134. Express in ounces ·14 ton + 1·56 cwt. + 2·7 qrs. + ·51 lbs.

135. How many acres are there in a parish which occupies $\frac{1}{4}$ sq. in. of paper on a map made on the scale of $\frac{1}{10}$ of an inch to a mile?

136. A garrison of 2400 men having provisions for 48 days is reinforced after 31 days by 800 men; in how many days after this will the provisions be exhausted?

137. Find to the nearest penny, by Decimalized Practice, the cost of repairing 3 mi. 3 fur. 33 po. of road at £17, 12*s.* 6*d.* per mile.

138. If a money-lender charges 2*d.* a week for the loan of £1, what rate per cent per annum does he obtain?

139. Find, to the nearest penny, the difference between the compound interest on £500 for 2 years at 5 per cent (i) payable yearly; and (ii) payable half-yearly.

140. If a crew can row $3\frac{1}{2}$ miles down stream in 15 minutes, and $1\frac{1}{2}$ miles up stream in 24 minutes, how many miles an hour can they row on still water?

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141. If the President of the United States received 5 dollars a year more than he does, he would have 137 dollars a day; what is his salary in £ *s. d.*, a dollar being worth 4*s.* 2*d.*?

142. Simplify $\dfrac{5\frac{1}{3}+4\frac{3}{4}}{3\frac{5}{6}+2\frac{7}{8}} \times \dfrac{5\frac{1}{3}-4\frac{3}{4}}{3\frac{5}{6}-2\frac{7}{8}} \div \dfrac{28\frac{4}{9}-22\frac{9}{16}}{14\frac{25}{36}-8\frac{17}{64}}$.

143. Find, *by inspection*, which of the following fractions would produce terminating decimals, and the number of figures in each decimal of that kind:—$\frac{11}{625}$, $\frac{7}{256}$, $\frac{8}{375}$, $\frac{13}{888}$, $\frac{9}{7500}$.

144. Find the value of $\cdot 00\dot{2}8571\dot{4}$ of £1, 9*s.* 2*d.*, and express in simplest form $10\cdot 89 \times \cdot 48\dot{6}\dot{4} - \cdot 12\dot{8}\dot{1} \times \cdot 55$.

145. A brick wall is to be built 53 ft. 4 in. long, 8 ft. high, and 1 ft. 8 in. thick. If the volume of each brick is 48 cubic inches, how many will be required, allowing $3\frac{1}{8}$ per cent of the whole space occupied by the wall for mortar?

146. If a centime is the interest on a franc from Aug. 7 to Oct. 19, find the rate per cent.

147. If $37\frac{1}{2}$ per cent of the candidates in an examination are girls, and if 75 per cent of the boys, and $62\frac{1}{2}$ per cent of the girls pass, and 342 girls fail, how many boys fail?

148. The face-value of a bill, drawn April 7th at 9 months, is £530, 5*s.* It is discounted at 5 per cent on June 17th; what cash is then obtained for it?

149. A clock which was 3 minutes slow at 2 P.M. on June 1, was 7 minutes fast at 2 A.M. on July 14; when was it right time?

150. The cost of cementing the walls of a rectangular tank 21 feet long 15 ft. 9 in. wide, and 11 ft. 8 in. deep is £9, 12*s.* 6*d.*; find the cost of cementing the floor at the same rate.

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151. April 14, 1897, was Wednesday, what day of the week was April 14, 1851?

152. Find, by *inspection*, the value of $\dfrac{\frac{117}{143}+\frac{153}{187}}{\frac{161}{253}+\frac{217}{341}} \times \frac{133}{171}$.

153. Find all the possible denominators of vulgar fractions in their lowest terms, which produce pure circulating decimals each having a period of four figures.

154. Express the difference between a sovereign and a penny as the decimal of a shilling; and find the value of $\cdot 1875$ of 2 tons $+ 1\cdot 875$ of 2 cwt. $+ 18\cdot 75$ of 2 qrs. $+ 187\cdot 5$ of 2 lbs.

155. At what time between 5 and 6 o'clock are the directions of the hands of a watch equally distant from the figure V?

156. How many eggs are sold for a shilling if a rise of 20 per cent in price would reduce the number obtained for a florin by three?

157. Find the compound interest on £221, 6*s.* 8*d.* for $2\frac{1}{2}$ years at 3 per cent.

158. Five tenders were sent in for building a school, viz.: £14,372, 14*s.* 4*d.*, £13,589, 12*s.* 8*d.*, £11,876, 13*s.* 10*d.*, £15,346, 15*s.* 4*d.*, £12,610, 0*s.* 6*d.*; by how much per cent did the first exceed the average of the five tenders?

159. Find, to a penny, parts of £28, 17*s.* 6*d.* proportional to 7, 9, 13.

160. Smith scores at the rate of 2 runs in 3 minutes; Brown at the rate of 3 runs in 2 minutes. Smith's score is 15 when Brown comes in; how many runs will each have made when their scores are equal?

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161. Find the sum of five-hundred-and-thirty-two thousand; five-hundred-and-thirty-two thousandths; five hundred, and thirty-two thousandths; and five-hundred-and-thirty two-thousandths.

162. Find the sum of all the proper fractions, having a single figure for denominator and differing in value, which can be formed with the figures 1, 3, 5, 7, 9.

163. Find the number of cubic decimetres of wood in a box (with lid) measuring externally 1 metre in length, 50 centimetres in width, and 50 centimetres in height, the wood being 2 centimetres thick.

164. Simplify (i) $\dfrac{(1\cdot005 + \cdot201) \times (1\cdot005 - \cdot201)}{1\cdot005 \times \cdot201}$; (ii) $\dfrac{\cdot009\dot{1} \div \cdot05\dot{6}\dot{7}}{41 \div 281}$;
and express the sum of $£\cdot9958\dot{3}$ and $£\cdot0041\dot{6}$ in account money.

165. When hens lay at the rate of an egg and a half in a day and a half by a hen and a half, how many eggs will be obtained from 21 hens in a week?

166. In what time would £1260 amount to £1496, 5*s.* at $3\frac{3}{4}$ per cent simple interest?

167. If selling an article for 15*s.* 10*d.* would involve a loss of 5 per cent, what should it be sold for to make a profit of 10 per cent?

168. Find the compound interest on £7500 for 1 year at 4 per cent per annum payable quarterly.

169. A quantity of brandy and water in the ratio 2 : 1 is mixed with double the quantity of brandy and water in the ratio 3 : 1; find the strength of the mixture.

170. A and B start from the same point and run, in opposite directions, round a circular track half a mile in circumference, A at the rate of 11, and B at the rate of 13 miles an hour; when and where will they meet for the first time? Also, what interval must elapse before they meet at the starting-point?

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171. By what number less than 1000 must 30107 be divided if the remainder is 567?

172. A sum of money amounting to £123, 19*s.* 6*d.* is made up of a certain number of crowns, three times as many half-crowns, five times as many florins, twelve times as many shillings, and eighteen times as many sixpences. How many are there of each?

173. Express the product of $\dfrac{\frac{108}{111}+\frac{1}{37}}{1\frac{46}{1113}-\frac{53}{371}}$ and $\dfrac{\frac{1067}{1111}+\frac{4}{101}}{1\frac{97}{1001}-\frac{14}{143}}$ as a decimal.

174. The circumferences of three wheels belonging to the engine of a train are 40, 11, and 8 feet respectively. How many times, in travelling a mile, do they complete revolutions together?

175. A contractor engaged to remove 4725 cubic yards of earth in 75 days; he employed 60 men, but at the end of $52\frac{1}{2}$ days he finds only 2700 cubic yards gone; find the least number of extra men he must put on to complete the work within the specified time?

176. Find, to the nearest penny, the rent of a rectangular field 11 chains 60 links long, and 8 chains 40 links wide, at 35*s.* per acre.

177. A cubic fathom of water weighs six tons, and a kilogram is the weight of a cubic decimetre of water. If 100 decimetres be equal to 11 yards, express a kilogram in pounds and fractions of a pound.

178. What sum of money will amount to £889, 8*s.* 4*d.* in 9 months at $3\frac{1}{2}$ per cent per annum?

179. A man invests £1980 in the $3\frac{1}{2}$ per cents at 99 and £3220 in the $4\frac{1}{2}$ per cents at 105; find the average rate of interest on his whole investment.

180. How many posts set 8 feet apart would surround a square field containing 1 ac. 2 ro. 16 po.?

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181. A certain number when divided by 209 leaves remainder 43; what is the remainder when the same number is divided by 19?

182. A dealer imports equal weights of tea, coffee, and cocoa; the value per lb. of the tea is half as much again as that of the coffee, and $1\frac{4}{5}$ as much as that of the cocoa; the whole weighs 54 cwts. and costs £336. At what price per lb. is each article imported?

183. Simplify $\dfrac{1\frac{2\frac{1}{2}}{3\frac{1}{2}}}{1\frac{3\frac{1}{2}}{4\frac{1}{2}}} \div \dfrac{5\frac{2\frac{1}{2}}{3\frac{1}{2}}}{5\frac{3\frac{1}{2}}{4\frac{1}{2}}}$, and $\dfrac{(\frac{11}{111})^2-(\frac{1}{11})^2}{\frac{11}{111}+\frac{1}{11}}$.

184. A train travels from A to B at a rate of ·625 mile per minute, and performs the journey in 7·4461 hours; how long does another train take, travelling at the rate of ·3919 mile per minute.

185. Two cogwheels, one of 15 and the other of 28 teeth, work together; if the smaller makes 16 revolutions in $1\frac{1}{2}$ seconds how many revolutions will the larger make in 21 seconds?

186. If 7 men earn as much in a day as 12 women, and 1 woman as much as 2 boys, and if 7 men, 6 women, and 12 boys working together for 10 days earn £21, what will be the earnings of 8 men and 8 women working together for 6 days?

187. A buys an article and sells it to B, gaining 10 per cent on the sale price; B sells it to C, gaining 10 per cent on what he paid for it. If C paid £550, what did A give?

188. Find the banker's discount on a bill for £2240 drawn June 8, at 3 months, discounted June 30 at 4 per cent.

189. If a man possesses £7600 stock in the $2\frac{3}{4}$ per cent consols, what is his net annual income when the income-tax is 6*d.* in the pound?

190. If 5 kilometres exceed 3 miles by as much as 2 miles exceed 3 kilometres, find the length of a kilometre in furlongs.

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191. The sum of five consecutive numbers is 1705; find them.

192. Resolve 53361 and 94864 into factors, and thence obtain their square roots and the square root of their sum.

193. Find the smallest (i) number, (ii) integer, which contains $6\frac{1}{4}$ and $4\frac{1}{6}$ each an exact number of times.

194. Use contracted methods to (i) multiply (ii) divide ·4263518 by ·2236054, each correct to 3 places of decimals.

195. An noon I had completed $\frac{2}{9}$ of my journey; at 2·15 P.M. I had completed $\frac{5}{9}$ of it, and had travelled 63 miles since noon. Find the length of my journey, the uniform rate at which I travelled, and the times of starting and arriving.

196. Three equal sums are invested at 3, 4, and 5 per cent respectively. The total income which results is £261; find the sums invested.

197. How much water must be mixed with 50 gallons of spirit costing 24*s.* per gallon, so that by selling the mixture at 25*s.* per gallon 15 per cent profit may be made?

198. The length of a rectangular field of 15 acres is to its breadth as 3 : 2; how much fencing would go round it?

199. What sum must be invested in 3 per cent stock at 104 to yield sufficient income to provide for an annual prize of five guineas? (Brokerage $\frac{1}{4}$ per cent.)

200. A could do a piece of work in 10 days with B's help on 3 of the days. B could do it in 8 days with A's help on 3 of the days. How long would it take them working together continuously?

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201. Find the number of days from the beginning of the present century until Mar. 11, 1897, inclusive.

202. Supply the missing figure in place of the asterisk, which will make the expression $\dfrac{2\frac{1}{13} \text{ of } \frac{*}{9} - 1\frac{1}{8} \text{ of } \frac{2}{3}}{1\frac{1}{21} \div 5\frac{4}{7}}$ equal to $2\frac{13}{88}$.

203. Find the greatest and smallest numbers each of four digits which are perfect squares.

204. Find, correct within one thousandth of the whole, the cube of 4·253253...

205. The circumferences of the front wheels of two bicycles are 7 ft. 4 in. and 9 ft. 2 in. respectively: how often in 5 miles will the same points, which were in contact with the ground at starting, be again simultaneously in contact with the ground?

206. Find the exact difference between the simple and compound interest on £1500 for 4 years at 5 per cent.

207. In what ratio must teas costing 1*s.* 6*d.* and 1*s.* 9*d.* per lb. be mixed so that (i) the mixture may be worth 1*s.* 7*d.* per lb., (ii) 10 per cent profit may be made by selling the mixture at 1*s.* 10*d.* per lb.?

208. A quart bottle of spirit contains 85 per cent of alcohol. If $\frac{1}{4}$ pint be removed and the bottle filled up with water, what percentage of alcohol is there in the bottle?

209. How many £20 shares must be sold at (i) 5 per cent discount, (ii) 5 discount, to realize £14,820.

210. If 7 men, 6 women, and 5 boys earn £13, 8*s.* in 8 days, and 2 men earn £2, 14*s.* in the time that 3 women earn £2, 5*s.*, and 10 women earn £5 in the time that 11 boys earn £1, 13*s.*, find the daily wages of a man, of a woman, and of a boy.

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211. The sum of 19 consecutive numbers is 34048; find the middle number.

212. The sum of the ages of a man and his son is now 67 years: sixteen years ago the man was six times as old as his son: find their present ages.

213. Simplify $\sqrt{\{(3\frac{5}{12})^2 - 5\}} \div \sqrt{\{(3\frac{5}{12})^2 + 5\}}$.

214. Find the integral part of

(i) the product of $8567{\cdot}425 \times {\cdot}093625$;

(ii) the quotient of $754{\cdot}683275 \div {\cdot}367804$.

215. For what sum must goods costing £735 be insured, the premium being 2 per cent, so that in case of loss the owner may recover both the value of the goods and the amount of premium paid?

216. A man walked a certain distance at the rate of $3\frac{1}{2}$ miles an hour, and rode back at $8\frac{1}{2}$ miles an hour; the total time occupied was 6 hours; how far did he walk?

217. Ten men begin a work which they could finish in 100 days, but at the end of every 10 days, 10 additional men are put on; how long does the work take?

218. If the population of a country is 2,560,000 and it decreases 6 per thousand annually, what will be the population in 3 years?

219. How many pounds of tea costing 2*s.* 3*d.* per lb. must be mixed with 60 lbs. costing 1*s.* 10*d.* per lb., so that a profit of 20 per cent may be made by selling the mixture at 2*s.* 6*d.* per lb.?

220. One-fifth of the capital of a company consists of 4 per cent Debenture, two-fifths of $4\frac{1}{2}$ per cent Preference, and the remainder of Ordinary, stock. The total capital of the company is £1,000,000. What dividend is paid on Ordinary stock in a year when profits to the amount of £50,000 are divided?

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221. A number divided successively by 3, 8, 5, 7, and 12 gives remainders 2, 0, 4, 3, and 7, and the last quotient is 49. Find the original number and the total remainder.

222. Find the perfect square nearest to 333333.

223. Find a multiplier which will convert miles per hour into inches per second.

224. If 7 cubic inches of lead weigh as much as 11 cubic inches of iron, and the price per ton of iron is £4, and of lead £15, find the value of a mass of iron twice the size of a mass of lead worth £4, 16*s*. 3*d*.

225. Find by Practice the value, to the nearest penny, of £$\frac{6}{25}$ + £$\frac{17}{28}$ + £$\frac{25}{36}$.

226. If 1·56 grammes of a certain solid afford on analysis ·991 grammes of A, ·441 grammes of B, and ·128 grammes of C; calculate its percentage composition.

227. At what time between 9 and 10 do the hands of an astronomical clock coincide? [One hand goes round the face in an hour, and the other in 24 hours.]

228. Find the present worth of £1000 due 3 years hence, allowing compound interest at 5 per cent.

229. Taking £1 as equal to 25·17 francs, and 1 kilogramme as equal to 2·204 lbs.; find, to the nearest farthing, the price per lb. of coffee costing 3 francs 25 centimes per kilogramme.

230. A man invested £3600 in stock paying $4\frac{1}{4}$ per cent, and on a rise of 5 in the price of the stock he sold out and lent the proceeds on mortgage at 5 per cent, thus increasing his annual income by £20. At what price did he buy the stock?

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231. The continued product of three consecutive numbers is 262080; find them.

232. Show that if the number of farthings in the price of a pound of goods be multiplied by 7 and divided by 3, the result gives the price per ton in sovereigns.

233. There are three fields of 10, 12, 18 acres respectively. A and B together can mow the first in 2 days; B and C, the second, in 4 days; C and A, the third, in 6 days; find the time in which (i) all three working together; (ii) each man alone, can mow the three fields.

234. If an 18-carat gold chain weighs $2\frac{1}{2}$ ozs., how many grains of alloy does it contain? [18-carat gold contains 18 parts by weight out of 24 of pure gold.]

235. A, B, and C start together to walk in the same direction round a circular course 280 yards in circumference, their rates being 4, 5, 6 miles an hour respectively; after what time will they be all together again at the starting point, and how many rounds will each of them have then made?

236. Find to the nearest tenth of a mile the rate per hour at which each of the following journeys is performed:—

Railway.	Depart from		Arrive at		Distance.
L.N.W.R.	London at	11.50 P.M.	Glasgow	7.50 A.M.	396 mi.
G.N.R.	....	11.30 P.M.	York	2.58 A.M.	188 ...
G.W.R.	....	10.30 A.M.	Exeter	2.20 P.M.	194 ...
G.E.R.	....	8.30 P.M.	Harwich	9.55 P.M.	69 ...

237. The "true" discount on a bill is £15. The "commercial" discount on it at the same rate per cent would be £16, 16*s*. Find the amount of the bill.

238. Find in sterling the value of 1353 francs 50 centimes when the exchange is 25·30.

239. Simplify $\sqrt{108900} + \sqrt{1089} + \sqrt{10 \cdot 89} + \sqrt{\cdot 1089}$;

and $\sqrt[3]{\frac{66}{175} \text{ of } \frac{176}{343} \text{ of } \frac{126}{605}}$.

240. When income-tax was 9*d.* in the pound a man's net income, obtained from $2\frac{3}{4}$ per cent stock, was £1450, 9*s.* 9*d.* He sold out at $98\frac{1}{2}$, and re-invested his capital in 3 per cent stock at $102\frac{5}{8}$, the income-tax meanwhile having been reduced to 7*d.* in the pound. What change was effected in his net income? (Brokerage $\frac{1}{8}$ per cent.)

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241. The sum of 953 consecutive numbers is 2256704, find (i) the middle number, (ii) the least of them.

242. Simplify $\sqrt[3]{\frac{196}{363}}$ of $\sqrt[3]{\frac{154}{231}}$ of $\sqrt[3]{\frac{121}{147}}$; and $\frac{\sqrt{\cdot 9} + \sqrt{\cdot 4}}{\sqrt{\cdot 9} - \sqrt{\cdot 4}}$.

243. What is the gain per cent in buying at half the price per dozen at which one sells per score?

244. Three watches are set together. The first gains 5 minutes a week, the second gains 8 minutes a week, whilst the third loses 4 minutes a week. When will they all again indicate the same time?

245. A can walk round a rectangular field, whose sides are 411·6 yds. and 548·8 yds., in the same time that B takes to walk from one corner to the opposite corner and back. Find the ratio of their rates of walking.

246. A bookseller sells books at a discount of 25 p.c. on the published price. The publisher supplies him at $33\frac{1}{3}$ per cent discount on the published price, and also gives 13 to the dozen. Find the bookseller's profit per cent on cash sales.

247. I sell some 4 per cent stock at $115\frac{5}{8}$, and some 5 per cent stock at $134\frac{3}{8}$. I invest the proceeds in $4\frac{1}{2}$ per cent stock at $123\frac{1}{2}$. The change does not alter my income. Find the ratio of 4 per cent to 5 per cent stock sold.

248. If £5000 be borrowed at 5 per cent, compound interest, to be repaid, principal and interest, in three equal annual instalments, what is the amount of one instalment?

249. Find three numbers which are to one another as 1 : 2 : 3, and such that the sum of their cubes is 4500.

250. Express $\frac{5}{12}$ as a radix fraction in each of the scales 2, 6, and 12.

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251. What day of the week was June 20, 1837?

252. Find two numbers whose product is $15336\frac{3}{4}$ and one of which is treble of the other.

253. Standard gold is worth £3, 17*s.* $10\frac{1}{2}$*d.* per oz.; and sovereigns are worth their weight. Find, correct to one-thousandth of a grain, the weight of a new sovereign.

254. Find, *by inspection*, the number of places of non-recurring decimal figures, and the number of figures in the recurring period, of the decimal equivalent to $\frac{21}{3120}$.

255. A and B have the same number of shots at pigeons and kill altogether 62. A kills, on the average, 3 in 4 shots, and B kills 4 in 5 shots. How many shots did each fire?

256. If £1 = 25·17 francs, and 1 kilogramme = 2·204 lbs., find, to the nearest farthing, the price per lb. of butter worth 275 francs per quintal.

257. Two trains pass each other, a passenger in the first, which is travelling at the rate of 48 miles an hour, finds that the second train takes $2\frac{1}{2}$ seconds to pass him completely. A passenger in the second, which is travelling at the rate of 42 miles an hour, finds that the first train passes him in $2\frac{1}{4}$ seconds. Find the length of each train.

258. Find the difference between the amount of £1205 for 2 years at 5 per cent compound interest, and the present value of the same sum due 2 years hence at 5 per cent compound interest.

259. What length of wood, 3 inches wide, and $\frac{1}{2}$ an inch thick, would make a lidless box measuring inside 18 in. in length, 14 in. in width, and 3 in. in depth?

Also, if the wood were $3\frac{1}{2}$ in. wide, what length would be required?

260. Find, by duodecimals, the value to the nearest halfpenny of a rectangular block of ore 11 feet $4\frac{1}{3}$ inches long, 3 feet $3\frac{5}{12}$ inches broad, and 2 feet $4\frac{1}{4}$ inches thick, at 8*s.* 9*d.* per cubic foot.

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261. A man was born at 10 P.M. on June 3, MDCCCVI and died at 6 A.M. on Feb. 1, MDCCCLXIX; find the number of hours he lived.

262. A's salary contains as many half-crowns as B's contains florins, and A's salary exceeds B's by twelve guineas; find their salaries.

263. Simplify

$$\frac{10\frac{12}{19} \text{ of } (\frac{2}{3}+\frac{3}{4}+\frac{4}{5}) \div 1\frac{6}{13} \text{ of } \left(\dfrac{1}{1\frac{2}{3}}+\dfrac{2}{2\frac{3}{4}}+\dfrac{3}{3\frac{4}{5}}\right)}{(2\frac{13}{15})^2-(1\frac{11}{20})^2+4\frac{5}{12} \text{ of } [1-\frac{2}{3}\{1-(\frac{4}{5}-\frac{3}{4})\}]}+\frac{31\times 40}{53\times 553}.$$

264. If the hands of a watch coincide every $65\frac{1}{4}$ minutes, how much does it gain or lose per day?

265. Find the smallest sum which can be divided between three persons in parts proportional to $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$ so that each may receive an even number of florins.

266. Calculate, correct to 8 places of decimals, the series

$$\frac{1}{5}-\frac{1}{3.5^3}+\frac{1}{5.5^5}-\frac{1}{7.5^7}+\ldots\ldots$$

267. A hare is pursued by a greyhound and is 40 of her leaps ahead of him. She takes 5 leaps while he takes 3, but he goes as far in 4 leaps as she goes in 7. How many leaps must the greyhound take to catch the hare?

268. What will it cost to cover a cubical box all over at 3*d.* per square foot, if it cost 12*s.* to cover, at 6*d.* per square foot, another box of the same shape but double the length?

269. Find the cost of paving the floor of a room $12\frac{1}{2}$ feet square with black and red tiles each 6 inches square, the black costing 3*s.*, and the red 2*s.* 6*d.*, per dozen, the tiles being so arranged that no black tiles touch each other or the walls.

270. At what rate per cent will £46, 10*s.* be the compound interest on £337, 10*s.* for 2 years?

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271. A certain number is divided by 385 by short division, the factors 5, 7, 11 being taken in order, and the remainders are respectively 4, 3, 9. What would the remainders be if these factors were taken as divisors in the reverse order?

272. When 87792 is divided by long division by a certain number the first remainder is 183, the second 104, and the last 1. Find the divisor and the quotient.

273. Three years ago a man's age was treble, now it is double, the united ages of his three children; find his present age.

274. A man's age is such that in 17 years' time he will be three times as old as he was 17 years ago. What is his present age?

275. The average age of 34 old men is 63; excluding the eldest the average is 62; find his age.

276. A certain sum of money divided equally between a certain number of persons gives each £1, 8*s.* 4*d.* If there were three persons more, each share would be £1, 2*s.* 8*d.* Find the sum of money and the number of persons.

277. When a sum of money was divided between two persons in the ratio of 5 to 7, one of them obtained £60 more than the other: what was the sum?

278. The value of 20 lbs. of a mixture of black and green tea is worth £1, 18*s.* 5*d.*; but if the quantities were interchanged it would be worth £2, 18*s.* 3*d.* The black tea is worth 1*s.* 10*d.* per lb. Find the value of a pound of the green tea.

279. In finding the G. C. M. of two numbers by the ordinary process the successive quotients were 1, 3, and 4, and the corresponding remainders 188, 47, and 0: what were the numbers?

280. Find two numbers whose product is 864, and whose sum is 60.

281. Find the smallest multiple of 7, which, added to 4538, makes the sum a multiple of 12.

282. Two clocks begin to strike 12 together; one takes 33 seconds and the other $27\frac{1}{2}$ seconds to finish; what strokes will occur simultaneously?

283. Divide 31 into two parts, so that three times the one and six times the other are together equal to 162.

284. Divide 99 into two parts such that the sum of seven times one of them and eleven times the other may be 945.

285. Divide 100 into two parts such that thirteen times the greater may exceed twenty-three times the less by 40.

286. Divide 87 into two parts such that $\frac{3}{8}$ of one part shall be equal to $\frac{5}{6}$ of the other.

287. Find a complex fraction equal to $\frac{3}{8}$ whose denominator exceeds its numerator by 7.

288. A certain number increased by 7 bears to the number decreased by 7 the ratio $3 : 2$; find the number.

289. Reduce $\frac{1171875}{8984375}$ to its lowest terms without *dividing*.

290. A woman took a basket of eggs to market. She sold three more than half of them to A, three more than half of what remained to B, three more than half of what then remained to C, and had seven left. How many did she take to market?

291. Two cogged wheels work together, there being 32 cogs in one, and 36 in the other. The larger wheel makes 64 revolutions in a minute. How often will the same cogs be in contact in the same position every minute?

292. A, B, C, D are four cog-wheels; A has 64 teeth, B 8, C 60, D 8. A works in B, C is attached to the same axle as B, and works in D; how many revolutions will D make while A makes one?

293. If 4 apples cost $1\frac{1}{2}d.$, and if 6 apples cost as much as 5 pears, 7 pears as much as 2 peaches, and 3 peaches as much as 28 plums, what is the smallest sum of money capable of being expressed in English coins with which I can purchase equal numbers of each kind of fruit?

294. If 16 men can build a wall 30 yards long in 20 days, working 6 hours a day, find how many men would be required to build a similar wall 48 yards long in 15 days working 8 hours a day, supposing that 8 extra men are put on the last 3 days.

295. If 20 giants with 15 dwarfs could do a piece of work in 20 days, and 15 giants with 20 dwarfs could do it in 25 days, compare the strength of a giant and a dwarf.

296. If 3 men and 4 boys can reap 5 acres in two days, and 5 men and 3 boys can reap 13 acres in four days; how long will it take 4 men and 6 boys to reap 21 acres?

297. If 8 men and 3 boys can hoe 5 acres in 2 days, and 6 men and 5 boys can hoe 7 acres in 3 days, how long would 4 men and 4 boys take to hoe $17\frac{1}{2}$ acres?

298. Two men, A and B, working alone can finish a piece of work in 7 and 8 hours respectively. If they work at it for an hour alternately, A beginning, in how many hours will the work be finished?

299. The wages of 30 men and boys amount to £7, 11*s.* per day, the men receiving 9*s.* each and the boys 2*s.* each. What would the wages amount to per day if the men received 6*d.* a day more and the boys 6*d.* a day less?

300. A crew which can row from A to B in 30 minutes can row from B to A in 24 minutes. Their rate on still water is 6 miles an hour. Find the distance, and the rate of the stream.

301. I have to be at a certain place at a certain time, and I find that if I walk at the rate of 4 miles per hour I shall be 10 minutes too late, if at the rate of 5 miles per hour, I shall be 5 minutes too soon: how far have I to go?

302. A person left A at 10.15 A.M. and rode, at $7\frac{1}{2}$ miles an hour, to B; stayed 3 hours 40 min. at B, and came back by train at 30 miles an hour, reaching A at 4.35 P.M. Find the distance between A and B.

303. Two bicyclists start together from the same point in the same direction on a circular track $\frac{1}{4}$ mile in circumference, their rates being 14 and 16 miles an hour respectively; when and where will one first overtake the other?

If they rode in opposite directions, when and where would they meet for (i) the first time? (ii) the 5th time? and (iii) after what interval would they meet at the starting point?

304. A, B, and C ride in the same direction round a circular course 270 yards in circuit, A at the rate of 18 yards, B at the rate of 14 yards, and C at the rate of 10 yards, per second. After what interval will they be all together for the first time, and how far from the starting point will they then be?

305. A bicyclist travelling with uniform velocity along a straight level road on a foggy day, when objects were just visible a furlong off, saw a milestone in front of him for half a minute. Afterwards he overtook a carriage, the back of which he saw for a minute and a half; with what velocity (supposed uniform) was the carriage travelling?

306. A ship and a steamer are both making directly for a river when a fort at its mouth begins firing minute guns. On board the ship the first gun is heard 6 seconds after the flash is seen and the others are heard every $59\frac{1}{2}$ seconds. On the steamer the first gun is heard 14 seconds after the flash is seen and the others are heard every $58\frac{3}{4}$ seconds. Which will reach the fort first, and how many feet ahead of the other? Sound travels 1120 feet per second.

307. A, B and C rent a house jointly for £150 per annum. C leaves at the end of $4\frac{1}{2}$ months, B at the end of 16 months; A stays 2 years. What shares of rent should each pay?

308. A, B, C meet in a desert; A has 5 loaves, B has 3, and C has only 10 shillings. C agrees to give all his money for a third share of the food; how must A and B divide the money?

309. A sportsman one season used 370 cartridges in killing 261 hares and birds. He used on an average 3 cartridges to kill 2 hares, and 7 to kill 5 birds. How many hares did he kill?

310. A mass of counterfeit metal contains gold, silver, and copper in the proportion by weight of 15 : 4 : 3, and equal volumes of gold, silver, copper, and water are as 20 : 10 : 9 : 1 in weight. If one cubic foot of water weighs 1000 oz., what is the volume of 44 lbs. of the composition?

311. How many oranges are sold for a shilling when by a fall of 16 per cent in the price 56 more would be obtained for a guinea?

312. A man for 5 years spends £40 a year more than his income. If he, at the end of that time reduce his expenditure 10 per cent, in 4 years he will have paid off his debts and saved £120. Find his income.

313. A banker, by discounting a bill at 4 per cent, obtained interest on the money he advanced at the rate of $4\frac{4}{31}$ per cent; how long had the bill to run?

314. A grocer having three kinds of sugar worth $2\frac{1}{2}d.$, $3d.$, and $4d.$ per lb. respectively wishes tc make a mixture of 120 lbs. worth $3\frac{1}{2}d.$ per lb.: how can he do this?

315. How can spirits at $18s.$ and $15s.$ a gallon respectively be mixed with water so as to yield a mixture worth $12s.$ a gallon?

316. How may teas at $2s.\ 8d.$, $3s.$, $3s.\ 6d.$ per lb. be mixed so as to sell at $3s.\ 4d.$ per lb. with a profit of 20 per cent?

317. In a mixture of 570 gallons of spirit and water, the ratio of spirit to water is $18 : 1$; how many gallons of water must be added to reduce the ratio to $15 : 1$?

318. How much gold 18 carats fine, other gold 15 carats fine, and alloy of no value, should be mixed with 8 ozs. of pure gold, so that the result may be 16 carats fine?

319. A wine glass is filled with a mixture of spirit and water in the ratio of 1 of spirit to 4 of water; another glass of the same size is filled in the ratio of 2 of spirit to 5 of water. Find the ratio of spirit to water when the contents of the two glasses are mixed in a tumbler.

320. Two pint measures are full of mingled milk and water; in the first the ratio of milk to water is $6 : 1$, in the second $7 : 1$. If both measures are emptied into a third vessel, what is the ratio of milk to water in it?

321. From a cask of wine, containing 64 gallons, 8 gallons are drawn out, and the cask is filled up with water: if the same process is repeated a second and then a third time, what will be the number of gallons of wine left in the cask?

322. A vessel is full of spirit; half of it is drawn off, and the vessel is filled up with water. The process is repeated a second, third, and fourth time. What is the final strength of the mixture?

323. There are two vessels, one containing 4 gallons of wine, and the other 3 gallons of water. Three pints are taken out of each, and then put into the other. This is done a second time. Find the final strength of each mixture.

324. Lead is 11·4 times and zinc is 7·2 times as heavy as water; if lead and zinc be mixed in the ratio of $8 : 7$, how many times as heavy as water is the compound metal?

325. Find a number which is the same multiple of 17 that it is measure of 612.

326. The product of two numbers is 151722163; their quotient is 307; find them.

327. The product of two numbers, whose difference is 2, is 901800899; find them.

328. The perimeter of a room is five times its height, the height is $\frac{3}{8}$ of the length, and the area of the walls is 1125 sq. ft.: find its dimensions.

329. A room 11 ft. high is half as long again as it is wide, and its cubical content is $4768\frac{1}{2}$ feet: find its length.

330. A square plot of ground is measured, and by an error in deficiency of 2 feet in one side and 3 feet in the other, is calculated to contain 74 square feet less than it really does; find the area of the plot.

331. The dimensions of a box are as 2 : 3 : 4; the difference between the cost of covering it with sheet lead at 8*d.* and $8\frac{1}{2}$*d.* per square foot is 4*s.* $10\frac{1}{2}$*d.*; find its dimensions.

332. A holder of railway stock receives for one year a 4 per cent dividend, and spends $\frac{7}{8}$ of his net income after paying an income tax of 6*d.* in the £. The next year, the dividend being $4\frac{3}{8}$ per cent, and the income tax being raised to 8*d.*, he only spends $\frac{4}{5}$ of his net income, and finds that he has a surplus income £22, 7*s.* 11*d.* greater than in the previous year. How much stock did he possess?

333. A has a capital of £666, 13*s.* 4*d.* in a business for 12 months. B has £500 in the same business for 7 months, which he raises to £766, 13*s.* 4*d.* for the remaining 5 months. C has £800 in the same business for the first 3 months, and only £300 for the last 9 months. If B's share of the profits is £53, 0*s.* 10*d.* more than C's, what should A's share be?

334. Find the equated time of payment of £750, half of which is due in 4 months, $\frac{2}{5}$ of it in 5 months, and the rest in 6 months.

335. The equated time of payment of three sums due in 4, 6, and 11 months respectively is $7\frac{1}{2}$ months. The first sum is £200, the second is £700: find the third.

336. Divide 48 into two parts, so that when one part is expressed in the senary and the other in the octenary scale, they may be represented by the same figures.

337. If 10 oxen were turned into a field which has remained empty for some time they would consume the grass in 12 days; 15 oxen would consume it in 6 days: how many oxen would consume it in 2 days, the grass being supposed to grow uniformly?

338. The areas of three adjacent grass fields which have remained empty for the same length of time are 6, 8, and 12 acres respectively; 12 oxen are put into the first, and the grass lasts them for 8 days; 22 oxen are put into the second, the grass of which lasts them for 4 days: how long would the grass of the third field last 19 oxen, the grass being supposed to grow uniformly?

339. Ash saplings after five years' growth are worth 1*s.* 3*d.* each, and increase in value by 1*s.* 3*d.* each year afterwards. They require twice as many square yards each as the number of years they are intended to grow before cutting. A plantation is so arranged that

each year the same number shall be ready to cut. Find the largest annual yield which can be obtained per acre, allowing one-fifth of the gross receipts for expenses.

340. Two cyclists, A and B, ride in opposite directions round concentric circular tracks 220 and 308 yards in circumference respectively, starting at the same time from opposite sides. A's rate is 32 feet per second, and 16 seconds after the start he for the second time hides B for an instant from the view of a spectator placed at the centre. Find B's rate in miles per hour.

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341. Prove that the quotient of an odd, divided by an even, number cannot be an integer.

342. Prove that the product of a number of four digits by a number of three digits cannot consist of less than six or more than seven digits.

343. Prove that the continued product of any five consecutive numbers is divisible by 60.

344. Prove that the difference between any two numbers formed of the same set of digits arranged differently is a multiple of 3.

345. State and prove a Test of Divisibility by 99.

346. Prove that if a number be separated into periods as if for extracting its cube root and the sum of these periods is divisible by 999, the number itself is divisible by 999.

347. Prove that the square of a prime greater than 5 differs by 1 from a multiple of 10.

348. Prove that if an integer be added to its square the sum can only end in 0, 2 or 6.

349. Prove that if an integer be added to its cube the sum can only end in 0, 2 or 8.

350. Prove that if the sum of two numbers be a multiple of 5, their product cannot differ from same multiple of 5 by more than unity.

351. Prove that every perfect fifth power ends in the same digit as its fifth root.

352. Prove that the square of the difference of two numbers is equal to the sum of their squares decreased by twice their product.

353. Prove that the sum of any two consecutive numbers is equal to the difference of their squares.

354. Prove that if a perfect square end in 5 the tens' digits is 2.

355. Prove that an even number which is a perfect cube must be divisible by 8.

356. Prove that $5^{40}-1$ is divisible by 264.

357. Prove that the difference between the eighth power of any odd number and unity is a multiple of 16.

358. Convert $\frac{17}{27}$ into a continued fraction, and find all the convergents.

359. Find the vulgar fraction, with numerator less than 1000, which is nearest to $3{\cdot}14159$.

360. Express $\sqrt{8}$ in the form of a continued fraction,

Money.

4 farthings (*f.*) = 1 penny (*d.*).
12 pence = 1 shilling (*s.*).
20 shillings, or 240 pence = 1 pound (£).

Note.—A *florin* = 2*s.*; a *crown* = 5*s.*; a *guinea* = 21*s.*
A *sovereign* = 20*s.*, or 8 half-crowns, or 10 florins, or 40 sixpences, or 80 threepences, or 480 half-pence, or 960 farthings.

Time.

60 seconds (*sec.*) = 1 minute (*min.*).
60 minutes = 1 hour (*hr.*).
24 hours = 1 day.
7 days = 1 week (*wk.*).
365 days = 1 (common) year (*yr.*).
366 days = 1 leap year.

Note.—A *common year* = 52 weeks + 1 day; a *century* contains 100 years; a *lunar* month contains about 4 weeks.

The year is divided into 12 *calendar* months, of which February, in common years, contains 28 days; and, in leap years, 29 days;
"Thirty days hath September, April, June, and November;"
and the remaining seven calendar months each contain 31 days.

Leap year occurs once in four years (except at the end of a century). In order to discover whether any year (not the last in a century) is a leap year, *divide the number of the year by* 4, *and if there is* no *remainder it is leap year.* But if the year ends a century it is *not* leap year *unless the first two figures divide by* 4 *without remainder.*

Avoirdupois Weight.

16 drams (*dr.*) = 1 ounce (*oz.*).
16 ounces, or 7000 grains = 1 pound (*lb.*).
14 pounds = 1 stone (*st.*).
28 pounds, or 2 stones = 1 quarter (*qr.*).
4 quarters, or 8 stones, or 112 pounds = 1 hundredweight (*cwt.*).
20 hundredweights = 1 ton.

Note.—This weight is used for all common substances; *e.g.* coal, meat, &c.

Troy Weight.

24 grains (*gr.*) = 1 pennyweight (*dwt.*).
20 pennyweights, or 480 grains = 1 ounce Troy (*oz. Tr.*).
12 ounces Troy = 1 pound Troy (*lb. Tr.*).

Note.—Troy weight is only used for gold, silver and jewellery.
The *grain* alone is the same in both Avoirdupois and Troy weights.

Long Measure.

12 inches (*in.*) = 1 foot (*ft.*).
3 feet, or 36 inches = 1 yard (*yd.*)
1760 yards = 1 mile (*mi.*).
$5\frac{1}{2}$ yards, or 11 half-yards = 1 pole (*po.*), rod, or perch (*per.*).
40 poles, or 220 yards = 1 furlong (*fur.*).
8 furlongs = 1 mile.

Note.—A *chain* = 22 yards, or 4 poles; 100 links = 1 chain.
A *fathom* = 6 feet; a *cable-length* = 120 fathoms; a *knot* = 6080 feet.
A *hand* = 4 inches. A *league* = 3 miles.
$2\frac{1}{4}$ inches = 1 *nail*, 4 nails = 1 quarter, 4 quarters = 1 yard.

Square Measure.

144 square inches (*sq. in.*) = 1 square foot (*sq. ft.*).
9 square feet = 1 square yard (*sq. yd.*).
$30\frac{1}{4}$ (*i.e.* $5\frac{1}{2} \times 5\frac{1}{2}$) square yards, or 121 square quarter-yards = 1 square pole (*sq. po.*, or P.), or square perch.
40 square poles = 1 rood (*ro.* or R.).
4 roods, or 4840 square yards = 1 acre (*ac.*, or A.).
640 acres or 1760×1760 square yards = 1 square mile (*sq. mi.*).

Note.—A *square chain* = 100×100 square links, or 22×22 square yards; 10 square chains = 1 acre.

Cubic Measure.

1728 (*i.e.* $12 \times 12 \times 12$) cubic inches (*cub. in.*) = 1 cubic foot (*cub. ft.*).
27 (*i.e.* $3 \times 3 \times 3$) cubic feet = 1 cubic yard (*cub. yd.*).

Capacity.

2 (imperial) pints (*pt.*) = 1 (imperial) quart (*qt.*)
4 quarts, or 8 pints = 1 gallon (*gal.*) } Liquids.

2 gallons = 1 peck (*pk.*)
4 pecks, or 8 gallons = 1 bushel (*bush.*)
8 bushels = 1 quarter (*qr.*) } Dry goods; *e.g.* corn.

Note.—A *hogshead of beer* = 54 gallons; a *hogshead of wine* = 63 gallons; a *pipe* of wine = 2 hogsheads. 4 *gills* = 1 imperial pint.
6 "reputed" quarts (*i.e.* common wine bottles) contain a gallon.
"A pint of pure water weighs a pound and a quarter."
A cubic foot of water weighs 1000 ounces.

Number.—12 units = 1 dozen; 12 dozen = 1 gross. 20 units = 1 score.

Paper.—24 sheets = 1 quire; 20 quires, or 480 sheets = 1 ream.

EXAMINATION PAPERS.

CAMBRIDGE PRELIMINARY LOCAL EXAMINATION, 1896. (2 hours.)

Part I.

1. Multiply sixty thousand and twenty-seven by seven thousand and nine, and state the result in words.

2. How many farthings are there in £159, 17*s.* $11\frac{3}{4}$*d.*?

3. Find the total cost of three dozen pocket handkerchiefs at 1*s.* 10*d.* each, 29 collars at $9\frac{1}{2}$*d.* each, and 13 neckties at 1*s.* $11\frac{3}{4}$*d.* each.

4. Reduce 735934 ounces to tons, &c.

5. Find the value of $4\frac{5}{42} + 6\frac{23}{91} + 11\frac{51}{52} + 1\frac{23}{156}$.

6. Simplify $4\frac{2}{3} \times 5\frac{4}{7} \div 3\frac{1}{4}$.

7. Express $\frac{821}{128}$ as a decimal; and find the value of 23·8 kilogrammes of copper at 2 francs 5 centimes per kilogramme. (1 franc = 100 centimes.)

8. Find the value of $\frac{13}{15}$ of £1 + $1\frac{4}{7}$ of 10*s.* 6*d.* + $3\frac{15}{16}$ of £1, 1*s.* 4*d.*

9. If $\frac{7}{11}$ of an estate be worth £4655, what is the value of the remainder?

10. At what rate per cent per annum will £12 amount to £13 in $2\frac{1}{2}$ years at simple interest?

Part II.

11. Extract the square root of 45954841 and of 457·5321.

12. Find the compound interest on £125 for 3 years at 5 per cent per annum.

13. Find the cost of carpeting a room 25 ft. 6 ins. long, and 15 ft. 9 ins. wide, with carpet 2 ft. 3 ins. broad at 3*s.* 6*d.* a yard.

14. A man's income is reduced to £522 after he has paid an income-tax of 8*d.* in the £1. Find what would have been his net income if the income-tax had been 7*d.* in the £1.

15. A tradesman marks his goods at prices which would yield him 25 per cent profit on his outlay, but he allows his customers 10 per cent discount. If his receipts are £88, 2*s.* 6*d.*, find what his profits are.

CAMBRIDGE LOCAL EXAMINATIONS.

JUNIORS, 1896. (2 hours.)

A 1. Divide three million four hundred and eighty-two thousand five hundred and two by two hundred and fifty-seven.

A 2. Find the Greatest Common Measure of 168543 and 314675.

A 3. Find the value of 17 tons 6 cwts. 1 qr. 20 lbs. at £11, 4*s.* a ton.

A 4. What weight of copper is worth 723 francs 69 centimes if 1 kilogramme be worth 1 franc 29 centimes? [1 franc = 100 centimes.]

A 5. Find the square root of 190·817 correct to five significant figures [*i.e.* the answer is to contain five figures and to differ as little as possible from the exact value of the square root].

A 6. Find the simple interest on £518, 19*s.* 2*d.* for 170 days at $9\frac{1}{8}$ per cent per annum.

B 1. Simplify (1) $\dfrac{5\frac{1}{2} \times 1\frac{2}{33} \div 1\frac{3}{4} - \frac{5}{18} \div 1\frac{1}{18}}{1\frac{1}{3} \times \frac{2}{3} - \frac{22}{57} + \frac{1}{4} \div 2\frac{1}{4}}$;

(2) $\frac{3}{10}$ of £2, 7*s.* 11*d.* + $\frac{4}{17}$ of $\cdot 1\dot{8}$ of 1*s.* 3*d.* − $\frac{1}{6}$ of 2*s.* 7*d.*

B 2. (1) How many pieces each exactly ·00189 yds. long can be cut from a rod ·0976 yds. long, and what is the length of the piece left over expressed as a decimal of a yard?

(2) Add together $1\cdot\dot{8}\dot{1}$, $\cdot 0\dot{3}2\dot{1}$, and $3\cdot 20\dot{7}4638\dot{2}$.

B 3. How far will a man have to walk while ploughing ten acres if the distance from the middle of one furrow to the middle of the next be 15 inches? Do not make any allowance for turning.

B 4. A bicycle manufacturer prints in his lists a price which would give him a profit of 60 per cent on the cost of manufacture, and then allows for "cash down" a discount of 22 per cent off the list price: what profit per cent does he actually make?

B 5. Find the compound interest on £1250 for 4 years at 4 per cent per annum correct to the nearest penny.

B 6. The driving-wheels of a locomotive are 7 ft. 8 ins. in diameter, the small wheels 3 ft. 8 ins. in diameter, and one spoke in each wheel is painted red. At the start all the painted spokes are upright and above the axle; how many more times in the first 5 miles will they all be upright and above the axle at the same instant?

The circumference of a wheel is to be taken as $\frac{22}{7}$ of its diameter.

SENIORS, 1896. (2 hours.)

A 1. If the total revenue of a country, whose population is 34,908,560, is £107,780,179, find the average amount of revenue raised per head of the population.

A 2. Simplify the expressions—

(i) $(\frac{3}{5} + \frac{4}{7} + \frac{5}{9}) \times (17\frac{6}{17} - 11\frac{9}{16})$; (ii) $\dfrac{3\frac{1}{12} - 1\frac{5}{8} \text{ of } 1\frac{2}{7}}{4\frac{1}{7} - 2\frac{4}{5} \text{ of } 1\frac{1}{6}}$ of £39, 17*s.* 4*d.*

A 3. Divide ·0100947 by 39·9,
and find the value of $1\cdot\dot{0}5\dot{4} \times 1\cdot\dot{1}7460\dot{3} \times 1\cdot\dot{6}1538\dot{4}$.

A 4. The fare for a certain railway journey is 18 francs 70 centimes; while for another journey, 87 kilometres longer than the former, the fare is 24 francs 20 centimes. Find the length of each journey, the rate per kilometre being the same for both.

A 5. Find the interest on £19,314, 11*s.* 8*d.* for 19 days at the rate of 4 per cent per annum.

A 6. A greengrocer sells at sixpence per peck potatoes which he bought at £3, 2*s.* 6*d.* per ton, thereby making a profit of 28 per cent. Find the weight of a peck of potatoes.

B 1. Find the L.C.M. of 3127, 3551, and 3953.

Find the greatest number which is such that, when 12288, 19139, and 28200 are divided by it, the remainders are all the same.

B 2. The railway between two stations A and B ascends at a gradient of 1 in 275 from A to a place C, distant $16\frac{1}{4}$ miles from A, and then descends at a gradient of 1 in 165 from C to B. If the station at B is 28 feet lower than that at A, find the distance from A to B.

B 3. A man invests £10,000 in $2\frac{3}{4}$ per cent Consols at $111\frac{3}{4}$, inclusive of commission. Find within a penny (1) the amount of Consols he will obtain, (2) the sum he will receive each quarter for interest.

B 4. A man in his will bequeathed £1000 to his widow, and £1500 in legacies to various institutions, and directed that one-third of the residue of his property should go to his widow, and that the remaining two-thirds should be equally divided amongst his three children. If the widow received altogether £4000 more than each child, what was the value of the property which the man left?

B 5. If the children born in the United Kingdom consist of males and females in the ratio of 81 to 80, while the population always consists of males and females in the ratio of 27 to 28, find the average length of life of all the females born, assuming that 45 years is the average length of life of the males, and neglecting the effects of immigration and emigration.

CAMBRIDGE HIGHER LOCAL EXAMINATIONS.

December, 1896. (2 hours.)

PART I.

1. In a certain town there are 15933 white and 50055 black inhabitants. Express in its simplest form the ratio of the number of the white inhabitants to the whole population.

2. Simplify $\dfrac{2 \times 1\frac{6}{11} \times (6\frac{2}{3} - 1\frac{1}{6})}{1\frac{35}{36} - \frac{1}{12}} \div \dfrac{3\frac{3}{4} - 1\frac{13}{20}}{3\frac{5}{12} + 1\frac{29}{60}}$.

3. Assuming that a gramme is $\cdot 0022$ of a pound avoirdupois, find the value of a ton of metal at the price of 2*s*. 1*d*. per kilogramme.

4. State the rule for the division of one decimal fraction by another, and prove its truth.

Divide $4\cdot 953$ by $\cdot 1524$, and from this result deduce the quotient when $495\cdot 3$ is divided by $\cdot 0001524$.

5. A man can afford £125 for rent, rates, and house-tax. If the house-tax is 9*d*. per pound upon the whole rent, and the rates are 3*s*. 10*d*. per pound upon $\frac{3}{4}$ of the rent, find, to the nearest shilling, how much rent he can pay.

6. Show that the difference between true discount and bankers' discount is the interest on the true discount.

Upon what sum, due 6 months hence, is the difference between true discount and bankers' discount equal to £1, 0*s*. 3*d*., interest being reckoned at $4\frac{1}{2}$ per cent per annum?

7. A grocer buys two kinds of tea costing respectively 1*s*. $1\frac{1}{2}$*d*. and 1*s*. 7*d*. per lb. In what proportion must he mix them so as to make a profit of 15 per cent by selling the mixture at the rate of 4 lbs. for 5*s*. 9*d*.?

8. A man invests £1000 in 3 per cent stock at the price of 105. At the end of the first year he invests the year's dividend in the same stock at 107; at the end of the second year he invests the whole dividend for

that year in the same stock at 108. What is his dividend for the third year, to the nearest penny?

PART II.

9. In a race B starts a certain distance ahead of A, and C an equal distance ahead of B. If A passes B in 12 minutes, and 9 minutes afterwards passes C, in what time from the start will B pass C?

10. In an orphan asylum for a certain quarter of the year in which the average number of boys was 104, and that of girls 72, the whole cost of keep was £669, 12*s.* 0*d.*, while for a single month in which the number of boys was 91, and that of girls 60, the cost was £191, 17*s.* 0*d.* What is the ratio of the cost of a girl to that of a boy?

11. Three persons A, B, C share a cask of beer. B and C would finish it in 20 days, C and A in 24 days, and B and A in 30 days. After they have shared it together for 8 days, C goes away and A and B share it for 12 days more; if then B goes away, how much longer will it last for A?

12. If I walk to the station at the rate of 4 miles an hour I shall arrive one minute too late for a certain train, but if I run at the rate of 9 miles an hour I shall arrive 3 minutes too early; find the distance, in yards, of the station from my starting-point.

CAMBRIDGE UNIVERSITY PREVIOUS EXAMINATION.

October, 1896. ($2\frac{1}{2}$ hours.)

1. Find the cost of $67\frac{3}{4}$ yards of cloth at 4*s.* 5*d.* per yard.

2. Find the highest common factor of 102765 and 82654.

What is the least number, which, when divided by 96 and 112, has in each case 5 as remainder?

3. Multiply 25·025 by ·404. Add together $4\cdot\dot{7}5\dot{6}$, $\cdot2\dot{2}\dot{7}$, and $\frac{5}{7}$, giving the sum as a decimal.

4. Find in yards the length of a side of a square field, whose area is 1 acre 2 roods 30 poles 22 sq. yds. 6 sq. ft. 108 sq. ins.

5. Find the amount (correct to a penny) of £4750 in 3 years at $3\frac{1}{2}$ per cent compound interest.

6. If a man gain 4 per cent by selling articles at 10*s.* 10*d.* each, what price must he charge for 100 that he may gain £4?

7. 11 cubic inches of iron weigh as much as 7 cubic inches of lead, and the price per ton of lead is £16 and of iron £5. Find the value of a block of lead of half the size of a block of iron worth £4, 4*s.* 7*d.*

8. A train from London to Cambridge ($55\frac{1}{2}$ miles) is timed to run throughout at 45 miles per hour. If it is 3 minutes behind time when it has been running for 30 minutes, at what rate must it travel for the remaining distance if it is to arrive punctually?

9. What sum must be invested in $4\frac{1}{2}$ per cent stock at $102\frac{1}{2}$ to produce an income of £363?

10. A cistern has two taps A and B by which water runs in, and another C fixed in the bottom by which water runs out. When A and C are open the cistern is filled in 24 min.; when B, C are open in 18 min.; when A, B, and C are open in 6 min. Find how long it will take to fill the cistern when A, B are open and C is closed.

OXFORD LOCAL EXAMINATIONS.

JUNIORS, 1897. (2 hours.)

1. Simplify (1) $\frac{3}{5}\frac{5}{6}\frac{5}{4}\frac{3}{8}$; (2) $\dfrac{3\frac{1}{2}}{4\frac{2}{3}} - \dfrac{2\frac{1}{8}}{1\frac{1}{2}} + \dfrac{1\frac{2}{3}}{2\frac{1}{2}}$.

2. Divide $\cdot021$ by the product of $1\cdot5$ and $\cdot28$: and express as a decimal of a hundredweight the difference between $\cdot063$ of a quarter and $\cdot42$ of a pound.

3. Multiply $5\cdot6\dot{2}\dot{1}$ by $3\frac{1}{7}$; and find the value of $\cdot058\dot{3}$ of £1.

4. Find the cost of 5 ozs. 6 dwts. 16 grs. at £3, 7*s.* 6*d.* an oz.

5. If $\frac{7}{10}$ of a ton of metal is worth £68, what is the value of $5\frac{1}{4}$ tons?

6. In how many days will the simple interest on £2433, 6*s.* 8*d.* at 5 per cent per annum amount to exactly £100?

7. A can do a piece of work in 2 days, which B can do in $3\frac{1}{2}$ days. If A's wages are 38*s.* 6*d.* a week and B's wages 27*s.* 6*d.* a week, what would A receive for doing a piece of work which B would do for £5, 10*s.*?

8. A train takes 20 minutes longer to do a journey when it is running 27 miles an hour than when it is running 30 miles an hour. Determine the length of the journey.

9. On what sum of money would the compound interest for 3 years at 5 per cent per annum exceed the simple interest by £38, 2*s.* 6*d.*?

10. A rectangular tank, with sides and a bottom 1 inch thick, is 21 ft. long, 7 ft. 8 ins. broad, and 11 ft. 1 in. deep, when measured externally. How many ounces of water will it contain, if 1 cubic foot of water weighs 1000 ozs.?

11. Two mixtures of milk and water, containing 34 per cent and 46 per cent of milk respectively, are mixed together in the proportion of 2 gallons of the latter to 1 gallon of the former, and the resulting mixture is sold at 4*d.* a quart. If 9 gallons of milk cost 10*s.*, what percentage is gained on the original outlay, after deducting from the profits the cost of delivering the mixture to customers, which amounts to 2*d.* a gallon?

12. A and B enter into partnership on the following terms: (1) B is to put in half as much capital again as A; (2) A, who is the working partner, is to receive 8 per cent on his capital, and B is to receive 4 per cent on his capital, out of the profits; (3) the rest of the profits are to be divided between them in the ratio of their respective capitals. At the end of the first year A receives in all £2456, and B £3084. What capital did each put into the business?

SENIORS, 1896. (2 hours.)

1. Simplify:

(1) $\{1\frac{29}{36} + 4\frac{1}{8} \times 1\frac{7}{11}\} \div \{5\frac{1}{9} - 7\frac{7}{8} \div 9\frac{9}{20}\}$;

(2) $\cdot\dot{4}\dot{5}$ of 16*s.* 6*d.* + $\cdot2\dot{2}\dot{7}$ of 9*s.* 2*d.*

2. Express $\frac{5}{6}$ of £3, 2*s.* $7\frac{1}{2}$*d.* as the decimal of £5; and reduce $\cdot24140625$ of a ton to cwts., qrs., &c.

3. If gold is worth £3, 17*s.* 6*d.* per ounce, what is the value of a lump weighing 2 ozs. 3 dwts. 16 grs.?

4. Extract the square root of $5\frac{1}{7}$ correct to 4 decimal places.

5. Find the sum of money on which the simple interest for 9 months at 6 per cent per annum is £3, 7*s.* 6*d.*

Either, 6. Between two railway-stations the first, second, and third class fares were at first fixed in the ratio of 8 : 6 : 3, but afterwards the first class fare was reduced by 2*d.* in the shilling and the second by

1*d*. in the shilling. In a year the numbers of first, second, and third class passengers were respectively as 9 : 12 : 26, and the money taken at the booking-offices was £1088; how much was paid by first class passengers?

or, 6*. A, B, and C rent land; A puts 500 sheep on it for 4 months, B 600 for 3 months, and C 1200 for 2 months. If the whole rent is £279, how much should each pay?

Either, 7. A man invested a sum of money partly in 3 per cent stock at 99 and partly in 4 per cent stock at 101. When the 4 per cents had risen to 114, he sold out this stock and invested the money realized in $2\frac{1}{2}$ per cent stock at 76. He thus diminished his income by £12, 10*s*., but increased his amount of stock by 25 per cent; what was the sum first invested?

or, 7*. What income would be obtained from the investment of £10,500, half in 4 per cent stock at 105, and half in 5 per cent stock at 125?

Either, 8. Determine, in ounces, the weight of an open tank made of iron $1\frac{1}{2}$ inches thick, which, when full, holds $23\frac{237}{256}$ cwts. of water; the length of the interior being twice the breadth, and the breadth twice the depth. A cubic foot of water weighs 1000 ozs., and iron is 8 times as heavy as water.

or, 8*. A rectangular grass plot 45 ft. long and 15 ft. broad is surrounded by a path 5 ft. wide; what is the area of the path?

Either, 9. A merchant bought a number of eggs, of which 25 per cent were broken in transit and the remainder were sold at 1*s*. 4*d*. per dozen; the expenses were 10 per cent of the receipts, and the net gain on the transaction £5. He calculated that if 10 per cent of the eggs had been broken, and his expenses had been $16\frac{2}{3}$ per cent of his receipts, his profit would have been $33\frac{1}{3}$ per cent. How many eggs were bought, and what sum did they cost?

or, 9*. A tradesman bought a quantity of tea, and sold it at 2*s*. per lb., thereby gaining $33\frac{1}{3}$ per cent; if his total gain was £14 what weight of tea did he buy?

Either, 10. A contractor undertook to make 15 miles of roadway in 40 weeks for £11,500; 180 men working 8 hours a day completed 3 miles in 10 weeks; the men then agreed to work one hour a day overtime, and, some boys being engaged to assist them, the work was finished in the stipulated time. The men received 6*d*. an hour, with 1*d*. an hour extra for overtime, and the boys received 5*d*. an hour. Assuming that a boy did as much work in 3 hours as a man in 2, and that there were 6 working days in a week, find the profit made by the contractor.

or, 10*. A farmer engaged a number of men and boys to reap corn, the men were to receive 4*s*. a day and the boys 2*s*. 6*d*.; if the work of 3 boys was equal to that of 2 men, and 4 men and 5 boys could reap 22 acres in 4 days, what sum should 6 men and 7 boys receive for reaping 48 acres?

OXFORD AND CAMBRIDGE SCHOOL EXAMINATIONS.

LOWER CERTIFICATES, 1897. ($1\frac{1}{2}$ hours.)

1. Find the sum of the quotient and the remainder when 38789 is divided by 3478.

2. How much money must be divided among 37 men so that each may receive £4, 7*s*. 9*d*.?

3. Reduce 231507 ozs. to tons, &c.; and 5 miles 3 furlongs 17 poles to inches.

4. Find the greatest common factor of the numbers 6254 and 10653, and also find their least common multiple.

5. Simplify:

$$(1)\ \left(\tfrac{2}{3} + \tfrac{1}{4}\right) \div \left(6\tfrac{2}{9} - 1\tfrac{2}{3}\right);$$

$$(2)\ \frac{\cdot 00843 \times (1\cdot 0351 - \cdot 9726)}{1\cdot 405}.$$

6. Find, correct to four decimal places, what fractional part 8*s.* 10½*d.* is of £11, 7*s.* 9½*d.*?

7. Find by practice the value of 127 things at £1, 9*s.* 4½*d.* each.

Find to the nearest franc the cost of making a railway 27345 metres long: the cost per kilometre being 303625 francs.

8. A wall is 54 yards long, 6 feet high, and 18 inches thick; find how many bricks it will contain if the dimensions of each brick be 9 inches by 4½ inches by 3 inches, and no allowance be made for the space occupied by the mortar.

9. Find the value of ·5 of £560, 4*s.* 7¼*d.* + 1·2 of 2·1875 pence.

10. Find the simple interest on £1368, 15*s.* for 106 days at 4¾ per cent per annum.

11. A watch which gains one second in every minute was set right at 6 A.M.; what was the true time in the evening of the same day when the watch indicated a quarter past 9 o'clock?

HIGHER CERTIFICATES, 1897. (2 hours.)

1. What weight of coal at 16*s.* 11*d.* per ton can be bought for £416, 4*s.* 2½*d.*?

2. Two sets of telegraph wires are carried on opposite sides of a railway on posts whose distances apart are 275 feet in one case, and 135 feet in the other. An engine starts from a point where two posts are exactly opposite one another, runs an exact number of quarter-miles, and stops at a point where two posts are again exactly opposite one another. Find the least distance which the engine can have travelled.

3. Simplify $\dfrac{5\frac{1}{5} - 4\frac{1}{4} + 1\frac{3}{16}}{2\frac{2}{5} - \frac{5}{6}} \div \dfrac{5}{12 + \dfrac{7}{3 + \dfrac{3}{2 - \frac{2}{3}}}}.$

4. Multiply together 25·63 and $2\cdot 4\dot{6}\dot{3}$.

Find the quotient of ·247329 by ·00574 correct to 5 places of decimals.

5. Assuming that a square metre is 1·196 of a square yard, find the number of acres in a square kilometre, neglecting fractions of an acre.

6. Find correct to the nearest farthing the compound interest on £2440 for three years at 2½ per cent, interest being payable yearly.

7. A gives B a bill for £1035, 3*s.* due 8 months hence in exchange for a bill for £1075, 18*s.* due 16 months hence. If the true present values correspond to a rate of interest of 4½ per cent per annum, determine which of the parties to the transaction is the gainer, and by how much.

8. Two casks originally contain 60 gallons of wine and 30 gallons of water respectively. On three successive occasions 12 gallons of liquid are

drawn from each cask and placed in the other. Express in gallons and decimals of a gallon the quantity of wine now in each cask.

9. A piece of work can be done by 3 men and 4 boys in 6 days, by 3 men and 1 boy in 8 days, and by 4 women and 8 boys in 5 days. How long would a woman take to complete the work single-handed?

10. Assuming that a cubic foot of water weighs 1000 ozs., determine what fraction of a ton of water is required to fill a cistern 3 ft. 8 ins. long, 2 ft. 8 ins. wide, and 2 ft. deep.

Assuming the above cistern to have no lid and to be made of wood 2 ins. thick, find the cost of *completely* covering the wood with paint at 9*d.* per square foot.

11. A banker invests half the money deposited by his clients in 4 per cent preference stock at 150, and lends a quarter of it on mortgages at 5 per cent, keeping the remainder in cash at the bank. After paying 2 per cent interest on deposits, he has an average annual balance of £1000 in his favour. Find the average amount of the deposits to the nearest pound.

12. At the beginning of a year a man invests £9000 in 3 per cent stock at 117, and £7200 in $2\frac{3}{4}$ per cent stock at 108. At the beginning of the next year when both stocks are at 111 he sells them. Assuming him to have received one year's interest on each stock, and to have paid a brokerage of 5*s.* per £100 of stock on the latter transaction but not on the former, find how much he has gained on the whole.

[*Any number of the following six questions may be substituted for an equal number of the questions 7–12. The marks for questions A–F are lower than those for 7–12.*]

A. What is the cost of 311 articles at £3, 11*s.* $7\frac{1}{2}$*d.* each?

B. Find to the nearest farthing the value of ·8*d.* + ·18*s.* + ·118 of a guinea.

C. Find the square root of 9·0707 correct to 6 places of decimals.

D. Beer is bought wholesale at 15*s.* 4*d.* the 18 gallon cask, and sold retail at 2*d.* a pint. What is the profit per cent on the outlay?

E. Find the cost of papering a room 18 ft. 6 ins. long, 15 ft. wide, and 10 ft. 6 ins. high with paper 2 ft. 6 ins. wide at 5*d.* a yard.

F. A man sells out £1225 of $2\frac{3}{4}$ per cent stock at 109 and invests the proceeds in 4 per cent stock at 140. What is the resulting change in his income?

COLLEGE OF PRECEPTORS EXAMINATIONS.

CERTIFICATE—THIRD CLASS. *Xmas,* 1896. (2 hours.)

1. Multiply 14830201 by 3851, and verify your answer by division.

2. Divide 135 tons 4 cwts. 3 qrs. 14 lbs. by 42; and reduce 9 miles 3 fur. 30 po. 1 yd. 2 ft. to feet.

3. From 271 times £35, 4*s.* 2*d.* take £9441, 6*s.* 8*d.*, and divide the remainder by 89.

4. I buy 500 oranges at 2 for three-halfpence, and again 500 more at 2 for threepence; after which 60 of the better sort are eaten; I then sell the remainder at five farthings each. How much do I gain or lose?

5. Calculate, by Practice, the cost of 1896 boxes of cigars at £1, 19*s.* $5\frac{1}{2}$*d.* each.

6. In what time will a careful workman save £45, 14*s.* 3*d.* who earns weekly £2, 10*s.* $9\frac{1}{2}$*d.* and expends in five weeks £7, 12*s.* $4\frac{1}{2}$*d.*?

7. Find the Greatest Common Measure of 75582 and 42237, and the Least Common Multiple of 12, 20, 24, 54, 81, 63.

8. Find the value of $6\frac{2}{3} + 7\frac{3}{4} + 8\frac{4}{5} + 9\frac{5}{6}$. Find also the difference between $6\frac{3}{8}$ and $4\frac{1}{2}$.

9. Reduce to their simplest values—

$$\text{(i.) } (3\tfrac{2}{5} \times 5\tfrac{1}{2} \times \tfrac{7}{9}) - (\tfrac{1}{3} \times \tfrac{5}{12}); \text{ (ii.) } \tfrac{3}{14} \times \frac{4\frac{5}{9}}{6\frac{1}{6}} \times \frac{6\frac{8}{11}}{11\frac{5}{7}}.$$

10. A clock, which loses 6 minutes in 15 hours, is 5 minutes too fast at midnight on Monday. What time will it indicate on the following Thursday at 5 o'clock in the afternoon, true time?

CERTIFICATE—Second Class. *Xmas*, 1896. (2 hours.)

1. Determine the value of seven million one hundred and twenty-eight trinkets at 1*s.* $4\frac{1}{2}$*d.* the twelve dozens.

2. How many telegraph poles will be required for wires extending 73 miles 3 furlongs 16 rods, supposing the interval between any two poles to be 99 feet?

3. If 3 sheep per acre are sufficient for prairie land, how many may feed upon a plot 240 miles long by 18 miles broad?

4. Reduce $6\frac{1}{2} - \{13\frac{3}{5} \times \frac{7}{10} \text{ of } \frac{55}{84} + \frac{4}{7} \text{ of } \frac{91}{204}\}$ to a single fraction in its lowest terms.

5. If in a Division sum the dividend is 483·758 and the quotient is 99·95, what is the divisor?

6. Add together $4{\cdot}\dot{5}$, $5{\cdot}\dot{6}$, $7{\cdot}1\dot{6}$, and $8{\cdot}\dot{2}7\dot{9}$.

7. Find (by Practice), to the nearest penny, the value of 11 tons 17 cwts. 3 qrs. 21 lbs. at £4, 17*s.* 6*d.* a ton. (*Most easily done by Decimals.*)

8. Obtain the amount of £72,552, 10*s.*, when put out at $4\frac{1}{4}$ per cent simple interest for $5\frac{1}{2}$ years.

9. Square 11·111.

10. Obtain the square root of 49381·7284.

11. If a French kilometre be taken as containing 1093·4 English yards, what fraction will 7 decametres be of 49 miles?

12. If a litre equals a cubic decimetre and contains 1·76 pints, how many (1) hectolitres, (2) pints will be contained in a tank whose capacity is 22 cubic hectometres?

13. An exhibition has 17,525 visitors. Of these a fifth hold season-tickets, each costing a certain number of half-sovereigns. Of the remainder a fifth purchase day-tickets costing the like number of half-crowns, and the rest pay the same number of shillings for their evening-tickets. The proceeds of the exhibition are £7991, 8*s.* What were the three entrance charges?

CERTIFICATE—First Class. *Xmas*, 1896. (2 hours.)

Part I.

1. How long will a man take to walk 8 miles 165 yards if he walks at the uniform rate of 105 steps per minute, and each step is 2 ft. 9 ins. long?

2. Multiply $\dfrac{4\frac{7}{9} - 2\frac{13}{15}}{3\frac{9}{20} + 1\frac{17}{30}}$ by $\{2\frac{3}{7} + \frac{2}{3} \text{ of } 2\frac{1}{16} - 1\frac{5}{28}\}$.

3. Divide $\cdot00015573$ by the square of $\cdot23$ increased by one tenth of the cube of $\cdot2$.

4. The value of $\cdot1625$ of a field at £17 per acre is £40, 1*s.* $1\frac{1}{2}$*d.* How much is the field worth?

If the area of the field is $\cdot080\dot{5}$ of the area of the farm to which it belongs, what is the area of the farm?

5. Express 2 gals. 3 qts. 1 pt. as a decimal of a cubic foot, correct to 5 places, assuming that a gallon contains $277\cdot274$ cub. ins.

6. Taking a sovereign to be worth $20\cdot42$ marks, find, correct to pence, the value of 1200 marks, and the rate per cent of the profit made by selling for a shilling an article which cost a mark.

7. Find the simple interest on £494, 10*s.* for 3 years at $2\frac{1}{2}$ per cent per annum; also, by how much it exceeds the true discount on that sum, due in 3 years, at $2\frac{1}{2}$ per cent per annum simple interest.

8. A man bought £1000 stock in $2\frac{3}{4}$ per cent Consols at $105\frac{3}{4}$. What is the lowest price at which he may sell the stock so as to make not less than 20 guineas profit? [Brokerage $\frac{1}{8}$ per cent.]

9. A rectangular walled yard is 50 yds. long and 35 yds. wide. All round the yard next the wall is a path 6 ft. wide. Find the area of the path in square feet.

In the middle of the yard is a square, each of whose sides is parallel to a wall and 5 yds. long, and from the middle of each side of the square a path $4\frac{1}{2}$ ft. wide leads directly to the outer path. How many cubic yards of gravel will be required to cover all the paths and the square to a depth of 4 ins.?

Part II.

10. Alter the number 873029067 by putting significant figures in place of the noughts, and thus obtain a number exactly divisible by 99.

11. If 12 men can do a certain amount of hoeing in 9 days, working 10 hours per day, how many men should a farmer set to work to do 3 times that amount of hoeing in 15 days, working 8 hours per day, he being aware that 3 men will join them at the beginning of the fourth day, and 4 more men at the beginning of the tenth day?

12. A bought lambs at a certain price per lamb and sold one-third of them to B and the rest to C. His actual profit on those sold to C was three times as great as that made on those sold to B, and his whole profit was at the rate of $10\frac{5}{8}$ per cent. The price paid per lamb by C exceeded that paid by B by one shilling. How much did A pay for one lamb?

13. A's income was to B's income as 5 to 4. A's income was increased 5 per cent, and B's income increased at a different rate per cent. Later on, their incomes were increased at the same rates, when A's income was to B's income as 1125 to 1024. At what rate per cent was B's income increased?

PROFESSIONAL PRELIMINARY EXAMINATIONS.

Second Class, 1896. (2 hours.)

1. In a division sum the divisor is 164600, the quotient 3854, and the remainder 26167. Find the dividend.

2. How many steps does a soldier take in marching $4\frac{3}{4}$ miles, each of his steps being 30 inches in length?

3. Reduce £14789, 19*s.* $11\frac{3}{4}$*d.* to farthings; and divide £23596, 11*s.* $8\frac{1}{4}$*d.* by 63.

4. Divide £430, 9*s.* $5\frac{1}{4}$*d.* between 4 women and 9 men, giving each woman twice as much as a man.

5. Find the cost of 5 cwts. 2 qrs. 24 lbs. at $7\frac{1}{2}$*d.* per pound.

6. Find the values of—

$$\text{(i.) } \left(7\tfrac{1}{2} + \tfrac{2}{3} + \tfrac{7}{12} + 2\tfrac{5}{24}\right) - \tfrac{7}{12}; \quad \text{(ii.) } \frac{\frac{4}{7} + \frac{3}{11}}{\left(1 - \frac{4}{7}\right) \times \frac{3}{11}}.$$

7. Reduce $\frac{5}{8}$ of $\frac{12}{25} + 5\frac{1}{2}$ to a decimal.

8. A person in 87 days spends £38, 19*s.* $4\frac{1}{2}$*d.* In how many days will he spend £163, 9*s.* $9\frac{1}{2}$*d.* at the same rate?

9. Find the simple interest on £645, 6*s.* for $10\frac{1}{4}$ years at $3\frac{1}{2}$ per cent per annum.

10. A room is 26 feet 3 inches long and 15 feet 9 inches broad. Find the cost of covering it with carpet which is three-quarters of a yard wide, at 4*s.* 6*d.* per yard.

11. Find the square root of 730·0804, and also that of $4\frac{289}{529}$.

12. One metre is equal to 3·28 English feet, and one French foot is equal to 1·066 of an English foot. Express one French foot in metres.

First Class, 1896. (2 hours.)

Part I.

1. The distance between two wickets was marked out for 22 yards; but the yard-measure made use of was $\frac{7}{12}$ of an inch too short. Find the true distance which was marked out.

2. Find the value of $2\frac{13}{45}$ of £32, 5*s.* 8*d.*, and of £36, 2*s.* 9*d.* divided by $4\frac{1}{5}$.

3. Simplify the expressions:—

$$\text{(i.) } \tfrac{7}{8} \times 2\tfrac{1}{7} \times \tfrac{3}{4} \times 1\tfrac{7}{9} \times \tfrac{2}{5}; \quad \text{(ii.) } \frac{1\frac{1}{7} \times \dfrac{4\frac{2}{3}}{3\frac{3}{7}} \times \dfrac{2 - \frac{2}{7}}{1\frac{3}{5}}}{1 + \dfrac{1}{1 + \frac{1}{2}}}.$$

4. Divide 23·78 by 62·5, and 6400 by ·04096; and reduce 3 hours 42 minutes 39 seconds to the decimal of a year.

5. Find (by Practice) the cost of $365\frac{1}{2}$ tons of coals at 13*s.* $5\frac{3}{4}$*d.* a ton.

6. If 7 oxen eat an acre of grass in 6 days, how long will it take 17 oxen to eat 34 acres?

7. The rate of interest being 7 per cent per annum, what is the discount on £1356, 13*s.* 4*d.* due 3 months hence; and what is the simple interest on the same sum for 9 months?

8. Extract the square root of 3·14515926535, or that of $\frac{7}{5}$, to seven places of decimals.

9. Given that a French metre is 39·37079 inches, show that the difference between 5 miles and 8 kilometres is very nearly 51 yards. [A kilometre is 1000 metres.]

Part II.

10. Two large steam ships travel, respectively, due north and due west from the same port, one at the rate of 16 and the other at the rate of 12 miles an hour. Find how far they are apart at the end of 6 hours.

11. 40 English navvies, each earning 7*s.* a day, do the same amount of work in 30 days which 56 Belgian workmen, each earning 6 francs a day, require 40 days to complete. Taking the value of the franc at

10*d.* in English money, determine which class of workmen is most profitable to a contractor. If a contract carried out by navvies cost £6000, what would be the cost of the same contract when executed by Belgian workmen?

12. An investor has £90,000 in India Stock, and has the choice given him by the Government of being paid off at par or of receiving £110 of Consols at $2\frac{1}{2}$ per cent for each £100 of his present holding. He chooses to be paid off at par, and then invests his money in the German 3 per cents at 92. Find the amount of his stock in German 3 per cents, and the excess of his income above what it would have been if he had accepted the proposed conversion.

13. A grazier buys 92 head of cattle for £635; he pays 100 guineas in rent, and his expenses in labour are £15, 10*s.* At the end of the year he sells off his stock. How much per head should he gain by the sale in order that he may realize his rent, his expenses, and obtain 10 per cent upon the money paid for his stock?

SCOTCH LEAVING CERTIFICATE, 1896.

Lower Grade. ($2\frac{1}{2}$ hours.)

1. By what number must 4766 be multiplied to obtain 2759514 as product?

2. What sum will remain out of 25 guineas, after paying 8*s.* 6*d.* for bread, £2, 15*s.* $3\frac{1}{2}d.$ for meat, 19*s.* 6*d.* for fish, 8*s.* $10\frac{1}{2}d.$ for vegetables, £1, 16*s.* 8*d.* for groceries, £1, 4*s.* $6\frac{1}{2}d.$ for milk, 14*s.* 8*d.* for washing, £3, 14*s.* 6*d.* for dress, £4, 13*s.* 6*d.* for wages, and £5, 8*s.* 4*d.* for rent?

3. A steam-engine, working steadily day and night, consumes a hundred-weight of coal in $7\frac{1}{2}$ hours. How many tons will be used in 100 days?

4. An army of 145,600 men loses $12\frac{1}{2}$ per cent of its number through disease, and 15 per cent of the remainder in battle; how many men are left?

5. (1) Show that the numbers 111 and 11111 are prime to each other.
   (2) Find the least common multiple of the even numbers from 2 to 20, inclusive.

6. A grain of gold if beaten into gold-leaf will cover $67\frac{1}{2}$ square inches. Find, in yards, the length of the side of a square that can be covered by an ounce Troy of gold-leaf.

7. Simplify (1) $\frac{17}{20}+\frac{18}{24}+\frac{10}{25}+\frac{12}{40}$; (2) $\frac{6\frac{1}{2}-3\frac{1}{4}}{7\frac{1}{3}+1\frac{4}{7}} \div \frac{4\frac{1}{3}\text{ of }1\frac{1}{4}}{2\frac{3}{7}\text{ of }3\frac{2}{3}}$.

8. Express (1) 5 yds. 2 ft. 4 ins. as a vulgar fraction of a mile.
   (2) £2, 17*s.* 4*d.* as a decimal fraction of £13, 8*s.* 9*d.*

9. (1) Multiply $\cdot02475$ by $\cdot64$, and divide the result by $\cdot000125$.
   (2) Reduce to a single vulgar fraction $\frac{\cdot6\dot{8}\dot{1} \times \cdot6\dot{1}}{1\cdot\dot{1}\dot{8} \div \cdot\dot{3}\dot{6}}$.

10. Find the total rental of a town if a tax of 9*d.* in the pound on rents brings in £2163, 14*s.* 3*d.*

11. A rectangular tank is 18 feet long, 14 feet broad, and 10 feet deep. Find the cost of painting the sides and bottom of the inside at 1*s.* $1\frac{1}{2}d.$ the square yard.

12. At what rate must simple interest be charged in order that £2375 may increase to £2826, 5*s.* in four years?

Higher Grade, 1896. ($2\frac{1}{2}$ hours.)

1. Find the values of—

(1) $\frac{13}{15}$ of 3 furlongs 4 chains 3 yards 2 feet;
(2) $1 \cdot 9375$ of 10 cubic yards.

2. A can do a piece of work in 14 days of 9 hours, and B can do the same piece in 12 days of 7 hours. How long will they take to do it, working together for 8 hours a day?

3. Give the two rules for the determination of leap-year.

On what day will a child, born 1st June, 1897, have lived exactly 4000 days?

4. The average passenger fare on a railway is $1\frac{1}{4}d.$ a mile, the average journey is 12 miles, and the average weekly revenue is £710306. Find the average number of passengers in a week.

5. Simplify the expressions—

$$(1)\ \left(\frac{4844}{5536} + \frac{7623}{8316}\right) \div 1\tfrac{19}{24};$$
$$(2)\ \cdot 61\dot{9}3\dot{6} \times 9 \cdot 68\dot{7}\dot{2} \times 20 \cdot 8\dot{3}.$$

6. The sum of £189, 6*s.* 9*d.* is to be divided between 7 men, 11 women, 5 boys, and 6 girls, so that for every 5*s.* a man receives a woman may get 2*s.* 11*d.*, and for every 3*s.* 6*d.* a woman receives a boy may get 2*s.* 6*d.* and a girl 1*s.* 6*d.* Find how much each person receives.

7. What sum will amount to £8334, 18*s.* in 3 years at 5 per cent compound interest?

8. A square enclosure is surrounded by a roadway whose breadth is one-eleventh of the side of the enclosure. The area of the roadway is 1 acre 2 roods 28 square poles 5 square yards. Find the length of the side of the enclosure in feet.

9. A man invests in Government stock paying $2\frac{3}{4}$ per cent interest, and finds that after paying 6*d.* in the pound income-tax, his net income is exactly $2\frac{1}{2}$ per cent of the sum invested. At what price did he purchase the stock?

IRISH INTERMEDIATE EXAMINATIONS.

Junior Grade, 1896. (3 hours.)

1. At a railway station during the year 1895 the average daily loading of waggons, excluding Sundays, amounted to 327 tons 13 cwts. 2 qrs. 12 lbs.; find the total loading for the year.

2. Find the cost of painting the walls of a room 43 feet 4 inches by 24 feet 10 inches, and 10 feet 6 inches high, at 1*s.* 6*d.* per square yard.

3. The ordinary process of finding the Greatest Common Measure of two numbers can be simplified when a factor common to the two is evident, or when an obvious factor of one is clearly not a factor in the other. Explain how you would proceed in each case.

4. If a clock gains 2 minutes 24 seconds in 24 hours of true time, and you wish to have it right at 6 P.M. on Thursday, how would you arrange the hands at 11 A.M. on the previous Tuesday?

5. If 24 men can excavate 840 cubic yards in 5 days, working 7 hours a day, how many hours a day will 20 men have to work to excavate 750 cubic yards in 4 days? Any 5 of the former can do as much work in 3 days as any 4 of the latter in 4 days.

6. In making gold thread a cylinder of silver weighing 7 lbs. avoirdupois, cased with 3 ozs. avoirdupois of gold, is drawn through holes continually diminishing in diameter until 200 ft. of the thread weighs only 1 drachm; find the entire length of the thread in miles, &c.

7. A piece of work can be done by 4 men in 6 days, or by 5 women in 8 days; 3 men and 3 women are employed; what is the total expense if a man's daily wage is 2*s.* 8*d.*, and a woman's 1*s.* 8*d.*?

8. Simplify $\dfrac{18\cdot\dot{8}5714\dot{2}}{\cdot03} \times \dfrac{4\cdot48\dot{6} \times 2\cdot25}{2\cdot\dot{3}}$.

9. Find by Practice the price of 192 casks of sugar, each weighing 29 stones 11 lbs. 14 ozs., at the rate of 11*s.* 8*d.* per cwt.

10. A sum of £116, 13*s.* 4*d.* is lent on the 11th March, 1892, and on the 16th October following the simple interest is found to be a guinea and a half. What is the rate of interest?

11. If £121, 13*s.* 4*d.* be placed at simple interest at $3\frac{1}{8}$ per cent on the 10th December, 1893, find the exact day on which the sum lent would amount to £127, 8*s.* 4*d.*

12. A bowling green whose length is 10 times its breadth covers an area of 5 acres 3 roods 8 perches 18 sq. yards; find its length.

13. Express, in its simplest form, the value of $\dfrac{\sqrt[3]{9\cdot663597}}{\sqrt{4\cdot5369}}$.

14. Find the area of one surface of a solid cube which contains 1 foot 30·416743 inches, cubic measure.

MIDDLE GRADE, 1897. (3 hours.)

1. Ten per cent of a battalion were absent from drill on a certain day, and of those 75 per cent were sick; find how many men were absent who were not sick, the number present at drill on the occasion being 576.

2. Simplify:

$$\tfrac{11}{16} \text{ of } \sqrt{\frac{(1\cdot345)^2 - (\cdot905)^2}{1\cdot\dot{2} + \cdot\dot{2}\dot{3}}} \times \sqrt[3]{\cdot\dot{0}3\dot{7}}\cdot$$

3. One-fourth of the subscribers to a fund gave one guinea each, one-third a pound each, one-fifth ten shillings each, one-twelfth a crown each, while the subscriptions of the rest at half a crown each came to £20: find the total sum subscribed, and the number of subscribers.

4. A contractor having engaged to complete a work in a certain time employs 15 men who work 9 hours a day, but when $\frac{2}{3}$ of the time is expired it is found that only $\frac{3}{5}$ of the work is done; if 3 additional men be then employed, how many hours a day must all work to finish the contract in the required time?

5. A car and a cyclist start together on a journey of $5\frac{1}{2}$ miles, the car travelling uniformly. The cyclist while on the footpath travels at the rate of 38 yards to 33 of the car, but on the road only at the rate of 10 yards to 11 of the car. If they finish together, what distance of the journey did the cyclist ride on the road?

6. The width of a rectangular cistern is twice its height, and its length is five times its height; what are its dimensions in feet if it holds $6\frac{1}{8}$ tons of water, a cubic foot of water weighing $62\frac{1}{2}$ lbs.?

7. A city square containing 37 acres 1 rood 39 perches $6\frac{1}{4}$ square yds. is to be enclosed by a railing; calculate the cost at 15*s.* 6*d.* per yard.

8. A person obtained a loan at 6 per cent per annum simple interest; but being unable to pay the debt at the appointed time, the loan was continued for 3 months more at 10 per cent per annum. The interest for the whole time would have been but for the increased rate £7, 2*s.* less than what it was. Find the amount of the loan.

9. At what rate per cent per annum did a banker discount a bill for £7460 due in 6 months for which he paid £7254, 17*s.* as its present worth?

10. A sum of £6375 is to be divided between A and B, who are respectively 18 and 19 years old, in such a way that, if their portions be invested at 4 per cent per annum compound interest, they shall receive equal amounts on reaching 21 years of age. How must the sum be now divided, and how much will each receive when 21 years old?

11. Apples are bought at the rate of 7 for 3*d.*; half of them are sold at 5 for 2*d.*, one-fifth at 7 for 4*d.*, and the rest at 7 for 5*d.* What is the smallest number of apples that can be so dealt in, and what is the gain or loss per cent?

12. Calculate the price of the $2\frac{1}{2}$ per cent Consols when £8670 can be purchased for £8279, 17*s.* (brokerage $\frac{1}{8}$ per cent).

13. A person purchased stock amounting to £3000 in the $2\frac{1}{2}$ per cents at 88. He sold out when these funds fell 3 per cent and invested the proceeds in 6 per cent railway preference shares at £180 per £100 share. Find the alteration in his income (brokerage to be neglected).

14. The sum of the true discounts at 6 per cent per annum and at 4 per cent per annum, on a bill having 6 months to run, is £1 less than the true discount at 5 per cent per annum on a bill of twice the amount, which also has 6 months to run. Find the amount of the first bill.

LONDON UNIVERSITY MATRICULATION EXAMINATIONS. *June*, 1896.

1. Simplify $10\cdot\dot{9}\dot{0} \times \cdot 9\dot{4}2857\dot{1} \div \dot{4}7619\dot{0}$.

2. In what years in the present century has the 29th of February fallen on a Sunday?

3. A tradesman buys milk at 11*d.* per gallon, and, after diluting it with water, sells it at $3\frac{1}{2}d.$ per quart, making a profit of 50 per cent. What proportion of water did he add to the milk?

4. The difference between the banker's discount and the true discount on a bill, due three months hence, is 2*s.* 1*d.*, interest being reckoned at 3 per cent per annum; find the amount of the bill (to the nearest penny).

January, 1897.

1. The metre was originally determined so that forty million metres should be equal to the earth's circumference: a metre is 39·37 inches. Compute from these data the mean radius of the earth in miles, being given that the circumference of a circle is to its diameter as 355 to 113.

2. (i.) A landlord's net rental, after paying 5 per cent for collection and 8*d.* in the pound income-tax *on the remainder*, is £580; find his gross rental.

(ii.) Selling articles at 6*s.* 6*d.* each, a dealer makes a certain percentage of profit; on increasing the price to 7*s.* 1*d.* his percentage of profit is increased by 10. What profit is he making at the latter price?

SCIENCE AND ART DEPARTMENT, *May*, 1896.

1. (*a*) State the rule for the division of one decimal fraction by another, and find the first five decimal places of the quotient of $0 \cdot 1373$ divided by $0 \cdot 021$.

(*b*) Reduce the following fraction to its simplest form:—

$$\frac{(3\tfrac{4}{11}) \times (1\tfrac{2}{9} \text{ of } 1 \cdot 08)}{(1\tfrac{11}{13}) \times (0 \cdot 6 + \tfrac{2}{9})}.$$

2. (*a*) Find the number that is to $7\tfrac{2}{3}$ in the ratio of £3, 1*s.* 3*d.* to £4, 13*s.* 11*d.*

(*b*) Divide 204 into three parts, proportional to the numbers 7, 8, 9.

3. A square field contains 6 acres 2 roods 19 poles $6\tfrac{1}{4}$ square yards; find the length of one side in yards.

4. Find by Practice the value of 12 tons 2 cwts. 3 qrs. 11 lbs. of material at £17, 11*s.* 8*d.* per ton.

5. (*a*) Find the difference between the simple interest and the compound interest on £2343, 15*s.* for two years at 4 per cent per annum.

(*b*) Find the present value of £84, 10*s.* due two years hence, compound interest being reckoned at 4 per cent per annum.

6. A man buys 45 bushels of apples at 2*s.* 6*d.* a bushel; a third of them is spoilt; of the remainder he sells a third at 2*d.* a pound, and two-thirds at 4*s.* a bushel: a bushel of apples weighs 40 lbs. What profit does he make? And what is his profit per cent of his outlay?

CIVIL SERVICE EXAMINATIONS.

BOY COPYISTS*—*February*, 1897. ($1\tfrac{1}{2}$ hours.)

1. Under what condition is a vulgar fraction equivalent to a terminating decimal? Prove that if the decimal does not terminate it must recur, and show, when the denominator is a prime number, what is the limit to the number of figures in the recurring part.

2. A publican buys beer at 10*d.* per gallon. To every 2 gallons of beer he adds a pint of water, and he sells the mixture at 2*d.* per pint. What is his gain per cent?

3. In 100 yards race A can give B 10 yards start and B can give C 20 yards, a dead heat resulting in both cases. How many yards start can A give C?

4. After deducting life insurance premium £100 and paying income tax at 8*d.* in the £ on the remainder, a man's nett income is £870. What is his gross income?

5. In 1871 the Census showed that the population of a certain town had increased 5 per cent on that of 1861. In 1881 the increase was 10 per cent on the population of 1871, and in 1891 it was 15 per cent on that of 1881. What was the increase per cent in the 30 years from 1861 to 1891?

6. A grocer has three kinds of tea. He sells some of one kind at 1*s.* 3*d.* per lb., and half that quantity of another kind at 1*s.* 6*d.* per lb. His sales of the third kind equal in weight those of the other two kinds together. The average price of all the tea sold being 1*s.* 2*d.* per lb., find the price of the third kind.

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* There were also 25 elementary questions similar to those set to candidates for the Excise.

ENGINEER STUDENTS.* 1896. ($1\frac{1}{2}$ hours.)

1. In a colonial town 28 per cent of the inhabitants were born in Europe, and the rest were born in the colony. Of the former four-sevenths are of British origin, and of the latter 81 persons are of British origin. What is the total number of inhabitants of the town if 18 per cent of them are of British origin?

2. A reservoir, supplied by two pipes A and B, could be filled by A in 16 hours, and by B in 24 hours; it could be emptied by another pipe, C, in 18 hours. The reservoir being empty all the pipes were opened, and the water was allowed to flow out for 6 hours; then pipe C was closed, but pipes A and B were left open. In what time was the reservoir filled?

3. Explain clearly the meaning of the term Scale of Notation. Multiply 65·65 in the scale of 7 by the number which is represented by 49 in the decimal scale; convert 65650 from the decimal scale to the scale of 7; and find the sum of your two results in the scale of 7.

4. Define the terms fraction, vulgar fraction, decimal fraction, improper fraction, compound fraction. Find the L.C.M. and the G.C.M. of the fractions $\frac{9}{34}$, $\frac{15}{68}$, and $\frac{21}{85}$, explaining clearly the reasons for your working in each case.

5. A field of 70 acres was rolled by a roller 11 ft. in circumference and 3 ft. 6 ins. in width, and another field of 63 acres was rolled by a roller 10 ft. 6 ins. in circumference and 3 ft. 4 ins. in width; the larger roller revolved at the uniform rate of 1 revolution in 3 seconds, and the smaller at the uniform rate of 9 revolutions in 25 seconds. Assuming that all the ground was rolled only once, in how much less time did one roller do its work than the other?

6. Prove the rule for finding the square root of any whole number, taking any number between 1000 and 10,000 as an example. Can any other method be employed to find the square root of whole numbers? If so, state in what cases it may be employed.

FEMALE CLERKSHIPS IN THE POST-OFFICE.

October, 1896. ($2\frac{1}{2}$ hours.)

1. Define Greatest Common Measure, and find that of 11781, 13923, and 17017.

If the L.C.M. of 68, 170, and another number is 2380, and the G.C.M. of the three numbers is 17, what may the other number be?

2. Two lots of coal were bought, one weighing 26 tons 3 cwts. 3 qrs., and the other 15 tons 14 cwts. 1 qr.; the price of the former was 16*s.* 3*d.* per ton, and the average price of the two together was 16*s.* $9\frac{3}{4}$*d.* per ton; what was the price per ton of the other?

3. Two clocks are set right simultaneously at 12 noon, one of which loses 6 seconds per hour, and the other gains 3 seconds in 50 minutes. How long will it be before the minute hands are again in the same direction, and what will then be the time, A.M. or P.M., by each clock?

4. Explain the terms *brokerage*, *at par*, *stock*.

I invest in $2\frac{3}{4}$ per cent stock at 110; after two years the stock is converted into $2\frac{1}{2}$ per cent stock; after two years more I sell out at par.

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* There were also 25 elementary questions similar to those set to candidates for the Excise.

Show that if brokerage of $\frac{1}{8}$ per cent be paid on investing and selling out, my capital, with accumulated simple interest, has increased only by about 4*s.* 6½*d.* per £100.

5. Define a *scale of notation* and its *radix*.

What is the condition among the digits in a scale of notation of which the radix is an odd number, that a given number is (1) odd, (2) even?

6. Calculate, to six places of decimals, the value of the following expression:—

$$16\left\{\frac{1}{5}-\frac{1}{5^3\times 3}+\frac{1}{5^5\times 5}-\frac{1}{5^7\times 7}\right\}-\frac{4}{239}.$$

7. Explain the contracted method employed to determine approximately the product of two decimals. By this method multiply $\cdot 111\dot{6}$ by $43\cdot 74\dot{2}85\dot{3}$, bringing out the product, correct to 8 places of decimals.

8. Define Present Worth, Discount, and Amount, giving simple examples to illustrate your definitions.

The discount upon £220, due at the end of a given number of years, is £44. Find the interest upon the same sum (£220) for the same number of years, and at the same rate of interest.

In the above problem determine the rate per cent and the number of years, given that these are indicated by whole numbers greater than unity.

9. Two posts are offered at the same initial salary. In one the salary increases at the end of every year by 5 per cent of that year's salary, in the other it increases at the end of every two years by 10 per cent of the salary for either of those years. Which is the better paid post? Show that for the fifth year one salary is slightly more than $\frac{5}{11}$ per cent better than the other.

10. Show that the sum of any numbers (each less than 100) will be exactly divisible by 11, if the difference between the sum of the units-digits and the sum of the tens-digits vanishes, or is exactly divisible by 11. Take any set of numbers—35, 67, 89, 8, 32, for instance—and base your reasoning upon them.

ASSISTANTS OF EXCISE—*May*, 1896. (1½ hours.)

1. What is the total weight of 3 lbs. 7 ozs. 9 dwts.; 17 ozs. 4 dwts. 6 grs.; and 4 ozs. 6¾ dwts.?

2. A cask, which originally contained 63 gallons, has lost by leakage 4 gallons 1 quart 3 gills; what quantity remains?

3. How many miles, furlongs, and poles are there in 5698 yards?

4. Reduce 3 acres 2 roods 4 perches to square feet.

5. Find the highest factor common to 1363 and 1551.

6. What is the least number into which 45, 54, 63, and 72 will divide without remainder?

7. Reduce $\frac{1584}{1760}$ to its most simple form.

8. Add together $4\frac{11}{16}$ and $3\frac{9}{14}$.

9. Subtract $2\frac{23}{24}$ from $5\frac{15}{16}$.

10. Multiply $4\frac{8}{11}$ by $3\frac{5}{13}$.

11. Divide $3\frac{1}{8}$ by $10\frac{5}{12}$.

12. Add together ·946307, 34·842, 9·0042 and ·000493.

13. Subtract 4·05632 from 10·001.

14. Multiply 63·432 by ·23075.

15. Divide 21·8775 by ·625.

16. Reduce $\frac{3563}{4375}$ to its decimal value.

17. Express as a vulgar fraction the ratio of 5 guineas to 35 crowns.

18. Reduce ·05637 of a cwt. to pounds, ounces, and the decimal of an ounce.

19. Find the value of 365 articles at 2*s.* 10½*d.* each.

20. What must be paid for 25 dozen articles at £4, 8*s.* 8*d.* per 100?

21. If a watch gains 25 seconds in 10 hours, in how many days will it gain exactly a quarter of an hour?

22. A man pays £12, 10*s.* for the oranges in 25 boxes, and £5 for oranges of the same quality filling 30 baskets each with the same number of fruit; if the boxes each contain 300 oranges, how many should be in each basket?

23. A dealer purchases goods and sells them at a profit of 12½ per cent; how much does he receive for each £ of his outlay?

24. The income-tax being 8*d.* in the £, how much per cent does this represent of a man's income?

25. A picture frame, 3 ft. 6 ins. long and 2 ft. 3 ins. broad, is made of wood 3 inches wide; how many square feet of wood does it contain?

ASSISTANTS OF EXCISE—*May*, 1896.

Higher Arithmetic. (3 hours.)

1. If a whole number be added to its square, show that their sum can only end in 0, 2, or 6; and if a whole number be added to its cube, the sum can only end in 0, 2, or 8.

2. A person writes down all the numbers from 21 to 40, both included, and proceeds to form the product of every two of them. Without attempting to form the products, determine what percentage of the whole number of them will be divisible by 6.

3. Find by the contracted method the square root of 3121, correct to within a hundred-thousandth; and thereby obtain $(55)^2$, $(5586)^2$, and the first seven figures of the square of 5586591. Show that $\sqrt{10}$ is less than $\frac{721}{228}$ and greater than $\frac{4443}{1405}$; and that it differs from the latter by a quantity less than $\dfrac{1}{2 \times 228 \times 1405}$.

4. A capitalist, having a sum of money to invest, places half in a bank which offers depositors 4½ per cent compound interest added yearly to the deposits; and the other half in a bank, which also allows compound interest, where the rate is ½ per cent less, but the additions of interest are made every six months. At the end of two years he finds there is a difference of £249, 16*s.* 3¼*d.* in the two accounts. Find the sum originally invested.

7. In 1892, Jan. 1 was a Friday. When does this occur again in this century, and when in the next, 1900 not being a leap-year? Give all the steps of your reasoning.

8. The present values on the 1st of May, 1895, of two bills drawn for the same amount were in the ratio of 4638 : 4639. Interest was reckoned in the case of the first bill at the rate of 4¼ per cent per annum, and in the case of the second at the rate of 4 per cent per annum. If the first was fully due on the 22nd of last September, when did the second become due?

9. Three points are kept moving at uniform rates of 5 ft. per second, 16 ft. in 3 seconds, and 23 ft. in 4 seconds, in concentric circles

having diameters 32 ins., 33 ins., and $33\frac{3}{4}$ ins. respectively. They start simultaneously from positions seen in the same direction from the centre, the last moving in the opposite direction to the other two. Find how long after the start they will next be seen in the same direction from the centre, and how many revolutions each will then have made.

(Nos. 5, 6, 10, 11, 12, *were questions on Mensuration.*)

SECOND DIVISION CLERKSHIPS—*February*, 1896.

(3 hours.)

1. Simplify $\dfrac{3\sqrt{2} - 2\sqrt{3}}{3\sqrt{2} + 2\sqrt{3}} + \dfrac{3\sqrt{2} + 2\sqrt{3}}{3\sqrt{2} - 2\sqrt{3}}$; and express $\sqrt{50}$ in the form of a continued fraction.

2. The process of finding the G.C.M. of two numbers may sometimes be abbreviated by the consideration of factors, which inspection shows to be (i.) common, (ii.) not common, at any stage of the work. Explain the principles upon which such factors must be dealt with, and apply them to determine the G.C.M. of 3237975 and 7047300.

3. Show how you would adapt the tests by casting out nines and elevens (usually applied to multiplication sums) to simple addition and subtraction sums.

An addition sum is incorrectly performed, but the tests by casting out nines and elevens both fail to detect the error. Show that the difference between the answer obtained and the correct answer must be a common multiple of 9 and 11.

4. £240 are invested in Vienna bills at the rate of 12 florins for £1, brokerage being $\frac{1}{10}$ per cent. These bills are sold in Paris, and bring in —after deduction there of $\frac{1}{10}$ per cent brokerage, $\frac{1}{8}$ per cent commission, and $\frac{1}{40}$ per cent for stamps—the sum of 6056 francs. Find to three places of decimals the rate of exchange between Vienna and Paris, *i.e.* the equivalent in francs of 100 Austrian florins.

5. On a circular railway connecting three towns, A, B, C, two trains are constantly running in opposite directions. Every four hours they leave A simultaneously, and travelling uniformly at the same rate, cover the distances A–B and B–C in 65 minutes and 75 minutes respectively. If the trains pass at a point which divides the distance from B to C in the ratio of 7 : 5, and stop the same time at each station, determine what this time must be, and also the time occupied in traversing the distance between C and A.

6. Prove (with the aid of a diagram, or in any arithmetical manner) that the difference of the squares of any two consecutive whole numbers is equal to the sum of the numbers.

A sum of money is lent at compound interest, and the increase in its amount for two years which would result from an increase of 1 in the rate per cent is 6*s.* 8*d.* more than it would have been if the initial rate had been 1 less. Determine the sum, and show that the rate cannot be determined.

7. Define a decimal and explain the use of the 0 in the decimal scale of notation.

The sum of a number of decimals which are all different from one another is to be less than unity, and each decimal is to consist of two figures. What is the greatest number of decimals which can be selected so as to fulfil these conditions?

Find also the greatest number of different decimals, each consisting of *three* figures, if their sum is to be equal to unity.

8. Solve by contracted division, giving all the steps, the following problem:—The old wine-gallon contained 231 cubic inches; the present or imperial gallon contains 277·274 cubic inches. Find how many imperial gallons are contained in the old wine-hogshead of 63 wine gallons, old measure.

9. Convert *te*4·*te*4 from the scale of twelve to the scale of eight. Explain the principle which governs each part of the process. To what extent would *a priori* reasoning enable you to predict the form of the part of the answer which lies to the right of the point (*i.e.* the part of the answer which is expressed in eighths and powers of eighths).

10. Tram-cars start up-hill from A to B (a distance of 10 miles), and return down-hill from B to A, at intervals of $7\frac{1}{2}$ minutes in each case. The cars running in the same direction travel at a uniform rate, but their speed from B to A is greater than from A to B. A cyclist riding at a uniform speed from A to B passes tram-cars at the rate of 8 in 3 hours, and meets them at the rate of 30 in $1\frac{3}{4}$ hours. He returns at a uniform, but different, speed of 15 miles an hour, and completes the whole journey from A to B and back in 115 minutes. Will the distance between the points at which he *passes* each car be greater on the outward or on the return journey, and by how much? Answer the same question in regard to the cars which he *meets*.

11. A person set to calculate the decimal equivalent of $\frac{130000}{837247}$ inadvertently transposes the 2 and 4 in the divisor, and in his third line of subtraction writes 20 . . where he should write 19 . . If he makes no other error, show (by the simplest approximations that will yield the result) that his answer will fall short of the true answer by more than ·00001 and less than ·00003.

12. (*This question involved Logarithms.*)

MILITARY ENTRANCE EXAMINATION.

ALL CLASSES—*November*, 1896. ($1\frac{1}{2}$ hours.)

1. Simplify $\dfrac{7\frac{1}{3}+3\frac{1}{7}}{11\frac{1}{9}-9\frac{1}{11}} \times \left(2\frac{3}{4} - \dfrac{3}{8\frac{1}{2}+6\frac{1}{6}}\right)$.

2. Divide 26·751 by ·000925.

3. Find the least common multiple of 34, 42, 119, and 255; and obtain the sum of the fractions $\frac{3}{34}$, $\frac{1}{42}$, $\frac{10}{119}$, and $\frac{1}{255}$, by reducing them to a common denominator.

4. Find the value of $\frac{4}{15}$ of £1, 19*s.* $8\frac{1}{4}$*d.*

5. Find the cost of 25 quarters 3 bushels of oats at 19*s.* $3\frac{1}{2}$*d.* a quarter.

6. One side of a rectangular field is $\cdot 0\dot{5}\dot{4}$ of a mile, and the adjacent side is $\cdot 1\dot{3}$ of a furlong. Find the length of each side in yards and feet, and express the area of the field as a fraction of an acre.

7. A sum of £377, 13*s.* 4*d.* lent at simple interest amounts in a year and a half to £396, 11*s.* 0*d.* What is the rate of interest?

8. A garden whose length is 67 ft. 9 ins. has a path 4 ft. wide on the two sides and at one end: if it costs £4, 10*s.* $3\frac{3}{4}$*d.* to turf the remainder at 6*d.* a square yard, what is the width of the garden?

9. A, B, and C, who are engaged on piece-work, do amounts in the same time which bear to one another the proportion of 10, 9, and 14

respectively. What ought each to receive if the amount paid for the whole is £24, 15*s.*?

10. What is meant by an *odd* number and an *even* number?

Show that it is only necessary to look at the digit in the unit's place to ascertain if a number is odd or even.

Several numbers have to be added together. What is the condition that their sum should be odd?

11. A cistern is filled in $3\frac{1}{2}$ hours by a pipe 3 sq. ins. in cross section through which water flows at the rate of 6·4 miles an hour. What is the volume of the cistern?

12. If coffee and chicory cost £8, 10*s.* and £2, 10*s.* per cwt. respectively, what is the proportion of coffee and chicory in a mixture of which 7 lbs. are worth 7*s.* 6*d.*?

13. A man buys goods and finds that the cost of carriage is 4 per cent on the cost of the goods. He is compelled to sell at a loss of 5 per cent on his total outlay; if, however, he had received £3, 5*s.* more than he did, he would have gained $2\frac{1}{2}$ per cent. What was the original cost of the goods?

14. A and B set out from the same place in the same direction, and travel uniformly; after 9 days' travelling A finds he is 72 miles ahead of B: he then turns and travels back the distance B would travel in 9 days, he then turns again and overtakes B in $22\frac{1}{2}$ days from the start. What is the rate of travelling of each?

QUEEN'S SCHOLARSHIP EXAMINATIONS.

ENGLAND—MALES AND FEMALES, *December*, 1896. (3 hours.)

SECTION I.

1. Divide 9173 by 39, and show, in the course of the working, as to a class of young children, how to find—

(*a*) The remainder after 39 has been subtracted from the dividend 200 times;

(*b*) How many times 39 has been subtracted, when the remainder is 203;

(*c*) The product of 235 and 39.

2. Explain the terms: Average, Measure, Quotient, Stock, Percentage, Present Worth.

3. Show, as to a class of children beginning fractions, how to subtract $\frac{1}{8}$ from $\frac{1}{6}$, explaining each step clearly.

4. Give short notes of a first lesson in decimal fractions.

SECTION II.

5. A sum of £657, 9*s.* $9\frac{1}{2}$*d.* is divided among 68 persons, so that one receives four times as much as each of the others; find the shares.

6. Find, by Practice, the cost of fencing a railway for 17 miles 5 furlongs 185 yards, at the cost of £5, 8*s.* 2*d.* per furlong.

7. The following items occurred in a bill:—

	s.	*d.*
To $23\frac{1}{4}$ lbs. of sugar, at 3*d.* per lb., ...	6	$1\frac{1}{2}$
To $5\frac{1}{2}$ lbs. of tea, at 1*s.* 9*d.* per lb., ...	9	11
To 35 lbs. of flour, at 1*s.* 10*d.* per stone, ...	3	11
To $3\frac{1}{2}$ ounces of pepper, at 2*s.* per lb., ...	0	$6\frac{1}{2}$

By how much was the bill incorrect?

8. Simplify $\dfrac{2}{3 + \dfrac{4}{5 + \frac{6}{7}}}$.

9. Divide (a) 3217 by ·00625; (b) ·9651 by 187·5; (c) $\cdot\dot{1}0\dot{0}$ by ·005.

10. If mackerel be bought at the rate of 16 for a shilling, and sold so as to gain $33\frac{1}{3}$ per cent, what is the selling price of each?

11. Find the discount on a bill for £2828 due in 2 years 146 days, at 5 per cent per annum.

12. Find the greatest common measure of 1 hour 7 minutes 47 seconds, and 7 hours 32 seconds.

Section III.

13. If $\frac{1}{18}$ of a certain sum of money be added to $\frac{1}{5}$ of the same sum, the total is found to be less than $\frac{1}{2}$ of the sum by $5\frac{1}{2}d.$; what is the sum?

14. If ·2 of one number be equal to ·04 of another, what is the ratio of the two numbers?

15. What sum will amount to £1, 16*s.* 9*d.* in two years, at 5 per cent per annum compound interest?

16. A man for a wager travels three miles in seventeen minutes, walking the first mile, running the second, and cycling the third; he runs twice as fast as he walks, cycles twice as fast as he runs, and loses five seconds in getting on his cycle. Find the time occupied in each mile.

17. If 2·23 be taken as the square root of five, show that the error must be less than ·01.

18. One-half of a certain sum of money is invested so as to return 5 per cent, one-third so as to return 6 per cent, and the remainder so as to return 8 per cent interest; what is the average rate of interest on the whole?

19. By buying 3 per cent Consols at a certain price, I find that I obtain $3\frac{5}{8}$ per cent for my money, and derive a net income therefrom, after paying an income-tax of 6*d.* in the £, of £421, 4*s.*; find the amount of stock bought and the price at which I bought it.

SCOTLAND—Males, *December*, 1896. (3 hours.)

1. Simplify $\dfrac{1\frac{1}{5} - 5\cdot25 \text{ of } \frac{6}{7} + 4\frac{1}{3}}{3\cdot5 + 4\frac{1}{5} \div 2\frac{1}{3} - 4\frac{1}{3} \times \frac{7}{26}}$.

2. Express the sum of $6\frac{1}{2}$ of $\frac{1}{605}$ acre and 1·2 of 20 poles 3·75 sq. yds. as the decimal of 1 ro. 1 po. $11\frac{3}{4}$ yds.

3. The Least Common Multiple of two numbers is 8211, and their Greatest Common Measure is 17. If one of the numbers is 391, find the other.

4. A contractor, having engaged to make a railway $58\frac{1}{2}$ miles long in 40 weeks, employed 2160 men upon the work. At the end of 13 weeks $19\frac{1}{2}$ miles were finished. How many men had he then to pay off so that the work might not be completed before the stipulated time?

5. A banker gains £1, 0*s.* 8*d.* by deducting interest instead of discount on a bill due 10 months hence at 5 per cent. What is the amount of the bill?

6. At what time between 6 and 7 o'clock will the hands of a watch be together?

7. What must a person have invested in the 3 per cents at 87, if the

transfer of three-fourths of his capital to the 4 per cents at 120 would diminish his income by £4? (*No brokerage allowed.*)

8. A grocer sells coffee, containing 25 per cent of chicory, at 1*s.* 4*d.* per lb. What does he gain or lose per cent if the cost price of the pure coffee be 1*s.*, and that of the chicory 4*d.* per lb.?

9. What sum put out for $1\frac{1}{2}$ years at 4 per cent compound interest, payable half-yearly, will amount to £6632, 11*s.*?

10. A does $\frac{3}{5}$ of a work in 6 hours, and his place is then taken by B, who does $\frac{3}{4}$ of the remainder in $3\frac{3}{5}$ hours. How long will C take to finish what is left, if A, B, and C can jointly do the whole in 4 hours?

11. Simplify $\dfrac{\sqrt[3]{12568\frac{5}{64}}}{\sqrt{\cdot 000625}}$.

12. The sum of £23 is made up of half-sovereigns, half-crowns, and sixpences. There are three times as many half-crowns as half-sovereigns, and the number of half-sovereigns is to the number of sixpences as 3 : 10. How many are there of each coin?

13. A, B, and C together purchase an estate, their shares being as $1\frac{1}{2} : 2\frac{1}{2} : 4$. C sells one-third of his share to A, who sells 96 acres to B. If A and C have then equal portions, how many acres had each at first?

14. A person standing on a railway platform, 264 yards long, noticed that a train passed the platform in 20 seconds and himself in 8 seconds. Find the length of the train and its rate per hour.

SCOTLAND—Females, *December*, 1896. (3 hours.)

1. Simplify $\dfrac{5\cdot 75 - \frac{3}{7} \times 15\frac{3}{4} + 2\frac{2}{35} \div 1\cdot 44}{\frac{3}{4} \text{ of } 7\frac{3}{7} - 5\cdot 6 \div 3\frac{4}{15}}$.

2. Express $2\cdot 625$ of $125\frac{5}{7}$ yards as the decimal of the sum of $\cdot 1875$ of $1\frac{3}{5}$ mile and $\frac{3}{13}$ of $\cdot 4\dot{3}$ furlong.

3. If 30 men can perform a piece of work in 13 days of $8\frac{1}{2}$ hours each, how many men working the same number of hours a day, can do one-third as much again in one-sixth of the time, three of the second set being equal to four of the first?

4. What is the least number, which, when divided by 124, 341, and 372, gives no remainder?

5. After paying income-tax at 8*d.* in the £, and his other rates which exceed the income-tax rate by 3*d.* in the pound, a man has £1105 left. What is his gross income?

6. What sum of money paid just now would discharge a debt of £206, 8*s.* 7*d.*, due 8 months hence, at $4\frac{1}{2}$ per cent?

7. A merchant buys oranges at the rate of 2 for $1\frac{1}{2}d.$ How many should he sell for £1, 19*s.* that he may gain 30 per cent?

8. A ship captain owns $\frac{3}{8}$ of his own vessel. In virtue of his command he receives $\frac{1}{25}$ of the profits, and of the remainder his share as proprietor. What proportion of the whole does he receive?

9. What sum should be invested in the $3\frac{1}{2}$ per cents at 98, to yield a net annual income of £413, after payment of income-tax at 4*d.* in the £?

10. The sum of £65, 5*s.* consists of half-sovereigns, half-crowns, and sixpences, the number of these coins being as 3 : 4 : 7. How many are there of each?

11. How much coffee, at 1*s.* 8*d.* per lb., must be mixed with 45 lbs. of chicory, at 7*d.* per lb., so that by selling the mixture at 1*s.* 8*d.* per lb., $33\frac{1}{3}$ per cent on the outlay may be gained?

EDUCATION DEPARTMENT.

CERTIFICATE EXAMINATIONS—ENGLAND.

MALES—FIRST YEAR, *July*, 1896. ($2\frac{1}{2}$ hours.)

1. Answer three of the following:—

(*a*) Define a Mixed Number, an Improper Fraction, Prime Number, Factor, Present Worth, and give an example in each case.

(*b*) In dividing by Short Division by a number made up of two factors (*e.g.* 35), explain and justify the rule for ascertaining the true remainder.

(*c*) How is it that a fraction may be divided by such a quantity that the quotient is greater than the dividend?

(*d*) Discuss the advantages of the duodenary as compared with the decimal system of notation. How many additional symbols would be required if the former were adopted?

2. Show the shortest method of calculating mentally—

(i.) The value of $(\frac{1}{51} + \frac{1}{85})$ of fifteen times $4\frac{1}{4}d.$;

(ii.) The yearly income arising from the investment of £672 in 3 per cent stock at 84, after paying an income-tax of 10*d.* in the pound;

(iii.) The square root of 7921;

and give the answer in each case.

3. Subtract 3·2061 of 9 ozs. 13 dwts. from 1·0672 of 2 lbs. 5 ozs. Troy, and give the answer in grains and decimals of a grain.

4. A man pays a poor-rate which exceeds his income-tax, at 7*d.* in the pound, by £22, 10*s.*, and after paying both his net income is £486 per annum. Find his gross income.

5. A person left his property to five persons in the following proportions:—$\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{7}$, $\frac{1}{8}$, $\frac{1}{9}$, and the residue to Guy's and St. Thomas' Hospitals in the ratio of 2 : 3. Guy's receives £769, 4*s.* What sum did he leave?

6. Find the discount on £808, 1*s.* 4*d.*, due 3 years and 9 months hence, at 4 per cent per annum.

7. Find the square root of—

$$\frac{10\frac{3}{5} - 9\frac{5}{7} + 4\frac{5}{14}}{8\frac{1}{8} + 9\frac{1}{9} - 10\frac{1}{10}}.$$

8. In walking, A covers 3 feet at each step, and B covers $2\frac{1}{2}$ feet. If both start together, and keep step, until A has walked a mile, how far behind will B be? and if they keep abreast, how often will they be in step in a mile?

9. A runs 100 yards in 10 seconds, and beats B by 10 yards. If B had a start of 2 seconds, what would be the exact result of the race?

10. A man holds £5000 Stock in the $2\frac{3}{4}$ per cents at $112\frac{1}{8}$, and £4000 Stock in the $2\frac{1}{2}$ per cents at $106\frac{3}{8}$. He sells out and invests the proceeds in a Debenture 3 per cent Stock at $98\frac{3}{8}$. What is the alteration in his income, allowing $\frac{1}{8}$ per cent brokerage for each transaction of buying and selling?

11. A is indebted to B in the sum of £1050, and to C in the sum of £1175; in how many years will the debts be equal, B's debt increasing at 5 per cent and C's at 4 per cent per annum, simple interest?

12. A rectangular tank, measuring internally 3 feet in width by 6 feet in length, receives all the rain that falls on a roof, whose area is 7200 square feet, and it was estimated that in one day a pint of water fell on

each square foot of roof. If the tank was empty to begin with, what was the depth of water in it at the end of the day, if a pint of water fills 12 cubic inches?

13. A person started at half-past two and walked to a village, arriving there when the church clock indicated a quarter-past three. After staying 25 minutes, he drove back by a road one-fourth as long again, at a rate twice as fast as he had walked, and reached home by four o'clock. How far wrong was the church clock?

FEMALES—FIRST YEAR, *July*, 1896. (3 hours.)

1. Answer three of the following:—

(*a*) Define a Fraction, a Decimal Fraction, a Common Measure, a Multiple, Discount, Present Worth, and give an example of each.

(*b*) In dividing a number by 20 we usually cut off the last figure of the dividend, and divide by 2; explain and justify this process.

(*c*) Explain clearly how you would give to children a first idea of ratio.

(*d*) What practical difficulties impede the adoption of the Decimal System of weights and measures in England?

2. Show the shortest methods of calculating mentally—

(i.) The total wages of 150 men at 1*s.* 4*d.* an hour, if they work 8 hours a day for 18 days;

(ii.) The interest on £360 for 146 days at $1\frac{1}{4}$ per cent per annum;

(iii.) The cost of the carpet, 27 inches wide, required to cover the floor of a rectangular room, 8 yards long and 6 yards wide, at 1*s.* 8*d.* a linear yard.

3. Reduce 3,256,794 drams to tons, cwts., quarters, &c.

4. In the reprint of a book, consisting of 810 pages, 50 lines are contained in a page instead of 40, and 72 letters in a line instead of 60; of how many pages will the new edition consist?

5. Find by Practice the cost of 5 cwts. 2 qrs. 23 lbs., at £4, 15*s.* 8*d.* per cwt.

6. Reduce $\frac{7}{11}$ of £2, 10*s.* to the fraction of $3\frac{1}{2}$ guineas, and find the value of ·68125 of £3.

7. Find the yearly income arising from the investment of £4788 in the $2\frac{1}{2}$ per cents at 105. Neglect brokerage.

8. Find the square root of £36,372,961.

9. If oranges can be bought at the rate of 18 for a shilling, how many should be sold for £1, 6*s.*, so as to gain 30 per cent?

10. What is the price of 4 per cent Stock, if an investment in it is as profitable as one in $3\frac{1}{2}$ per cent Stock at 98? Find the loss of income when £5200 of the above 4 per cent Stock is sold at the ascertained price and the money reinvested in $2\frac{3}{4}$ per cent Stock at 96. Neglect brokerage.

11. A cistern is filled by two pipes A and B, and emptied by a pipe C. A can fill it in 20 minutes, and B in 30 minutes, while C can empty it in 48 minutes. If all three pipes are open together, in what time will the cistern be half full?

12. Two men undertake to do a piece of work for $1\frac{1}{2}$ guineas. One could do it alone in 7 days, the other in 8 days. With the help of a third man they do it in 3 days. How much ought this man to receive?

13. If A runs a mile in 7·5 minutes, and B runs at the rate of 7·5 miles an hour, how far behind will the loser be in a race which the faster wins in 6 minutes?

FEMALES—SECOND YEAR, *July*, 1896. (3 hours.)

1. Answer three of the following:—

(*a*) Define Interest, Simple Interest, Compound Interest, Present Worth, a Common Measure, an Aliquot Part.

(*b*) How would you give to children who understood Ratio a first idea of Proportion?

(*c*) Give a short sketch of a first lesson in Decimals.

(*d*) Show, by a diagram, that

$$\tfrac{1}{2} - \tfrac{1}{4} - \tfrac{1}{5} = \tfrac{9}{20}.$$

2. In a bag of 2760 coins there is an equal number of sovereigns, half-sovereigns, crowns, half-crowns, florins, shillings, sixpences, and threepenny-pieces. Find the total value of the coins.

3. Simplify $\dfrac{7\frac{1}{2} + 1\frac{17}{22}}{8\frac{2}{3} + 3\frac{23}{33}} - \dfrac{2\frac{1}{9} - 1\frac{2}{3}}{1\frac{2}{5} \text{ of } 1\frac{3}{7}}$, and express the answer as a decimal.

4. How often must I run round a square field of 10 acres to run a mile?

5. If 40 men can mow a field of 19 acres in $8\frac{1}{2}$ days of 10 hours each, how many acres can 17 men mow in 50 days of 8 hours each?

6. If oranges can be bought at the rate of 20 for a shilling, how many should be sold for £1, 8*s.* so as to gain 40 per cent?

7. If a watch, which is 10 minutes too fast at noon on Monday, gains 3 minutes and 12 seconds every 24 hours, what will be the time by it at a quarter-past ten on the following Saturday morning?

8. If 48 lbs. of tea are worth 55 gals. of ale, and 63 gals. of ale 24 bottles of wine, and 11 bottles of wine 9 pairs of gloves, how many pounds of tea must be given for 20 pairs of gloves?

9. How many pounds of tea, at 1*s.* 6*d.* a pound, should be added to 25 pounds, at 2*s.* 6*d.* a pound, so that by selling the mixture at 2*s.* 1*d.* a pound, a gain of 25 per cent may be made?

10. I hold £6000 $2\frac{3}{4}$ per cent Stock at $112\frac{1}{2}$, and decide to transfer it to the $2\frac{1}{2}$ per cents. What must the price of the latter Stock be so that my income may be increased by £3, 15*s.*? Neglect brokerage.

11. A person in his will directed that $\frac{1}{2}$ of his property should be given to A, $\frac{1}{4}$ to B, $\frac{1}{5}$ to C, and $\frac{1}{8}$ to D; show that these directions cannot be carried out; and if his property amounted to £536, 18*s.* $1\frac{3}{4}$*d.*, show how it should be divided so that the shares may have to one another the ratio he intended.

CERTIFICATE EXAMINATIONS—SCOTLAND, 1896.

MALES—FIRST YEAR. ($2\frac{1}{2}$ hours.)

1. (*a*) Reduce $\frac{4489}{3125}$ to a decimal, and $2{\cdot}48\dot{0}2\dot{7}$ to a vulgar fraction in its lowest terms.

(*b*) Simplify $5\frac{1}{3}$ of $\dfrac{1}{1\frac{1}{3} + \dfrac{1}{2\frac{1}{4}}} \div \dfrac{4\frac{1}{5} + 5\frac{1}{4}}{4\frac{1}{4} + 3\frac{2}{8}}$.

2. (*a*) Find the square root of $27\frac{1}{25}$, and $90018{\cdot}0009$.

(*b*) £9, 16*s.* is spent on a certain number of persons, such that each receives as many shillings as there are persons. How many are there?

3. Three men working all day can plant a field in 10 days; but one of them having other employment can only work half-time: how long will it take them to complete the work?

4. A room is 19 feet 4 inches long, 12 feet 2 inches wide, and 10 feet 6 inches high. Find the difference of cost of papering the walls with English paper, which is 21 inches wide, and costs 7*d.* a yard, or with French paper, which is 18 inches wide, and costs $6\frac{1}{2}$*d.* a yard, allowing 13 square yards for windows, &c.

5. A railway which used to pay 5 per cent duty on all its passenger traffic has now to pay nothing on its third-class traffic, and only two per cent on the remainder. If the whole amount of duty payable is now one-tenth of what it was before, find what fraction the third-class traffic is of the whole.

6. A bill for £1812 drawn July 13th, at 5 months, is discounted October 4th, at $3\frac{1}{2}$ per cent. Find the discount, allowing the usual three days of grace.

7. A starts a business with a capital of £3000. After five months he takes B, with a capital of £2000, into partnership, and one month later C with a capital of £4500. At the end of the year the profits are divided, and B finds that he has to pay 5 guineas as income-tax. Find the total profits of the concern, the tax being at the rate of 6*d.* in the pound.

8. A person invests £13,650 in a 4 per cent Stock at 91. On the Stock falling to 75 he sells out, and, investing the proceeds in an 8 per cent Stock, he finds that he thereby loses in interest £60. What is the price of the latter Stock?

9. A grocer mixes 5 cwts. of tea, which cost 12 gns. per cwt., with 2 cwts. at 14 gns. per cwt. What is his gain or loss per cent if he sells the mixture at half a crown per lb.?

10. One liquid contains $22\frac{1}{2}$ per cent of water, another 27 per cent. A glass is filled 5 parts of the one liquid and 7 parts of the other. What percentage of water is in the glass?

Females—First Year. (3 hours.)

1. (*a*) Define Fraction, Factor, Numerator, Denominator.

(*b*) Explain as clearly as you can why the value of a fraction is increased if its denominator is diminished.

2. Find the greatest and the least of the following fractions:—$\frac{11}{12}$, $\frac{29}{30}$, $\frac{7}{16}$, $\frac{17}{18}$, $\frac{47}{48}$.

3. A man whose weekly earnings are 17*s.* 6*d.* saves a fifth part of that sum every fortnight; find how many weeks it will take him to save as much as he spends in 40 weeks.

4. If 5 cwts. 3 qrs. 14 lbs. of a certain article cost £6 per cwt., find what will be the cost per lb., when the cost of the whole has been reduced by £7, 16*s.* 8*d.*

5. A can finish a piece of work in 12 days of 8 hours; B can finish it in 7 days of 9 hours; find in how many days they can finish it together, if they work 6 hours a day.

6. Two men bought an ox weighing 5 cwts. 2 qrs. 20 lbs., and sold it at the rate of $7\frac{1}{2}$*d.* per lb.; find their shares of the selling price, the one having given half a crown for every two shillings of the other in the purchase-money.

7. A party consisting of 27 persons set out to travel. The fare for each was to be £1, 16*s.* 5*d.*, but some of them having no money, the others agreed to pay their fares in equal proportions, and so had to pay £2, 2*s.* 9*d.* each; find how many of the party did not pay.

8. A, B, and C entered into partnership; A had £175 and B had £210; their profits amounted to £422, 10*s.* of which C's share was £172, 5*s.*; find (1) what sum C put into the business, (2) what share of the profits was due to A and B respectively.

Females—Second Year. (3 hours.)

1. Explain fully:—

(*a*) Why $\frac{3}{4}$ multiplied by $\frac{1}{2}$ gives the same result as $\frac{3}{4}$ divided by 2.
(*b*) How Simple Proportion differs from Compound Proportion.

2. What number divided by the product of $\frac{11}{12}$, $\frac{1}{3}$, and $\frac{7}{2}$ will give the difference between $3\frac{1}{3}$ and $2\frac{5}{8}$?

3. A grocer buys 3 cwts. of sugar at 5*d.* per lb., and 7 cwts. at $6\frac{1}{2}$*d.* per lb.; he sells $5\frac{1}{2}$ cwts. at $5\frac{1}{2}$*d.* per lb.; find at what price per lb. he must sell the rest in order to gain 25 per cent upon his outlay.

4. For a monthly return ticket 25 per cent more is paid than for a single ticket. At the end of the month an extension of the time for a week is obtained by paying 5 per cent on the monthly ticket. The whole sum paid is £2, 12*s.* 6*d.*; find the price of the single ticket.

5. A watch shows the true time at 6 o'clock on Sunday morning, and at noon on Tuesday has gained 24 minutes; find what the true time will be when the watch shows 1 o'clock on Friday afternoon.

6. A train starts from A for B, 72 miles distant. Its proper speed is 20 miles an hour, but after going 24 miles it meets with an accident which delays it 10 minutes, and reduces its speed to 15 miles an hour; find how many minutes it will be behind time when it reaches B.

7. A certain sum invested in the Three per Cents yields an annual income of £75. The same sum invested in a $5\frac{1}{4}$ per cent stock at par would yield an annual income of £126, 13*s.* $1\frac{1}{2}$*d.*; find (1) the sum invested, (2) the price of the Three per Cents.

ANSWERS

TO THE EXERCISES OF PART I.

I. **1.** Thirty *thousand*, eight hundred. **2.** Fifty-one *thousand*, and twenty.
3. Eighteen *thousand*, two hundred. **4.** Seventy *thousand*, and twenty.
5. Three-hundred-and-eighty-four *thousand*, six hundred-and-ten.
6. Four-hundred-and-twenty-three *thousand*, six-hundred-and-fifty-four.
7. Two *million*. **8.** Three *million*, three *thousand*.
9. Seven-hundred *thousand*, one-hundred.
10. One-hundred-and-sixty *thousand*, and eleven.
11. Five-hundred-and-three *thousand*, and ten.
12. Six-hundred-and-seventy-five *thousand*, and thirty-two.
13. Three *million*, five-hundred-and-three *thousand*, and seventy-one.
14. Four *million*, four *thousand*, and four.
15. Thirty-two *million*, five *thousand*, seven-hundred-and-one.
16. Sixty *million*, six-hundred *thousand*, seven-hundred-and-fifty-two.
17. Ten *million*, one-hundred-and-one *thousand*, and ten.
18. One-hundred-and-twenty-two *thousand*, and twelve.
19. Thirty-three *million*, three-hundred-and-thirty-three *thousand*, three-hundred-and-thirty-three. **20.** Fifty *million*.
21. Eighteen *million*, eight-hundred *thousand*, and eight.
22. Nineteen *million*, ninety-nine *thousand*, and ninety.
23. Five-hundred-and-sixty-seven *million*, eight-hundred-and-two *thousand*, and five. **24.** Ten *million*, seven *thousand*, and sixty-three.
25. Three-thousand-seven-hundred-and-fifty-six *million*, four-hundred-and-twenty-one *thousand*, eight-hundred-and-seventy-one.
26. Eight-thousand-three-hundred *million*, two-hundred-and-thirty-five *thousand*, and seven.
27. Three-hundred-and-eight-thousand-and-fifty-six *million*, three-hundred *thousand*, and seventy-two. **28.** One *billion*.
29. Two-hundred-and-thirty-six-thousand-seven-hundred-and-fifty *million*, eight-hundred-and-forty-six *thousand*, three-hundred-and-seventy-four.
30. Three *billion*, eight-thousand-seven-hundred *million*, fifty *thousand*.
31. 17,020. **32.** 102,700. **33.** 650,000. **34.** 543,011.
35. 3,700,070. **36.** 13,001,300. **37.** 5,694,387. **38.** 23,000,110.
39. 401,040,440. **40.** 768,675,586. **41.** 80,018,000. **42.** 7011,060,300.
43. 1003,001,003. **44.** 49281,550,660. **45.** 100070,014,008.
46. 360000,306,036. **47.** 200020,020,200. **48.** 1,000001,001,001.
49. 17,890045,796,016. **50.** 19,001090,100,900. **51.** 19. **52.** 24.
53. 58. **54.** 45. **55.** 76. **56.** 68. **57.** 49. **58.** 83.
59. 104. **60.** 160. **61.** 89. **62.** 95. **63.** 112. **64.** 204.
65. 166. **66.** 144. **67.** 284. **68.** 540. **69.** 390. **70.** 555.
71. 790. **72.** 679. **73.** 1764. **74.** 1647. **75.** 1885. **76.** 1566.
77. 1588. **78.** 1666. **79.** 1789. **80.** 1899. **81.** XXXIV.
82. XLVI. **83.** LXXIII. **84.** LXXXVII. **85.** XCIX.

86. CCIII. **87.** DLV. **88.** CCCCXXIII., or CDXXIII. **89.** CCCXXIV. **90.** DLX. **91.** MCCXLI. **92.** MCCCXXXVI. **93.** MDCCLIX. **94.** MDCCCLXVII. **95.** MDCCCXCI. **96.** MC. **97.** MCCCCXIII., or MCDXIII. **98.** MDCLXXXIX. **99.** MDCCCLXXXVIII. **100.** $\overline{\text{V}}$.

II. **1.** 74040. **2.** 258704. **3.** 295856. **4.** 673753. **5.** 676758. **6.** 309361. **7.** 450192. **8.** 489952. **9.** 516526. **10.** 1866945. **11.** 8423335. **12.** 14492458. **13.** 43520. **14.** 292112. **15.** 405452. **16.** 58569. **17.** 1691716. **18.** 969785. **19.** 7051968. **20.** 88126046. **21.** 181. **22.** 338. **23.** 2206. **24.** 1172. **25.** 334. **26.** 318. **27.** 251. **28.** 971. **29.** 5280. **30.** 8689. **31.** 1331. **32.** 1459. **33.** 8535. **34.** 4127. **35.** 1583. **36.** 15513. **37.** 9077. **38.** 591405. **39.** 15293. **40.** 5652. **41.** 65711. **42.** 48392. **43.** 44080. **44.** 90195. **45.** 24947. **46.** 41439. **47.** 18011. **48.** 45116. **49.** 39867. **50.** 971004. **51.** 6271. **52.** 2356. **53.** 569. **54.** 22456. **55.** 10270. **56.** 16440. **57.** 58024. **58.** 864198. **59.** 656546. **60.** 6498. **61.** 714285. **62.** 666556. **63.** 89683. **64.** 296355. **65.** 882. **66.** 15336. **67.** 125. **68.** 36971. **69.** 9628. **70.** 11107. **71.** 24482. **72.** 17224. **73.** 2899. **74.** 83805. **75.** 3254. **76.** 35647. **77.** 634. **78.** 9268. **79.** 20988. **80.** 1797. **81.** 74812. **82.** 13605. **83.** 42287. **84.** 38007. **85.** 55162. **86.** 121144. **87.** 214303. **88.** 968000. **89.** 99593496. **90.** 2998899993. **91.** 427. **92.** 17. **93.** 2. **94.** 2. **95.** 2. **96.** 64. **97.** 24. **98.** 29. **99.** 6. **100.** 2. **101.** 23. **102.** 8. **103.** 263. **104.** 9. **105.** 13. **106.** 42. **107.** 156. **108.** 478. **109.** 5271. **110.** 5231. **111.** 1. **112.** 6. **113.** 1633. **114.** 848. **115.** 359315. **116.** 21717. **117.** 216129. **118.** 9278. **119.** 418828. **120.** 8007. **121.** 870. **122.** 5600. **123.** 19000. **124.** 1750. **125.** 3000. **126.** 70000. **127.** 15000. **128.** 3080. **129.** 43000. **130.** 10020. **131.** 170000. **132.** 40300. **133.** 10740. **134.** 703000. **135.** 50000. **136.** 710000. **137.** 2700000. **138.** 8000000. **139.** 1111111101. **140.** 8888888889. **141.** 11999988. **142.** 6999993. **143.** 7777777707. **144.** 35555555556. **145.** 682670832. **146.** 266634058. **147.** 103351500. **148.** 1636529400. **149.** 80612690400. **150.** 26876148300. **151.** 91132497000. **152.** 38518802300. **153.** 1676312100. **154.** 7454185680. **155.** 19723000000. **156.** 291251870100. **157.** 70179941000. **158.** 80683213020. **159.** 12014402400000. **160.** 606000134241. **161.** 7544676; 1234000. **162.** 60768396; 520426776. **163.** 27964417. **164.** 45808416423. **165.** 1700. **166.** 24000. **167.** 80000. **168.** 75000. **169.** 30000. **170.** 810000. **171.** 470. **172.** 890. **173.** 7900. **174.** 100. **175.** 700. **176.** 2300. **177.** 23868. **178.** 57855. **179.** 40905480. **180.** 13838590116. **181.** 226834247640. **182.** 375645200000. **183.** 361. **184.** 3025. **185.** 10609. **186.** 44100. **187.** 25921. **188.** 55225 **189.** 9659664.

190. 21996100. 191. 29506624. 192. 60481729. 193. 4913.
194. 9261. 195. 262144. 196. 941192. 197. 9938375.
198. 87528384. 199. 247673152000. 200. 1371330631.

	quotient. remr.		quotient. remr.		quotient. remr.
201.	42 + 3.	207.	350.	213.	301 + 1.
202.	76 + 81.	208.	700 + 6.	214.	270 + 70.
203.	4 + 865.	209.	8 + 46.	215.	58000.
204.	203.	210.	760 + 8.	216.	34 + 50.
205.	51 + 40.	211.	360 + 1.	217.	4 + 6321.
206.	470.	212.	2340.	218.	120 + 8.
219.	62321 + 4.	225.	4321 + 11.	231.	1854 + 11.
220.	86730 + 5.	226.	1931 + 12.	232.	19026 + 13.
221.	295064 + 1.	227.	1429 + 27.	233.	7981 + 40.
222.	61864 + 3.	228.	6094 + 92.	234.	1219 + 77.
223.	705442 + 3.	229.	9736 + 851.	235.	657 + 6643.
224.	55733399 + 7.	230.	7610 + 407.	236.	9704 + 3081.
237.	3800 + 1.	245.	1882 + 1982.	253.	411 + 72700.
238.	7300 + 14.	246.	17337 + 1288.	254.	12 + 51524.
239.	183003.	247.	285340 + 4322.	255.	30 + 51.
240.	80307.	248.	3759 + 29556.	256.	2016 + 15000.
241.	19759 + 2.	249.	160401.	257.	24 + 214302.
242.	24582 + 41.	250.	14000014.	258.	1263 + 1100.
243.	4560.	251.	1680 + 51.	259.	178 + 2728.
244.	30079.	252.	18 + 113.	260.	788 + 2177.
261.	582203 + 13.	265.	26317 + 29.	269.	39597 + 3.
262.	161674 + 9.	266.	15885 + 22.	270.	28957 + 38.
263.	38231 + 7.	267.	187323 + 27.	271.	1721 + 13.
264.	2192 + 9.	268.	2219 + 1.	272.	5910 + 42.
273.	8433 + 15.	281.	200 + 11.	288.	41 + 815.
274.	124955 + 32.	282.	258 + 43.	289.	232 + 911.
275.	188033 + 63.	283.	396 + 332.	290.	251 + 1341.
276.	268 + 7.	284.	1089 + 212.	291.	62 + 341.
277.	1128 + 53.	285.	127 + 245.	292.	153 + 523.
278.	1758 + 125.	286.	46 + 1206.	293.	1818 + 4887.
279.	29403977 + 74.	287.	752 + 178.	294.	1447 + 2693.
280.	6730243 + 106.				

295.	77.	302.	207.	309.	130130.	315.	86.
296.	304.	303.	45067 times.	310.	7575.	316.	6758.
297.	34.	304.	83205 times.	311.	609.	317.	74274.
298.	72.	305.	8.	312.	70905.	318.	317450.
299.	312 times.	306.	325.	313.	763.	319.	689902.
300.	340 times.	307.	15607.	314.	560.	320.	66579051.
301.	1858.	308.	6702.				

III. **1.** Seven *million*, seven hundred and seven *thousand*, and seventy.
2. 13,050,011. **3.** 43795. **4.** 33018.
5. 20536900. **6.** 1412884 + 500 remr. **7.** 11210 + 4 remr.
8. 12. **9.** 428 pages. **10.** 80 nuts.
11. Forty-one *million*, three hundred and five *thousand*, and eighty-seven.
12. 17,450,302. **13.** 2999997. **14.** 295654.
15. 3482569401472. **16.** 1388. **17.** 27 remr.
18. 2270. **19.** 732 years. **20.** 63840 letters.
21. One thousand seven hundred and seventy-seven. **22.** MDCCCLI.
23. 107170. **24.** 198000. **25.** 14395553. **26.** 7007 times.
27. 226981. **28.** 4578. **29.** 28 years. **30.** 81 seats.
31. 1413. **32.** MDCIII. **33.** 26545. **34.** 550440033.
35. 76649903. **36.** 3003 times. **37.** 49885969. **38.** 30081.
39. 65 years. **40.** 8581. **41.** Five hundred thousand. Five hundred.
42. 12. **43.** 742481. **44.** 21206628450. **45.** 1055841.
46. 74000. **47.** 3. **48.** 650 + 56803 remr. **49.** 2499.
50. 380. **51.** Thirty thousand. Thirty. **52.** 26.
53. 74137. **54.** 305365476. **55.** 92055. **56.** 230.
57. 5. **58.** 1677200. **59.** 8. **60.** 8421.
61. Two thousand three hundred *million*, twenty-five *thousand*, and fifteen. **62.** 390. **63.** 4227321; 385470, and 34264000.
64. 61146 + 92 remr. **65.** 7903000. **66.** 312. **67.** 145 times.
68. $40^2 = 1600$; $9^2 = 81$; $41^2 = 1681$. **69.** 189 figures. **70.** 17 marbles.
71. 13,400,057. **72.** 106. **73.** 4474728. **74.** By 120.
75. 370 + 53 remr. **76.** 42310; 4231, and 33848. **77.** 43 times.
79. 9999. **80.** 294. **81.** DCCCLXXIV.
82. 3,500,000. **83.** 593496. **84.** 5865436 + 2067 remr.
85. 39906 + 11 remr; 399 + 113 remr. **86.** 15. **87.** 49788.
88. By 4. **89.** 43 years. **90.** 1080 yds. **91.** 23,030,005.
92. 40. **93.** 10675. **94.** 2308256. **95.** 740 + 221 remr.
96. By 507. **97.** 456 + 123 remr. **98.** 174.
99. 32 pages. **100.** 312 animals. **101.** Seven thousand three hundred and six *million*, two *thousand*, and fifteen.
102. 15 + 30432 remr. **103.** By 457327. **104.** 13.
105. 71 years old. **106.** $41^3 = 68921$; $236^2 = 55696$; $115^2 = 13225$.
107. 6113. **108.** 107. **109.** 867537. **110.** 929325.
111. Ten thousand eight hundred and sixty-one. **112.** 492 figures, including 1 and 200. **113.** 11441476. **114.** 578 boxes.
115. 740480 letters. **116.** 6 marbles. **117.** 38 years old. **118.** 1.
119. 306504. **120.** 1080375. **121.** Three *sets of ten units* + four *units*.
122. Seventy thousand and seven *million*, seven *thousand*, and seventy-seven. **123.** 410921984. **124.** 20. **125.** 86 years old.
126. 210 and 30. **127.** 2100. **128.** 442. **129.** 7856136. **130.** 1795164.
131. By 49950. **132.** 75 and 68. **133.** 209. **134.** 90 ladies.
135. 131. **137.** 707594 + 770 remr. **138.** 58. **139.** 42 and 55.
140. 992817. **141.** Ten thousand seven hundred and forty-nine.
142. 109999. **143.** By 888. **144.** Divisor 7087; remr. 1569.
145. 6708. **146.** 2454. **147.** 2 more. **148.** 86. **149.** 37990092.
150. 376728. **151.** 448847. **152.** 2662. **153.** 623.

154. (i) 137; (ii) 5421 and 1807. 155. 937. 156. 93000. 157. 23.
158. 2037 and 679. 159. 125. 160. 116571375. 161. 83952.
162. 83205. 163. 25000. 164. 19 boys. 165. 3974 and 3423.
166. 76 and 33. 167. 3615 and 2892. 168. 4. 169. 10379355.
170. 376731. 171. 29732. 172. 412776. 173. By 97.
174. 24 men. 175. 73 and 37. 176. 7. 177. 13 and 10.
178. 19. 179. 1171000. 180. 1338776. 181. 2,000,020,000,200.
182. 302. 183. 862. 184. 2375776242. 185. 2478. 186. 6 games.
187. 16. 188. 336. 189. 89. 190. 24429215144. 191. VIDCLXVI.
192. 656100810001. 193. 61. 194. 337. 195. 8 girls.
196. 3034 and 2081. 197. A has 36; B has 7; C has 46. 198. 1400.
199. 2362266. 200. 63. 201. 6140. 202. 7700.
203. 4763; 763; 63. 204. 6715. 205. 47. 206. 9.
207. 2449 persons. 208. 34635330. 209. 43. 210. 22653 + 2 rem^r.
211. 54255219. 212. 2259159. 213. 57980. 214. 650470.
215. 6543 and 2222. 216. 33 and 18. 217. 56. 218. 4378444.
219. 375 + 857 rem^r. 220. 144094291992. 221. 4 groups of *thirty-six* + 2 groups of *six* + 3 units, *i.e.*, one hundred and fifty-nine units. 222. 98980001. 223. 3227 + 40 rem^r.

224.
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47 ) 3189 ( 67.
     282
     ---
      369
      329
      ---
       40
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225. 857142 and 142857.
226. $21\frac{1}{2}$ years old.
227. 3905; 3344, and 2203.

228. 364893507.
229. 4679 + 64 rem^r.
230. 88808256828.

231. 1 group of *one hundred and twenty-five* + 2 groups of *twenty-five* + 3 groups of *five* + 4 units, *i.e.* one hundred and ninety-four units.

232. 10219567.
233. 118. See Example on p. 14 of Theory.
234. 173042.

235.
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   457
    38
  ----
  3656
 1371
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 17366
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236. 17 boys; 52 girls.
237. 117.
238. Either 23475, or 513.
239. 35130711336.
240. 181170.

IV. 1. 109339 farthings. 2. 786926 farthings. 3. 1067133 farthings.
4. 959991 farthings. 5. 17373 halfpence. 6. 22817 halfpence.
7. 96849 halfpence. 8. 106613 halfpence. 9. 2399 threepences.
10. 3494 threepences. 11. 8709 threepences. 12. 35307 threepences.
13. 1073 fourpences. 14. 5306 fourpences. 15. 8758 fourpences.
16. 18127 fourpences. 17. 2071 sixpences. 18. 3437 sixpences.
19. 10303 sixpences. 20. 26653 sixpences. 21. 5729 florins.
22. 6317 florins. 23. 30518 florins. 24. 42354 florins.
25. 103 crowns. 26. 273 crowns. 27. 1890 crowns.
28. 3334 crowns. 29. 1883 half-cr. 30. 5145 half-cr.
31. 3661 half-cr. 32. 2610 half-cr. 33. 5775 half-cr.
34. 6644 half-cr. 35. 3379 three halfpences. 36. 3419 threepences.
37. 154863 half-sovereigns. 38. 4819 double florins.

39. £175, 2s. 11d. **40.** £153, 18s. 5d. **41.** £104, 3s. 4d.
42. £462, 19s. 3d. **43.** £10, 8s. 4d. **44.** £15, 19s. 8½d.
45. £125, 12s. 6½d. **46.** £118, 15s. 1½d. **47.** £57, 0s. 10d.
48. £161, 19s. 8d. **49.** £277, 15s. 6d. **50.** £972, 4s. 3d.
51. £938, 12s. 6d. **52.** £215, 1s. 6d. **53.** £1237, 2s.
54. £1765, 18s. **55.** £121, 15s. **56.** £138, 15s.
57. £154, 5s. **58.** £540, 2s. 6d. **59.** £954, 15s.
60. £1213, 10s. **61.** £1275, 17s. 6d. **62.** £2500, 12s. 6d.
63. £948, 4s. **64.** £15410, 10s. **65.** £17, 6s. 4½d.
66. £117, 6s. 1½d. **67.** 37401s. **68.** 97251s.
69. 119196d. **70.** 176400f. **71.** 11772 twopences.
72. 5104 sixpences. **73.** 14610d. **74.** 104040 halfpence.
75. 148260 threepences. **76.** 16475 sixpences. **77.** 844 hf.-cr. + 2s.
78. 37320f. **79.** 157 fl. + 1s. 1½d. **80.** 374 hf.-cr. + 1s. 2d.
81. 116 gs. **82.** 32 hf.-sovs. + 2s. 11d. **83.** 9655 fl.
84. 184 cr. + 4d. **85.** 99 gs. + 4s. 4d. **86.** 17 gs. + 13s. 4d.
87. £4662. **88.** £8757. **89.** £3016, 13s. **90.** £13027, 7s.
91. 13320 gs. **92.** 15540 gs. **93.** 38696 gs. + 4s.
94. 34288 gs. + 12s. **95.** 13143 fourpences. **96.** 3388 sixpences + 4d.
97. 17577s. 6d. **98.** 7110 hf.-cr. + 1s. **99.** 95 hf.-gs. + 6d.
100. 7740 fourpences. **101.** 843140 fl. **102.** 529376 hf.-cr. + 1s.
103. 7310 hf.-gs. **104.** 2704 hf.-gs. **105.** 796110 hf.-cr. **106.** 579065 fl.
107. £2, 1s. 8d. **108.** £6, 5s. **109.** 1200. **110.** 180.
111. £41, 4s. **112.** 1141 people. **113.** 39 five-pound notes.
114. 76 pencils. **115.** £13, 15s. **116.** £15, 17s. 8d. **117.** 70422 secs.
118. 85265 secs. **119.** 25307 min. **120.** 33819 min. **121.** 1074 hrs.
122. 927 hrs. **123.** 31505 hrs. **124.** 22069 hrs.
125. 1104587 min. **126.** 2368813 secs. **127.** 8 hrs. 43 min. 43 secs.
128. 15 hrs. 50 min. 29 secs. **129.** 12 days 11 hrs. 22 min.
130. 14 days 10 hrs. 5 min. **131.** 3 days 17 hrs. 7 min. 21 secs.
132. 5 days 13 hrs. 4 min. 33 secs. **133.** 47 wks. 1 day 11 hrs. 45 min.
134. 45 wks. 4 days 19 hrs. 13 min. 40 secs.
135. 2 yrs. 223 days 4 hrs. 13 min. **136.** 3 yrs. 299 days 4 hrs. 3 min.
137. 1464 hrs. **138.** 131040 min. **139.** 56 hrs. **140.** 108 hrs.
141. 585 min. **142.** 9600 secs. **143.** 243 days. **144.** 274 days.
145. 1826 days. **146.** 17544 hrs. **147.** 1945 lbs. **148.** 1116 lbs.
149. 631 ozs. **150.** 1021 ozs. **151.** 23517 ozs. **152.** 30604 ozs.
153. 38436 lbs. **154.** 53439 lbs. **155.** 1511 ozs. **156.** 5047 drs.
157. 95671 ozs. **158.** 272347 ozs. **159.** 7654567 drs. **160.** 30009615 drs.
161. 31 stones. **162.** 1242 stones. **163.** 3583 lbs. **164.** 9369 lbs.
165. 10454 ozs. **166.** 33691 ozs. **167.** 1799000 grs. **168.** 4431000 grs.
169. 2 qrs. 13 lbs. 7 ozs. **170.** 3 qrs. 9 lbs. 15 ozs. **171.** 17 lbs. 1 oz. 3 drs.
172. 19 lbs. 10 ozs. 5 drs. **173.** 4 tons 15 cwt. 3 qrs. 5 lbs.
174. 21 tons 4 cwt. 2 qrs. 24 lbs. **175.** 9 cwt. 0 qrs. 18 lbs. 1 oz.
176. 10 cwt. 3 qrs. 7 lbs. 6 ozs. **177.** 19 cwt. 3 qrs. 8 lbs. 6 oz. 14 drs.
178. 27 cwt. 0 qr. 24 lbs. 10 ozs. 3 drs. **179.** 12 cwt. 3 qrs. 14 lbs. 3 oz.
180. 18 cwt. 2 qrs. 11 lbs. 5 oz. **181.** 1 ton 4 cwt. 2 qrs. 4 lbs. 2 ozs.
182. 2 tons 4 cwts. 3 qrs. 7 lbs. 3 ozs. **183.** 18 tons 1 cwt. 3 qrs. 4 lbs. 7 ozs.
184. 132 tons 19 cwt. 0 qr. 15 lbs. 15 oz. **185.** 213 tons 13 cwts. 1 qr. 17 lbs. 2 oz.

186. 2 tons 0 cwts. 0 qr. 1 lb. 8 ozs. 10 drs. **187.** 9 tons 13 cwt. 3 qrs. 1 lb. 6 ozs. 3 drs. **188.** 16 tons 0 cwt. 0 qr. 15 lbs. 11 ozs. 9 drs.
189. 20981 grs. **190.** 43855 grs. **191.** 34512 grs. **192.** 14155 grs.
193. 18100 grs. **194.** 22017 grs. **195.** 1 lb. Tr. 3 ozs. 13 dwt. 13 grs.
196. 4 lb. Tr. 5 oz. 3 dwt. 13 grs. **197.** 15 lb. Tr. 3 ozs. 0 dwt. 23 grs.
198. 182 ozs. Tr. 416 grs. **199.** 132 ozs. Tr. 385 grs.
200. 188 ozs. Tr. 431 grs. **201.** 647 in. **202.** 1063 in.
203. 2037 in. **204.** 3202 in. **205.** 7420 yds. **206.** 23936 yds.
207. 18635 ft. **208.** 29776 ft. **209.** 41764 ft. **210.** 56000 ft.
211. 12 yds. 0 ft. 9 in. **212.** 19 yds. 0 ft. 9 in. **213.** 36 yds. 0 ft. 9 in.
214. 57 yds. 1 ft. 7 in. **215.** 43 mi. 861 yds. **216.** 48 mi. 83 yds.
217. 6 mi. 1633 yds. 2 ft. **218.** 9 mi. 123 yds. 2 ft.
219. 5 mi. 1153 yds. 1 ft. 3 in. **220.** 12 mi. 131 yds. 0 ft. 9 in.
221. 132 yds. **222.** 209 yds. **223.** 151 yds. **224.** 219 yds.
225. 307 ft. **226.** 380 ft. **227.** 341 hf.-yds. **228.** 407 hf.-yds.
229. 217 hf.-yds. **230.** 314 hf.-yds. **231.** 7128 in. **232.** 7632 in.
233. 4635 in. **234.** 7373 in. **235.** 3529 in. **236.** 5954 in.
237. 125½ ft. **238.** 162½ ft. **239.** 228 ft. **240.** 292 ft.
241. 15283 yds. **242.** 58741 ft. **243.** 94381 ft. **244.** 33779 yds.
245. 316827 in. **246.** 13278 yds. **247.** 13600 yds. **248.** 36711 ft.
249. 643548 in. **250.** 117851 ft. **251.** 233094 in. **252.** 94578 in.
253. 205862 in. **254.** 1104786 in. **255.** 1635033 in. **256.** 2768645 in.
257. 93 po. 5 yds. **258.** 89 po. 3½ yds. **259.** 326 po. ½ yd.
260. 239 po. 2½ yds. **261.** 484 hf.-yds. **262.** 1284 hf.-yds.
263. 1320 hf.-yds. **264.** 3268 hf.-yds. **265.** 186 po. **266.** 126 po.
267. 133 po. 3½ yds. **268.** 294 po. **269.** 155 po. 3½ yds.
270. 224 po. 2 yds. **271.** 8 po. 4 yds. 2 ft. 11 in.
272. 14 po. 3 yds. 2 ft. **273.** 18 po. 5 yds. 1 ft. 3 in.
274. 34 po. 4 yds. 1 ft. **275.** 16 po. 3 yds. **276.** 23 po. 0 yd. 1 ft. 6 in.
277. 30 po. 0 yd. 1 ft. **278.** 30 po. 2 yds. 2 ft. **279.** 34 po. 3 yds. 1 ft.
280. 41 po. 3 yds. 1 ft. 6 in. **281.** 2 mi. 3 fur. 21 po. 2 yds. 2 ft. 3 in.
282. 3 mi. 6 fur. 22 po. 0 yd. 2 ft. 5 in. **283.** 4 mi. 1 fur. 36 po. 1 yd. 1 ft. 7 in.
284. 8 mi. 2 fur. 26 po. 0 yd. 2 ft. 3 in. **285.** 10 mi. 5 fur. 5 po. 5 yds. 0 ft. 11 in. **286.** 49 mi. 2 fur. 31 po. 3 yds. 1 ft. 11 in.
287. 147 mi. 6 fur. 32 po. 2 yds. 2 ft. 3 in. **288.** 59 mi. 0 fur. 32 po.
289. 5 mi. 5 fur. 18 po. 1 yd. **290.** 25 mi. 2 fur. 0 po. 4 yds.
291. 32 mi. 4 fur. 33 po. 1½ yds. **292.** 701 mi. 3 fur. 26 po. 4 yds.
293. 4419 mi. 1 fur. 21 po. 1½ yd. **294.** 21 mi. 2 fur. 22 po. 1 yd.
295. 189 mi. 3 fur. 6 po. 0 yd. 1 ft. **296.** 19 mi. 5 fur. 20 po. 4 yds.
297. 349 mi. 7 fur. 18 po. 0 yd. 1 ft. **298.** 1279 mi. 1 fur. 32 po. 4 yds. 1 ft.
299. 1594 mi. 1 fur. 32 po. 0 yd. 0 ft. 5 in. **300.** 1578 mi. 2 fur. 10 po. 2 yds. 2 ft. 4 in. **301.** 4915 sq. in. **302.** 23263 sq. in.
303. 2864 sq. in. **304.** 20013 sq. in. **305.** 11659 sq. po.
306. 21037 sq. po. **307.** 22945 sq. po. **308.** 14381 sq. po.
309. 4 sq. yds. 8 sq. ft. 117 sq. in. **310.** 6 sq. yds. 5 sq. ft. 11 sq. in.
311. 54 sq. yds. 3 sq. ft. 122 sq. in. **312.** 95 sq. yds. 4 sq. ft. 60 sq. in.
313. 22 ac. 3 rds. 11 po. **314.** 46 ac. 1 rd. 8 po. **315.** 457 ac. 2 rds. 17 po.
316. 566 ac. 0 rds. 13 po. **317.** 726 sq. yds. **318.** 1089 sq. yds.
319. 623 sq. yds. **320.** 995 sq. yds. **321.** 544½ sq. yds.

322. 665½ sq. yds. **323.** 514¼ sq. yds. **324.** 877¼ sq. yds.
325. 441 sq. yds. **326.** 648¼ sq. yds. **327.** 1296 sq. ft.
328. 2448 sq. ft. **329.** 1638 sq. ft. **330.** 2965½ sq. ft.
331. 4680 sq. ft. **332.** 5283 sq. ft. **333.** 1176120 sq. in.
334. 823284 sq. in. **335.** 1058508 sq. in. **336.** 1528956 sq. in.
337. 218052 sq. in. **338.** 310716 sq. in. **339.** 37268 sq. yds.
340. 24079 sq. yds. **341.** 10254¾ sq. yds. **342.** 20479¼ sq. yds.
343. 25070 sq. yds. **344.** 45563 sq. yds. **345.** 82418 sq. yds.
346. 131130 sq. yds. **347.** 23048773 sq. in. **348.** 54650895 sq. in.
349. 333274481 sq. in. **350.** 24820485 sq. in. **351.** 20 sq. po. 11 yds.
352. 28 sq. po. 4 yds. **353.** 8 sq. po. 25 yds. **354.** 11 sq. po. 20¼ yds.
355. 30 sq. po. 9½ yds. **356.** 35 sq. po. 2¼ yds.
357. 3 sq. po. 7 yds. 2¼ ft. **358.** 4 sq. po. 16 yds. 1 ft.
359. 4 sq. po. 17 yds. 8 ft. 123 in. **360.** 6 sq. po. 8 yds. 8 ft. 99 in.
361. 2 A. 1 R. 20 P. 0 yds. **362.** 17 A. 0 R. 25 P. 24¾ yds.
363. 7 A. 2 R. 39 P. 15¼ yds. **364.** 15 A. 0 R. 14 P. 14 yds. 7½ ft.
365. 12 A. 3 R. 27 P. 21 yds. 6¼ ft. **366.** 22 A. 3 R. 33 P. 2 yds. 7¾ ft.
367. 1 A. 1 R. 0 P. 19 yds. 0 ft. 8 in. **368.** 2 A. 2 R. 8 P. 18 yds. 5 ft. 120 in.
369. 4 A. 0 R. 13 P. 5 yds. 7 ft. 109 in. **370.** 8 A. 2 R. 34 P. 0 yds. 3 ft. 87 in.
371. 185 A. 0 R. 2 P. 26½ yds. **372.** 76 A. 2 R. 3 P. 19 yds. 5¼ ft.
373. 53 A. 0 R. 21 P. 0 yds. 8 ft. 125 in. **374.** 15 A. 3 R. 20 P. 15 yds. 2 ft. 0 in.
375. 30711 cub. in. **376.** 32177 cub. in. **377.** 88888 cub. in.
378. 78391 cub. in. **379.** 209093 cub. in. **380.** 457067 cub. in.
381. 3 cub. yds. 2 ft. 138 in. **382.** 16 cub. yds. 10 ft. 815 in.
383. 18 cub. yds. 4 ft. 605 in. **384.** 21 cub. yds. 4 ft. 719 in.
385. 103 cub. yds. 15 ft. 362 in. **386.** 171 cub. yds. 13 ft. 664 in.
387. 111 pints. **388.** 155 pints. **389.** 146 pints.
390. 185 pints. **391.** 109 gal. 1 qt. 1 pt. **392.** 84 gal. 3 qts. 0 pt.
393. 154 gal. 0 qt. 0 pt. **394.** 225 gal. 2 qts. 1 pt. **395.** 238 pecks.
396. 314 pecks. **397.** 31017 qts. **398.** 45817 pts.
399. 4 qrs. 7 bush. 3 pks. 1 gal. 3 qts. 1 pt. **400.** 130 qrs. 1 bush. 2 pks. 1 gal. 1 qt.
401. 15 lbs. Av. + 3000 grs.
402. 23136 links. **403.** 141⅜ in. **404.** 21683200 sq. yds.
405. 864 pints. **406.** 8483 sheets. **407.** 114449″. **408.** 33 knots.
409. 1452 acres. **410.** 9800 ozs. Troy.

V. **1.** £7800, 3*s.* 0*d.* **2.** £888, 3*s.* 7¼*d.* **3.** £22703, 0*s.* 1¼*d.*
4. £6186, 1*s.* 11*d.* **5.** £17625, 6*s.* 8*d.* **6.** £28910, 13*s.* 0*d.*
7. £36343, 3*s.* 0*d.* **8.** £57447, 10*s.* 11*d.* **9.** £42450, 5*s.* 2*d.*
10. £36711, 11*s.* 2*d.* **11.** £44375, 2*s.* 8*d.* **12.** £48237, 16*s.* 11*d.*
13. £6381709, 13*s.* 11*d.* **14.** £4278818, 3*s.* 11*d.* **15.** £5253431, 8*s.* 9*d.*
16. £40418, 4*s.* 0¼*d.* **17.** £25925, 16*s.* 7*d.* **18.** £885299, 19*s.* 11*d.*
19. 92 days 12 hr. 16 min. 37 secs. **20.** 13 days 7 hrs. 8 min. 37 secs.
21. 33 yrs. 72 days 2 hrs. **22.** 77 yrs. 15 days 1 hr.
23. 22 cwt. 1 qr. 10 lbs. 9 ozs. **24.** 2 cwt. 1 qr. 17 lbs. 11 ozs. 13 drs.
25. 43 tons 3 cwt. 2 qrs. 4 lbs. **26.** 27 tons 10 cwt. 3 qrs. 2 lbs.
27. 22 cwt. 77 lbs. 6 ozs. **28.** 24 tons 6 cwt. 105 lbs. 11 ozs.
29. 36 lbs. Tr. 9 ozs. 8 dwt. 15 grs. **30.** 78 ozs. Tr. 34 grs.

31. 16 yds. 0 ft. 9 in. **32.** 32 yds. 2 ft. 7 in. **33.** 17 mi. 774 yds.
34. 25 mi. 449 yds. 1 ft. 7 in. **35.** 90 po. 3 yds. 0 ft. 3 in.
36. 33 mi. 1 fur. 38 po. 5 yds. 1 ft. **37.** 19 fur. 20 po. 5 yds. 1 ft. 9 in.
38. 24 mi. 30 chs. 41 lks. **39.** 30 sq. yds. 4 ft. 3 in.
40. 20 sq. yds. 1 ft. 95 in. **41.** 307A. 3R. 2P. **42.** 122A. 1R. 38P.
43. 10A. 1R. 21P. $20\frac{1}{2}$ sq. yds. **44.** 16A. 1R. 24P. $16\frac{1}{4}$ sq. yds.
45. 25A. 0R. 39P. 12 sq. yds. 5 ft. 36 in. **46.** 2R. 32P. 8 sq. yds. 0 ft. 37 in.
47. 47 cub. yds. 23 ft. 542 in. **48.** 444 cub. yds. 8 ft. 1259 in.
49. 47 gal. 3 qts. 1 pt. **50.** 84 qrs. 1 bush. 2 pks. 0 gal. **51.** £8, 9s. $2\frac{3}{4}$d.
52. £4, 4s. $4\frac{1}{4}$d. **53.** £3188, 8s. $10\frac{3}{4}$d. **54.** £7599, 17s. $9\frac{3}{4}$d.
55. £1, 2s. $1\frac{1}{2}$d. **56.** £15, 11s. $5\frac{1}{4}$d. **57.** £1, 18s. $8\frac{1}{2}$d.
58. £7, 16s. $9\frac{3}{4}$d. **59.** £91, 1s. 4d. **60.** £21, 0s. 10d.
61. 8 hrs. 27 min. 50 secs. **62.** 6 hrs. 27 min. 44 secs.
63. 3 days 9 hrs. 36 min. **64.** 2 days 17 hrs. 24 min. 13 secs.
65. 4 lbs. 7 ozs. 10 drs. **66.** 1 qr. 15 lbs. 8 ozs. **67.** 7 tons 11 cwt. 3 qrs.
68. 11 tons 15 cwt. 1 qr. 9 lbs. **69.** 1 ton 16 cwt. 3 qrs. 23 lbs.
70. 16 cwt. 3 qrs. 11 lbs. 12 ozs. **71.** 5 tons 4 cwt. 2 qrs. 3 lbs. 6 ozs.
72. 6 tons 2 cwt. 2 qrs. 8 lbs. 6 ozs. 3 drs. **73.** 7 lbs. Tr. 7 ozs. 16 dwt. 5 grs.
74. 13 ozs. Tr. 344 grs. **75.** 13 yds. 1 ft. 7 in. **76.** 8 yds. 1 ft. 9 in.
77. 7 mi. 1033 yds. 2 ft. **78.** 21 mi. 427 yds. 1 ft. 5 in.
79. 34 m. 5 fur. 6 yds. **80.** 4 fur. 24 po. 2 yds. **81.** 28 po. 1 yd. 2 ft. 11 in.
82. 3 mi. 4 fur. 18 po. 4 yds. 1 ft. 5 in. **83.** 4 sq. yds. 3 ft. 27 in.
84. 7 sq. yds. 0 ft. 101 in. **85.** 47A. 2R. 24P. **86.** 106A. 3R. 17P.
87. 37 sq. po. $20\frac{1}{4}$ sq. yds. **88.** 2A. 2R. 21P. $0\frac{1}{4}$ sq. yd.
89. 5 ac. 3470 sq. yds. **90.** 1 sq. mi. 130 ac. 2967 sq. yds.
91. 1A. 2R. 10P. 2 sq. yds. 3 ft. 36 in. **92.** 1A. 0R. 0P. 13 sq. yds. 2 ft. 82 in.
93. 1A. 1R. 7P. 17 sq. yds. 5 ft. 26 in. **94.** 1A. 0R. 6P. 26 sq. yds. 8 ft. 3 in.
95. 8 cub. yds. 6 ft. 727 in. **96.** 180 cub. yds. 16 ft. 741 in.
97. 19 gals. 1 qt. 1 pt. **98.** 3 bush. 2 pks. 1 gal. **99.** 44 qrs. 4 bush.
100. 67 qrs. 5 bush. 3 pks. **101.** 5s. 2d. **102.** 1s. $8\frac{1}{2}$d.
103. £5, 18s. 3d. **104.** 18s. **105.** £2019, 16s. $8\frac{3}{4}$d. **106.** £8114, 11s. $8\frac{1}{4}$d.
107. 10s. $3\frac{1}{4}$d. **108.** £1, 2s. $0\frac{1}{4}$d. **109.** £2, 1s. 8d. **110.** 18s. $11\frac{1}{4}$d.
111. £3, 17s. $1\frac{1}{4}$d. **112.** £5, 7s. 8d. **113.** 11s. $4\frac{1}{4}$d. **114.** £7, 1s. $8\frac{3}{4}$d.
115. £2, 9s. **116.** £15, 7s. $8\frac{3}{4}$d. **117.** 16s. 8d. **118.** £24, 11s. 3d.
119. 14s. 6d. **120.** £1, 8s. 1d. **121.** £159, 1s. $1\frac{1}{2}$d. **122.** £206, 14s. $4\frac{3}{4}$d.
123. £1083, 17s. $1\frac{1}{4}$d. **124.** £754, 13s. 6d. **125.** £1402, 4s. $4\frac{1}{2}$d.
126. £2695, 2s. $1\frac{1}{2}$d. **127.** £1046, 17s. 6d. **128.** £2157, 19s. $4\frac{1}{2}$d.
129. £178, 12s. $7\frac{1}{2}$d. **130.** £535, 8s. **131.** £16544. **132.** £15851.
133. £2277, 5s. 10d. **134.** £2411, 8s. **135.** £2499, 3s. 4d.
136. £5976, 5s. **137.** £7687, 10s. **138.** £7687, 10s. **139.** £907, 14s. 7d.
140. £841, 18s. $10\frac{1}{2}$d. **141.** £201, 9s. $5\frac{1}{2}$d. **142.** £143, 9s. $9\frac{3}{4}$d.
143. £5526, 15s. 8d. **144.** £5240, 19s. 8d. **145.** £974, 9s. 1d.
146. £1072, 11s. 7d. **147.** £19679, 1s. $10\frac{1}{2}$d. **148.** £68867, 16s. 11d.
149. £2568, 0s. 10d. **150.** £4324, 0s. 5d. **151.** £1255, 3s. 4d.
152. £8704, 10s. **153.** £5655, 19s. 8d. **154.** £17877, 14s. 2d.
155. £2363, 11s. $2\frac{1}{2}$d. **156.** £262, 16s. $11\frac{1}{4}$d. **157.** £11270, 10s. $11\frac{1}{2}$d.
158. £78697, 0s. $10\frac{1}{2}$d. **159.** £4495, 11s. 1d. **160.** £1645, 0s. 4d.
161. £28231, 5s. 3d. **162.** £22240, 19s. $4\frac{1}{2}$d. **163.** £446, 19s. 4d.
164. £600, 7s. 11d. **165.** £974, 17s. 1d. **166.** £1254, 13s. 9d.

167. £728700. 168. £89723, 2s. 6d. 169. £3123, 8s.
170. £16340, 13s. 8d. 171. £9545, 4s. 6d. 172. £12057, 9s. 9d.
173. £69, 3s. 4½d. 174. £92, 10s. 6¼d. 175. £310, 13s. 6¼d.
176. £504, 8s. 4½d. 177. £8115, 16s. 178. £65988.
179. £4606, 17s. 6d. 180. £5406, 9s. 181. £1753, 5s. 6d.
182. £5395, 16s. 4d. 183. £2273, 16s. 10d. 184. £2173, 3s. 9d.
185. £3602, 7s. 6d. 186. £12859, 15s. 9d. 187. £5733, 12s. 3d.
188. £12882, 19s. 5d. 189. £43955, 8s. 4d. 190. £36627, 11s. 2½d.
191. 5 dys. 10 hrs. 44 min. 39 secs. 192. 5 dys. 17 hrs. 45 min. 45 secs.
193. 154 tons 7 cwts. 0 qr. 194. 3 cwts. 2 qrs. 16 lbs. 12 ozs.
195. 3 qrs. 20 lbs. 2 ozs. 14 drs. 196. 10 tons 19 cwts. 1 qr. 20 lbs.
197. 73 tons 4 cwts. 0 qr. 1 lb. 198. 1 tn. 1 cwt. 0 qr. 26 lbs. 15 oz.
199. 418 tns. 0 cwts. 2 qrs. 12 lbs. 15 ozs. 200. 27 tns. 6 cwt. 3 qr. 1 lb. 8 oz.
201. 933 tons 9 cwts. 2 qrs. 1 lb. 202. 103 tons 7 cwts. 1 qr. 9 lbs. 5 ozs.
203. 121 yds. 2 ft. 3 in. 204. 65 mi. 4 fur. 4 po. 205. 81 mi. 6 fur. 39 po.
206. 421 yds. 0 ft. 2 in. 207. 211 mi. 1 fur. 13 po. 3½ yds.
208. 3 fur. 24 po. 4 yds. 1 ft. 6 in. 209. 1020 mi. 2 fur. 10 po. 4½ yds.
210. 157 mi. 3 fur. 37 po. 3 yds. 1 ft. 211. 409 mi. 2 fur. 9 po. 1 yd.
212. 6 mi. 2 fur. 39 po. 5 yds. 213. 94A. 2R. 0P. 214. 112A. 0R. 8P.
215. 13 sq. yds. 6 ft. 54 in. 216. 13 sq. yds. 2 ft. 108 in.
217. 16A. 2R. 34P. 19 sq. yd. 7 ft. 108 in. 218. 23A. 2R. 14P. 8 sq. yd. 6 ft. 72 in.
219. 30A. 2R. 1P. 7½ sq. yds. 220. 2728A. 3R. 6P. 8¾ sq. yds.
221. 119A. 1R. 26P. 12 sq. yds. 2 ft. 36 in. 222. 10A. 0R. 28P. 10 sq. yds. 8 ft. 96 in.
223. 118 cub. yds. 13 ft. 708 in.
224. 15587 qrs. 0 bush. 225. 11s. 9¼d. 226. £17, 17s. 7¼d.
227. £33, 19s. 5½d. + 1¼d. remr. 228. £225, 15s. 3¾d. + 1¾d. remr.
229. £579, 3s. 8½d. 230. £8118, 14s. 4½d. + ½d. remr.
231. £2607, 17s. 9¼d. + ¾d. remr. 232. £86878, 17s. 6d.
233. £3, 3s. 3¾d. 234. £4, 4s. 4¼d. 235. £13, 2s. 0¾d. + 10d. remr.
236. £4, 5s. 9¼d. + 4d. remr. 237. £21, 5s. 5d. + 2s. 0½d. remr.
238. £27, 18s. 6½d. + 5¾d. remr. 239. £37, 13s. 2½d.
240. £149, 17s. 10d. 241. £100, 10s. 10¾d. 242. £147, 12s. 4½d.
243. £9723, 1s. 2¼d. 244. £12447, 5s. 8¾d. 245. £2, 1s. 7¾d.
246. £19, 19s. 11¾d. 247. £9, 0s. 11¾d. 248. £89, 16s. 6d.
249. £5, 5s. 6¾d. 250. £3, 3s. 3¾d. 251. £2, 15s. 4½d.
252. £432, 6s. 2½d. 253. 7s. 1½d. 254. 19s. 10¼d.
255. £201, 17s. 6¼d. + 6¾d. remr. 256. £1079, 6s. 8¾d. + 6¼d. remr.
257. £222, 2s. 2½d. 258. £1100, 11s. 9¼d. 259. £79, 16s. 4½d.
260. £625, 10s. 7d. 261. £1386, 0s. 5¾d. + 1s. 8¼d. remr.
262. £2187, 16s. 6¾d. 263. £368, 12s. 9¼d.
264. £274, 18s. 9¾d. + 6s. 2¾d. remr. 265. £3, 17s. 4½d.
266. £29, 13s. 11¼d. 267. £730, 18s. 0¾d. + 7s. 2¼d. remr.
268. £2534, 13s. 8½d. + 12s. 11d. remr. 269. £12, 12s. 7d.
270. £30, 1s. 11½d. 271. 6s. 2d. + £3, 7s. 10d. remr.
272. £8, 13s. 7d. + £5, 15s. 11d. remr. 273. £1737, 3s. 9d.
274. £375623, 17s. 11¾d. 275. £1, 15s. 0¾d. + £3, 3s. 9d. remr.
276. £11, 15s. 10¾d. + £5, 11s. 1d. remr. 277. £19, 19s. 9¼d.
278. £65, 4s. 3½d. 279. £117, 1s. 1½d. 280. £320, 13s. 1½d. + 1s. 8½d. remr.
281. £23, 14s. 4½d. + 3½d. remr. 282. £403, 2s. 6¼d. + 1s. 3¼d. remr.

283. £1, 2*s.* 9¼*d.* + 10*d.* remʳ. **284.** 1*s.* 11¼*d.* + 18*s.* 6½*d.* remʳ. **285.** 17*s.* 6¼*d.* + 11*s.* 0½*d.* remʳ. **286.** £279, 1*s.* 4*d.* **287.** 30 dys. 10 hrs. 29 min. 4 sec. **288.** 2 dys. 9 hrs. 10 min. 10 secs. **289.** 11 dys. 11 hrs. 11 min. 11 sec. **290.** 400 dys. 6 hrs. 22 min. 42 sec. **291.** 2 tons 5 cwts. 2 qrs. 25 lbs. 6 ozs. **292.** 16 cwts. 1 qr. 8 lbs. 2 ozs. **293.** 3 tns. 11 cwts. 3 qrs. 10 lbs. 8 ozs. **294.** 3 tns. 3 cwts. 0 qr. 2 lbs. 4 oz. **295.** 6 cwts. 2 qrs. 9 lbs. **296.** 6 tons 11 cwts. 3 qrs. 1 lb. **297.** 3 cwts. 0 qr. 3 lbs. 0 oz. 3 drs. **298.** 3 cwts. 2 qrs. 12 lbs. 5 ozs. **299.** 3 cwts. 2 qrs. 1 lb. **300.** 3 tons 0 cwts. 1 qr. 4 lbs. 15 ozs. 10 drs. + 154 drs. remʳ. **301.** 7 yds. 1 ft. 6 in. **302.** 224 yds. 0 ft. 10 in. **303.** 1 mi. 3 fur. 10 po. 5 yds. 0 ft. 6 in. **304.** 1 mi. 4 fur. 34 po. 3 yds. 2 ft. 8 in. + 5 in. remʳ. **305.** 25 po. 3 yds. 1 ft. 7 in. **306.** 4 fur. 8 po. 3 yds. 2 ft. **307.** 1 mi. 0 fur. 1 po. 0 yds. 1 ft. **308.** 3 mi. 1119 yds. 2 ft. 11 in. **309.** 2 mi. 525 yds. 0 ft. 8 in. + 404 in. remʳ. **310.** 2 po. 1 yd. 1 ft. **311.** 1 sq. yd. 5 ft. 132 in. **312.** 2 sq. ft. 32 in. **313.** 2 A. 1 R. 31 P. 10 sq. yds. 8 ft. 15 in. **314.** 1 A. 3 R. 15 P. 11 sq. yds. 6 ft. 92 in. + 5 in. remʳ. **315.** 51 A. 2 R. 18 P. 20 sq. yds. 8 ft. 6 in. + 12 in. remʳ. **316.** 1 A. 0 R. 10 P. 4 sq. yds. 2 ft. **317.** 20 sq. yds. 4 ft. 36 in. **318.** 160 A. 2 R. 10 P. 1 sq. yd. **319.** 16 cub. yds. 0 ft. 48 in. **320.** 1 bush. 3 pks. 1 gal. 3 qts. 1 pt. **321.** 84. **322.** 93. **323.** 182. **324.** 1364. **325.** 1001. **326.** 15. **327.** 903. **328.** 474 + 2*f.* over. **329.** 13. **330.** 15. **331.** 6. **332.** 16. **333.** 60. **334.** 14. **335.** 24. **336.** 20. **337.** 71. **338.** 72. **339.** 16. **340.** 18. **341.** 1753. **342.** 1000. **343.** 16. **344.** 64. **345.** 35. **346.** 61. **347.** 37. **348.** 103. **349.** 9. **350.** 39 + 5 yds. over. **351.** 18026 + 7 ft. over. **352.** 800. **353.** 38 + 3 in. over. **354.** 5. **355.** 119 payments. **356.** 483 lbs. **357.** 601 tons. **358.** 40 persons. **359.** 96 bits. **360.** 17 plots.

VI.

1. 227749*f.* **2.** 86400 sec. **3.** £239, 7*s.* 0¼*d.* **4.** £2, 19*s.* 11¾*d.* **5.** £41656, 7*s.* 2¼*d.* **6.** £1777, 4*s.* 11½*d.* **7.** £843, 17*s.* 6*d.* **8.** £98, 2*s.* 4½*d.* **9.** 48 stamps. **10.** 7 times. **11.** 27283 lbs. **12.** 264960 mins. **13.** £1111, 11*s.* 11¼*d.* **14.** £1, 17*s.* 1½*d.* **15.** £43824, 19*s.* 6*d.* **16.** £2, 0*s.* 5*d.* + 2½*d.* remʳ. **17.** £2, 19*s.* 10¼*d.* **18.** £2, 1*s.* 5¼*d.* **19.** 105 eggs. **20.** 26 times + 11 in. over. **21.** 57223 of each. **22.** 7752*d.* **23.** £120. **24.** 66 tons 6 cwts. 3 qrs. 0 lbs. 3 oz. **25.** £457, 10*s.* 5*d.* **26.** £621. **27.** 10*s.* 4¾*d.* **28.** ¾*d.* **29.** 605 balls. **30.** 2160 times. **31.** 2 wks. 6 days 17 hrs. 46 min. 40 secs. **32.** £1055, 4*s.* 2*d.* **33.** 2694384*f.* **34.** 331 tons 16 cwt. 8 oz. **35.** £2, 2*s.* 2¼*d.* **36.** £363, 19*s.* 6½*d.* **37.** 7½ lbs. **38.** £6, 5*s.* **39.** 13. **40.** 96. **41.** 1248 hf.-crs. **42.** £13, 8*s.* 8½*d.* **43.** £19, 14*s.* 4*d.* **44.** £1, 2*s.* 6*d.* **45.** £6, 19*s.* 6*d.* **46.** £13, 12*s.* 11¾*d.* **47.** £3447, 10*s.* 2*d.* **48.** 2*s.* 10½*d.* **49.** 216 post-cards. **50.** £10. **51.** 17 hf.-crs. **52.** £5, 14*s.* 6*d.* **53.** £61, 12*s.* 6*d.* **54.** £52, 13*s.* 9*d.* **55.** £49, 19*s.* 11¾*d.* **56.** £944, 0*s.* 4*d.* **57.** £12439, 0*s.* 6¾*d.* **58.** £4, 13*s.* 1½*d.* **59.** 777 persons. **60.** £27, 8*s.* 4*d.* **61.** 8784 hrs.

62. 474539 half-ozs. **63.** 57 days 13 hrs. 9 min. 25 secs.
64. 273 cwt. 1 qr. 21 lbs. **65.** 1524 yds. 0 ft. 5 in.
66. 10 oz. 10 dwt. 10 grs. **67.** 2 tons 13 cwts. 1 qr. 23 lbs. 7 oz.
68. 2688 pennies. **69.** 11 yrs. 4 mo. **70.** 9216 steps. **71.** 13 cwt.
72. 3553 half-mins. **73.** 69 mi. 131 yds. **74.** 130 ac. 1 ro.
75. 1 gross 8 doz. and 9. **76.** 137 yds. 1 ft. 5 in.
77. 2 cwt. 3 qrs. 1 lb. 8 oz. **78.** 2 tons 4 cwt. 2 qrs. 16 lbs.
79. 5 ft. 1 in. **80.** 72 medals. **81.** 2316 gs. + 14*s.*
82. £38, 11*s.* 4½*d.* **83.** £38936, 8*s.* 8½*d.* **84.** £5759.
85. £781328, 4*s.* 4½*d.* **86.** £21, 0*s.* 6*d.* **87.** £6, 9*s.* 2¾*d.* + ½*d.* over.
88. £387387, 8*s.* 7½*d.* **89.** 31. **90.** 88. **91.** The latter by 1.
92. £1, 5*s.* 11½*d.* **93.** £31, 5*s.* **94.** £2480, 16*s.* 4*d.*
95. £5265, 11*s.* 3*d.* **96.** £21, 11*s.* 4½*d.* **97.** £93, 0*s.* 6¼*d.*
98. £10, 10*s.* 10¼*d.* **99.** 231 persons. **100.** 41 persons. **101.** 14532 ft.
102. 523 mi. **103.** 19025 com. yrs. 319 days 10 hrs. 40 min.
104. 22 tons 5 cwt. 2 qrs. **105.** £1, 1*s.* 1*d.* **106.** £72, 7*s.* 11*d.*
107. 2 mi. 2 fur. 2 po. 2 yds. 2 ft. 2 in. **108.** 16. **109.** £1, 15*s.*, and 5*s.*
110. 32640. **111.** 980064 in. **112.** 21 mi. 3 fur. 23 po. 3 yds. 2 ft. 7 in.
113. £511, 4*s.* 8*d.* **114.** 1101600 pins. **115.** £405, 11*s.* 2½*d.*
116. £199, 17*s.* 11*d.* **117.** £1, 4*s.* **118.** 54 allotments.
119. £30, 19*s.* 4*d.*; £3, 17*s.* 5*d.* **120.** 12 lbs. Avoir.
121. 54 tons 11 cwts. 3 qrs. 20 lbs. 6 ozs. **122.** The farthings.
123. 416 acres. **124.** £9, 3*s.* 9*d.* **125.** 1 fur. 5 po. 2 yds. 2 ft. 8 in. + 138 in. over. **126.** £20, 3*s.* 9*d.* **127.** 100 times.
128. £3, 2*s.* 1*d.* for A; £1, 2*s.* 3*d.* for B. **129.** A, 6*s.*; B, 7*s.*; C, 8*s.*
130. £49, 11*s.* 8*d.* **131.** 1718 of each. **132.** 51 mi. 1 fur. 13 po. 3 yds. 1 ft. 11 in. **133.** £1, 12*s.* 7½*d.* **134.** £3, 11*s.* 10½*d.*
135. 9 tons 5 cwts. 1 qr. 4 lbs.; 4 tons 12 cwts. 2 qrs. 16 lbs. **136.** 7*s.* 1½*d.*
137. 15 allotments and 4 po. over. **138.** £2, 1*s.* 10*d.* for one; £1, 11*s.* 10*d.* for each of the others. **139.** Coat, £4, 4*s.*; hat, 17*s.*; umbrella, 19*s.*
140. £6, 19*s.* and £17, 7*s.* 6*d.* **141.** £1. **142.** 17695260 sq. in.
143. £25446, 8*s.* 4*d.* **144.** £12, 14*s.* 10½*d.* **145.** 13 cwts. 2 qrs. 27 lbs.; 1 cwt. 2 qrs. 3 lbs. **146.** £7, 2*s.* 1*d.*; £5, 14*s.* 8*d.*
147. 4½*d.* per lb. **148.** 64 times. **149.** 13 times. **150.** Man, 6*s.* 6*d.*; woman, 4*s.* 6*d.*; boy, 1*s.* 6*d.* **151.** 116160 sheets; 5 cwts. 3 qrs. 16 lbs.
152. 29947000 hf.-cr. **153.** £724, 5*s.* 11¼*d.* **154.** £2, 10*s.* 2¾*d.*
155. A, £81, 3*s.* 5*d.*; B, £1, 1*s.* 1*d.* **156.** £4, 17*s.* 6*d.* **157.** 720.
158. 1*d.* per lb. **159.** 37 persons. **160.** Man, 8*s.* 3*d.*; woman, 2*s.* 9*d.*; boy, 11*d.* **161.** 2557 days.
162. 17995 ozs. Tr. 400 grs. **163.** £6, 10*s.* **164.** 11 cwts. 3 qrs. 4 lbs.
165. Wednesday. **166.** 18*s.* 8*d.* to one; 9*s.* 4*d.* to each of the others.
167. 1*s.* 9*d.* per lb. **168.** 55 yards. **169.** Man, 10*s.* 6*d.*; woman, 7*s.*; boy, 4*s.* 8*d.* **170.** 39 persons. **171.** 4924 days.
172. 1092 times. **173.** £725, 0*s.* 2*d.* **174.** The 11th. **175.** £40 and £5.
176. 7*s.* 9*d.* per gal. **177.** 480 yds. **178.** Man, 5*s.*; woman, 3*s.*; boy, 2*s.* **179.** 38 men. **180.** 1835.
181. £3, 18*s.* (There were 53 Sundays in that year.) **182.** 8*s.* per yd.
183. £3600. **184.** Each man, £3, 1*s.*; each woman, £2, 1*s.*
185. 2742 shils., and 3 dwts. over. **186.** 1 inch. **187.** 5*d.* loss.

188. 5 lbs. **189.** 12 dozen of each. **190.** Tuesday.
191. 6*s.* 2*d.* (Omit 12 Sundays.) **192.** 1 oz. Tr. 6 dwts. 10 grs.
193. 2*s.* 3*d.* **194.** 4 gallons. **195.** The latter by 40 ft.
196. 7 horses, 7 pigs, 21 cows, 105 sheep. **197.** January, 1885.
198. 8 dozen of each. **199.** 632 eggs. **200.** April 14, 1852.

VII. **1.** 2, 4, 3, 9. **2.** 2, 5, 10, 25, 3, 9. **3.** 2.
4. 5, 25, 3, 9. **5.** None. **6.** 2, 4, 3. **7.** 2, 4.
8. 2, 4, 5, 10, 3, 9. **9.** 2, 3, 9. **10.** 5, 25, 3. **11.** 2, 4, 3, 9.
12. 2, 5, 10, 4. **13.** None. **14.** 3. **15.** All. **16.** 5, 3.
17. 2, 5, 10, 4, 25, 3. **18.** 2, 4, 3. **19.** 2, 5, 10, 25, 3. **20.** 3, 9.
21. 38964035. **22.** 709082649. **23.** 2; 11; 1; 8. **24.** 1; 2; 1; 2.
25. $2 \times 2 \times 3$. **26.** $2 \times 3 \times 3$. **27.** $2 \times 2 \times 5$. **28.** $2 \times 2 \times 2 \times 3$.
29. $3 \times 3 \times 3$. **30.** 2×17. **31.** 3×13. **32.** $2 \times 3 \times 7$.
33. 3×17. **34.** 3×19. **35.** $3 \times 3 \times 7$. **36.** 5×13.
37. $2 \times 5 \times 7$. **38.** $2 \times 2 \times 2 \times 11$. **39.** $2 \times 3 \times 3 \times 5$.
40. $2 \times 2 \times 5 \times 5$. **41.** $2 \times 2 \times 7$. **42.** $2 \times 2 \times 3 \times 3$.
43. $2 \times 2 \times 2 \times 5$. **44.** $3 \times 3 \times 5$. **45.** $2 \times 2 \times 13$. **46.** 3×23.
47. $3 \times 3 \times 3 \times 3$. **48.** 7×13. **49.** $3 \times 3 \times 11$. **50.** $2 \times 5 \times 11$.
51. 11×11. **52.** $5 \times 5 \times 5$. **53.** $2 \times 5 \times 13$.
54. $2 \times 2 \times 2 \times 13$. **55.** $2 \times 2 \times 3 \times 3 \times 3$. **56.** $2 \times 2 \times 2 \times 2 \times 3 \times 3$.
57. $2^4 \times 7$. **58.** $2^3 \times 3 \times 5$. **59.** 3×37. **60.** 2^7. **61.** $2^3 \times 17$.
62. $2^2 \times 37$. **63.** 7×23. **64.** $3^2 \times 19$. **65.** $2^2 \times 3^2 \times 7$.
66. $3 \times 5 \times 17$. **67.** $3^2 \times 41$. **68.** $2^6 \times 7$. **69.** $3^2 \times 7^2$.
70. $3^4 \times 7$. **71.** $3^2 \times 7 \times 11$. **72.** $2 \times 3^2 \times 7^2$. **73.** $3^3 \times 37$.
74. $2^3 \times 5^3$. **75.** 11^3. **76.** $2^5 \times 5 \times 11$. **77.** $2^6 \times 3^3$.
78. $2^6 \times 5 \times 7$. **79.** $2^3 \times 5 \times 11^2$. **80.** $2^2 \times 11 \times 43$. **81.** 3×631.
82. $2^7 \times 3^2 \times 5$. **83.** $2^4 \times 3^3 \times 7$. **84.** $3^2 \times 5 \times 11 \times 13$.
85. $3^2 \times 11 \times 29$. **86.** $7 \times 11 \times 101$. **87.** $2 \times 3^3 \times 5 \times 11 \times 19$.
88. $2 \times 5 \times 13^3$. **89.** $11 \times 7 \times 13^2$. **90.** 11^4.
91. $3 \times 7 \times 11 \times 13 \times 37$. **92.** $2 \times 3^3 \times 11 \times 37$. **93.** $3 \times 7^2 \times 101$.
94. $3^2 \times 7 \times 11 \times 13 \times 97$. **95.** $2^6 \times 3 \times 643$. **96.** $3^3 \times 7 \times 11 \times 13 \times 37$.
97. Twenty-five. **98.** Twenty-one. **99.** Yes. **100.** No.

VIII. **1.** 7. **2.** 10. **3.** 5. **4.** 25. **5.** 4. **6.** 8.
7. 8. **8.** 12. **9.** 17. **10.** 13. **11.** 20. **12.** 15.
13. 41. **14.** 23. **15.** 19. **16.** 14. **17.** 33. **18.** 22.
19. 8. **20.** 200. **21.** 1. **22.** 3. **23.** 3. **24.** 31.
25. 101. **26.** 3. **27.** 111. **28.** 202. **29.** 101. **30.** 102.
31. 35. **32.** 85. **33.** 4. **34.** 108. **35.** 432. **36.** 96.
37. 27. **38.** 283. **39.** 77. **40.** 677. **41.** 73. **42.** 23.
43. 28. **44.** 320. **45.** 440. **46.** 119. **47.** 35. **48.** 267.
49. 729. **50.** 693. **51.** 113. **52.** 21. **53.** 87. **54.** 27.
55. 49. **56.** 51. **57.** 1. **58.** 53. **59.** 141. **60.** 551.
61. 495. **62.** 8371. **63.** 198. **64.** 61. **65.** 741. **66.** 71017.
67. 43. **68.** 18. **69.** 28. **70.** 17. **71.** 12. **72.** 37.
73. 207. **74.** 207. **75.** 581. **76.** 1017. **77.** 12. **78.** 17.
79. 16. **80.** 23.

IX. **1.** 12. **2.** 24. **3.** 30. **4.** 18. **5.** 16. **6.** 36. **7.** 60. **8.** 60. **9.** 100. **10.** 66. **11.** 12. **12.** 8. **13.** 60. **14.** 60. **15.** 40. **16.** 18. **17.** 42. **18.** 24. **19.** 385. **20.** 440. **21.** 105. **22.** 693. **23.** 672. **24.** 588. **25.** 130. **26.** 102. **27.** 114. **28.** 222. **29.** 595. **30.** 667. **31.** 1207. **32.** 91. **33.** 1680. **34.** 896. **35.** 630. **36.** 330. **37.** 864. **38.** 630. **39.** 1755. **40.** 105. **41.** 112. **42.** 2448. **43.** 336. **44.** 300. **45.** 360. **46.** 660. **47.** 360. **48.** 180. **49.** 252. **50.** 1260. **51.** 144. **52.** 1260. **53.** 1680. **54.** 1680. **55.** 462. **56.** 4320. **57.** 6720. **58.** 2520. **59.** 3640. **60.** 20449. **61.** 1680. **62.** 240. **63.** 2940. **64.** 12000. **65.** 600. **66.** 10080. **67.** 554400. **68.** 1260. **69.** 660660. **70.** 7140. **71.** 17700. **72.** 82861. **73.** 95040. **74.** 12649. **75.** 1196421. **76.** 92939. **77.** 9367. **78.** 9702. **79.** 3764640. **80.** 1052205.

X. **1.** $2^3 \times 3 \times 7 \times 11 \times 17$. **2.** —. **3.** 59. **4.** 1680. **5.** 5031. **6.** $2^6 \times 3 \times 13 \times 53$. **7.** —. **8.** 101. **9.** 1155. **10.** —. **11.** 881. **12.** 13. **13.** 707707. **14.** 5, 7, 13, 35, 65, 91. **15.** 2*s.* 2*d.* **16.** 5612. **17.** 114. **18.** 748000. **19.** 1575. **20.** 1*s.* 1*d.* **21.** $3^2 \times 7^2 \times 11^2 \times 13$. **22.** Their G.C.F. is 1. **23.** 3. **24.** 5863. **25.** 3. **26.** $2^2 \times 3^2 \times 5 \times 7^2 \times 13$. **27.** 17. **28.** 27720. **29.** 19. **30.** 3. **31.** 886. **32.** Yes. **33.** 84 times. **34.** £21. **35.** 76529. **36.** Their G.C.F. is 1. **37.** No. **38.** 3848. **39.** 14 ton 10 cwt. 2 qrs. **40.** 100. **41.** 1517. **42.** 3 hrs. 35 min. **43.** 3961. **44.** 56700. **45.** 629. **46.** 123. **47.** 1 qr. 1 lb. 1 oz. **48.** 187. **49.** 101. **50.** 2155 and 3017. **51.** 999. **52.** 4176. **53.** 72. **54.** 7 mins. **55.** 330 × 330. **56.** 984. **57.** 1053. **58.** 873. **59.** 3 francs. **60.** 40 grs. **61.** 10406. **62.** 3 yds. 2 ft. **63.** 5543. **64.** 273 and 637; or 91 and 1911. **65.** 84 and 42. **66.** 999663 and 100203. **67.** 12 days 2 hrs. 16 min. **68.** 10011. **69.** 137 and 822; or, 274 and 685; or, 411 and 548. **70.** 13. **71.** 83. **72.** 7, 11, 13, 77, 91, 143, 1001. **73.** 4 cwt. 2 qrs. **74.** 678. **75.** 21, 22, 23.

XI. **1.** One-*seventh*; Two-*fifths*; Three-*eighths*; Five-*ninths*; One-*eleventh*; Four-*thirteenths*.

2. One-*twentieth*; Three-*sevenths*; Nine-*tenths*; Four-*nineteenths*; Three-*fortieths*; Seventeen-*thirtieths*.

3. Two-*fifteenths*; Ten-*elevenths*; Nineteen-*twentieths*; Three-*thousandths*; Nineteen-*eight-hundredths*.

4. Twelve-*thirteenths*; One-*hundredth*; Sixty-nine-*seventieths*; Two-hundred-and-eleven-*three-hundred-and-fiftieths*.

5. $\frac{1}{10}$; $\frac{1}{12}$; $\frac{7}{20}$; $\frac{13}{16}$. **6.** $\frac{5}{9}$; $\frac{12}{19}$; $\frac{23}{100}$. **7.** $\frac{2}{5}$; $\frac{73}{700}$; $\frac{3}{1000}$. **8.** $\frac{3}{7}$; $\frac{19}{99}$; $\frac{8}{81}$.
9. 10*s.*; 5*s.*; 4*s.*; 2*s.*; 1*s.* **10.** 6*d.*; 4*d.*; 3*d.*; 2*d.*; 1*d.*
11. 8 ozs.; 4 ozs.; 2 ozs.; 1 oz. **12.** 14 lbs.; 4 lbs.; 2 lbs.; 1 lb.
13. 3*s.*; 7*s.*; 11*s.*; 13*s.*; 19*s.* **14.** 5*d.*; 7*d.*; 3*d.*; 11*d.*; 8*d.*
15. 3 lbs.; 5 lbs.; 7 lbs.; 13 lbs.; 11 lbs. **16.** 3 lbs.; 11 lbs.; 19 lbs.; 5 lbs.; 23 lbs. **17.** 12; 8; 6; 4; 3; 2; 1; hours. **18.** 30; 20; 15; 12; 10; 6; 5; 4; 3; 2; mins. **19.** 7; 11; 9; 13; 23; hours.

20. 17; 19; 13; 29; 51; mins. **21.** 34; 31; 146. **22.** 16; 49; 85.
23. 15 cwts.; £16; 3*d.* **24.** 45 min.; £27; 1*s.* 6*d.*
25. 6*s.*; 14*s.*; 18*s.*; 8*s.*; 16*s.*; 12*s.*; 1*d.*; 7*d.*; 1*s.* 11*d.*
26. 15*s.*; 2*s.*; 22*s.*; 9*s.*; 18*s.*; 6*d.*; 11*d.*; 1*s.* 7*d.*; 2*s.* 5*d.*
27. 9*d.*; 8*d.*; 10*d.*; $\frac{1}{2}$*d.*; $3\frac{1}{2}$*d.*; $5\frac{1}{2}$*d.*; $1\frac{3}{4}$*d.*; $2\frac{3}{4}$*d.*; $1\frac{1}{2}$*d.*; $7\frac{1}{2}$*d.*
28. 45 secs.; 10 hrs.; 6 cwts.; 18 mins.; 7 in.; 40 sq. in.
29. 4*s.* 4*d.*; $3\frac{3}{4}$*d.*; 34 min.; 6 cwts. 2 qrs.; 10 in.; 1 ro. 8 sq. po.
30. 12*s.* 8*d.*; $9\frac{3}{4}$*d.*; 146 days; 40 lbs.; 800 yds.; $1\frac{1}{2}$ pints.

XII.

1. Five and three-eighths; Seven and four-fifths; Nine and one-eleventh; Twelve and three-thirteenths; Fifteen and seven-twentieths.

2. One and two-thirty-thirds; Twelve and seventeen-fiftieths; Ten and five-ninths; Twenty-eight and seven-twenty-fifths.

3. $2\frac{3}{5}$; $7\frac{1}{3}$. **4.** $8\frac{7}{10}$; $1\frac{1}{12}$. **5.** $11\frac{3}{4}$; $17\frac{1}{4}$. **6.** $20\frac{3}{20}$; $9\frac{8}{9}$.
7. 11; 3; 12; 17; 2. **8.** 20; 4; 8; 27; 4. **9.** $1\frac{2}{3}$; $1\frac{3}{5}$; $5\frac{1}{2}$; $1\frac{1}{6}$; $1\frac{3}{7}$; $4\frac{1}{4}$.
10. $3\frac{1}{2}$; $1\frac{4}{5}$; $1\frac{3}{11}$; $2\frac{5}{8}$; $2\frac{2}{7}$; $4\frac{1}{3}$. **11.** $3\frac{1}{7}$; $3\frac{3}{5}$; $2\frac{8}{11}$; $3\frac{5}{12}$; $8\frac{8}{9}$. **12.** $6\frac{1}{6}$; $4\frac{4}{11}$; $7\frac{7}{12}$; $11\frac{1}{9}$; $12\frac{3}{10}$. **13.** $\frac{6}{5}$; $\frac{7}{3}$; $\frac{15}{4}$; $\frac{39}{7}$; $\frac{53}{8}$. **14.** $\frac{10}{9}$; $\frac{17}{5}$; $\frac{29}{6}$; $\frac{11}{2}$; $\frac{29}{12}$.
15. $\frac{19}{4}$; $\frac{35}{6}$; $\frac{23}{7}$; $\frac{23}{2}$; $\frac{85}{8}$. **16.** $\frac{50}{7}$; $\frac{38}{9}$; $\frac{27}{20}$; $\frac{43}{2}$; $\frac{88}{9}$. **17.** $\frac{88}{25}$. **18.** $\frac{81}{38}$.

19. $\frac{65}{24}$.	**20.** $\frac{79}{20}$.	**21.** $\frac{128}{27}$.	**22.** $\frac{197}{47}$.	**23.** $\frac{167}{13}$.
24. $\frac{144}{71}$.	**25.** $\frac{142}{15}$.	**26.** $\frac{142}{9}$.	**27.** $\frac{155}{12}$.	**28.** $\frac{83}{19}$.
29. $\frac{101}{17}$.	**30.** $\frac{131}{11}$.	**31.** $\frac{181}{13}$.	**32.** $\frac{208}{29}$.	**33.** $\frac{298}{33}$.
34. $\frac{280}{13}$.	**35.** $\frac{157}{53}$.	**36.** $\frac{250}{9}$.	**37.** $\frac{474}{41}$.	**38.** $\frac{425}{12}$.
39. $\frac{629}{50}$.	**40.** $\frac{1217}{40}$.	**41.** $\frac{803}{8}$.	**42.** $\frac{643}{108}$.	**43.** $\frac{4948}{51}$.
44. $\frac{520}{19}$.	**45.** $\frac{10779}{200}$.	**46.** $\frac{42271}{831}$.	**47.** $\frac{6286}{1197}$.	**48.** $\frac{13989}{2359}$.
49. $142\frac{6}{7}$.	**50.** $111\frac{2}{9}$.	**51.** $336\frac{2}{3}$.	**52.** $90\frac{10}{11}$.	**53.** $70\frac{11}{12}$.
54. $97\frac{1}{8}$.	**55.** $236\frac{1}{10}$.	**56.** $28\frac{19}{20}$.	**57.** $42\frac{31}{100}$.	**58.** $53\frac{267}{1000}$.
59. $46\frac{5}{13}$.	**60.** $15\frac{5}{14}$.	**61.** $46\frac{11}{15}$.	**62.** $49\frac{10}{17}$.	**63.** $34\frac{7}{23}$.
64. $45\frac{20}{29}$.	**65.** $19\frac{8}{35}$.	**66.** $18\frac{17}{45}$.	**67.** $24\frac{223}{300}$.	**68.** $2\frac{573}{700}$.
69. $7\frac{103}{110}$.	**70.** $1\frac{3573}{5000}$.	**71.** $1\frac{265}{552}$.	**72.** $1\frac{281}{324}$.	**73.** $75\frac{47}{50}$.
74. $145\frac{11}{40}$.	**75.** $276\frac{9}{41}$.	**76.** $1354\frac{29}{31}$.	**77.** $24\frac{92}{147}$.	**78.** $38\frac{94}{207}$.
79. $76\frac{245}{558}$.	**80.** $22\frac{11}{884}$.			

XIII.

1. Six *ninths.*	**2.** Three *twelfths.*	**3.** 35 *fiftieths.*
4. 27 *sixty-thirds.*	**5.** 35 *fortieths.*	**6.** 20 *fifty-fifths.*
7. 33 *fifty-fifths.*	**8.** 25 *thirtieths.*	**9.** 65 *seventieths.*
10. 69 *sixtieths.*	**11.** Three *fourths.*	**12.** Three *fifths.*
13. One *sixth.*	**14.** Two *tenths.*	**15.** Three *fifteenths.*
16. Three *sevenths.*	**17.** Three *fifths.*	**18.** Two *fifths.*
19. Six *tenths.*	**20.** Five *eighths.*	**21.** 4.

22. 9.	**23.** 10.	**24.** 20.	**25.** 45.	**26.** 28.	**27.** 7.
28. 35.	**29.** 33.	**30.** 40.	**31.** 84.	**32.** 70.	**33.** 34.
34. 44.	**35.** 72.	**36.** 57.	**37.** 100.	**38.** 93.	**39.** 85.
40. 57.	**41.** 6.	**42.** 7.	**43.** 6.	**44.** 11.	**45.** 5.
46. 3.	**47.** 3.	**48.** 4.	**49.** 8.	**50.** 7.	**51.** 10.
52. 7.	**53.** 3.	**54.** 6.	**55.** 11.	**56.** 8.	**57.** 4.
58. 19.	**59.** 5.	**60.** 19.	**61.** $\frac{35}{55}$.	**62.** $\frac{45}{65}$.	**63.** $\frac{68}{84}$.

64. $\frac{85}{119}$. 65. $\frac{91}{156}$. 66. $\frac{123}{159}$. 67. $\frac{40}{56}$. 68. $\frac{108}{132}$. 69. $\frac{100}{850}$.
70. $\frac{169}{182}$. 71. $\frac{6}{8}$. 72. $\frac{3}{4}$. 73. $\frac{2}{3}$. 74. $\frac{8}{12}$. 75. $\frac{15}{25}$.
76. $\frac{22}{24}$. 77. $\frac{9}{10}$. 78. $\frac{6}{9}$. 79. $\frac{7}{13}$. 80. $\frac{2}{3}$.

XIV.

1. $\frac{1}{2}$. 2. $\frac{2}{3}$. 3. $\frac{3}{4}$. 4. $\frac{5}{6}$. 5. $\frac{3}{4}$.
6. $\frac{9}{10}$. 7. $\frac{3}{5}$. 8. $\frac{4}{5}$. 9. $\frac{1}{3}$. 10. $\frac{3}{8}$. 11. $\frac{5}{8}$.
12. $\frac{2}{3}$. 13. $\frac{2}{7}$. 14. $\frac{2}{3}$. 15. $\frac{2}{5}$. 16. $\frac{3}{5}$. 17. $\frac{14}{19}$.
18. $\frac{3}{4}$. 19. $\frac{5}{7}$. 20. $\frac{2}{3}$. 21. $\frac{2}{3}$. 22. $\frac{1}{2}$. 23. $\frac{5}{6}$.
24. $\frac{3}{4}$. 25. $\frac{1}{2}$. 26. $\frac{2}{3}$. 27. $\frac{2}{3}$. 28. $\frac{1}{3}$. 29. $\frac{1}{3}$.
30. $\frac{2}{3}$. 31. $\frac{7}{16}$. 32. $\frac{3}{4}$. 33. $\frac{3}{4}$. 34. $\frac{4}{5}$. 35. $\frac{1}{4}$.
36. $\frac{5}{8}$. 37. $\frac{7}{9}$. 38. $\frac{8}{11}$. 39. $\frac{9}{11}$. 40. $\frac{9}{11}$. 41. $\frac{3}{8}$.
42. $\frac{5}{9}$. 43. $\frac{2}{3}$. 44. $\frac{3}{8}$. 45. $\frac{3}{5}$. 46. $\frac{2}{3}$. 47. $\frac{3}{5}$.
48. $\frac{3}{5}$. 49. $\frac{3}{4}$. 50. $\frac{3}{7}$. 51. $\frac{2}{3}$. 52. $\frac{2}{3}$. 53. $\frac{1}{7}$.
54. $\frac{1}{4}$. 55. $\frac{1}{7}$. 56. $\frac{2}{3}$. 57. $\frac{3}{8}$. 58. $\frac{7}{9}$. 59. $\frac{13}{17}$.
60. $\frac{3}{5}$. 61. $\frac{7}{9}$. 62. $\frac{9}{13}$. 63. $\frac{7}{11}$. 64. $\frac{3}{7}$. 65. $\frac{3}{4}$.
66. $\frac{4}{5}$. 67. $\frac{5}{6}$. 68. $\frac{4}{7}$. 69. $\frac{8}{9}$. 70. $\frac{13}{17}$. 71. $\frac{29}{31}$.
72. $\frac{23}{29}$. 73. $\frac{43}{88}$. 74. $\frac{46}{49}$. 75. $\frac{31}{41}$. 76. $\frac{3}{13}$. 77. $\frac{5}{9}$.
78. $\frac{3}{11}$. 79. $\frac{2}{7}$. 80. $\frac{13}{19}$. 81. $\frac{9}{107}$. 82. $\frac{37}{367}$. 83. $\frac{5}{8}$.
84. $\frac{7}{8}$. 85. $\frac{5}{7}$. 86. $\frac{11}{12}$. 87. $\frac{11}{13}$. 88. $\frac{121}{130}$. 89. $\frac{71}{81}$.
90. $\frac{3}{4}$. 91. $\frac{43}{44}$. 92. $\frac{68}{123}$. 93. $\frac{21}{22}$. 94. $\frac{101}{117}$. 95. $\frac{9}{55}$.
96. $\frac{29}{31}$. 97. $\frac{1009}{1734}$. 98. $\frac{263}{277}$. 99. $\frac{11}{17}$. 100. $\frac{15}{29}$. 101. $57\frac{2}{7}$.
102. $44\frac{2}{3}$. 103. $48\frac{1}{3}$. 104. $41\frac{1}{2}$. 105. $6\frac{1}{8}$. 106. $7\frac{8}{13}$. 107. $5\frac{16}{43}$.
108. $9\frac{2}{5}$. 109. $40\frac{4}{11}$. 110. $70\frac{1}{4}$. 111. $17\frac{1}{7}$. 112. $14\frac{3}{7}$. 113. $1\frac{2}{137}$.
114. $1\frac{1}{200}$. 115. $5\frac{112}{257}$. 116. $4\frac{23}{41}$. 117. $10\frac{4}{35}$. 118. $10\frac{11}{46}$. 119. $1\frac{29}{85}$.
120. $3\frac{16}{113}$.

XV.

1. $\frac{20}{24}, \frac{21}{24}$. 2. $\frac{21}{36}, \frac{22}{36}$. 3. $\frac{9}{60}, \frac{14}{60}$. 4. $\frac{44}{60}, \frac{39}{60}$.
5. $\frac{15}{48}, \frac{22}{48}$. 6. $\frac{27}{150}, \frac{26}{150}$. 7. $\frac{68}{192}, \frac{63}{192}$. 8. $\frac{64}{252}, \frac{57}{252}$.
9. $\frac{169}{312}, \frac{214}{312}$. 10. $\frac{119}{663}, \frac{52}{663}$. 11. $\frac{33}{108}, \frac{34}{108}$. 12. $\frac{110}{285}, \frac{123}{285}$.
13. $\frac{4}{24}, \frac{9}{24}, \frac{10}{24}$. 14. $\frac{18}{24}, \frac{20}{24}, \frac{21}{24}$. 15. $\frac{18}{48}, \frac{28}{48}, \frac{27}{48}$. 16. $\frac{40}{60}, \frac{45}{60}, \frac{48}{60}$.
17. $\frac{30}{84}, \frac{32}{84}, \frac{33}{84}$. 18. $\frac{66}{144}, \frac{60}{144}, \frac{57}{144}$. 19. $\frac{40}{300}, \frac{45}{300}, \frac{48}{300}$.
20. $\frac{140}{600}, \frac{135}{600}, \frac{132}{600}$. 21. $\frac{861}{1008}, \frac{918}{1008}, \frac{854}{1008}$. 22. $\frac{66}{252}, \frac{44}{252}, \frac{33}{252}$.
23. $\frac{15}{1020}, \frac{12}{1020}, \frac{10}{1020}$. 24. $\frac{140}{1365}, \frac{84}{1365}, \frac{60}{1365}$. 25. $\frac{30,\ 20,\ 15,\ 12,\ 10}{60}$.
26. $\frac{56,\ 20,\ 6,\ 1}{64}$. 27. $\frac{30,\ 40,\ 45,\ 48}{300}$. 28. $\frac{630,\ 400,\ 285,\ 216}{660}$.
29. $\frac{35,\ 32,\ 33,\ 34}{84}$. 30. $\frac{900,\ 672,\ 784,\ 675}{2520}$. 31. $\frac{284,\ 282,\ 281}{808}$.
32. $\frac{498,\ 501,\ 502}{1998}$. 33. $\frac{1848,\ 1155,\ 840,\ 660}{15400}$. 34. $\frac{429,\ 484,\ 455,\ 440}{7865}$.
35. $\frac{16578,\ 16555}{95040}$. 36. $\frac{195,\ 192,\ 190}{18180}$. 37. $\frac{7}{9}$. 38. $\frac{13}{20}$.
39. $\frac{93}{104}$. 40. $\frac{83}{108}$. 41. $\frac{79}{90}$. 42. $\frac{23}{84}$. 43. $\frac{1}{9}$. 44. $\frac{51}{71}$.
45. $\frac{11}{156}$. 46. $\frac{16}{19}$. 47. $\frac{52}{73}$. 48. $\frac{23}{153}$. 49. $\frac{2}{3}, \frac{8}{11}, \frac{17}{22}, \frac{5}{6}$.
50. $\frac{5}{12}, \frac{3}{7}, \frac{19}{42}, \frac{10}{21}$. 51. $\frac{19}{42}, \frac{16}{35}, \frac{13}{28}$. 52. $\frac{15}{22}, \frac{7}{10}, \frac{39}{55}, \frac{8}{11}$.
53. $\frac{25}{144}, \frac{5}{27}, \frac{3}{16}, \frac{7}{36}$. 54. $\frac{5}{119}, \frac{3}{68}, \frac{4}{85}$. 55. $\frac{80}{99}, \frac{53}{66}, \frac{26}{33}$.
56. $\frac{11}{18}, \frac{7}{12}, \frac{9}{16}, \frac{13}{24}$. 57. $\frac{4}{5}, \frac{27}{35}, \frac{19}{25}, \frac{11}{15}$. 58. $\frac{87}{130}, \frac{17}{26}, \frac{42}{65}, \frac{33}{52}$.
59. $\frac{19}{50}$ is greatest; $\frac{23}{70}$ is least. 60. $\frac{111}{1111}$ is greatest; $\frac{1}{11}$ is least.

XVI.

1. $\frac{8}{11}$. 2. $\frac{11}{12}$. 3. $\frac{23}{30}$. 4. $\frac{32}{90}$. 5. $\frac{13}{24}$.
6. $\frac{12}{17}$. 7. $\frac{19}{29}$. 8. $\frac{26}{43}$. 9. $\frac{33}{100}$. 10. $\frac{22}{27}$. 11. $\frac{5}{6}$.
12. $\frac{7}{12}$. 13. $\frac{11}{12}$. 14. $\frac{13}{20}$. 15. $\frac{11}{12}$. 16. $\frac{5}{6}$. 17. $\frac{11}{12}$.

18. $\frac{11}{12}$. 19. 1. 20. 1. 21. $\frac{3}{4}$. 22. $\frac{3}{4}$. 23. $\frac{23}{24}$.
24. $\frac{31}{84}$. 25. $1\frac{19}{99}$. 26. $1\frac{19}{56}$. 27. $1\frac{13}{54}$. 28. $1\frac{26}{105}$. 29. $\frac{3}{7}$.
30. $\frac{5}{12}$. 31. $\frac{5}{7}$. 32. $\frac{8}{15}$. 33. $1\frac{1}{3}$. 34. $1\frac{1}{4}$. 35. $\frac{5}{6}$.
36. $\frac{7}{12}$. 37. $\frac{13}{48}$. 38. $\frac{23}{24}$. 39. $\frac{15}{16}$. 40. $1\frac{3}{20}$. 41. $1\frac{1}{3}$.
42. $2\frac{23}{60}$. 43. $1\frac{1}{60}$. 44. $1\frac{11}{54}$. 45. $1\frac{17}{60}$. 46. $1\frac{13}{30}$. 47. $\frac{227}{1000}$.
48. $\frac{3333}{10000}$. 49. $1\frac{9}{20}$. 50. 1. 51. $1\frac{59}{60}$. 52. $1\frac{267}{280}$. 53. $4\frac{11}{40}$.
54. $4\frac{57}{140}$. 55. $1\frac{189}{200}$. 56. $1\frac{17}{168}$. 57. $1\frac{9}{17}$. 58. $\frac{43}{102}$. 59. $1\frac{1}{78}$.
60. $\frac{158}{195}$. 61. $9\frac{47}{60}$. 62. $5\frac{13}{60}$. 63. $39\frac{2}{3}$. 64. $20\frac{2}{105}$. 65. 5.
66. $9\frac{10}{21}$. 67. $924\frac{3}{4}$. 68. $1000\frac{7}{24}$. 69. $270\frac{329}{600}$. 70. $129\frac{103}{300}$.
71. $8\frac{11}{18}$. 72. $45\frac{31}{60}$. 73. $10\frac{62}{81}$. 74. $22\frac{19}{32}$. 75. $27\frac{37}{114}$.
76. $6\frac{41}{132}$. 77. $10\frac{1}{12}$. 78. 11. 79. $4\frac{29}{50}$. 80. $28\frac{9}{40}$.
81. $900\frac{13}{2400}$. 82. $1408\frac{74}{225}$. 83. $48294\frac{17}{77}$. 84. $9255\frac{7}{24}$.
85. $11\frac{8}{21}$. 86. $8\frac{296}{315}$. 87. $6\frac{53}{56}$. 88. $7\frac{17}{40}$.
89. $2085\frac{9}{50}$. 90. $19\frac{43}{90}$. 91. $3\frac{1265}{13104}$. 92. $123\frac{573}{625}$.
93. $6\frac{3}{25}$. 94. $4\frac{1931}{4380}$. 95. $37\frac{33}{272}$. 96. $30\frac{480}{93439}$.
97. $11\frac{5}{8}$. 98. $28\frac{55}{63}$. 99. $1222666\frac{1}{6}$. 100. $24\frac{13}{15252}$.

XVII.

1. $\frac{5}{11}$. 2. $\frac{4}{17}$. 3. $\frac{13}{25}$. 4. $\frac{5}{49}$. 5. $\frac{1}{8}$.
6. $\frac{5}{24}$. 7. $\frac{1}{16}$. 8. $\frac{7}{30}$. 9. $\frac{27}{32}$. 10. $\frac{31}{56}$. 11. $\frac{9}{28}$.
12. $\frac{7}{64}$. 13. $\frac{1}{2}$. 14. $\frac{5}{48}$. 15. $\frac{99}{112}$. 16. $\frac{73}{80}$. 17. $6\frac{1}{3}$.
18. $3\frac{3}{8}$. 19. 5. 20. 8. 21. $2\frac{1}{6}$. 22. $2\frac{5}{12}$. 23. $4\frac{3}{7}$.
24. $6\frac{5}{8}$. 25. $10\frac{13}{16}$. 26. $14\frac{11}{20}$. 27. $11\frac{3}{14}$. 28. $16\frac{6}{25}$. 29. $4\frac{3}{5}$.
30. $5\frac{1}{6}$. 31. $5\frac{6}{7}$. 32. $1\frac{7}{9}$. 33. $9\frac{10}{17}$. 34. $17\frac{18}{19}$. 35. $15\frac{13}{28}$.
36. $8\frac{31}{40}$. 37. $5\frac{1}{4}$. 38. $7\frac{1}{8}$. 39. $5\frac{3}{5}$. 40. $5\frac{17}{28}$. 41. $3\frac{1}{12}$.
42. $4\frac{3}{20}$. 43. $\frac{43}{180}$. 44. $\frac{5}{126}$. 45. $4\frac{1}{96}$. 46. $5\frac{1}{84}$. 47. $\frac{1}{96}$.
48. $\frac{1}{70}$. 49. $\frac{1}{156}$. 50. $\frac{6}{455}$. 51. $\frac{5}{99}$. 52. $\frac{1}{84}$. 53. $1\frac{5}{72}$.
54. $1\frac{7}{90}$. 55. $1\frac{33}{35}$. 56. $4\frac{31}{40}$. 57. $3\frac{13}{20}$. 58. $8\frac{16}{21}$. 59. $13\frac{5}{8}$.
60. $12\frac{5}{8}$. 61. $3\frac{54}{55}$. 62. $8\frac{51}{56}$. 63. $74\frac{11}{12}$. 64. $3\frac{119}{180}$. 65. $3\frac{29}{78}$.
66. $4\frac{67}{90}$. 67. $95\frac{211}{360}$. 68. $471\frac{17}{30}$. 69. $\frac{127}{154}$. 70. $\frac{241}{280}$. 71. $4\frac{29}{42}$.
72. $110\frac{152}{165}$. 73. $28\frac{59}{228}$. 74. $1\frac{157}{306}$. 75. $103\frac{385}{403}$. 76. $18\frac{349}{432}$.
77. $14\frac{50}{561}$. 78. $10\frac{10}{11211}$. 79. $4\frac{551}{1200}$. 80. $507\frac{27}{35}$. 81. $\frac{5}{14014}$.
82. $\frac{7}{1881}$. 83. $111\frac{1087}{1210}$. 84. $8568\frac{19}{120}$. 85. $9875\frac{1}{9875}$.
86. $12344\frac{1}{12345}$. 87. $\frac{3}{46189}$. 88. $\frac{1}{56}$. 89. $\frac{17}{54}$. 90. $\frac{1}{42}$.
91. $\frac{13}{252}$. 92. 0. 93. $\frac{1}{28}$. 94. $2\frac{71}{80}$. 95. $\frac{1}{255}$. 96. $\frac{5}{24}$.
97. 0. 98. $\frac{1}{154}$. 99. 1. 100. $\frac{1}{3}$. 101. $3\frac{27}{40}$. 102. $2\frac{1}{15}$.
103. 5. 104. $10\frac{6}{7}$. 105. $\frac{1}{24}$. 106. $\frac{9}{10}$. 107. $5\frac{23}{60}$. 108. $1\frac{1}{312}$.
109. $1\frac{1}{48}$. 110. $\frac{33}{35}$. 111. $16\frac{19}{20}$. 112. $7\frac{12}{35}$. 113. $\frac{439}{4096}$. 114. $\frac{41}{243}$.
115. $1\frac{13}{72}$. 116. $2\frac{44}{75}$. 117. $11\frac{61}{72}$. 118. $3\frac{491}{512}$. 119. $34\frac{51}{70}$. 120. $336\frac{4}{343}$.

XVIII.

1. $\frac{6}{7}$. 2. $\frac{10}{11}$. 3. $\frac{14}{15}$. 4. $\frac{16}{17}$. 5. $\frac{8}{15}$.
6. $\frac{9}{16}$. 7. $\frac{7}{11}$. 8. $\frac{20}{23}$. 9. $\frac{96}{97}$. 10. $\frac{91}{100}$. 11. $\frac{3}{4}$.
12. $\frac{5}{6}$. 13. $\frac{1}{5}$. 14. $\frac{3}{7}$. 15. $\frac{5}{7}$. 16. $\frac{2}{5}$. 17. $\frac{7}{25}$.
18. $\frac{11}{20}$. 19. $\frac{2}{5}$. 20. $\frac{3}{4}$. 21. $\frac{3}{11}$. 22. $\frac{4}{15}$. 23. $\frac{5}{17}$.
24. $\frac{7}{19}$. 25. $\frac{4}{13}$. 26. $\frac{5}{16}$. 27. $\frac{4}{21}$. 28. $\frac{7}{45}$. 29. $\frac{5}{26}$.
30. $\frac{2}{41}$. 31. $\frac{7}{32}$. 32. $\frac{3}{20}$. 33. $\frac{5}{52}$. 34. $\frac{11}{68}$. 35. $\frac{7}{64}$.
36. $\frac{23}{75}$. 37. $\frac{5}{96}$. 38. $\frac{19}{96}$. 39. $\frac{11}{120}$. 40. $\frac{10}{77}$. 41. $3\frac{1}{3}$.

42. $2\frac{2}{3}$. 43. $5\frac{3}{5}$. 44. $14\frac{2}{3}$. 45. $14\frac{2}{3}$. 46. $9\frac{6}{7}$. 47. $7\frac{14}{19}$.
48. $30\frac{10}{17}$. 49. 10. 50. $40\frac{1}{2}$. 51. $13\frac{1}{2}$. 52. $22\frac{2}{3}$. 53. $39\frac{3}{4}$.
54. $34\frac{2}{3}$. 55. $95\frac{1}{3}$. 56. $116\frac{2}{3}$. 57. $376\frac{1}{2}$. 58. $297\frac{1}{3}$. 59. $155\frac{1}{3}$.
60. $703\frac{1}{2}$. 61. $317\frac{1}{2}$. 62. $154\frac{4}{5}$. 63. 22875. 64. $13368\frac{5}{6}$.
65. $\frac{4}{51}$. 66. $\frac{2}{63}$. 67. $\frac{2}{125}$. 68. $\frac{2}{99}$. 69. $\frac{20}{63}$. 70. $\frac{17}{100}$.
71. $\frac{2}{217}$. 72. $\frac{4}{285}$. 73. $\frac{17}{650}$. 74. $\frac{23}{625}$. 75. $\frac{1}{6}$. 76. $\frac{2}{7}$.
77. $\frac{5}{16}$. 78. $\frac{2}{9}$. 79. $\frac{3}{16}$. 80. $1\frac{7}{12}$. 81. $\frac{3}{7}$. 82. $\frac{4}{13}$.
83. $\frac{11}{72}$. 84. $\frac{2}{27}$. 85. $1\frac{5}{33}$. 86. $\frac{5}{16}$. 87. $1\frac{7}{18}$. 88. $1\frac{68}{133}$.
89. 16173. 90. $112\frac{5}{16}$.

XIX. 1. $\frac{7}{24}$. 2. $\frac{6}{35}$. 3. $\frac{6}{35}$. 4. $\frac{9}{20}$. 5. $\frac{117}{187}$.
6. $\frac{128}{189}$. 7. 1. 8. $\frac{1}{2}$. 9. 1. 10. $\frac{1}{24}$. 11. $3\frac{1}{3}$.
12. $\frac{7}{12}$. 13. $\frac{32}{39}$. 14. $2\frac{5}{14}$. 15. 10. 16. 1. 17. $1\frac{6}{7}$.
18. 53. 19. $\frac{16}{25}$. 20. $8\frac{1}{3}$. 21. $\frac{2}{15}$. 22. $\frac{15}{64}$. 23. $\frac{2}{9}$.
24. $\frac{16}{63}$. 25. $\frac{2}{3}$. 26. $\frac{1}{3}$. 27. $\frac{5}{9}$. 28. $\frac{1}{5}$. 29. 1.
30. 1. 31. $2\frac{1}{2}$. 32. $5\frac{1}{7}$. 33. 30. 34. $12\frac{8}{9}$. 35. 15.
36. $61\frac{4}{9}$. 37. $14\frac{11}{25}$. 38. $51\frac{1}{49}$. 39. 42. 40. 12. 41. 2920.
42. $41\frac{1}{2}$. 43. $43\frac{1}{2}$. 44. $32\frac{1}{2}$. 45. $\frac{1}{25}$. 46. $\frac{15}{28}$. 47. $\frac{11}{4416}$.
48. $\frac{49}{165}$. 49. $\frac{3}{5}$. 50. $9\frac{1}{3}$. 51. $4\frac{1}{5}$. 52. $2\frac{170}{231}$. 53. $2\frac{1}{3}$.
54. 90. 55. $140\frac{1}{4}$. 56. 392. 57. $13\frac{1}{3}$. 58. 3. 59. $27\frac{1}{2}$.
60. $\frac{33}{169}$. 61. 1. 62. $1\frac{7}{9}$. 63. $46\frac{4}{5}$. 64. 70. 65. 1.
66. $8\frac{1}{4}$. 67. 1. 68. $\frac{2}{7}$. 69. $\frac{3}{121}$. 70. $\frac{1}{12}$. 71. $1\frac{7}{8}$.
72. 4. 73. $\frac{920}{1353}$. 74. $4\frac{2}{3}$. 75. $40\frac{1}{3}$. 76. $\frac{5}{8}$. 77. 6.
78. 1. 79. $\frac{2}{125}$. 80. $\frac{220}{279}$.

XX. 1. 12. 2. 10. 3. 15. 4. 39. 5. 84.
6. 160. 7. 1080. 8. 378. 9. 600. 10. 1015. 11. $\frac{8}{9}$.
12. $\frac{16}{25}$. 13. $1\frac{1}{6}$. 14. $2\frac{4}{9}$. 15. $1\frac{1}{3}$. 16. 22. 17. 1.
18. 1. 19. $1\frac{1}{8}$. 20. $1\frac{32}{45}$. 21. $9\frac{1}{3}$. 22. $16\frac{2}{3}$. 23. $17\frac{1}{2}$.
24. $17\frac{1}{3}$. 25. 40. 26. $\frac{1}{40}$. 27. $\frac{7}{40}$. 28. $\frac{8}{35}$. 29. 95.
30. $17\frac{1}{2}$. 31. $2\frac{4}{5}$. 32. $1\frac{1}{3}$. 33. $1\frac{5}{21}$. 34. $7\frac{6}{19}$. 35. $\frac{3}{8}$.
36. $6\frac{2}{3}$. 37. $4\frac{1}{2}$. 38. $9\frac{9}{80}$. 39. $\frac{99}{896}$. 40. $\frac{203}{951}$. 41. $2\frac{5}{8}$.
42. $1\frac{3}{32}$. 43. $\frac{21}{32}$. 44. $\frac{2}{3}$. 45. $2\frac{7}{16}$. 46. $4\frac{1}{2}$. 47. $\frac{16}{21}$.
48. $1\frac{7}{20}$. 49. $3\frac{3}{5}$. 50. $5\frac{5}{8}$. 51. $\frac{5}{18}$. 52. $\frac{5}{96}$. 53. $\frac{3}{35}$.
54. $\frac{2}{45}$. 55. $\frac{1}{24}$. 56. $\frac{1}{33}$. 57. $11\frac{1}{4}$. 58. $25\frac{2}{3}$. 59. $13\frac{1}{3}$.
60. $49\frac{1}{2}$. 61. $1\frac{1}{2}$. 62. $\frac{7}{8}$. 63. $\frac{13}{18}$. 64. $\frac{5}{16}$. 65. $5\frac{5}{11}$.
66. 12. 67. $3\frac{11}{15}$. 68. $2\frac{5}{8}$. 69. $\frac{11}{12}$. 70. $3\frac{1}{9}$. 71. $3\frac{39}{112}$.
72. $4\frac{2}{5}$. 73. $\frac{7}{9}$. 74. $\frac{2}{27}$. 75. 75. 76. $1\frac{46}{53}$. 77. $7\frac{88}{107}$.
78. $3\frac{1}{3}$. 79. $\frac{1}{4}$. 80. $\frac{28}{129}$. 81. $\frac{11}{12}$. 82. $3\frac{1}{2}$. 83. $10\frac{163}{182}$.
84. $14\frac{2}{15}$. 85. $6\frac{5}{12}$. 86. $\frac{2}{3}$. 87. $\frac{56}{117}$. 88. $10\frac{39}{50}$. 89. 68.
90. $8\frac{16}{21}$.

XXI. 1. $7\frac{1}{2}$. 2. 120. 3. $5\frac{11}{15}$. 4. 3. 5. $\frac{1}{20}$.
6. $44\frac{4}{5}$. 7. $7\frac{11}{19}$. 8. $1\frac{45}{53}$. 9. $\frac{28}{75}$. 10. $\frac{1}{3}$. 11. $9\frac{13}{20}$.
12. 12. 13. 1. 14. $\frac{9}{25}$. 15. $\frac{5}{6}$. 16. $\frac{49}{72}$. 17. $12\frac{13}{49}$.
18. $15\frac{17}{81}$. 19. $40\frac{31}{40}$. 20. $\frac{14}{15}$. 21. $19\frac{2}{3}$. 22. 11. 23. $22\frac{2}{3}$.

24. $9\frac{1}{8}$.	25. $55\frac{71}{128}$.	26. $83\frac{27}{104}$.	27. $3\frac{29}{30}$.	28. $1\frac{5}{8}$.	29. $2\frac{17}{120}$.
30. $\frac{1}{11}$.	31. $7\frac{1}{6}$.	32. $6\frac{3}{4}$.	33. $2\frac{15}{56}$.	34. $231\frac{5}{6}$.	35. $14\frac{11}{20}$.
36. $2\frac{1}{7}$.	37. $4\frac{5}{27}$.	38. 4.	39. 9.	40. $\frac{173}{882}$.	41. $18\frac{1}{18}$.
42. $5\frac{1}{3}$.	43. $\frac{1}{4}$.	44. $\frac{51}{76}$.	45. 1.	46. $\frac{53}{189}$.	47. 11.
48. $7\frac{41}{60}$.	49. 1.	50. 0.	51. $1\frac{1}{4}$.	52. $10\frac{11}{70}$.	53. $\frac{15}{16}$.
54. $\frac{2}{15}$.	55. $106\frac{1}{3}$.	56. $\frac{64}{85}$.	57. $\frac{5}{81}$.	58. $\frac{1}{47}$.	59. 10.
60. $\frac{23}{28}$.	61. $2\frac{13}{24}$.	62. $\frac{1}{24}$.	63. $\frac{1}{18000}$.	64. $\frac{9}{100}$.	65. $\frac{527}{1225}$.
66. $2\frac{2}{7}$.	67. $\frac{13}{23}$.	68. $1\frac{140}{549}$.	69. 1.	70. $6\frac{2}{3}$.	71. $2\frac{6}{25}$.
72. $1\frac{1}{5}$.	73. $8\frac{2}{3}$.	74. $1\frac{23}{105}$.	75. $\frac{1}{5}$.	76. 4.	77. 1.
78. $\frac{53}{122}$.	79. $\frac{273}{1315}$.	80. $\frac{95}{737}$.	81. $1\frac{5}{22}$.	82. 1.	83. $\frac{1}{4}$.
84. 3.	85. 1.	86. 10.	87. 2.	88. $\frac{1}{2}$.	89. $\frac{1}{8}$.
90. 1.	91. 76.	92. 75.	93. $\frac{1}{144}$.	94. 4.	95. $\frac{44}{105}$.
96. $\frac{11}{17}$.	97. $\frac{159}{3146}$.	98. 9.	99. 1.	100. $\frac{3}{187}$.	101. $\frac{3}{4}$.
102. $\frac{4}{5}$.	103. $\frac{13}{30}$.	104. $\frac{16}{37}$.	105. $\frac{16}{29}$.	106. $\frac{32}{33}$.	107. $\frac{5}{8}$.
108. $\frac{58}{69}$.	109. 2.	110. $\frac{57}{115}$.	111. $2\frac{13}{60}$.	112. $\frac{2}{5}$.	113. $\frac{37}{60}$.
114. $\frac{43}{60}$.	115. $1\frac{4}{15}$.	116. $1\frac{3}{10}$.	117. $\frac{43}{60}$.	118. $\frac{1}{15}$.	119. $\frac{5}{8}$.
120. $1\frac{1}{9}$.	121. $1\frac{29}{48}$.	122. $1\frac{31}{45}$.	123. $1\frac{1}{30}$.	124. $1\frac{2}{15}$.	125. $1\frac{37}{48}$.
126. $\frac{40}{93}$.	127. $\frac{32}{45}$.	128. $1\frac{1}{9}$.	129. $\frac{47}{48}$.	130. $\frac{10}{27}$.	131. 17.
132. 2.	133. $\frac{11}{221}$.	134. $6\frac{1}{4}$.	135. $1\frac{53}{121}$.	136. $1\frac{1}{2}$.	137. $15\frac{1}{2}$.
138. $1\frac{37}{72}$.	139. 1.	140. 5.	141. 2.	142. $1\frac{7}{87}$.	143. $\frac{1}{3}$.
144. $\frac{54}{175}$.	145. $4\frac{23}{74}$.	146. $2\frac{19}{30}$.	147. $1\frac{8}{195}$.	148. $\frac{348}{365}$.	149. $1\frac{445}{1612}$.
150. $1\frac{2}{3}$.	151. $\frac{288}{289}$.	152. $4\frac{17}{47}$.	153. $\frac{286}{633}$.	154. $96\frac{1}{4}$.	155. 10.
156. 0.	157. 0.	158. $5\frac{2}{9}$.	159. 2.	160. $\frac{1}{9}$.	161. 8.
162. $1\frac{5}{16}$.	163. $15\frac{1}{2}$.	164. $61\frac{10}{27}$.	165. $7\frac{25}{32}$.	166. $129\frac{37}{42}$.	167. $1\frac{8}{17}$.
168. 1.	169. $1\frac{11}{139}$.	170. 1.	171. 1.	172. $3\frac{81}{125}$.	173. $1\frac{289}{3600}$.
174. $2\frac{24}{317}$.	175. $2\frac{1}{6}$.	176. $4\frac{1}{6}$.	177. $\frac{5}{22}$.	178. $2\frac{7}{11}$.	179. 1.
180. $2\frac{5}{7}$.	181. $1\frac{40}{441}$.	182. 6.	183. 1.	184. $\frac{9}{25}$.	185. 1.
186. $\frac{15}{28}$.	187. $\frac{223}{360}$.	188. $1\frac{7}{18}$.	189. $11\frac{50}{111}$.	190. 1.	

XXII.

1. 6*d.*; 3*d.*; $1\frac{1}{2}$*d.*; $\frac{3}{4}$*d.* 2. 4*d.*; 2*d.*; 1*d.*; $\frac{1}{2}$*d.*; $\frac{1}{4}$*d.*
3. $4\frac{1}{2}$*d.*; $7\frac{1}{2}$*d.*; $10\frac{1}{2}$*d.*; $2\frac{1}{4}$*d.*; $3\frac{3}{4}$*d.* 4. 8*d.*; 10*d.*; $2\frac{1}{4}$*d.*; $5\frac{1}{2}$*d.*; $1\frac{1}{4}$*d.*
5. 10*s.*; 5*s.*; 2*s.* 6*d.*; 1*s.* 3*d.* 6. 6*s.* 8*d.*; 13*s.* 4*d.*; 3*s.* 4*d.*; 16*s.* 8*d.*; 1*s.* 8*d.*; 18*s.* 4*d.* 7. 15*s.*; 7*s.* 6*d.*; 12*s.* 6*d.*; 17*s.* 6*d.*; 3*s.* 9*d.*
8. 4*s.*; 8*s.*; 12*s.*; 16*s.*; 1*s.* 4*d.*; 9*s.* 6*d.* 9. 1*d.*; 1*s.* 5*d.*; 2*d.*; 4*d.*; 2*s.* 4*d.*; 3*d.*; 2*s.* 3*d.*; $\frac{1}{4}$*d.*; $4\frac{1}{4}$*d.*; $\frac{1}{2}$*d.* 10. 1*s.* 11*d.*; 3*s.* 1*d.*; 4*s.* 5*d.*; 1*s.* 10*d.*; 5*s.* 2*d.*; 1*s.* 9*d.*; $3\frac{1}{2}$*d.*; $3\frac{1}{4}$*d.*; $5\frac{3}{4}$*d.*; $9\frac{1}{4}$*d.* 11. £1, 18*s.* $1\frac{1}{4}$*d.*
12. £2, 7*s.* $4\frac{1}{5}$*d.* 13. 9*s.* $5\frac{2}{5}$*d.* 14. £1, 11*s.* $8\frac{5}{8}$*d.* 15. 3*s.* $7\frac{2}{7}$*d.*
16. 6*s.* $11\frac{4}{9}$*d.* 17. £1, 14*s.* $6\frac{1}{4}$*d.* 18. £2, 3*s.* $8\frac{13}{24}$*d.* 19. 8*s.* 4*d.*
20. 15*s.* 21. £2, 16*s.* 22. £2, 5*s.* 23. £4, 5*s.* $8\frac{4}{7}$*d.*
24. £5, 9*s.* $1\frac{1}{11}$*d.* 25. 8*s.* $3\frac{1}{9}$*d.* 26. 5*s.* $7\frac{1}{12}$*d.* 27. 17 hrs. 36 min.
28. 2 hrs. 21 min. 40 secs. 29. 3 ro. 30 sq. po. 30. 480 yds. (*or* 2 fur. 7 po. $1\frac{1}{2}$ yd.). 31. 3 yds. 2 ft. 8 in. 32. 13 cwts. 3 qrs.
33. 8 yds. 1 ft. $8\frac{4}{7}$ in. 34. 1 sq. yd. 8 sq. ft. $26\frac{2}{11}$ sq. in. 35. £10, 3*s.* $8\frac{55}{96}$*d.*
36. £34, 6*s.* $5\frac{11}{72}$*d.* 37. £4, 15*s.* $2\frac{6}{7}$*d.* 38. £11, 13*s.* $10\frac{9}{11}$*d.*
39. £3, 3*s.* $8\frac{3}{4}$*d.* 40. £1, 16*s.* 10*d.* 41. £2191, 3*s.* 8*d.*
42. £117, 6*s.* $5\frac{4}{5}$*d.* 43. 9 yds. 0 ft. $8\frac{1}{9}$ in. 44. 2 tons 5 cwts. 3 qrs. 21 lbs.
45. 1 ton 16 cwts. 0 qrs. $9\frac{4}{5}$ lbs. 46. 8 sq. po. $24\frac{3}{8}$ sq. yds. 47. $11\frac{1}{4}$*d.*
48. 8*s.* 9*d.* 49. 11*s.* $9\frac{3}{4}$*d.* 50. 3*s.* $9\frac{1}{3}$*d.* 51. 1*s.* $6\frac{27}{34}$*d.* 52. 2*s.* $0\frac{21}{76}$*d.*

53. £7, 10*s*. **54.** £2, 11*s*. **55.** 7 tons 8 cwts. 1 qr. $15\frac{11}{31}$ lbs.
56. 9 tons 19 cwts. 1 qr. $6\frac{22}{29}$ lbs. **57.** 1 ton 2 cwts. 2 qrs.
58. 9 hrs. 28 min. 45 secs. **59.** £11, 17*s*. $5\frac{1}{16}$*d*. **60.** £12, 10*s*. $10\frac{1}{2}$*d*.
61. £4042, 17*s*. $6\frac{3}{4}$*d*. **62.** £2025, 5*s*. $2\frac{2}{3}$*d*. **63.** £65, 6*s*. $1\frac{5}{7}$*d*.
64. £83, 19*s*. $2\frac{50}{63}$*d*. **65.** 16*s*. $10\frac{3}{5}$*d*. **66.** £1, 7*s*. 9*d*.
67. £1, 0*s*. $10\frac{113}{122}$*d*. **68.** £1, 6*s*. $8\frac{13}{71}$*d*. **69.** £17, 17*s*. $3\frac{1}{2}$*d*.
70. £9, 17*s*. $1\frac{7}{25}$*d*. **71.** 7*s*. **72.** £5. **73.** £1. **74.** £3.
75. £5. **76.** £7, 9*s*. $7\frac{1}{2}$*d*. **77.** 5 yds. 0 ft. 9 in. **78.** 3 ro. 10 sq. po.
79. $4\frac{1}{4}$*d*. **80.** 6*s*. $3\frac{1}{4}$*d*. **81.** 2*s*. $0\frac{1}{2}$*d*. **82.** 1*s*. $0\frac{3}{4}$*d*. **83.** £2, 16*s*. 11*d*.
84. 1*d*. **85.** 1 ton 3 cwts. 0 qrs. $21\frac{1}{3}$ lbs. **86.** 1 cwt. 0 qrs. 3 lbs.
87. £7, 7*s*. 7*d*. **88.** £14, 7*s*. **89.** 1 cwt. 0 qrs. 16 lbs.
90. 9 tons 6 cwts.

XXIII.

1. $\frac{1}{2}$; $\frac{1}{4}$; $\frac{1}{8}$; $\frac{1}{16}$. **2.** $\frac{3}{8}$; $\frac{5}{8}$; $\frac{7}{8}$. **3.** $\frac{1}{3}$; $\frac{1}{6}$; $\frac{1}{12}$; $\frac{1}{24}$; $\frac{1}{48}$.
4. $\frac{2}{3}$; $\frac{5}{6}$; $\frac{7}{12}$; $\frac{5}{24}$; $\frac{5}{48}$. **5.** $\frac{1}{2}$; $\frac{1}{4}$; $\frac{1}{8}$; $\frac{1}{16}$. **6.** $\frac{3}{4}$; $\frac{3}{8}$; $\frac{5}{8}$; $\frac{7}{8}$. **7.** $\frac{1}{3}$; $\frac{2}{3}$; $\frac{1}{6}$; $\frac{1}{12}$.
8. $\frac{1}{5}$; $\frac{2}{5}$; $\frac{3}{5}$; $\frac{4}{5}$; $\frac{1}{15}$. **9.** $\frac{1}{240}$; $\frac{1}{120}$; $\frac{1}{60}$; $\frac{1}{80}$; $\frac{1}{480}$; $\frac{1}{960}$. **10.** $\frac{7}{240}$; $\frac{11}{240}$; $\frac{7}{480}$; $\frac{17}{960}$; $\frac{31}{960}$. **11.** $\frac{5}{18}$. **12.** $\frac{1}{5}$. **13.** $\frac{5}{16}$. **14.** $\frac{3}{26}$. **15.** $\frac{1}{5}$.
16. $\frac{11}{37}$. **17.** $\frac{101}{276}$. **18.** $\frac{19}{141}$. **19.** $\frac{11}{32}$. **20.** $\frac{11}{14}$.
21. $\frac{35}{104}$. **22.** $\frac{20}{99}$. **23.** $\frac{47}{290}$. **24.** $\frac{125}{228}$. **25.** $\frac{77}{160}$.
26. $\frac{31}{320}$. **27.** $\frac{1}{10}$. **28.** $\frac{1}{100}$. **29.** $\frac{5}{51}$. **30.** $\frac{112}{285}$.
31. $\frac{9}{16}$. **32.** $\frac{39}{224}$. **33.** $\frac{9}{160}$. **34.** $\frac{237}{320}$. **35.** $\frac{7}{352}$.
36. $\frac{109}{160}$. **37.** $\frac{21}{320}$. **38.** $\frac{1}{36}$. **39.** $\frac{139}{200}$. **40.** $\frac{7}{8}$.
41. $48\frac{3}{4}$*s*. **42.** $111\frac{5}{8}$*s*. **43.** $63\frac{5}{16}$*s*. **44.** $57\frac{3}{16}$*s*. **45.** £$14\frac{13}{15}$.
46. £$21\frac{67}{160}$. **47.** £$7\frac{7}{16}$. **48.** £$7\frac{3}{32}$. **49.** $31\frac{3}{4}$ qrs. **50.** $33\frac{3}{4}$ cwts.
51. $192\frac{1}{7}$ cwts. **52.** $13\frac{15}{56}$ qrs. **53.** $17\frac{11}{12}$ yds. **54.** $127\frac{1}{2}$ po.
55. $12\frac{7}{20}$ ac. **56.** $7\frac{9}{10}$ ac. **57.** 46 times. **58.** 62 times.
59. $\frac{1}{6}$. **60.** $\frac{1}{12}$. **61.** $\frac{4}{9}$. **62.** $\frac{1}{39}$. **63.** $\frac{6}{5}$.
64. $\frac{6}{11}$. **65.** $\frac{9}{28}$. **66.** $\frac{1}{8}$. **67.** $\frac{1}{6}$. **68.** $\frac{1}{924}$.
69. $\frac{1}{880}$. **70.** $\frac{1}{160}$. **71.** $\frac{3}{7040}$. **72.** $\frac{5}{192}$. **73.** $\frac{1}{11}$.
74. $\frac{24}{25}$. **75.** $\frac{7}{11}$. **76.** $\frac{30}{49}$. **77.** $\frac{3}{4}$. **78.** $\frac{4}{33}$.
79. $\frac{13}{125}$. **80.** $\frac{73}{252}$.

XXIV.

1. Nineteen *ninety-eighths*. **2.** $\frac{91}{149}$. **3.** $\frac{126}{17}$. **4.** $4\frac{9}{17}$.
5. $\frac{147}{777}$. **6.** $\frac{12}{19}$. **7.** $\frac{64}{616}$, $\frac{49}{616}$. **8.** $\frac{137}{140}$. **9.** $8\frac{2}{315}$. **10.** 2*s*.
11. Ninety-one *ninetieths*. **12.** $700\frac{19}{91}$. **13.** $\frac{1890}{19}$. **14.** $52\frac{11}{19}$.
15. $\frac{63}{153}$. **16.** $\frac{2}{9}$. **17.** $\frac{17}{38}$. **18.** $17\frac{133}{162}$. **19.** $46\frac{17}{19}$. **20.** 5*s*. 3*d*.
21. 101 *tenths*. **22.** 34 *fifty-oneths*. **23.** $\frac{25}{83}$. **24.** $7\frac{161}{250}$. **25.** $3\frac{11}{185}$.
26. $\frac{3}{20}$. **27.** $1\frac{1}{1460}$. **28.** 0. **29.** $\frac{11}{210}$. **30.** $\frac{1}{3}$ of it. **31.** 49 *units*.
32. 4 *ninths*. **33.** $\frac{9}{10}$. **34.** $1\frac{11}{45}$. **35.** $20\frac{501}{1001}$. **36.** $8\frac{7}{16}$. **37.** $14\frac{2}{3}$.
38. $1\frac{3}{4}$. **39.** $3\frac{4}{57}$. **40.** 30 acres. **41.** $\frac{442}{33}$. **42.** $\frac{19}{83}$. **43.** $\frac{19}{81}$.
44. $10\frac{23}{102}$. **45.** $3\frac{431}{448}$. **46.** $1\frac{7}{9}$. **47.** $\frac{10}{11}$. **48.** $\frac{5}{19}$. **49.** 2.
50. $\frac{1}{60}$ of the field. **51.** $19\frac{99}{101}$. **52.** $\frac{123}{231}$. **53.** $\frac{19}{82}$. **54.** $14\frac{97}{195}$.
55. $3\frac{268}{285}$. **56.** 6. **57.** $1\frac{61}{65}$. **58.** $\frac{9}{10}$. **59.** 30 bits. **60.** 3*s*. $1\frac{1}{2}$*d*.
61. 500 *seventieths*. **62.** $\frac{12}{89}$. **63.** $8\frac{13}{44}$. **64.** $13708\frac{23}{24}$. **65.** 5.
66. 1. **67.** $1\frac{1}{27}$. **68.** 33. **69.** The latter. **70.** 3 hrs. 54 m.
71. 820 *ninetieths*. **72.** $\frac{94}{129}$. **73.** $\frac{1}{15}$. **74.** $\frac{1}{180}$. **75.** 0.
76. $6\frac{2}{9}$. **77.** $163\frac{26}{45}$. **78.** 240. **79.** £3, 11*s*. 3*d*. **80.** 9 hrs. 24 m.

81. $\frac{999}{1147}$. **82.** $\frac{6}{7}$ is greatest; $\frac{5}{6}$ is least. **83.** $9\frac{4}{7}$. **84.** 81884.
85. 42. **86.** 0. **87.** 1. **88.** £2, 10*s.* 9*d.* **89.** $\frac{1}{512}$.
90. $\frac{6}{7}$ of the book. **91.** $\frac{259}{777}$. **92.** $\frac{11 \div 9}{12 \div 10}$. **93.** $\frac{23}{120}$. **94.** 7.
95. $82\frac{85}{126}$. **96.** $\frac{8}{27}$. **97.** £1, 11*s.* $4\frac{7}{8}$*d.* **98.** 2*s.* **99.** $\frac{7}{192}$.
100. $\frac{1}{100}$ of the pole. **101.** 19. **102.** $\frac{1}{5}$. **103.** 36. **104.** $\frac{2}{17}$.
105. $\frac{1}{4}$. **106.** 2. **107.** $1\frac{281}{1000}$. **108.** £7, 6*s.* $0\frac{3}{4}$*d.* **109.** $\frac{1}{308}$.
110. A, £2, 12*s.* 4*d.*; B, £1, 19*s.* 3*d.* **111.** $\frac{1404}{1712}$. **112.** $13\frac{1}{3}$. **113.** $58\frac{155}{396}$.
114. 0. **115.** 7. **116.** $7\frac{8}{15}$. **117.** 15. **118.** £5, 11*s.* $8\frac{1}{2}$*d.* **119.** $\frac{1}{27}$.
120. A, £11, 19*s.* $7\frac{1}{2}$*d.*; B, £7, 19*s.* 9*d.* **121.** £4, 3*s.* $1\frac{1}{2}$*d.* **122.** The latter.
123. $1\frac{1}{20}$. **124.** 15 times. **125.** 16 times; rem^{r} $\frac{1}{35}$. **126.** $2\frac{15}{49}$.
127. $2\frac{13}{88}$. **128.** $\frac{63}{515}$*d.* **129.** 8 marbles. **130.** $\frac{7}{10}$ of the work.
131. £5381, 11*s.* 3*d.* **132.** B. **133.** $34\frac{79}{312}$. **134.** $5\frac{91}{102}$. **135.** $\frac{3}{4}$.
136. $158\frac{351}{2450}$. **137.** $\frac{1}{9}$. **138.** £40, 19*s.* **139.** $\frac{9}{20}$ of the property.
140. $\frac{6}{35}$ of the field. **141.** £281, 0*s.* $11\frac{3}{7}$*d.* **142.** G.C.M. $\frac{1}{24}$; L.C.M. $26\frac{1}{4}$.
143. $\frac{7}{37}$. **144.** $\frac{157}{1728}$. **145.** $\frac{72}{175}$. **146.** 21 dys. 3 hrs. 30 m.
147. 3 dwts. $8\frac{19}{40}$ grs. **148.** 2 ft. 6 in. **149.** $\frac{4}{57}$ of his journey.
150. £6, 14*s.* $4\frac{1}{2}$*d.* for one; 2*s.* 1*d.* for each of the others. **151.** £4.
152. G.C.M. $\frac{12}{35}$; L.C.M. 24. **153.** $\frac{1}{12}$. **154.** $\frac{26}{1155}$. **155.** 3.
156. $\frac{9}{806}$. **157.** 17 cwts. 3 qrs. 10 lbs. **158.** 67584 stamps. **159.** $\frac{1}{10}$.
160. 1 hr. 12 min. **161.** $\frac{1}{1\frac{12}{23}}$. **162.** 233 tons 10 cwts. 2 qrs. $10\frac{2}{3}$ lbs.
163. 77 times. **164.** $\frac{7+4}{11+4}$ is greatest; $\frac{7-4}{11-4}$ is least. **165.** $286999\frac{72}{359}$.
166. $1\frac{217}{221}$. **167.** $\frac{42}{43}$. **168.** $\frac{11}{15}$. **169.** 4 inches. **170.** $\frac{69}{160}$.
171. $\frac{47619}{333333}$. **172.** 144 tons 3 cwts. 2 qrs. $4\frac{26}{55}$ lbs. **173.** 52560 times.
174. $\frac{31}{91}$, $\frac{41}{119}$, $\frac{51}{143}$. **175.** $16699\frac{107}{274}$. **176.** 5. **177.** $1\frac{30}{187}$. **178.** $1\frac{114113}{1000000}$.
179. 955 feet. **180.** $\frac{91}{240}$ of the day. **181.** 1219. **182.** 7245.
183. $\frac{10}{31}$, $\frac{9}{28}$, $\frac{8}{25}$, $\frac{7}{22}$. **184.** $1\frac{3}{17}$. **185.** 1510. **186.** 1126 lengths, 4 inches over. **187.** 1. **188.** $\frac{11}{18}$. **189.** $\frac{17}{367}$. **190.** $1\frac{51}{145}$.
191. 3213. **192.** 11088. **193.** $\frac{2}{17}$, $\frac{3}{25}$, $\frac{4}{33}$, $\frac{5}{41}$. **194.** $2\frac{113}{204}$. **195.** $12\frac{47}{90}$.
196. 9 times, $1\frac{3}{40}$ rem. **197.** £49. **198.** $\frac{7}{8}$. **199.** 999000 grains.
200. $\frac{7}{8}$.

XXV.

1. 6*d.*; 4*d.*; 3*d.*; 2*d.*; $1\frac{1}{2}$*d.*; 1*d.*; $\frac{3}{4}$*d.* **2.** 10*s.*; 6*s.* 8*d.*; 5*s.*; 4*s.*; 3*s.* 4*d.*; 2*s.* 6*d.*; 2*s.*; 1*s.* 8*d.*; 1*s.* 4*d.*; 1*s.* 3*d.*; 1*s.*; 8*d.*; 6*d.*; 4*d.*; 3*d.*
3. $\frac{1}{4}$; $\frac{1}{5}$; $\frac{1}{10}$; $\frac{1}{3}$; $\frac{1}{6}$; $\frac{1}{12}$; $\frac{1}{8}$; $\frac{1}{16}$. **4.** $\frac{1}{2}$; $\frac{1}{4}$; $\frac{1}{8}$; $\frac{1}{3}$; $\frac{1}{6}$; $\frac{1}{12}$; $\frac{1}{5}$; $\frac{1}{10}$; $\frac{1}{20}$.
5. $\frac{1}{2}$; $\frac{1}{4}$; $\frac{1}{8}$; $\frac{1}{5}$; $\frac{1}{10}$; $\frac{1}{20}$; $\frac{1}{12}$; $\frac{1}{15}$; $\frac{1}{30}$; $\frac{1}{60}$. **6.** $\frac{1}{2}$; $\frac{1}{4}$; $\frac{1}{8}$; $\frac{1}{5}$; $\frac{1}{10}$; $\frac{1}{20}$; $\frac{1}{3}$; $\frac{1}{6}$; $\frac{1}{12}$. **7.** $\frac{1}{2}$; $\frac{1}{4}$; $\frac{1}{3}$; $\frac{1}{6}$; $\frac{1}{12}$; $\frac{1}{5}$; $\frac{1}{10}$; $\frac{1}{20}$. **8.** $\frac{1}{2}$; $\frac{1}{4}$; $\frac{1}{8}$; $\frac{1}{16}$; $\frac{1}{3}$; $\frac{1}{6}$; $\frac{1}{12}$. **9.** $\frac{1}{2}$; $\frac{1}{4}$; $\frac{1}{8}$; $\frac{1}{16}$; $\frac{1}{5}$; $\frac{1}{10}$; $\frac{1}{20}$; $\frac{1}{40}$; $\frac{1}{80}$. **10.** $\frac{1}{2}$; $\frac{1}{4}$; $\frac{1}{8}$; $\frac{1}{16}$; $\frac{1}{5}$; $\frac{1}{10}$; $\frac{1}{20}$; $\frac{1}{40}$. **11.** £115, 10*s.* **12.** £96, 15*s.*
13. £148, 8*s.* **14.** £127. **15.** £142, 6*s.* **16.** £197, 16*s.*
17. £68, 2*s.* 6*d.* **18.** £84, 2*s.* 6*d.* **19.** £218, 17*s.* 6*d.*
20. £237, 17*s.* 6*d.* **21.** £143, 6*s.* 8*d.* **22.** £177, 6*s.* 8*d.*
23. £513, 13*s.* 4*d.* **24.** £417, 13*s.* 4*d.* **25.** £14, 16*s.* 8*d.*
26. £16, 3*s.* 4*d.* **27.** £32, 1*s.* 8*d.* **28.** £64, 1*s.* 8*d.*
29. £42, 12*s.* 6*d.* **30.** £59, 5*s.* 6*d.* **31.** £2, 18*s.* 6*d.*
32. £2, 4*s.* 9*d.* **33.** £14, 10*s.* 4*d.* **34.** £7, 17*s.* 8*d.*
35. £10, 11*s.* 2*d.* **36.** £21, 9*s.* 8*d.* **37.** £8, 19*s.*
38. £15, 6*s.* $7\frac{1}{2}$*d.* **39.** £6, 2*s.* 3*d.* **40.** £6, 7*s.* $10\frac{1}{2}$*d.*

41. £430, 2s. 42. £557, 3s. 43. £296, 2s.
44. £528, 6s. 45. £55, 16s. 46. £64, 4s.
47. £303, 7s. 6d. 48. £451, 17s. 6d. 49. £2666, 5s.
50. £3569, 7s. 6d. 51. £16, 6s. 3d. 52. £35, 16s. 3d.
53. £245, 18s. 8d. 54. £206, 1s. 3d. 55. £34, 8s. 9d.
56. £43, 10s. 5d. 57. £6, 9s. 4½d. 58. £10, 3s. 7½d.
59. £31, 4s. 4½d. 60. £34, 14s. 4½d. 61. £2, 14s. 5¾d.
62. £3, 11s. 3d. 63. £1, 4s. 11¼d. 64. £2, 10s. 0¾d.
65. £65, 16s. 9½d. 66. £116, 17s. 3½d. 67. £3, 14s. 0¾d.
68. £8, 11s. 6¾d. 69. £22, 15s. 7½d. 70. £31, 0s. 5¼d.
71. £933. 72. £1128, 15s. 73. £879, 12s.
74. £298, 2s. 75. £475, 17s. 6d. 76. £615, 7s. 6d.
77. £125, 6s. 8d. 78. £96, 16s. 8d. 79. £164, 13s. 4d.
80. £222, 1s. 8d. 81. £152, 15s. 82. £149, 12s.
83. £799, 10s. 84. £1902, 7s. 6d. 85. £2442, 8s.
86. £1865, 10s. 87. £3805, 13s. 4d. 88. £9791, 13s. 4d.
89. £551, 9s. 6d. 90. £540, 11s. 3d. 91. £382, 2s. 8d.
92. £270, 15s. 8d. 93. £44, 2s. 94. £45, 10s. 1½d.
95. £2213, 8s. 3d. 96. £3238, 13s. 7d. 97. £45.
98. £76, 16s. 99. £53, 8s. 100. £198, 9s. 4½d. 101. £102, 15s.
102. £161, 5s. 103. £62, 8s. 104. £73, 16s. 105. £386, 2s.
106. £597, 12s. 6d. 107. £449, 15s. 108. £1402. 109. £1430, 12s. 6d.
110. £692, 10s. 111. £63, 15s. 112. £245, 2s. 113. £119, 3s. 4d.
114. £139, 18s. 4d. 115. £517, 10s. 116. £887, 5s. 117. £284, 4s.
118. £435. 119. £53, 15s. 6d. 120. £68, 18s. 121. £22, 6s. 8d.
122. £21, 6s. 1½d. 123. £7, 4s. 10d. 124. £8, 6s. 9d.
125. £9, 3s. 9d. 126. £21, 10s. 10d. 127. £61, 4s. 4½d.
128. £75, 16s. 10½d. 129. £86, 19s. 130. £77, 7s.
131. £42, 12s. 132. £37, 11s. 4d. 133. £29, 18s. 6d.
134. £38, 1s. 3d. 135. £609, 5s. 9d. 136. £957, 4s. 1½d.
137. £362, 14s. 138. £336, 7s. 139. £1340, 12s. 6d.
140. £2025, 7s. 6d. 141. £2479, 19s. 142. £3729, 13s. 9d.
143. £273, 14s. 1d. 144. £302, 18s. 7d. 145. £348, 3s.
146. £792, 11s. 6d. 147. £818, 0s. 8d. 148. £838, 15s.
149. £8746, 6s. 3d. 150. £7623, 8s. 4d. 151. £24581, 2s. 10d.
152. £38162, 11s. 153. £1657, 18s. 1½d. 154. £1766, 7s. 6d.
155. £19410, 4s. 4d. 156. £19295, 7s. 6d. 157. £90, 17s. 6¼d.
158. £46, 1s. 9¾d. 159. £23, 13s. 7½d. 160. £22, 18s. 1d.
161. £13, 11s. 11½d. 162. £13, 8s. 4d. 163. £10, 19s. 9¼d.
164. £12, 4s. 3¼d. 165. £13, 0s. 9d. 166. £24, 18s.
167. £23, 0s. 7½d. 168. £19, 15s. 3d. 169. £46, 19s. 10½d.
170. £85, 18s. 9½d. 171. £54, 18s. 10d. 172. £48, 9s. 4½d.
173. £312, 6s. 7½d. 174. £342, 3s. 0¼d. 175. £41227, 5s. 3¾d.
176. £48103, 11s. 9¼d. 177. £8469, 10s. 178. £10057, 19s.
179. £36039, 6s. 9½d. 180. £39467, 11s. 0¾d. 181. £10416, 14s. 2d.
182. £5211, 0s. 8¼d. 183. £903, 9s. 184. £941, 14s.
185. £801, 13s. 4d. 186. £783, 5s. 1d. 187. £564, 15s. 1½d.
188. £458, 13s. 9d. 189. £2901, 19s. 11¾d. 190. £8885, 8s. 1½d.
191. £1017, 6s. 8¼d. 192. £1289, 9s. 193. £1350, 1s. 0¾d.

194. £1226, 9s. $7\frac{1}{2}$d	**195.** £4424, 3s.	**196.** £4108, 13s.
197. £855, 11s.	**198.** £962, 7s. 8d.	**199.** £1586, 16s. 3d.
200. £1615, 9s. 8d.	**201.** £2186, 4s. $5\frac{3}{4}$d.	**202.** £2968, 9s. $0\frac{3}{4}$d.
203. £663, 8s. 1d.	**204.** £570.	**205.** £1999, 12s. 2d.
206. £2816, 9s. $0\frac{3}{4}$d.	**207.** £79, 14s. $8\frac{1}{4}$d.	**208.** £69, 8s. $3\frac{1}{2}$d.
209. £512, 15s. 6d.	**210.** £869, 18s. 9d.	**211.** £347, 11s. 9d.
212. £373, 1s. $0\frac{1}{2}$d.	**213.** £155, 0s. 11d.	**214.** £293, 7s. $6\frac{3}{4}$d.
215. £5588, 5s. $5\frac{1}{4}$d.	**216.** £4240, 9s. $3\frac{1}{4}$d.	**217.** £44, 14s. $5\frac{3}{4}$d.
218. £65, 6s. $6\frac{3}{4}$d.	**219.** £323, 18s. 4d.	**220.** £297, 14s. 1d.
221. £49, 4s. $10\frac{3}{4}$d.	**222.** £188, 17s. $4\frac{1}{4}$d.	**223.** £413, 18s. $10\frac{1}{2}$d.
224. £1105, 6s. $8\frac{3}{4}$d.	**225.** £1142, 12s. 3d.	**226.** £1455, 17s. $10\frac{1}{2}$d.
227. £378, 10s. $0\frac{3}{4}$d.	**228.** £502, 1s. $8\frac{3}{4}$d.	**229.** £8, 11s. $2\frac{1}{4}$d.
230. £8, 10s. 11d.	**231.** £25, 11s. $8\frac{3}{4}$d.	**232.** £42, 6s. $4\frac{1}{2}$d.
233. £32, 1s. $9\frac{3}{4}$d.	**234.** £49, 19s. $1\frac{1}{4}$d.	**235.** £362, 5s. $0\frac{3}{4}$d.
236. £363, 16s. $2\frac{1}{2}$d.	**237.** £140, 0s. $8\frac{1}{4}$d.	**238.** £96, 1s. $4\frac{1}{4}$d.
239. £6330, 12s. $2\frac{1}{4}$d.	**240.** £7193, 6s.	**241.** £17, 9s. $7\frac{5}{16}$d.
242. £102, 1s. $0\frac{7}{8}$d.	**243.** £146, 13s. $8\frac{5}{8}$d.	**244.** £312, 4s. $7\frac{5}{16}$d.
245. £104, 19s. $11\frac{1}{8}$d.	**246.** £337, 13s. $6\frac{39}{40}$d.	**247.** £128, 16s. $0\frac{7}{8}$d.
248. £656, 15s. $10\frac{13}{20}$d.	**249.** £876, 19s. $2\frac{5}{8}$d.	**250.** £784, 18s. $8\frac{5}{8}$d.
251. £87, 13s. $2\frac{2}{5}$d.	**252.** £54, 12s. $3\frac{1}{13}$d.	**253.** £1311, 19s. 7d.
254. £3735, 15s.	**255.** £631, 10s.	**256.** £153, 16s. $9\frac{1}{2}$d.
257. £584, 18s. 8d.	**258.** £2695, 14s. $0\frac{1}{2}$d.	**259.** £197, 9s. 2d.
260. £624, 17s. $9\frac{1}{2}$d.	**261.** £353, 17s. $3\frac{3}{5}$d.	**262.** £410, 17s. $11\frac{5}{8}$d.
263. £11, 12s. 9d.	**264.** £12, 7s. $2\frac{1}{2}$d.	**265.** £383, 5s.
266. £389, 17s. $10\frac{1}{4}$d.	**267.** £526, 1s. $5\frac{5}{8}$d.	**268.** £4327, 7s. $5\frac{15}{32}$d.
269. £9, 3s. $1\frac{1}{8}$d.	**270.** £38, 5s. $11\frac{1}{4}$d.	**271.** £5353, 1s. 6d.

272. £2732, 11s. 7d. **273.** 5 tons 11 cwts. 0 qrs. 1 lb. 6 ozs. 11 drs.
274. 37 mi. 1516 yds. 1 ft. 6 in. *or* 37 mi. 6 fur. 35 po. 4 yds. **275.** £397.
276. £464, 18s. $3\frac{1}{4}$d. **277.** £408, 16s. $1\frac{1}{8}$d. **278.** £2671, 11s. $2\frac{4}{5}$d.
279. 83 tons 10 cwts. 0 qrs. 4 lbs. **280.** 561 tons 18 cwts. 0 qrs. 24 lbs.

XXVI.

1. 10 cwts.; 5 cwts.; 4 cwts.; 2 cwts.; 2 qrs.; 1 qr.; 16 lbs.; 14 lbs. **2.** 14 lbs.; 7 lbs.; 4 lbs.; 8 ozs.; 4 ozs.; 2 ozs.; 1 oz. **3.** 2 ro.; 1 ro.; 20 sq. po.; 10 sq. po.; 20 sq. po.; 10 sq. po.; 8 sq. po.; 5 sq. po. **4.** 1 ft. 6 in.; 1 ft.; 9 in.; 6 in.; 4 in.; 3 in.; 6 in.; 4 in.; 3 in.; 2 in.; 1 in. **5.** $\frac{1}{2}$; $\frac{1}{4}$; $\frac{1}{5}$; $\frac{1}{10}$. $\frac{1}{2}$; $\frac{1}{4}$; $\frac{1}{8}$; $\frac{1}{7}$. **6.** $\frac{1}{2}$; $\frac{1}{10}$. $\frac{1}{4}$; $\frac{1}{8}$; $\frac{1}{7}$; $\frac{1}{14}$. **7.** $\frac{1}{2}$; $\frac{1}{4}$; $\frac{1}{8}$; $\frac{1}{5}$; $\frac{1}{10}$. $\frac{1}{2}$; $\frac{1}{4}$; $\frac{1}{5}$; $\frac{1}{10}$. **8.** $\frac{1}{2}$; $\frac{1}{4}$; $\frac{1}{5}$; $\frac{1}{10}$. $\frac{1}{2}$; $\frac{1}{3}$; $\frac{1}{4}$; $\frac{1}{6}$; $\frac{1}{8}$; $\frac{1}{12}$. **9.** $\frac{1}{2}$; $\frac{1}{3}$; $\frac{1}{4}$; $\frac{1}{6}$; $\frac{1}{9}$; $\frac{1}{12}$. $\frac{1}{2}$; $\frac{1}{3}$; $\frac{1}{6}$; $\frac{1}{12}$. **10.** $\frac{1}{2}$; $\frac{1}{4}$; $\frac{1}{5}$; $\frac{1}{8}$. $\frac{1}{8}$; $\frac{1}{10}$; $\frac{1}{2}$; $\frac{1}{11}$. **11.** £6, 15s. $11\frac{1}{4}$d.

12. £5, 4s. $6\frac{1}{4}$d.	**13.** £161, 7s. 3d.	**14.** £173, 12s. $10\frac{1}{2}$d.
15. £40, 4s. 2d.	**16.** £46, 8s. $1\frac{3}{4}$d.	**17.** £4, 17s. $7\frac{1}{2}$d.
18. £4, 4s. $8\frac{3}{4}$d.	**19.** £24, 10s. $5\frac{1}{2}$d.	**20.** £24, 15s. 10d.
21. £564, 13s. 9d.	**22.** £468, 15s.	**23.** £1350, 14s. $0\frac{3}{4}$d.
24. £1183, 18s. 3d.	**25.** £14, 11s. 10d.	**26.** £19, 10s.
27. £31, 18s. 7d.	**28.** £51, 18s. $6\frac{3}{4}$d.	**29.** £33, 9s.
30. £95, 17s. 1d.	**31.** £3, 14s. $5\frac{3}{4}$d.	**32.** £4, 6s. $4\frac{3}{4}$d.
33. £63, 5s. $2\frac{1}{4}$d.	**34.** £60, 6s. 5d.	**35.** £7, 6s. 3d.
36. £6, 9s. $11\frac{3}{4}$d.	**37.** £12, 2s. $1\frac{1}{2}$d.	**38.** £10, 11s. 6d.

39. £7, 18s. $0\frac{1}{4}$d.
40. £8, 10s. $11\frac{1}{2}$d.
41. £104, 3s. 1d.
42. £71, 18s. $10\frac{1}{4}$d.
43. £122, 6s. $4\frac{1}{4}$d.
44. £87, 16s. $1\frac{1}{4}$d.
45. 14s. $9\frac{3}{4}$d.
46. 3s. $2\frac{3}{4}$d.
47. £18, 3s.
48. £14, 16s. $3\frac{3}{4}$d.
49. £160, 6s. $5\frac{1}{2}$d.
50. £250, 11s. $3\frac{1}{4}$d.
51. £11, 11s. $6\frac{3}{4}$d.
52. £12, 8s. $2\frac{1}{4}$d.
53. £185, 12s. $10\frac{3}{4}$d.
54. £340, 12s. $9\frac{3}{4}$d.
55. £204, 14s. $11\frac{5}{8}$d.
56. £38, 2s. $5\frac{11}{14}$d.
57. £76, 12s. $11\frac{7}{16}$d.
58. £12, 4s. $2\frac{41}{56}$d.
59. £3, 0s. $11\frac{1}{12}$d.
60. £7, 19s. $8\frac{2}{3}$d.
61. £54, 15s. $1\frac{2}{3}$d.
62. £5, 19s. $0\frac{37}{72}$d.
63. £519, 18s. $6\frac{27}{32}$d.
64. £2493, 4s. $3\frac{27}{64}$d.
65. £11, 6s. $3\frac{5}{8}$d.
66. £59, 11s. $10\frac{2}{7}$d.
67. £50, 12s. $6\frac{7}{16}$d.
68. £45, 2s. $10\frac{103}{224}$d.
69. £4, 2s. $1\frac{5}{7}$d.
70. 14s. $8\frac{11}{14}$d.
71. £179, 16s. $7\frac{2}{7}$d.
72. £155, 12s. $3\frac{6}{7}$d.
73. £134, 5s. $1\frac{4}{5}$d.
74. £128, 6s. $5\frac{13}{16}$d.
75. £24, 11s. $6\frac{3}{4}$d.
76. £20, 14s. $4\frac{10}{11}$d.
77. £145, 2s. $3\frac{3}{11}$d.
78. £451, 3s. $2\frac{47}{110}$d.
79. £4, 14s. $9\frac{23}{54}$d.
80. £3, 11s. $9\frac{13}{24}$d.
81. £1704, 3s. $6\frac{3}{4}$d.
82. £2033, 15s. $3\frac{3}{4}$d.
83. £139, 17s. $3\frac{3}{4}$d.
84. £250, 15s. $10\frac{7}{8}$d.
85. £257, 6s. $11\frac{11}{32}$d.
86. £587, 7s. $6\frac{3}{8}$d.
87. £15527, 1s. 8d.
88. £19126, 16s. 6d.
89. £7117, 18s. $8\frac{3}{4}$d.
90. £2979, 0s. $7\frac{1}{2}$d.
91. £896, 12s. $6\frac{1}{2}$d.
92. £2085, 9s. $10\frac{1}{2}$d.
93. £353, 17s. $9\frac{23}{48}$d.
94. £44, 14s. $3\frac{3}{80}$d.
95. £146, 10s. 10d.
96. £6672, 5s. $1\frac{1}{2}$d.
97. £734, 7s. $8\frac{13}{16}$d.
98. £3262, 15s. $9\frac{4}{5}$d.
99. £2507, 11s. $3\frac{21}{80}$d.
100. £23498, 3s. $1\frac{1}{2}$d.

XXVII.

1. 12s. 1d.
2. 12s. $3\frac{1}{4}$d.
3. 10s. $11\frac{1}{2}$d.
4. 14s. 3d.
5. 13s. 10d.
6. 13s. $8\frac{1}{4}$d.
7. 10s. 11d.
8. 18s. $10\frac{1}{2}$d.
9. 19s. $5\frac{3}{4}$d.
10. 14s. $9\frac{1}{2}$d.
11. 15s. $0\frac{1}{2}$d.
12. 9s. 2d.
13. 8s. $3\frac{3}{4}$d.
14. 9s. $11\frac{1}{4}$d.
15. 8s. $8\frac{1}{2}$d.
16. 10s. $9\frac{1}{4}$d.
17. £6, 4s. 6d.
18. £7, 7s.
19. 13s. $6\frac{1}{2}$d.
20. £1, 14s. $4\frac{1}{2}$d.
21. £1, 9s. $9\frac{3}{4}$d.
22. £1, 3s.
23. £1, 16s. $1\frac{3}{4}$d.
24. £2, 2s. 1d.
25. £2, 7s. 10d.
26. £1, 11s. $1\frac{3}{4}$d.
27. £9, 8s. $10\frac{1}{4}$d.
28. £3, 2s. $9\frac{1}{2}$d.
29. £2, 12s. 7d.
30. £2, 6s. $3\frac{1}{4}$d.
31. $8\frac{3}{4}$d.
32. $4\frac{3}{4}$d.
33. $7\frac{3}{4}$d.
34. 3d.
35. 3s. $0\frac{1}{4}$d.
36. 2s. $0\frac{1}{2}$d.
37. 1s. $11\frac{1}{4}$d.
38. 4s. $4\frac{1}{2}$d.
39. 11s. 4d.
40. 2s. $1\frac{1}{2}$d.
41. 3s. $8\frac{1}{4}$d.
42. £1, 15s. $5\frac{1}{2}$d.
43. 14s. $3\frac{1}{2}$d.
44. 12s. 5d.
45. £1, 0s. $9\frac{3}{4}$d.
46. 18s. $9\frac{1}{2}$d.

47.

	£	s.	d.
		18	0
		2	$2\frac{1}{2}$
		9	$9\frac{1}{2}$
	1	10	0
Discount,			9
	£1	9	3 *Ans.*

48.

	£	s.	d.
		19	$10\frac{1}{2}$
		18	$9\frac{3}{4}$
	3	19	$7\frac{1}{4}$
	5	18	$3\frac{1}{2}$
Discount,		2	$10\frac{1}{2}$
	£5	15	5 *Ans.*

49.

	£	s.	d.
		11	5
	3	13	0
	2	12	1
	6	16	6
Discount,	1	0	4
	£5	16	2 *Ans.*

50.

	£	s.	d.
	3	9	9
	2	6	0
	2	18	2
	8	13	11
Discount,	1	13	6
	£7	0	5 *Ans.*

XXVIII. 1. Seven *tenths*; Seven *hundredths*; Seven *thousandths*; Seven *ten-thousandths*; Seven *hundred-thousandths*; Seven *millionths*; Seven *ten-millionths*.

2. Eight *hundredths*; Eight *ten-thousandths*; Eight *tenths*; Eight *hundred-thousandths*; Eight *thousandths*; Eight *ten-millionths*; Eight *millionths*.

3. Twenty-three *hundredths*; Forty-five *hundredths*; Four-hundred-and-eighty-six *thousandths*; Two-hundred-and-seventy-one *thousandths*; Eighty-three *thousandths*; Eight-hundred-and-three *thousandths*; Forty-one *ten-thousandths*; Six-hundred-and-thirty-two *ten-thousandths*.

4. Two, and three *tenths*; Six-hundred-and-one *thousandths*; Five, and three *hundredths*; Twenty-four, and five *hundredths*; Seventeen, and seventeen *hundredths*; Eighty-nine *thousandths*; Sixty-six, and sixty-six *hundredths*; Six-thousand-six-hundred-and-sixty-six *ten-thousandths*.

5. ·3; ·7; ·08; ·004. 6. ·1; ·06; ·09; ·005. 7. ·002; ·0007; ·000006. 8. ·0008; ·000004; ·009. 9. 2·3; 12·07. 10. 70·007; 10·0000001. 11. ·21; ·17; 1·1. 12. ·12; ·99; 3·3. 13. ·202; ·000014. 14. 1·09; ·0087. 15. 2·3. 16. 5·7. 17. 45. 18. 72. 19. ·8. 20. ·06. 21. 7·01. 22. 92·1. 23. 53·4. 24. 724·5. 25. 123·4. 26. 500·3. 27. 421. 28. 503. 29. 1210. 30. 205·7. 31. 2315. 32. 41030. 33. 5200. 34. 8. 35. ·23. 36. ·51. 37. ·05. 38. ·07. 39. ·063. 40. 6·1. 41. 2·9. 42. 6·03. 43. ·802. 44. ·0001. 45. 1·531. 46. 4·076. 47. ·437. 48. ·802. 49. ·023. 50. ·2361. 51. ·4009. 52. ·0213. 53. ·0805. 54. ·0012. 55. 472·8, 30·45, ·847, 2306·7, ·00703. 56. 4728, 304·5, 8·47, 23067, ·0703. 57. 47280, 3045, 84·7, 230670, ·703. 58. 234·07, 7030·057, 40·01, ·003, ·4367. 59. 23·407, 703·0057, 4·001. ·0003, ·04367. 60. ·23407, 7·030057, ·04001, ·000003, ·0004367.

XXIX. 1. 20·92. 2. 5·385. 3. 17·81. 4. 20·8. 5. 39·638. 6. 68·684. 7. 934·4794. 8. 467·1889. 9. 207·5191. 10. 3889·9611. 11. 4·53. 12. 3·072. 13. 31·8. 14. 42·7. 15. 10·5969. 16. 3·512. 17. 1·545. 18. ·632. 19. 15·3525. 20. 22·3022. 21. 4475·105045. 22. 23·4397464. 23. ·45. 24. ·063. 25. ·0044. 26. ·001. 27. 0. 28. ·01. 29. ·3897. 30. 39·483. 31. ·04202. 32. 59·0522. 33. ·88428. 34. ·41925. 35. 1·0129. 36. 3·5044. 37. 9·99. 38. 169·983. 39. 45·45. 40. 77·922. 41. 17·7082. 42. 2·61807. 43. 2·2111. 44. 313·95804. 45. 243·97578. 46. 362·60703. 47. 3·996. 48. 11·27263. 49. 2·662. 50. 1·111.

XXX. 1. 16·8. 2. 5·64. 3. 24·31. 4. ·805. 5. 569·24. 6. 2832·09. 7. 3·48. 8. ·371. 9. 959·1.

10. 48360. **11.** 305·3. **12.** 32·4. **13.** 3·65. **14.** 1·68.
15. ·0657. **16.** ·02781. **17.** ·08815. **18.** 1·1055. **19.** ·1281.
20. 1·8139. **21.** 4·03. **22.** 17·907. **23.** ·12879.
24. ·0009108. **25.** ·03003. **26.** ·337264. **27.** ·0690606.
28. 36·94581. **29.** 45·3. **30.** 509·427. **31.** ·000274855.
32. ·000185745. **33.** ·000105. **34.** ·000048. **35.** 2·023.
36. 2·9791. **37.** ·00000072. **38.** ·00016807. **39.** ·194481.
40. 27·9841.

XXXI.

1. 12·41. **2.** 23·014. **3.** ·5509. **4.** ·00221.
5. 138·5. **6.** ·0121. **7.** ·2126. **8.** 57·825. **9.** 5·2014.
10. 4·275. **11.** ·0425. **12.** ·1125. **13.** 23·218. **14.** ·1539.
15. ·0513. **16.** ·0868. **17.** ·88125. **18.** ·00548. **19.** ·115625.
20. ·0002265625. **21.** ·021875. **22.** ·004375. **23.** ·0118.
24. ·129875. **25.** ·72. **26.** ·033. **27.** ·0084. **28.** ·00016.
29. 9·6. **30.** ·175. **31.** ·004. **32.** ·03008. **33.** ·012.
34. 10·3. **35.** ·002272. **36.** ·1326171875. **37.** 5·976.
38. ·013. **39.** ·00305. **40.** ·001702.

XXXII.

1. 116·94. **2.** ·8393. **3.** 8·815. **4.** 53·934.
5. ·0121. **6.** 28·8. **7.** 193·19. **8.** 3636·4. **9.** 13648.
10. 5860. **11.** 10500. **12.** 6·40125. **13.** 2·5. **14.** 20.
15. 3750. **16.** 17500000. **17.** 15·07. **18.** 1·728. **19.** ·9268.
20. 242·156. **21.** 13·846875. **22.** 17·840625. **23.** ·0046875.
24. 152·546875. **25.** 8·75. **26.** 125. **27.** 2440. **28.** 1988.
29. 234·375. **30.** 21·875. **31.** ·022. **32.** 1·3. **33.** ·072.
34. ·0009. **35.** 210·3. **36.** 83·3. **37.** ·603. **38.** 6·073.
39. ·32. **40.** ·525. **41.** ·275. **42.** 2·25. **43.** ·002215.
44. 9·236. **45.** ·40105. **46.** 13·756. **47.** 605000. **48.** 7500.
49. 500. **50.** 640000. **51.** 2000. **52.** 22500. **53.** 600600.
54. 21432. **55.** ·1857... **56.** ·0037... **57.** ·0530... **58.** ·5952...
59. ·1263... **60.** ·0092... **61.** 11·3207... **62.** 344·8275...
63. ·0049... **64.** 6·59; 6590. **65.** 501000; ·00501. **66.** 3·25; 32500.
67. 1·2; 12; 12000; ·012. **68.** 1·3; ·13; 130000; 13; ·00013.
69. Quotient 32; remainder ·153. **70.** 518 times, and ·14 over.

XXXIII.

1. $\frac{3}{10}$, $\frac{7}{10}$, $\frac{9}{10}$, $\frac{1}{100}$. **2.** $\frac{1}{10}$, $\frac{7}{100}$, $\frac{1}{100}$, $\frac{9}{1000}$.
3. $\frac{3}{100}$, $\frac{1}{10000}$, $\frac{9}{100000}$. **4.** $\frac{7}{1000}$, $\frac{3}{100000}$, $\frac{7}{10000000}$. **5.** $\frac{13}{100}$, $\frac{23}{100}$, $\frac{17}{1000}$.
6. $\frac{59}{100}$, $\frac{509}{1000}$, $\frac{5009}{10000}$. **7.** $\frac{71}{100}$, $\frac{29}{1000}$, $\frac{413}{1000}$. **8.** $\frac{17}{1000}$, $\frac{113}{1000}$, $\frac{1003}{10000}$.
9. $\frac{871}{1000}$, $\frac{701}{10000}$, $\frac{4359}{10000}$. **10.** $\frac{6789}{10000}$, $\frac{813}{100000}$, $\frac{1793}{1000000}$. **11.** $3\frac{1}{10}$, $21\frac{7}{10}$, $1\frac{9}{1000}$.
12. $5\frac{3}{10}$, $50\frac{3}{100}$, $5\frac{3}{10000}$. **13.** $22\frac{7}{1000}$, $220\frac{7}{100}$, $2020\frac{7}{10}$.
14. $170\frac{3}{10}$, $17\frac{3}{100}$, $1\frac{7}{100000}$. **15.** $1\frac{23}{100}$, $12\frac{57}{100}$, $103\frac{31}{1000}$.
16. $8\frac{47}{100}$, $19\frac{351}{1000}$, $104\frac{401}{1000}$. **17.** $11\frac{11}{100}$, $1\frac{111}{1000}$, $11\frac{1011}{100000}$.
18. $70\frac{77}{100}$, $77\frac{7}{100}$, $7\frac{7007}{100000}$. **19.** $50\frac{571}{1000}$, $17\frac{3001}{10000}$, $1\frac{7593}{10000}$.
20. $41\frac{209}{10000}$, $8\frac{30459}{100000}$, $1\frac{405039}{1000000}$. **21.** $\frac{1}{5}$. **22.** $\frac{1}{2}$. **23.** $\frac{3}{5}$.
24. $\frac{4}{5}$. **25.** $\frac{1}{50}$. **26.** $\frac{1}{20}$. **27.** $\frac{2}{25}$. **28.** $\frac{1}{250}$. **29.** $\frac{3}{20}$.
30. $\frac{1}{4}$. **31.** $\frac{2}{125}$. **32.** $\frac{7}{250}$. **33.** $1\frac{7}{20}$. **34.** $11\frac{9}{20}$. **35.** $2\frac{1}{8}$.

36. $10\frac{9}{200}$. **37.** $6\frac{13}{40}$. **38.** $71\frac{5}{8}$. **39.** $\frac{3}{625}$. **40.** $\frac{3}{80}$.
41. $1\frac{29}{40}$. **42.** $3\frac{7}{80}$. **43.** $17\frac{117}{500}$. **44.** $50\frac{9}{125}$. **45.** $1\frac{39}{40}$.
46. $1\frac{21}{80}$. **47.** $\frac{3}{4000}$. **48.** $\frac{1}{15625}$. **49.** $\frac{5}{32}$. **50.** $1\frac{9}{16}$.
51. ·1, ·7, ·01. **52.** ·3, ·07, ·09. **53.** ·08, ·002, ·0007.
54. ·009, ·0005, ·000001. **55.** ·17, ·027, ·127. **56.** ·43, ·0174, ·02715.
57. 1·1, 20·3, 5·16. **58.** 2·03, 96·01. **59.** 57·1, ·571, ·00571.
60. 9·9, ·099, ·000099. **61.** 4·03, 7·5, 21·023.
62. 100·01, 7·009, 12·0014. **63.** 1·0001, 231·9, 4·073.
64. 51·03, 8·0008, 1000·019. **65.** 38·9, ·0047, 80·03.
66. ·023, 5·0715, 735·61. **67.** 43·256, 10·007, ·00153.
68. ·1753, 1·001, 1457·03. **69.** 10·1, 10·001, ·000101.
70. 7·0001, ·0071, 70·0001. **71.** ·2. **72.** ·8. **73.** ·5. **74.** ·25.
75. ·75. **76.** ·14. **77.** ·26. **78.** ·12. **79.** 1·45. **80.** ·85.
81. ·56. **82.** ·125. **83.** ·625. **84.** ·4375. **85.** ·824. **86.** 1·056.
87. 2·075. **88.** 7·462. **89.** 10·1875. **90.** 2·1375. **91.** ·05375.
92. ·764. **93.** ·0016. **94.** ·00048. **95.** 1·3224. **96.** ·09375.
97. ·084375. **98.** ·015625. **99.** ·5859375. **100.** ·17376.

XXXIV.

1. $1{\cdot}7\dot{3}$. **2.** $2{\cdot}2\dot{8}\dot{1}$. **3.** ${\cdot}379\dot{1}$. **4.** $5{\cdot}451\dot{6}$.
5. ${\cdot}041\dot{6}$. **6.** ${\cdot}0278\dot{3}$. **7.** ${\cdot}023\dot{2}$. **8.** ${\cdot}00358\dot{3}$. **9.** ${\cdot}0\dot{4}2857\dot{1}$.
10. ${\cdot}03\dot{5}7142\dot{8}$. **11.** ${\cdot}\dot{5}$. **12.** $93{\cdot}\dot{3}$. **13.** $36{\cdot}\dot{3}\dot{6}$. **14.** $2583{\cdot}\dot{3}$.
15. $1571{\cdot}\dot{4}2857\dot{1}$. **16.** $5714{\cdot}\dot{2}8571\dot{4}$. **17.** ${\cdot}02\dot{7}$. **18.** ${\cdot}188\dot{6}\dot{3}$.
19. ${\cdot}00958\dot{3}$. **20.** ${\cdot}480\dot{5}$. **21.** ${\cdot}03\dot{5}7142\dot{8}$. **22.** ${\cdot}00\dot{5}0617283\dot{9}$.
23. ${\cdot}005\dot{6}\dot{3}$. **24.** ${\cdot}00486\dot{1}$. **25.** $7{\cdot}\dot{4}0\dot{7}$. **26.** $60{\cdot}\dot{6}\dot{0}$. **27.** ${\cdot}013\dot{8}$.
28. ${\cdot}00\dot{9}2\dot{5}$. **29.** $14090{\cdot}\dot{9}\dot{0}$. **30.** $20{\cdot}8\dot{3}$. **31.** $1785{\cdot}\dot{7}1428\dot{5}$.
32. $9{\cdot}\dot{8}7654320\dot{9}$. **33.** ${\cdot}\dot{0}69\dot{3}$. **34.** ${\cdot}1\dot{4}8\dot{6}$. **35.** ${\cdot}0\dot{0}243\dot{9}$.
36. ${\cdot}\dot{0}7692\dot{3}$. **37.** ${\cdot}0\dot{4}0\dot{5}$. **38.** ${\cdot}00\dot{6}097\dot{5}$. **39.** ${\cdot}0\dot{5}44\dot{5}$. **40.** ${\cdot}2\dot{2}8205\dot{1}$.
41. ${\cdot}0016\dot{4}$. **42.** ${\cdot}0\dot{1}0\dot{8}$. **43.** $2{\cdot}\dot{4}\dot{2}$. **44.** $47{\cdot}6\dot{3}5\dot{1}$. **45.** ${\cdot}3652\dot{0}2\dot{7}$.
46. $1{\cdot}37\dot{1}951\dot{2}$. **47.** ${\cdot}\dot{1}3286\dot{7}$. **48.** ${\cdot}017\dot{2}0779\dot{2}$. **49.** ${\cdot}\dot{3}225806451612 9\dot{0}$.
50. ${\cdot}0\dot{2}35294117647058\dot{8}$.

XXXV.

1. ${\cdot}\dot{3}$. **2.** ${\cdot}\dot{4}$. **3.** ${\cdot}\dot{2}\dot{7}$. **4.** ${\cdot}\dot{6}\dot{3}$. **5.** ${\cdot}\dot{1}4285\dot{7}$.
6. ${\cdot}\dot{7}1428\dot{5}$. **7.** ${\cdot}0\dot{7}$. **8.** ${\cdot}07\dot{6}$. **9.** ${\cdot}4\dot{2}\dot{7}$. **10.** ${\cdot}51\dot{1}\dot{8}$.
11. ${\cdot}0\dot{4}2857\dot{1}$. **12.** ${\cdot}04\dot{4}2857\dot{1}$. **13.** ${\cdot}8\dot{3}$. **14.** ${\cdot}91\dot{6}$. **15.** ${\cdot}1\dot{3}$.
16. ${\cdot}791\dot{6}$. **17.** ${\cdot}1\dot{7}1428\dot{5}$. **18.** ${\cdot}17\dot{8}5714\dot{2}$. **19.** $1{\cdot}0208\dot{3}$.
20. $3{\cdot}00\dot{3}\dot{6}$. **21.** $4{\cdot}018\dot{6}$. **22.** $11{\cdot}007291\dot{6}$. **23.** ${\cdot}\dot{7}8\dot{3}$.
24. ${\cdot}\dot{1}951\dot{2}$. **25.** ${\cdot}\dot{1}5384\dot{6}$. **26.** ${\cdot}\dot{0}1234567\dot{9}$. **27.** $1{\cdot}0\dot{1}219\dot{5}$.
28. $3{\cdot}0\dot{6}7\dot{5}$. **29.** ${\cdot}\dot{0}1587\dot{3}$. **30.** ${\cdot}2\dot{9}2307\dot{6}$. **31.** $4{\cdot}00\dot{2}47\dot{5}$.
32. $4{\cdot}00\dot{2}2\dot{5}$. **33.** ${\cdot}036458\dot{3}$. **34.** ${\cdot}02556\dot{8}\dot{1}$. **35.** ${\cdot}0266\dot{1}\dot{8}$.
36. ${\cdot}03912\dot{8}$. **37.** $5{\cdot}19\dot{8}7179\dot{4}$. **38.** $2{\cdot}\dot{7}9894\dot{1}$.
39. ${\cdot}\dot{3}52941176470588\dot{2}$. **40.** ${\cdot}\dot{0}5263157894736842\dot{1}$. **41.** $\frac{3}{8}, \frac{7}{20}, \frac{1}{40}, \frac{19}{50}$.
42. $\frac{3}{16}, \frac{3}{25}, \frac{9}{32}, \frac{7}{64}$. **43.** $\frac{1}{80}, \frac{1}{100}, \frac{1}{125}$. **44.** $\frac{1}{160}, \frac{1}{200}, \frac{1}{250}$.
45. $\frac{7}{320}, \frac{13}{400}, \frac{17}{500}$. **46.** $\frac{1}{625}, \frac{1}{640}$. **47.** Two. **48.** Three.
49. Seven. **50.** Eight.

XXXVI.

1. $\frac{2}{9}$. **2.** $\frac{5}{9}$. **3.** $\frac{2}{3}$. **4.** $\frac{8}{9}$.
5. $4\frac{7}{9}$. **6.** $11\frac{1}{9}$. **7.** 1. **8.** 8. **9.** $\frac{8}{33}$. **10.** $\frac{5}{11}$.

11. $1\frac{8}{11}$. 12. $4\frac{32}{33}$. 13. $\frac{26}{111}$. 14. $\frac{10}{37}$. 15. $10\frac{35}{111}$. 16. $6\frac{31}{37}$.
17. $\frac{21}{101}$. 18. $\frac{1355}{3333}$. 19. $\frac{44}{369}$. 20. $\frac{7}{123}$. 21. $\frac{1}{7}$. 22. $\frac{3}{7}$.
23. $1\frac{4}{7}$. 24. $3\frac{6}{7}$. 25. $\frac{75}{91}$. 26. $\frac{69}{91}$. 27. $\frac{10}{21}$. 28. $\frac{12}{13}$.
29. $\frac{7}{81}$. 30. $\frac{5}{451}$. 31. $\frac{5}{6}$. 32. $\frac{11}{30}$. 33. $2\frac{26}{45}$. 34. $1\frac{1}{90}$.
35. $\frac{116}{495}$. 36. $\frac{23}{66}$. 37. $\frac{1}{66}$. 38. $\frac{35}{36}$. 39. $\frac{3}{275}$. 40. $\frac{1}{44}$.
41. $1\frac{29}{60}$. 42. $2\frac{17}{24}$. 43. $\frac{53}{222}$. 44. $\frac{71}{185}$. 45. $1\frac{17}{72}$. 46. $3\frac{1}{375}$.
47. $\frac{301}{2775}$. 48. $\frac{51}{148}$. 49. $\frac{168}{205}$. 50. $\frac{104}{205}$. 51. $\frac{1}{9375}$. 52. $\frac{11}{96}$.
53. $\frac{32083}{495000}$. 54. $\frac{297}{2960}$. 55. $\frac{13}{14}$. 56. $\frac{23}{35}$. 57. $\frac{6}{175}$.
58. $\frac{113}{875}$. 59. $\frac{47}{56}$. 60. $\frac{3}{52}$.

XXXVII.

1. $\cdot\dot{6}18\dot{3}$. 2. $\cdot\dot{9}1500\dot{5}$. 3. $\cdot\dot{2}1893\dot{3}$. 4. $\cdot\dot{5}6128\dot{8}$.
5. $\cdot9\dot{8}\dot{4}$. 6. $\cdot72\dot{4}\dot{7}$. 7. $2\cdot4\dot{2}\dot{1}$. 8. $6\cdot15\dot{6}\dot{3}$. 9. $\cdot51\dot{3}$.
10. $1\cdot2\dot{5}\dot{7}$. 11. $\cdot29\dot{2}\dot{3}$. 12. $\cdot74\dot{8}\dot{1}$. 13. $4\cdot00\dot{2}\dot{5}$. 14. $\cdot143\dot{8}\dot{2}$.
15. $\cdot12\dot{4}8\dot{5}$. 16. $2\cdot3\dot{2}5\dot{6}$. 17. $1\cdot0000\dot{2}47\dot{1}$. 18. $\cdot5189\dot{7}6\dot{5}$.
19. $16\cdot\dot{8}5714\dot{2}$. 20. $4\cdot\dot{9}2307\dot{6}$. 21. $3\cdot73\dot{4}5017\dot{7}$. 22. $\cdot203337\dot{0}6\dot{9}$.
23. $\cdot\dot{1}49788770\dot{6}$. 24. $\cdot\dot{4}2621108957\dot{7}$. 25. $\cdot\dot{9}$, *i.e.* 1. 26. $1\cdot\dot{9}$, *i.e.* 2.
27. $1\cdot1\dot{9}$, *i.e.* $1\cdot2$. 28. $1\cdot4\dot{9}$, *i.e.* $1\cdot5$. 29. $\cdot8\dot{9}$, *i.e.* $\cdot9$. 30. 0.
31. $228\cdot037\dot{4}71\dot{3}$. 32. $778\cdot1\dot{6}4391\dot{6}$. 33. $12\cdot00\dot{3}99435812\dot{5}$.
34. $\cdot\dot{9}$, *i.e.* 1. 35. $\cdot4\dot{9}$, *i.e.* $\cdot5$. 36. $6\cdot346\dot{2}8571\dot{4}$. 37. $2\cdot247\dot{3}82\dot{4}$.
38. $2\cdot3431\dot{7}6994546301\dot{4}$. 39. 1000. 40. $1\cdot0\dot{9}$, *i.e.* $1\cdot1$.

XXXVIII.

1. $1\cdot2\dot{8}$. 2. $10\cdot\dot{3}\dot{6}$. 3. $36\cdot\dot{2}\dot{1}$. 4. $5\cdot\dot{4}7\dot{7}$.
5. $32\cdot6\dot{8}\dot{1}$. 6. $7\cdot2\dot{4}\dot{8}$. 7. $1\cdot62\dot{1}8\dot{7}$. 8. $\cdot0057\dot{0}\dot{5}$. 9. $\cdot0071\dot{3}\dot{6}$.
10. $\cdot07\dot{6}9\dot{5}$. 11. $1\cdot8\dot{3}\dot{6}$. 12. $\cdot023258\dot{8}\dot{3}$. 13. $\cdot02940\dot{0}6\dot{5}$. 14. $\cdot\dot{0}3\dot{6}$.
15. $\cdot\dot{3}550\dot{1}$. 16. $1173\cdot2\dot{6}$. 17. $73\cdot\dot{6}5079\dot{3}$. 18. $138\cdot\dot{0}9523\dot{8}$.
19. $17135\cdot41\dot{6}$. 20. $23\cdot\dot{1}6498\dot{3}$. 21. $\cdot\dot{3}4\dot{3}$. 22. $\cdot01\dot{0}\dot{9}$. 23. $\cdot25$.
24. $7\cdot\dot{1}\dot{2}$. 25. $\cdot6\dot{6}\dot{3}$. 26. $\cdot37\dot{5}9\dot{2}$. 27. $\cdot01500\dot{1}\dot{8}$. 28. $\cdot2780\dot{0}6\dot{5}$.
29. $\cdot5$. 30. $\cdot309375$. 31. $19\cdot\dot{0}\dot{9}$. 32. $\cdot\dot{6}$. 33. $\cdot1\dot{6}2\dot{9}$.
34. $\cdot33\dot{8}5\dot{8}$. 35. $2\cdot646$. 36. 25. 37. $33\cdot33$. 38. $\cdot6249375$.
39. $3\cdot\dot{6}\dot{3}$. 40. $\cdot3\dot{6}4\dot{8}$. 41. $\cdot69$. 42. $16\cdot56$. 43. $11\cdot08\dot{3}$.
44. $6\cdot\dot{6}$. 45. $3\cdot99$. 46. $3\cdot3$. 47. $5\cdot2\dot{4}2857\dot{1}$. 48. $11\cdot68\dot{4}2857\dot{1}$.
49. $\cdot2\dot{4}$. 50. 4. 51. $3\cdot\dot{6}\dot{3}$. 52. $\cdot01\dot{6}$. 53. $\cdot9\dot{8}$. 54. $\cdot\dot{6}$.
55. 1. 56. $2\cdot88$. 57. 1. 58. 2. 59. $\cdot5$. 60. $\cdot01$.

XXXIX.

1. $4\frac{1}{2}$*d.* 2. 4*s.* $7\frac{1}{2}$*d.* 3. 17*s.* 4. £2, 3*s.* 6*d.*
5. 2*s.* 9*d.* 6. 18*s.* 9*d.* 7. $2\frac{1}{4}$*d.* 8. $11\frac{1}{4}$*d.* 9. 7*s.* $4\frac{1}{2}$*d.*
10. 5*s.* $3\frac{3}{4}$*d.* 11. 3*s.* $1\frac{1}{2}$*d.* 12. 2*s.* $7\frac{1}{2}$*d.* 13. £3, 7*s.* 6*d.*
14. £15, 3*s.* 3*d.* 15. 11*s.* $11\frac{1}{4}$*d.* 16. 15*s.* $6\frac{3}{4}$*d.* 17. £4, 15*s.* $9\frac{3}{4}$*d.*
18. 2*s.* $1\frac{1}{2}$*d.* 19. 2*s.* $3\frac{1}{2}$*d.* 20. 3*s.* $6\frac{1}{2}$*d.* 21. 2*s.* $9\frac{3}{4}$*d.* 22. 16*s.* $5\frac{3}{4}$*d.*
23. £1, 2*s.* $3\frac{3}{4}$*d.* 24. £3, 12*s.* $7\frac{1}{2}$*d.* 25. £1, 9*s.* $0\frac{3}{4}$*d.* 26. 15*s.* 6*d.*
27. 3*s.* $10\frac{3}{4}$*d.* 28. 4*d.* 29. 3 qrs. 7 lbs. 30. 21 lbs.
31. 1 cwt. 2 qrs. 8 lbs. 12 ozs. 32. 13 cwts. 1 qr. 12 lbs. 4 ozs.
33. 2 mi. 2 fur. 30 po. 34. 24 cub. ft. 810 in. 35. 1 ton 15 cwts. 3 qrs.
36. 1 ton 3 cwts. 2 qrs. 1 lb. 37. $15\frac{3}{4}$ poles, or 15 po. 4 yds. $4\frac{1}{2}$ in.
38. 2 ac. 2 ro. 39. £5, 15*s.* 6*d.* 40. £1, 0*s.* $8\frac{3}{4}$*d.*
41. 3 tons 7 cwts. 3 qrs. 22 lbs. 42. 5 yds. 1 ft. $7\frac{1}{2}$ in. 43. 10 poles.

44. 1 lb. 45. 2*s.* 1*d.* 46. $3\frac{1}{2}d.$ 47. 16*s.* $7\frac{1}{2}d.$ 48. 1*s.* $7\frac{1}{4}d.$
49. 1 ft. 10 in. 50. 3 in. 51. 110 yds. 0 ft. 8 in. 52. 2 mi. 622 yds. 8 in.
53. 6*d.* 54. 7*s.* 6*d.* 55. $4\frac{1}{4}d.$ 56. $3\frac{3}{4}d.$ 57. 13*s.* 58. 6*d.*
59. 1 lb. 60. 1 ton 13 cwt. 14 lbs.

XL.

1. ·4. 2. ·6. 3. ·15. 4. ·35. 5. ·25.
6. ·75. 7. ·125. 8. ·225. 9. ·1125. 10. ·0875. 11. ·75.
12. ·125. 13. ·625. 14. ·4375. 15. ·0625. 16. ·9375. 17. ·155.
18. ·175. 19. ·021875. 20. ·14625. 21. ·03125. 22. ·034375.
23. ·115625. 24. ·23125. 25. ·01171875. 26. ·001953125.
27. ·0125. 28. ·00875. 29. ·109375. 30. ·06525. 31. ·015625.
32. ·078125. 33. ·0588125. 34. ·38390625. 35. 5·16875.
36. 14·54375. 37. ·00063. 38. ·000096. 39. ·1390625.
40. ·03671875. 41. ·005625. 42. ·00204. 43. 642·4995 pence.
44. 108·864275 pence. 45. 1·19 ozs. 46. 724·57875 dwts.
47. 20·52 pints. 48. 17214·912 sq. yds. 49. 892·8 secs.
50. 162·504 mins. 51. $\cdot\dot{6}$. 52. $\cdot 41\dot{6}$. 53. $\cdot 08\dot{3}$. 54. $\cdot 1458\dot{3}$.
55. $\cdot 8958\dot{3}$. 56. $\cdot 958\dot{3}$. 57. $\cdot\dot{6}$. 58. $\cdot 8\dot{3}$. 59. $\cdot 358\dot{3}$.
60. $\cdot 2541\dot{6}$. 61. $\cdot 3\dot{7}\dot{5}$. 62. $\cdot 215\dot{9}\dot{0}$. 63. $\cdot 35069\dot{4}$. 64. $\cdot 1430\dot{5}$.
65. $\cdot 43\dot{2}926\dot{8}$. 66. $\cdot 300\dot{6}7\dot{5}$. 67. $\cdot 803\dot{5}7142\dot{8}$. 68. $\cdot 8\dot{3}0769\dot{2}$.
69. $\cdot 1306\dot{8}\dot{1}$. 70. $\cdot 06\dot{8}\dot{1}$. 71. $\cdot 0\dot{0}\dot{5}$. 72. $\cdot 0011\dot{3}\dot{6}$. 73. $\cdot 0\dot{0}\dot{3}$.
74. $\cdot 7\dot{3}$. 75. $\cdot 07102\dot{2}\dot{7}$. 76. $\cdot 23\dot{4}2857\dot{1}$. 77. $\cdot 82886\dot{9}0476\dot{1}$.
78. $\cdot 00948660\dot{7}1428\dot{5}$. 79. ·056465. 80. $\cdot 4097\dot{2}$.

XLI.

1. Three *hundredths.* 2. 3·003. 3. Seven *millionths.*
4. ·0123. 5. 345·26579. 6. 6·21004. 7. 6·9557137. 8. ·0462.
9. 8·0881. 10. 24000. 11. Neither; they are equal. 12. ·000037.
13. $\frac{47}{100000}$. 14. 619·62482. 15. 12·499897. 16. 364·406442.
17. 4·240671875. 18. 470·34. 19. 2·3681. 20. ·8075.
21. 470·31. 22. 2 in., 1 *tenth* in., and 6 *thousandths* in. 23. ·703.
24. $\frac{7}{625}$. 25. 9·293295. 26. ·00047089. 27. 49·0734375.
28. 2665. 29. ·012. 30. 249*d.* 31. ·02935. 32. $3\frac{671}{1000000}$.
33. 68·08792. 34. 33750. 35. ·0314574. 36. 506·9896925.
37. 5·5546875. 38. 1200. 39. 1245*d.* 40. ·0625. 41. 10·0121.
42. ·4783. 43. ·00065. 44. ·000012167. 45. ·0705. 46. 3·152.
47. ·00064. 48. 6 fur. 12 po. 49. 7 times. 50. ·61125.
51. 100·593. 52. 57830. 53. 133·6336. 54. ·04775. 55. 4670.
56. $\frac{23}{32}$. 57. ·00096875. 58. 8 ozs. 15 dwts. 5 grs. 59. ·065.
60. ·00275. 61. ·35. 62. ·04123. 63. ·1875. 64. 304·3069436.
65. ·02025. 66. 1. 67. 562·1. 68. 111. 69. 3600 times.
70. £1, 3*s.* 6*d.* 71. Nothing; they are equal. 72. ·0721. 73. ·6512.
74. 4·79. 75. 1·45. 76. 5616·1. 77. 2070000. 78. 31·79.
79. 7000. 80. ·0484375. 81. 1920·919. 82. ·0065. 83. ·24992.
84. ·02919189. 85. ·0575. 86. 0. 87. 1 ton 3 cwts. 2 qrs. 1 lb.
88. 308·7 inches. 89. £4, 3*s.* 4*d.* 90. A, £6, 1*s.* 3*d.*; B, 12*s.* $1\frac{1}{2}d.$; C, £3, 8*s.* $8\frac{1}{2}d.$ 91. $7\frac{14}{25}$. 92. ... 93. 8·91. 94. 199·991.

95. ·000125. **96.** 94 mi. 7 fur. 36 po. **97.** 670·68 mins. **98.** ·0625.
99. 18*s.* 8*d.* **100.** £106, 8*s.* **101.** $\cdot\dot{4}$. **102.** $\cdot 8\dot{5}\dot{4}$. **103.** $\cdot\dot{2}5\dot{9}$.
104. $\frac{27}{37}$. **105.** $7\cdot 25\dot{1}\dot{0}$. **106.** $2\cdot 552\dot{0}8\dot{4}$. **107.** 2. **108.** $1\cdot\dot{1}0\dot{8}$.
109. 3*d.* **110.** $\cdot\dot{1}4285\dot{7}$. **111.** $\cdot\dot{1}\dot{7}$. **112.** $\cdot 2\dot{2}195\dot{1}$. **113.** $\cdot 291\dot{6}$.
114. $\frac{73}{75}$. **115.** $3\cdot 45\dot{6}\dot{0}$. **116.** $\cdot 000\dot{5}3\dot{5}$. **117.** $\cdot\dot{3}4\dot{5}$. **118.** ·495.
119. 110 sq. yds. **120.** $\cdot 3\dot{5}7142\dot{8}$. **121.** 6299·37. **122.** 3753·2; ·0427.
123. $4\frac{13}{55}$. **124.** $\cdot 01458\dot{3}$. **125.** $\cdot 35763\dot{8}$. **126.** 4·2. **127.** 5·6.
128. $11\frac{1}{5}$*d.* **129.** 21421·62 mins. **130.** 1785 pieces, and ·001 of an inch over. **131.** $\cdot\dot{0}8\dot{3}$. **132.** $\frac{2}{125}$; $\frac{1}{60}$. **133.** ·04375; $22\cdot\dot{8}5714\dot{2}$.
134. ·000002355. **135.** 2·88. **136.** 6199. **137.** $199\cdot\dot{8}1\dot{4}$.
138. £16, 0*s.* $7\frac{1}{4}$*d.* **139.** 132·09 perches. **140.** 23 bits, and ·12 of an inch over. **141.** $67\cdot\dot{0}6\dot{7}$. **142.** $\cdot 1\dot{7}$ is the greater.
143. $\frac{2321}{2331}$. **144.** 4·8. **145.** $\cdot\dot{2}4462626442\dot{8}$. **146.** $\cdot 1864\dot{2}\dot{0}$.
147. $\cdot 005208\dot{3}$ of an inch. **148.** £123, 19*s.* 6*d.* **149.** $1\frac{1}{2}$*d.* **150.** $\cdot 16\dot{8}\dot{1}$.
151. $\cdot 03\dot{7}$ is greatest; $\cdot\dot{0}3\dot{7}$ is least. **152.** ·0030507. **153.** 90.
154. $\frac{19}{5400}$. **155.** $\cdot\dot{0}0573\dot{3}$. **156.** 13. **157.** 3906250 times.
158. 2559·318 mins. **159.** 3*s.* $2\frac{1}{2}$*d.* **160.** 497 acres. **161.** $\frac{233}{67500}$.
162. $6477\cdot\dot{7}$; $14\cdot 28571\dot{4}$. **163.** $3\cdot 3\dot{7}\dot{8}$. **164.** 600. **165.** 80; 79·92. **166.** $\cdot\dot{6}\dot{3}$. **167.** 4 cwts. 3 qrs. 14 lbs. 14 oz.
168. $\frac{1}{4}$*d.* **169.** 2381·0625 ozs. **170.** A. **171.** $\cdot\dot{3}5\dot{1}$. **172.** $\cdot\dot{7}1428\dot{5}$.
173. 760; 18·5. **174.** ·00167. **175.** $\frac{5}{7}$. **176.** ·6. **177.** ·00001; $\cdot\dot{0}1774\dot{5}$. **178.** ... **179.** $\cdot 1\dot{4}$. **180.** A. **181.** Four places.
182. $3\cdot 72\dot{6}\dot{9}$. **183.** $\frac{219}{11000}$. **184.** $153\cdot 0\dot{0}\dot{2}$. **185.** ... **186.** $\frac{75}{91}$ or $\cdot\dot{8}2417\dot{5}$. **187.** 19*s.* $7\frac{1}{4}$*d.* **188.** ·14375. **189.** 1171428 times, and $6\cdot\dot{8}5714\dot{2}$ pence over. **190.** 10·30 A.M. **191.** Six places.
192. $1\frac{2}{17}$. **193.** $\cdot 0\dot{6}$. **194.** ·0588235294... **195.** ·000625.
196. ·0009027 and 9·027. **197.** $11336\cdot 69204\dot{7}2900\dot{1}$. **198.** 3·45.
199. ·0625. **200.** Tuesday.

XLII.

1. 15*s.* **2.** £1, 8*s.* **3.** £20. **4.** £27. **5.** £2, 15*s.*
6. £4, 14*s.* 6*d.* **7.** 3*s.* 4*d.* **8.** 5*s.* 3*d.* **9.** £3, 15*s.* **10.** £5, 12*s.*
11. 2*s.* 8*d.* **12.** 12*s.* **13.** 10 lbs. **14.** 21 lbs. **15.** 21 tons.
16. 27 tons. **17.** 12 lbs. **18.** 8 lbs. **19.** 20 yds. **20.** 6 yds.
21. $5\frac{1}{3}$ miles. **22.** $13\frac{1}{3}$ miles. **23.** 45 seconds. **24.** $17\frac{1}{2}$ seconds.
25. 10 hours. **26.** $8\frac{2}{3}$ hours. **27.** 39 miles. **28.** 92 miles.
29. 35 weeks. **30.** 15 weeks. **31.** 4 days. **32.** 1 day. **33.** 5 days.
34. 18 days. **35.** 2 days. **36.** 2 days. **37.** 9 days. **38.** 56 days.
39. 4 days. **40.** 18 days. **41.** 10 men. **42.** 1107 men.
43. 21 men. **44.** 108 men. **45.** 14 days. **46.** 12 days.
47. 9 horses. **48.** 25 horses. **49.** 2 months. **50.** 52 weeks.
51. 1*s.* $5\frac{1}{2}$*d.* **52.** 13*s.* 4*d.* **53.** 11*s.* 1*d.* **54.** 8*s.* 8*d.* **55.** £1, 5*s.*
56. 7*s.* 1*d.* **57.** £143. **58.** £2800. **59.** 3*s.* 9*d.* **60.** 4*s.* $11\frac{1}{2}$*d.*
61. £1, 11*s.* 6*d.* **62.** £8, 15*s.* **63.** £3. **64.** 14*s.* 2*d.* **65.** £31, 7*s.*
66. £111, 16*s.* **67.** £1, 5*s.* $10\frac{1}{2}$*d.* **68.** 1*s.* $1\frac{1}{2}$*d.* **69.** £2075.
70. £1560. **71.** 9*s.* $4\frac{1}{2}$*d.* **72.** 13*s.* $10\frac{1}{2}$*d.* **73.** 6*s.* 6*d.* **74.** 12*s.*
75. £4, 13*s.* **76.** £5, 12*s.* 6*d.* **77.** £57, 10*s.* 6*d.* **78.** £284, 15*s.*
79. £11, 0*s.* 6*d.* **80.** £203, 10*s.* **81.** 11 weeks. **82.** 5 weeks.
83. 15 sheep. **84.** 19 sheep. **85.** 315 eggs. **86.** 755 eggs.

87. 12 mins. 88. 40 mins. 89. 14 miles. 90. $17\frac{1}{2}$ miles.
91. 15 days. 92. 16 days. 93. 15 miles. 94. 90 miles.
95. 28 days. 96. $27\frac{1}{2}$ weeks. 97. 108 miles. 98. 171 miles.
99. £1, 0s. 4d. 100. £1, 8s. 4d. 101. £31, 5s. 102. £42, 11s. 8d.
103. $5\frac{1}{4}$d. 104. 1s. $7\frac{1}{4}$d. 105. £10, 17s. 106. 15s. 6d. 107. 2s. 4d.
108. 2s. $0\frac{3}{4}$d. 109. £66. 110. £22, 15s. 10d. 111. $2\frac{1}{2}$d. 112. $3\frac{3}{4}$d.
113. 9s. $2\frac{1}{4}$d. 114. £2, 5s. 9d. 115. £4, 19s. $1\frac{1}{2}$d. 116. £5, 11s. 7d.
117. £7, 11s. 118. £6, 18s. 10d. 119. £810, 12s. 6d. 120. £332, 12s. $10\frac{1}{2}$d.
121. 6 ozs. 122. 4 ozs. 123. 324 bushels. 124. 138 bottles.
125. 45 weeks. 126. 62 qrs. 4 bush. 127. 15 yards. 128. 96 yards.
129. $16\frac{3}{5}$ inches. 130. $62\frac{1}{2}$ inches. 131. 63 sheep. 132. 28 horses.
133. £835, 13s. $1\frac{1}{2}$d. 134. £15213, 10s. 10d. 135. £107, 8s. 9d.
136. 34 qrs. 3 bus. 2 pks. 137. £70, 12s. 11d. 138. £109, 4s.
139. £71, 8s. $11\frac{1}{4}$d. 140. £95, 4s. $5\frac{1}{4}$d. 141. 121 loads. 142. $56\frac{1}{4}$ miles.
143. £1, 6s. $9\frac{3}{4}$d. 144. $9\frac{11}{16}$ hours (*or* 9 hrs. 41 min. 15 secs.).
145. 90 miles. 146. 11 inches. 147. £182, 15s. 148. £222, 1s. $1\frac{3}{4}$d.
149. 96 feet. 150. 93 feet 4 in. 151. 4 ozs. 152. £63.
153. £4, 5s. $8\frac{1}{2}$d. 154. £56, 18s. 8d. 155. 694 A. 3 R. 23 P.
156. £73073, 2s. 157. 6 P.M. on Oct. 19. 158. 4 hrs. 27 min. 18 secs.
159. 5 min. 1 sec. fast. 160. 19 min. 24 secs. past seven.

XLIII.

1. £25. 2. £63. 3. £39, 4s. 4. £41, 5s. 5. £27.
6. £44, 16s. 7. 42 acres. 8. 63 acres. 9. 81 acres. 10. $69\frac{3}{10}$ acres.
11. £8. 12. £3. 13. 12 acres. 14. 16 acres. 15. £13, 15s.
16. £26, 5s. 17. £12. 18. £7, 8s. 6d. 19. $22\frac{1}{2}$ acres. 20. 55 acres.
21. 5 men. 22. 8 men. 23. 12 men. 24. 36 men. 25. 24 men.
26. 30 men. 27. 4 men. 28. 35 men. 29. 16 men. 30. 21 men.
31. 4 days. 32. 1 day. 33. 8 days. 34. 4 days. 35. 32 miles.
36. 6s. 37. 1 day. 38. 6 days. 39. £8. 40. £54, 16s. 6d.
41. 60 days. 42. 36 days. 43. £6, 18s. 44. £109, 7s. 6d.
45. £106, 17s. 6d. 46. £3, 4s. $5\frac{1}{2}$d. 47. 60 days. 48. 28 horses.
49. 17 days. 50. $20\frac{1}{4}$ miles. 51. 9d. 52. 2 lbs. 10 ozs.
53. $10\frac{2}{3}$ hours. 54. 24 men. 55. 6 days. 56. 10 hours.
57. 8 days. 58. 171 men. 59. 81 men. 60. 16 days.
61. 30 days. 62. 14 days. 63. £198, 18s. 64. 21 days.
65. 432000 gallons. 66. $4\frac{1}{2}$ days. 67. 20 men. 68. 4 men.
69. 36 days. 70. $10\frac{1}{2}$ hours. 71. 9 days. 72. 20 men.
73. 18 days. 74. 20 days. 75. £2, 12s. 3d. 76. £13, 15s. $7\frac{1}{2}$d.
77. 2 men. 78. 66 days. 79. 750 men. 80. 50 men.

ANSWERS

TO THE EXERCISES OF PART II.

XLIV. **A.**—**1.** 12 days. **2.** 32 days. **3.** 4 days. **4.** 2 days. **5.** £5, 7*s.* 4*d.* **6.** 11 days. **7.** 5 mo. **8.** 26 boys. **9.** $1\frac{1}{4}$ hrs. **10.** 4 days. **11.** 15 days. **12.** 8 days. **13.** 11 days. **14.** 21 men. **15.** 45 days. **16.** 15 men. **17.** 2400 men. **18.** 177 days. **19.** 1 hour. **20.** $9\frac{3}{4}$ weeks; £341, 5*s.*

B.—**21.** $7\frac{1}{4}$*d.* **22.** 14*s.* 9*d.* **23.** £239, 7*s.* $7\frac{1}{2}$*d.* **24.** £101, 10*s.* **25.** £722, 13*s.* 4*d.* **26.** £545, 4*s.* 5*d.* **27.** 7*s.* $7\frac{1}{4}$*d.* **28.** £3840. **29.** 4*s.* 8*d.* **30.** £23, 16*s.* 8*d.*

C.—**31.** £19, 0*s.* 11*d.* **32.** £17, 9*s.* 3*d.* **33.** £666. **34.** £1456, 8*s.* **35.** £17, 0*s.* 6*d.* **36.** £34, 0*s.* 4*d.* **37.** £8, 18*s.* $1\frac{1}{2}$*d.* **38.** £5676, 13*s.* 4*d.* **39.** £65. **40.** £934, 10*s.* **41.** £496, 10*s.* 1*d.* **42.** £4979, 6*s.* 8*d.* **43.** £840. **44.** £650, 3*s.* 3*d.* **45.** £1725.

D.—**46.** 72. **47.** 91. **48.** 55. **49.** 35. **50.** 105. **51.** 315. **52.** 60. **53.** 504. **54.** £23, 12*s.* 6*d.* **55.** £1, 0*s.* $0\frac{3}{4}$*d.* **56.** £15, 16*s.* $6\frac{3}{4}$*d.* **57.** 39. **58.** 12*s.* 3*d.* **59.** 114 miles. **60.** 24 ft. **61.** 2*s.* **62.** £1. **63.** 2*s.* 8*d.* **64.** 63. **65.** 24. **66.** $1\frac{1}{2}$ days. **67.** $\frac{7}{10}$ day.

E.—**68.** 6 days. **69.** 4 days. **70.** 12 hrs. **71.** $13\frac{1}{3}$ min. **72.** $5\frac{1}{7}$ min. **73.** $2\frac{2}{3}$ days. **74.** 4 min. **75.** $1\frac{1}{2}$ days. **76.** $1\frac{1}{29}$ days. **77.** 12 days. **78.** 12 days. **79.** 9*s.* 9*d.* **80.** $37\frac{1}{2}$ min. **81.** $17\frac{1}{7}$ min. **82.** $14\frac{2}{5}$ days. **83.** $17\frac{1}{2}$ days. **84.** 30 days. **85.** $9\frac{1}{11}$ days. **86.** $5\frac{5}{6}$ hrs. **87.** 30 days. **88.** A, 6, B, 12, days. **89.** 20 days. **90.** 120 days.

F.—**91.** 88 ft. **92.** 66 ft. **93.** $15\frac{2}{5}$ ft. **94.** $5\frac{2}{15}$ ft. **95.** $117\frac{1}{3}$. **96.** 152. **97.** $7\frac{1}{2}$ mi. **98.** $18\frac{3}{4}$ mi. **99.** $21\frac{9}{11}$ mi. **100.** $426\frac{3}{22}$ mi. **101.** $22\frac{8}{11}$ mi. **102.** $4\frac{1}{2}$ mi. **103.** 36 mi. **104.** $22\frac{1}{2}$ mi. **105.** $16\frac{1}{2}$ secs. **106.** $26\frac{1}{4}$ mi. **107.** 228 yds. **108.** 8.49 A.M. **109.** 90 mi. **110.** $3\frac{1}{2}$ hrs. **111.** $8\frac{1}{4}$ mi. **112.** $2\frac{2}{3}$ mi. per hr. **113.** 3 hrs. **114.** $3\frac{1}{3}$ hrs. **115.** $104\frac{4}{11}$ mi. **116.** 9 mi. per hr.; $3\frac{3}{4}$ secs. **117.** (i) $3\frac{15}{16}$ secs.; (ii) $31\frac{1}{2}$ secs. **118.** 88 yds.; 132 yds. **119.** No. **120.** 5 mi. per hr. **121.** (i) 3 hrs.; (ii) $4\frac{1}{5}$ hrs. **122.** $1\frac{2}{7}$ hrs. **123.** $16\frac{4}{11}$ min. past 3. **124.** $38\frac{2}{11}$ min. past 7. **125.** $27\frac{3}{11}$ min. past 5. **126.** $21\frac{9}{11}$ min. past 4. **127.** $43\frac{7}{11}$ min. past 2. **128.** $21\frac{9}{11}$ min. past 10. **129.** $10\frac{10}{11}$ min. past 8. **130.** $49\frac{1}{11}$ min. past 3. **131.** $16\frac{4}{11}$ and $49\frac{1}{11}$ min. past 6. **132.** $21\frac{9}{11}$ and $54\frac{6}{11}$ min. past 1. **133.** $16\frac{4}{11}$ and $49\frac{1}{11}$ min. past 12. **134.** At 9, and at $32\frac{8}{11}$ min. past 9. **135.** $15\frac{3}{11}$ min. **136.** $57\frac{9}{11}$ min. past 9. **137.** $32\frac{8}{11}$ min. past 4. **138.** $54\frac{6}{11}$ min. past 11.

G.—**139.** $8\frac{4}{5}$ yds. **140.** 38 yds. **141.** 30 points. **142.** 271 points. **143.** 1 min. **144.** $10\frac{1}{3}$ secs.

H.—**145.** 10*s*. **146.** 595. **147.** 15. **148.** 99. **149.** 816. **150.** 3*s*. $4\frac{1}{2}$*d*.

XLV. **1.** 2 : 3. **2.** 11 : 1. **3.** 14 : 11. **4.** 2 : 21. **5.** 47 : 57. **6.** 8 : 13. **7.** 6 : 13. **8.** 5 : 16. **9.** 14 : 1. **10.** 37 : 57. **11.** The latter. **12.** The former. **13.** —. **14.** —. **15.** £1, 2*s*. 11*d*. **16.** 560. **17.** Decreased. **18.** Increased. **19.** $4\frac{2}{5}$. **20.** $2\frac{1}{3}$. **21.** $5\frac{2}{5}$. **22.** $10\frac{22}{65}$. **23.** —. **24.** —. **25.** —. **26.** 98. **27.** $6\frac{8}{15}$. **28.** £46, 13*s*. 4*d*. **29.** 1 cwt. 0 qr. 14 lbs. **30.** $1\frac{3}{5}$. **31.** 10*d*. **32.** 4*s*. 8*d*. **33.** $3\frac{1}{5}$. **34.** $\cdot\dot{5}\dot{4}$. **35.** $2\frac{1}{3}$ qts. **36.** $3\frac{1}{3}$. **37.** 12 tons. **38.** 28. **39.** 80. **40.** $11\frac{1}{3}$. **41.** —. **42.** 7·65. **43.** —. **44.** Yes. **45.** No. **46.** 54 ft. **47.** 15 yds. **48.** 88 ft. **49.** $11\frac{1}{4}$ and $13\frac{1}{2}$ chs. **50.** 28 chs. 50 lks.; 25 chs. 65 lks.; 39 chs. 90 lks.

XLVI. **1.** 99; 209. **2.** 315; 1287. **3.** 69; 161; 253. **4.** 3125; 12500; 12500; 18750. **5.** $12\frac{2}{7}$; $9\frac{5}{7}$. **6.** $7\frac{1}{4}$; $9\frac{1}{4}$; $11\frac{1}{2}$. **7.** 70; 63. **8.** 240; 476. **9.** 3125; 6250; 12500; 25000. **10.** 30; 48; 72. **11.** 1440; 144; 1017. **12.** 135; 36. **13.** 17; 19·5. **14.** £1700; £5100; £8500; £13600. **15.** 60; 100; 260; 340; 420; 580, yds. **16.** £1, 7*s*. 9*d*.; £3, 4*s*. 9*d*. **17.** £115, 15*s*.; £135, 0*s*. 10*d*.; £154, 6*s*. 8*d*. **18.** £23, 12*s*. $9\frac{1}{2}$*d*.; £30, 7*s*. $10\frac{1}{2}$*d*.; £43, 18*s*. $0\frac{1}{2}$*d*. **19.** £5, 10*s*. 6*d*.; £26, 19*s*. 6*d*. **20.** £2, 8*s*. 7*d*.; £1, 19*s*. 5*d*. **21.** £9, 4*s*.; £10; £10, 16*s*. **22.** £60; £40; £30; £24. **23.** £400; £450; £480. **24.** 88; 120; 130, ft. **25.** A, £666, 13*s*. 4*d*.; B, £1333, 6*s*. 8*d*.; C, £1000. **26.** A, £312, 10*s*.; B, £187, 10*s*.; C, £125. **27.** £306; £336; £366. **28.** 152; 171; 361. **29.** A, £202, 10*s*.; B, £225; C, £112, 10*s*.; D, £75. **30.** £760, 18*s*.; £384, 6*s*.; £959, 18*s*. 8*d*.; £584, 5*s*. 4*d*., respectively. **31.** £9736, 3*s*. $2\frac{3}{4}$*d*.; £6954, 8*s*. $0\frac{1}{4}$*d*. **32.** 186; 83; 231. **33.** A, £42; B, £48; C, £60. **34.** A, £28, 9*s*. $5\frac{1}{4}$*d*.; B, £18, 19*s*. $7\frac{1}{2}$*d*.; C, £12, 13*s*. 1*d*. **35.** A, £16, 13*s*. 4*d*.; B, £33, 6*s*. 8*d*.; C, £100. **36.** A, £1, 6*s*. 8*d*.; B, £1, 3*s*. 4*d*.; C, £1, 8*s*. 4*d*. **37.** £208; £104; £52; £156. **38.** £212, 2*s*.; £353, 10*s*.; £388, 17*s*. **39.** A, £700; B, £600; C, £480; D, £320. **40.** $15\frac{3}{4}$; $26\frac{1}{4}$; $33\frac{3}{4}$; $38\frac{1}{4}$, gal. **41.** £6, 8*s*. for each man; £3, 14*s*. 8*d*. for each woman; £2, 6*s*. 8*d*. for each child. **42.** £26, 2*s*. 6*d*. for each man; £16, 10*s*. for each woman; £12 for each boy. **43.** A, £13, 10*s*.; B, £14, 8*s*.; C, £14, 14*s*. **44.** A, £126; B, £112; C, £504. **45.** A, £105; B, £225; C, £240.

46. A, £300; B, £192. 47. A, £309, 2*s.* 5*d.*; B, £144, 5*s.* 1*d.*
48. £2266, 13*s.* 4*d.* 49. A, £160; B, £110. 50. £89, 9*s.* 9*d.*
51. 7 mo. after A. 52. Equally; B; 2 mo. before the division of profits.
53. A, £2, 5*s.* 5*d.*; B, £2, 14*s.* 7*d.* 54. £870. 55. A, £8750; B, £6250.
56. A, £146, 5*s.*; B, £232, 17*s.* 6*d.* 57. A, £94, 10*s.*; B, £168; C, £396.
58. £2018, 3*s.* 4*d.* nearly. 59. 6 shillings worth.
60. 4 cwt. 1 qr. 14 lbs. of 1st; 4 cwt. 0 qr. 14 lbs. of 2nd; 3 cwt. 1 qr. of 3rd.

XLVII.

1. 56 sq. ft. 2. 143 sq. ft. 3. 7 sq. ft. 126 sq. in.
4. 24 sq. ft. 128 sq. in. 5. 65 sq. ft. 38 sq. in.
6. 9 sq. yds. 1 sq. ft. 72 sq. in. 7. $13\frac{1}{30}$ sq. in. 8. $3\frac{7}{16}$ sq. in.
9. 47 sq. ft. $15\frac{3}{4}$ sq. in. 10. 7 sq. ft. $8\frac{3}{8}$ sq. in. 11. 4 ac. 12. 24 ac.
13. 3 ac. 3 ro. 14. 3 ac. 0 ro. 33 po. 15. 11 ac. 0 ro. 32 po.
16. 20 ac. 1 ro. 24 po. 17. 1 sq. ft. 25 sq. in. 18. $54\frac{25}{64}$ sq. in.
19. 8 sq. yds. 1 sq. ft. 97 sq. in. 20. 10 ac. 21. 1 ac. 0 ro. 36 po.
22. 26 ac. 3 ro. 4 po. 9 sq. yds. 23. 8 ac. 0 ro. 16 po.
24. 5 ac. 1 ro. 1 po. 25. 3 ac. 1 ro. 9 po. 26. 301 sq. ft. 18 sq. in.
27. $39\frac{3}{8}$ sq. in. 28. 5 sq. yds. 2 sq. ft. $76\frac{1}{2}$ sq. in. 29. 2 sq. ft. 36 sq. in.
30. 7 sq. yds. 31. 680 ac. 32. 1 ac. $13\frac{1}{8}$ po. 33. 5 ac. 1 ro.
34. 56 ac. 0 ro. 31 po. 35. 17 ft. 36. 15 yds. 37. 63 ft.
38. $3\frac{3}{5}$ sq. in. 39. 10 ft. 6 in. 40. 26 ft. 3 in. 41. 110 yds.
42. 572 yds. 43. 1 ft. 6 in. 44. 1 yd. 1 ft. 45. $10\frac{5}{7}$ in.
46. $\frac{3}{10}$ in. 47. 5 chains. 48. 8 chains. 49. 18 po. 1 yd.; or 100 yds.
50. 50 yds. 51. 18 ft. 6 in. 52. 8 ft. 53. 76 yds. 54. 4 ft.
55. 3 ft. $4\frac{1}{2}$ in. 56. $5\frac{1}{2}$ yds. 57. $7\frac{1}{2}$ in. 58. 14 yds.
59. 1 ft. 9 in. 60. $7\frac{1}{4}$ in. each. 61. 198. 62. 288. 63. 56.
64. 1584. 65. 36. 66. 432. 67. 2304. 68. 118800. 69. 1152.
70. 36. 71. 1024. 72. $74\frac{1}{9}$. 73. $32\frac{2}{3}$. 74. $61\frac{3}{4}$. 75. 81.
76. $52\frac{1}{2}$. 77. 64. 78. $149\frac{1}{3}$. 79. 48. 80. $24\frac{8}{9}$. 81. 32.
82. $43\frac{17}{18}$. 83. 33. 84. $80\frac{2}{9}$. 85. £3, 15*s.* $6\frac{1}{2}$*d.* 86. £9.
87. £10. 88. £23, 9*s.* 4*d.* 89. £4, 13*s.* 6*d.* 90. £8, 10*s.* $7\frac{1}{2}$*d.*
91. £26, 1*s.* 4*d.* 92. £39, 2*s.* 93. 9*s.* 7*d.* 94. 6*s.* 10*d.*
95. £7, 16*s.* 9*d.*; £1, 10*s.* 96. £1, 3*s.*; 64 yds.; £12. 97. £29, 6*s.* 8*d.*
98. £30, 1*s.* 11*d.* 99. £20, 9*s.* 6*d.*; £1, 16*s.* $11\frac{1}{2}$*d.* 100. £75, 7*s.* 4*d.*
101. £49, 2*s.* 6*d.* 102. £2, 9*s.* $4\frac{1}{2}$*d.* 103. 165 sq. ft. 104. 18 ft.
105. $3\frac{3}{4}$ ft. 106. 3*s.* 6*d.* 107. 7*s.* 108. 23 ft. 109. 9*d.*
110. 88 ft. 111. 63 ft. 112. $22\frac{1}{2}$ ft.; 45 ft. 113. (i) 64 ft.; (ii) 704 sq. ft.
114. (i) 60 ft.; (ii) 600 sq. ft. 115. (i) 74 ft.; (ii) 666 sq. ft.
116. (i) 74 ft.; (ii) 777 sq. ft. 117. (i) 70 ft.; (ii) 1225 sq. ft.
118. (i) 81 ft.; (ii) $992\frac{1}{4}$ sq. ft. 119. (i) 82 ft.; (ii) 943 sq. ft.
120. (i) 147 ft.; (ii) $2719\frac{1}{2}$ sq. ft. 121. (i) 79 ft.; (ii) $941\frac{5}{12}$ sq. ft.
122. (i) 72 ft.; (ii) 711 sq. ft. 123. 2 sq. ft. 62 sq. in.
124. 3 sq. ft. 27 sq. in. 125. $731\frac{1}{2}$ sq. ft. 126. 88. 127. 8.
128. $23\frac{1}{2}$ sq. ft. 129. $43\frac{3}{16}$ sq. ft. 130. 160. 131. 88. 132. 138.
133. 108. 134. 11. 135. 902. 136. 420. 137. £2, 12*s.* 1*d.*
138. £1, 16*s.* 3*d.* 139. £5, 5*s.* 140. 17*s.* 6*d.* 141. £1, 7*s.*
142. £8, 7*s.* 1*d.* 143. £1, 0*s.* 9*d.* 144. 978. 145. 1442. 146. $1211\frac{1}{3}$.

147. £2, 10*s*. **148.** £2, 3*s*. $1\frac{1}{2}$*d*. **149.** £2, 6*s*. 9*d*. **150.** £3, 18*s*.
151. 12 ft. **152.** 13 ft. 7 in. **153.** $6\frac{1}{2}$ in. **154.** 5 ft.
155. 6 ft. 6 in. **156.** 7 ft. 6 in. **157.** $11\frac{1}{2}$ ft. **158.** 37 in. **159.** 1*d*.
160. 15 ft. wide; 11 ft. high.

XLVIII.

1. 108 cub. ft. **2.** 595 cub. ft. **3.** 154 cub. ft.
4. 525 cub. ft. **5.** 7 cub. ft. 504 cub. in. **6.** 91 cub. ft. 936 cub. in.
7. 16 cub. ft. 864 cub. in. **8.** 1228 cub. ft. 432 cub. in.
9. $5\frac{23}{64}$ cub. in. **10.** 12 cub. ft. 1216 cub. in. **11.** 1 cub. ft. 469 cub. in.
12. 29·791 cub. in. **13.** 13 ft. **14.** 13 ft. 4 in. **15.** $1\frac{1}{2}$ in.
16. 3 yds. 2 ft. **17.** 1 yd. **18.** 10 in. **19.** 3111. **20.** 8.
21. 30. **22.** 11 ft. **23.** $15\frac{1}{4}$ ft. **24.** $4\frac{4}{5}$ ft. **25.** 108 ft.
26. $\frac{3}{4}$ in. **27.** 162. **28.** 24 ft. **29.** 30 ft. wide, 16 ft. 8 in. high.
30. ·0625. **31.** £1, 3*s*. **32.** 14*s*. $10\frac{1}{2}$*d*. **33.** $1633\frac{1}{2}$. **34.** $1633\frac{1}{2}$.
35. $47\frac{197}{224}$. **36.** 117 tons 3 cwts. 3 qrs. **37.** $6\frac{1}{8}$. **38.** £1, 17*s*. 4*d*.
39. £2, 14*s*. 2*d*. **40.** 317 yds. **41.** 18432. **42.** 440. **43.** 1120.
44. 16. **45.** 51200. **46.** 44352. **47.** 60. **48.** 4. **49.** 26800.
50. 11680. **51.** 350. **52.** 1000. **53.** 459, and $\frac{3}{4}$ pint over.
54. 6. **55.** 8. **56.** 16. **57.** 2376. **58.** 2700. **59.** 15770.
60. $5376\frac{3}{8}$. **61.** 8680. **62.** $1568\frac{1}{2}$. **63.** 37 ft. **64.** $6\frac{3}{16}$.
65. $136\frac{1}{3}$. **66.** 13 cwt. 59 lbs. **67.** 1 ton 1 lb. **68.** 1 grain.
69. £37, 10*s*. **70.** 3 ft. 2 in.

XLIX.

1. 5 ft. 10′. 6″. **2.** 6 ft. 7′. 6″. **3.** 4 ft. 3′. 3″.
4. 11′. 9″. **5.** 8′. 7″. 6‴. **6.** 2′. 1″. 6‴. **7.** 6 ft. 4′. 6″.
8. 14 ft. 9′. **9.** 16 ft. 2′. 3″. **10.** 3 sq. ft. 6′. **11.** 2 sq. ft. 3′.
12. 4 sq. ft. 7′. 4″. **13.** 11′. 4″. **14.** 11′. 11″. **15.** 7′. 5″. 6‴.
16. 5 sq. ft. 9′. **17.** 6 sq. ft. 7′. 6″. **18.** 46 sq. ft. 8′. 3″.
19. 10 cub. ft. 9′. **20.** 17 cub. ft. 4′. 6″. **21.** 3 cub. ft. 9′. 9″.
22. 3‴. 6^{iv}. **23.** 7″. 2‴. 9^{iv}. **24.** 9″. 9‴. 1^{iv}. 6^{v}.
25. 6 cub. ft. 3′. 0″. 4‴. **26.** 11 cub. ft. 6′. **27.** 2 cub. ft. 10′. 7″.
28. 2 ft. 5 in. **29.** 1 ft. $7\frac{1}{2}$ in. **30.** 5 ft. $4\frac{3}{4}$ in. **31.** 11 ft. $10\frac{2}{3}$ in.
32. 7 ft. $0\frac{1}{3}$ in. **33.** 16 ft. $2\frac{5}{8}$ in. **34.** 5 sq. ft. 108 sq. in.
35. 14 sq. ft. 44 sq. in. **36.** 6 sq. ft. 141 sq. in.
37. 1 sq. ft. $103\frac{1}{2}$ sq. in. **38.** $131\frac{2}{3}$ sq. in. **39.** $112\frac{11}{24}$ sq. in.
40. 12 cub. ft. 288 cub. in. **41.** 7 cub. ft. 60 cub. in.
42. 3 cub. ft. 807 cub. in. **43.** 1547 cub. in. **44.** $1246\frac{1}{2}$ cub. in.
45. $840\frac{35}{48}$ cub. in. **46.** 22 ft. 3′. 1″. 8‴. **47.** 17 sq. ft. 9′. 4″. 1‴. 6^{iv}.
48. 17 sq. ft. 3′. 10″. 4‴. **49.** 4 cub. ft. 5′. 4″. 9‴. 6^{iv}.
50. (i) 31 ft. 8′.; (ii) 5 ft. 3′. 4″.; (iii) 34 ft. 3′. 8″.
51. (i) 66 sq. ft. 9′. 4″.; (ii) 400 sq. ft. 8′.; (iii) 261 sq. ft. 6′. 6″. 8‴.
52. (i) 1 ft. 4′. 3″. 5‴.; (ii) 1 ft. 7′. 6″. 6‴.; (iii) 8′. 1″. 8‴. 6^{iv}.
53. £1, 17*s*. $1\frac{1}{4}$*d*. **54.** £12, 0*s*. $1\frac{2}{3}$*d*. **55.** 10 sq. ft. 1′. 6″.
56. 1 ft. 1′. 5″. 3‴. **57.** 1′. 1″. 8‴. 3^{iv}. **58.** 5″. 6‴. 8^{iv}.
59. 7 sq. ft. 8′. 3″. **60.** 11 sq. ft. 10′. 6″. **61.** 44 sq. ft. 4′. 8″.
62. 99 sq. ft. 3′. 11″. 8‴. **63.** 277 sq. ft. 11′. 2″. 3‴.
64. 10 sq. ft. 7′. 2″. 7‴. 6^{iv}. **65.** 104 sq. ft. 5′. 0″. 9‴. 11^{iv}.

66. 339 sq. ft. 1′. 6″. 7‴. 6^{iv}. **67.** 21 sq. ft. 0′. 1″. **68.** 2′. 5″. 4‴. 1^{iv}.
69. 2 sq. ft. 5′. 0″. 5‴. 4^{iv}. **70.** 190 sq. ft. 2′. 6″. 3‴.
71. 12 cub. ft. 4′. 6″. **72.** 3 cub. ft. 1′. 6″. **73.** 42 cub. ft. 0′. 2″.
74. 3 cub. ft. 4′. 6″. **75.** 57 cub. ft. 3′. 6″. **76.** 6 cub. ft. 9′. 10″. 6‴.
77. 7 cub. ft. 11′. 10″. 6‴. **78.** 56 cub. ft. 11′. 1″. 9‴.
79. 382 cub. ft. 11′. 5″. 3‴. 6^{iv}. 6^{v}. **80.** 216 cub. ft. 7′. 5″. 11‴. 9^{iv}. 4^{v}.
81. 2315 cub. ft. 7′. 11″. 2‴. 6^{iv}. **82.** 6185 cub. ft. 2′. 10″. 5‴. 9^{iv}.
83. 12 cub. ft. 8′. 5″. 4‴. **84.** 49 cub. ft. 3′. 6″. 8‴.
85. 4′. 6″. 2‴. 11^{iv}. 6^{v}. 8^{vi}. **86.** 444 cub. ft. 6′. 4″. 11‴. 10^{iv}. 1^{v}. 7^{vi}.
87. $136\frac{1}{8}$ sq. ft. **88.** $292\frac{17}{32}$ sq. ft. **89.** £57, 1*s.* $10\frac{1}{2}$*d.*
90. £92, 15*s.* **91.** £9, 8*s.* $3\frac{7}{12}$*d.* **92.** 62 sq. yd. 0 sq. ft. $58\frac{2}{3}$ sq. in.
93. 4231 sq. ft. $26\frac{2}{3}$ sq. in. **94.** £2622, 12*s.* **95.** 3375 cub. in.
96. £38, 8*s.* 8*d.* **97.** 5 ft. 7′. **98.** 16 ft. 8′. 6″. **99.** 2 ft. 4′. 3″.
100. 13 ft. 9′. 6″.

L. **1.** (i) 4·563; (ii) 4·56328. **2.** (i) ·033; (ii) ·03265.
3. (i) 15·248; (ii) 15·24763. **4.** (i) ·279; (ii) ·27893.
5. (i) 6·150; (ii) 6·14993. **6.** (i) ·423; (ii) ·42337.
7. (i) 2·068; (ii) 2·06833. **8.** (i) 473·667; (ii) 473·66667.
9. (i) ·059; (ii) ·05859. **10.** 47·67. **11.** 346·0. **12.** 49·642.
13. 99·783. **14.** 11·3092. **15.** 6·48007. **16.** 1327. **17.** 40,000000.
18. 170·9. **19.** 91·8. **20.** 152462·4. **21.** 59·91. **22.** 400·7397.
23. 116·510. **24.** 28·989. **25.** 5·228. **26.** 12·454.
27. 5296·627. **28.** 10·603. **29.** ·178. **30.** 13·627.
31. ·012. **32.** ·615. **33.** ·544. **34.** ·014. **35.** 5·014.
36. 9·376. **37.** ·774. **38.** 2·545. **39.** 2033·941. **40.** 30·906.
41. 144·383. **42.** 15·461. **43.** ·949. **44.** 12·294. **45.** 2·486.
46. 18·991. **47.** £2·821. **48.** £13·908. **49.** 1·829.
50. £43·654. **51.** £4·779. **52.** £·487, or £·488. **53.** £63·554.
54. £10·183. **55.** £3·175. **56.** £75·992. **57.** £2·096.
58. £·885. **59.** £1·49167. **60.** £16·62917. **61.** £·69583.
62. £·28333. **63.** £453·80833. **64.** £197·67083. **65.** £4·93542.
66. £87·16562. **67.** £142·08021. **68.** £4, 16*s.* 4*d.* **69.** £1, 11*s.* 8*d.*
70. £17, 18*s.* 2*d.* **71.** £13, 14*s.* 1*d.* **72.** 12*s.* 9*d.* **73.** £38, 7*s.* 2*d.*
74. £123, 17*s.* 8*d.* **75.** £1, 11*s.* 11*d.* **76.** £11, 2*s.* **77.** £47, 10*s.* $11\frac{1}{4}$*d.*
78. 13*s.* $8\frac{1}{2}$*d.* **79.** £1, 10*s.* $4\frac{1}{2}$*d.* **80.** £11, 0*s.* $11\frac{1}{4}$*d.*
81. £546, 9*s.* $2\frac{1}{2}$*d.* **82.** £3, 5*s.* $11\frac{1}{2}$*d.* **83.** £33, 10*s.* 5*d.*
84. £347, 9*s.* 4*d.* **85.** £45, 18*s.* 10*d.* **86.** £7, 18*s.* 10*d.*
87. £5989, 12*s.* 10*d.* **88.** £1672, 1*s.* 1*d.* **89.** £1255, 16*s.* 4*d.*
90. £3223, 15*s.* 11*d.* **91.** £3788, 10*s.* 2*d.* **92.** £411, 14*s.* 3*d.*
93. £517, 19*s.* 11*d.* **94.** £4, 8*s.* 8*d.* **95.** £50, 7*s.* 6*d.*
96. £4, 13*s.* 3*d.* **97.** £6, 15*s.* **98.** £42, 3*s.* 2*d.* **99.** £7, 4*s.* 6*d.*
100. £11, 7*s.* 2*d.* **101.** 727·23. **102.** 15·24. **103.** 7·79.
104. 29163·99. **105.** ·126. **106.** 27·201. **107.** 3·50. **108.** 230·29.
109. 46·916. **110.** 33·61. **111.** 1435·283. **112.** ·168. **113.** 24·94.
114. ·3573. **115.** ·0004. **116.** ·656. **117.** ·05. **118.** ·088808.
119. 6·7630. **120.** 65·261905. **121.** 17·228. **122.** 9·870. **123.** ·33.
124. 145·3833. **125.** ·000457. **126.** 10·22. **127.** 31·01. **128.** ·07.

129. ·00002. **130.** ·135. **131.** 1·137. **132.** 1·267. **133.** 1·13141.
134. 1·42331. **135.** 1·710. **136.** ·16. **137.** 12·077. **138.** 315·0677.
139. 281·524. **140.** 1368. **141.** 1713. **142.** 1700. **143.** 6984000.
144. 13·275. **145.** 9244. **146.** 8·297. **147.** ·351. **148.** 27426·54.
149. 3802. **150.** ·0002588. **151.** £1, 9*s.* 1*d.* **152.** £10, 3*s.* 11*d.*
153. £3032, 9*s.* 7*d.* **154.** £43741, 11*s.* 3*d.* **155.** £348019, 10*s.* 10*d.*
156. £168023, 19*s.* 3*d.* **157.** £9250, 4*s.* 1*d.* **158.** £713, 1*s.*
159. £1363, 5*s.* 5*d.* **160.** £9286, 10*s.* 10*d.* **161.** ·519. **162.** 17·16.
163. ·0219. **164.** ·629. **165.** ·180. **166.** ·237. **167.** 305·46.
168. 701·94. **169.** ·0014. **170.** ·0028. **171.** ·12947. **172.** 14·9491.
173. ·050193. **174.** 44·814217. **175.** 25·107. **176.** 9·9804.
177. ·02467. **178.** ·079091. **179.** 1·45. **180.** 15·94. **181.** 12·056.
182. ·375. **183.** ·192. **184.** 6·48. **185.** ·0041. **186.** 26.
187. 2525. **188.** ·4617. **189.** 216·0. **190.** 464·3642. **191.** £1, 8*s.*
192. 6*s.* 6*d.* **193.** £2, 10*s.* 9*d.* **194.** £107, 16*s.* 6*d.* **195.** £5, 3*s.* 11*d.*
196. £15, 8*s.* 1*d.* **197.** ·71828. **198.** ·36788. **199.** ·41069.
200. ·27522. **201.** £31, 4*s.* 2*d.* **202.** £256, 8*s.* 7*d.* **203.** 6*s.* 11*d.*
204. 14*s.* **205.** £318, 1*s.* 11*d.*; £155, 16*s.* 4*d.* **206.** 11*s.* 6*d.*
207. (i) £26, 12*s.* 5*d.*; £18, 13*s.* 6*d.*; £11, 10*s.* 5*d.*; (ii) £40, 8*s.* 5*d.*; £48, 3*s.* 10*d.*; £59, 1*s.* 6*d.*; (iii) £338, 14*s.* 10*d.*; £501, 12*s.*; £573, 5*s.* 2*d.*; (iv) 6 tons 12 cwt. 2 qr.; 16 tons 1 cwt. 2 qr.; 34 tons 19 cwt. 3 qr.; (v) 153 ac. 2 ro. 23 po.; 162 ac. 0 ro. 28 po.; 157 ac. 3 ro. 26 po.
208. 916·3 sq. ft. **209.** 2 sq. in. **210.** 47·94 mi.

LI. **1.** 50. **2.** 25. **3.** 20. **4.** 5. **5.** 4. **6.** 2.
7. 75. **8.** 40. **9.** 70. **10.** 45. **11.** 44. **12.** 34.
13. $33\frac{1}{3}$. **14.** $12\frac{1}{2}$. **15.** $8\frac{1}{3}$. **16.** $2\frac{1}{2}$. **17.** $7\frac{1}{2}$. **18.** $3\frac{1}{2}$.
19. $62\frac{1}{2}$. **20.** $12\frac{3}{4}$. **21.** $\frac{2}{25}$. **22.** $\frac{6}{25}$. **23.** $\frac{3}{20}$. **24.** $\frac{7}{20}$.
25. $\frac{39}{50}$. **26.** $\frac{3}{50}$. **27.** $\frac{9}{200}$. **28.** $\frac{1}{30}$. **29.** $\frac{1}{16}$. **30.** $\frac{2}{3}$.
31. $\frac{1}{12}$. **32.** $\frac{1}{8}$. **33.** $18\frac{3}{4}$. **34.** $26\frac{6}{19}$. **35.** $14\frac{1}{6}$. **36.** $41\frac{3}{17}$.
37. $3\frac{37}{61}$. **38.** $37\frac{1}{2}$. **39.** $\frac{1}{32}$. **40.** $\frac{2}{45}$. **41.** $\frac{11}{160}$. **42.** $\frac{7}{320}$.
43. $\frac{17}{40}$. **44.** $\frac{23}{125}$. **45.** 270. **46.** 453. **47.** 177. **48.** 82.
49. 40·5. **50.** 3551. **51.** 4. **52.** $31\frac{1}{3}$. **53.** $37\frac{1}{2}$. **54.** $81\frac{9}{11}$.
55. 5. **56.** 1·5. **57.** 920. **58.** 576. **59.** 915. **60.** £4, 14*s.*
61. £5, 16*s.* **62.** £13, 15*s.* **63.** £17, 7*s.* **64.** 12*s.* 6*d.*
65. £1, 3*s.* 3*d.* **66.** 8*s.* 9*d.* **67.** 17*s.* 6*d.* **68.** £1, 16*s.* $7\frac{1}{2}$*d.*
69. 4*s.* 4*d.* **70.** 9*s.* 8*d.* **71.** £2, 11*s.* $2\frac{1}{2}$*d.* **72.** £4, 6*s.* 5*d.*
73. £6, 12*s.* 10*d.* **74.** £9, 9*s.* 7*d.* **75.** £26, 17*s.* 11*d.* **76.** 10*s.* 1*d.*
77. 3*s.* 7*d.* **78.** £1, 6*s.* 11*d.* **79.** 14*s.* 8*d.* **80.** £2, 3*s.* 9*d.*
81. £3, 2*s.* 7*d.* **82.** £5, 7*s.* 4*d.* **83.** £9, 1*s.* 5*d.* **84.** £23, 7*s.* 8*d.*
85. 17*s.* **86.** £1, 8*s.* 6*d.* **87.** 6*s.* 3*d.* **88.** 3*s.* 9*d.* **89.** 7*s.* 3*d.*
90. 14*s.* 2*d.* **91.** 5*s.* 5*d.* **92.** 2*s.* 2*d.* **93.** £1, 9*s.* 10*d.* **94.** 6*s.*
95. £2, 17*s.* **96.** 3 cwts. 3 qrs. **97.** 16 ozs. 3 dwt.
98. 29·025 grs. **99.** £1, 15*s.* 7*d.* **100.** 12*s.* **101.** £9, 1*s.* 3*d.*
102. £9, 12*s.* $4\frac{1}{2}$*d.* **103.** £23, 16*s.* $7\frac{1}{2}$*d.* **104.** $16\frac{2}{3}$. **105.** $12\frac{1}{2}$.
106. $\frac{5}{8}$. **107.** $97\frac{1}{2}$. **108.** $6\frac{2}{3}$. **109.** $20\frac{5}{6}$. **110.** 5. **111.** $42\frac{6}{7}$.
112. $4\frac{1}{6}$. **113.** $18\frac{3}{4}$. **114.** $3\frac{1}{3}$. **115.** 2. **116.** 14*s.* 2*d.*
117. £2, 9*s.* 5*d.* **118.** £1, 7*s.* 1*d.* **119.** £3, 19*s.* 7*d.* **120.** £6, 9*s.* 10*d.*

121. £4, 19*s*. 9*d*. 122. £10, 16*s*. 5*d*. 123. £21, 11*s*. 10*d*.
124. £16, 15*s*. 8*d*. 125. £4, 15*s*. 2*d*. 126. £3, 4*s*. 3*d*.
127. £2, 10*s*. 3*d*. 128. £7, 4*s*. 9*d*. 129. £18, 1*s*. 6*d*.
130. £24, 19*s*. 6*d*. 131. £5, 11*s*. 132. £1, 18*s*. 2*d*.
133. £28, 7*s*. 134. 459. 135. 295. 136. £15, 4*s*. 6*d*.
137. £35, 2*s*. 138. £48, 19*s*. 6*d*. 139. £109, 4*s*. $3\frac{1}{2}$*d*.
140. £49, 8*s*. 141. £53, 4*s*. 4*d*. 142. £19, 8*s*. 1*d*.
143. £93, 13*s*. 6*d*. 144. £163, 19*s*. 11*d*. 145. £295, 0*s*. 1*d*.
146. 1701. 147. 3927. 148. £50, 4*s*. 3*d*. 149. £91, 15*s*. 6*d*.
150. 10 ton 3 cwt. 2 qr. 151. 8 ton 11 cwt. 2 qr. 152. £35, 16*s*. 7*d*.
153. £15, 5*s*. 154. £71, 7*s*. 8*d*. 155. £102, 16*s*. 7*d*.
156. £260, 10*s*. 7*d*. 157. £506, 9*s*. 11*d*. 158. £85, 10*s*.
159. 139195. 160. 5. 161. $8\frac{1}{3}$. 162. 11120. 163. £644, 3*s*. 4*d*.
164. 1435. 165. 3·06. 166. $37\frac{1}{2}$. 167. £5, 1*s*. 4*d*. 168. 165200.
169. $5\frac{1}{2}$. 170. 1683. 171. 22. 172. 700. 173. 88.
174. 20200. 175. 20% increase. 176. (i) $3\frac{1}{3}$; (ii) $3\frac{13}{29}$.
177. £1500. 178. 8600. 179. 3. 180. £5, 4*s*. 2*d*.
181. 4*s*. in the pound. 182. 60000. 183. £551. 184. £520.
185. £733, 19*s*. 186. 44% increase. 187. $2\frac{1}{4}$% decrease.
188. 4947·014 cub. ft. nitrogen; 1520·4456 cub. ft. oxygen; 80·5404 cub. ft. carbonic acid. 189. 1671 ozs. 190. $62\frac{1}{2}$.
191. 800. 192. £7400. 193. $535\frac{1}{2}$; $484\frac{1}{2}$. 194. £468,750000.
195. 34. 196. E., 72·9; W., 4·0; S., 10·7; I., 12·5.
197. 1st class, 3·5; 2nd, 6·6; 3rd, 2·2; total, ·8.
198. 1st class, 2·7 decrease; 2nd, 12·4 dec.; 3rd, 2·0 inc.; total, 1·1 inc.
199. 1st class, 3·8 decrease; 2nd, 10·0 dec.; 3rd, 2·3 inc.; total, ·8 inc.
200. 1st class, ·8 dec.; 2nd, ·3 inc.; 3rd, 4·9 inc.; S.T., 24·8 dec.; total, 4·3 inc.

LII. 1. 20% gain. 2. $12\frac{1}{2}$% gain. 3. $16\frac{2}{3}$% loss. 4. $9\frac{3}{8}$% loss.
5. 8% gain. 6. $66\frac{2}{3}$% loss. 7. 10% gain. 8. 18% gain.
9. 2% loss. 10. 20% gain. 11. £9, 18*s*. 12. 8*s*. 3*d*.
13. £2, 0*s*. 3*d*. 14. £6, 6*s*. 15. 2*s*. $11\frac{3}{4}$*d*. 16. 11*s*.
17. £8, 15*s*. 18. £1369. 19. £5, 8*s*. 20. £1, 4*s*. 10*d*.
21. 13*s*. 4*d*. 22. £1166, 13*s*. 4*d*. 23. 8*s*. 4*d*. 24. £1, 3*s*. 4*d*.
25. £44. 26. 8*s*. 4*d*. 27. £560. 28. 1*s*. $0\frac{1}{2}$*d*.
29. £5, 5*s*. 30. £60. 31. £33, 17*s*. 3*d*. 32. $5\frac{7}{8}$. 33. $12\frac{1}{2}$.
34. £37, 10*s*. 35. £90, 8*s*. 4*d*. 36. 16*s*. 8*d*. 37. £15. 38. £75.
39. $2\frac{3}{4}$*d*. 40. $2\frac{1}{2}$*d*. 41. $29\frac{17}{27}$. 42. $16\frac{2}{3}$. 43. £25.
44. $12\frac{1}{2}$. 45. 150. 46. 44. 47. $36\frac{1}{9}$. 48. 10. 49. 100.
50. 3*s*. $4\frac{1}{2}$*d*. 51. 2*s*. $10\frac{1}{2}$*d*. 52. $31\frac{1}{4}$. 53. £1, 8*s*. 9*d*.
54. £8. 55. £1642, 13*s*. 4*d*. 56. 25. 57. £59, 17*s*.
58. $7\frac{1}{2}$% gain. 59. 2% loss. 60. £21, 5*s*. 61. 3*s*. $7\frac{1}{2}$*d*.
62. Bought at 12*s*. 6*d*.; sold at 12*s*. per yd. 63. 3456. 64. 1% loss.
65. £1, 4*s*. loss. 66. £12, 10*s*. 67. £2, 4*s*. gain.
68. $4\frac{1}{6}$% gain. 69. 80% gain. 70. 48. 71. £225.
72. 14% gain. 73. £39, 6*s*. 74. 2*s*. 6*d*. 75. $15\frac{20}{27}$.
76. 1*s*. 10*d*. 77. $16\frac{2}{3}$. 78. 2*s*. 79. $21\frac{31}{39}$. 80. At 2*s*. 6*d*. per lb.

81. 20% gain. 82. 40. 83. $1\frac{3}{4}$ pints. 84. 9s. per gal.; $18\frac{14}{27}$%.
85. $12\frac{1}{2}$. 86. 8 gals. 87. 156 gals. 88. 4 gals. 89. £60.
90. 10s. 91. £133, 6s. 8d. 92. $9\frac{1}{4}$. 93. 13s. 9d. 94. $22\frac{1}{2}$% gain.
95. £60. 96. 8s. 4d. 97. £1, 13s. 4d. 98. £8, 6s. 8d.
99. 2s. $7\frac{1}{4}$d. 100. £808; £606; £404.

LIII. 1. £73, 12s. 9d. 2. £25, 12s. 3. £3, 4s. 1d. 4. £48, 11s. 5d.
5. £83, 13s. 7d. 6. £1, 7s. 6d. 7. £4, 6s. 3d. 8. £2, 8s. 5d.
9. £9, 9s. 9d. 10. £12, 14s. 6d. 11. £24, 12s. 8d. 12. £117, 11s. 11d.
13. £22, 2s. 9d. 14. £1757, 16s. 2d. 15. £12, 12s. 9d.
16. £57, 18s. 4d. 17. £1, 7s. 18. £2, 12s. 6d. 19. £3, 1s. 4d.
20. £30, 10s. 8d. 21. £117, 16s. 6d. 22. £326, 5s. 23. £71, 5s.
24. £733, 6s. 8d. 25. £4, 3s. 4d. 26. £976, 1s. 3d. 27. £7625.
28. £455, 3s. 29. £775. 30. £61, 19s. 9d.

LIV. 1. £21, 13s. 2. £12, 10s. 6d. 3. £19, 2s. 6d. 4. £93, 15s.
5. £28, 11s. 6d. 6. £2, 16s. 8d. 7. £72, 5s. 8. £35, 2s. 9d.
9. £34, 9s. 5d. 10. £561, 15s. 7d. 11. £179, 4s. 12. £78.
13. £649, 13s. 9d. 14. £26, 3s. 6d. 15. £28, 19s. 9d. 16. £123, 7s. 6d.
17. £369, 15s. 18. £96, 5s. 8d. 19. £1065, 17s. 3d. 20. £4249, 0s. 9d.
21. £4, 1s. 8d. 22. £50, 10s. 3d. 23. £19, 3s. 3d. 24. 15s. 10d.
25. £12, 15s. 6d. 26. 13s. 6d. 27. £43, 4s. 3d. 28. £2, 16s. 6d.
29. £6, 11s. 3d. 30. £784, 4s. 31. £7, 7s. 7d. 32. £4, 4s. 4d.
33. £1, 1s. 1d. 34. £138, 18s. 8d. 35. £239, 15s. 6d.
36. £8, 4s. 9d.; £282, 16s. 5d. 37. £1, 15s. 11d.; £91, 14s. 1d.
38. £10, 12s. 10d.; £1713, 18s. 8d. 39. £1, 12s.; £28, 5s. 7d.
40. £96, 14s. 10d.; £813, 7s. 3d. 41. 18s. 6d.; £70, 8s. 4d.
42. £23, 9s. 11d.; £1048, 17s. 11d. 43. £90, 4s. 9d.; £3474, 3s. 9d.
44. 3s. 9d.; £10, 14s. 7d. 45. £2, 1s. 11d.; £109, 15s. 4d.
46. £16, 5s. 47. £6, 8s. 48. £3, 11s. 6d. 49. £3, 12s. 9d.
50. 19s. 10d. 51. 7s. 1d. 52. 10s. 7d. 53. £6, 13s. 6d.
54. £31, 10s. 6d. 55. £4, 10s. 6d. 56. 5s. 11d. 57. 6d.
58. £1, 0s. 5d. 59. 8s. 9d. 60. 15s. 8d. 61. £2, 18s. 6d.
62. £7, 19s. 5d. 63. £213, 11s. 64. £14, 11s. 6d. 65. 6s. 8d.
66. £17, 12s. 7d. 67. £13, 0s. 8d. 68. 1s. 6d. 69. 2s. 5d.
70. £2, 12s. 9d. 71. 3. 72. $3\frac{1}{4}$. 73. $4\frac{3}{4}$. 74. $2\frac{1}{8}$. 75. $4\frac{1}{2}$.
76. $3\frac{1}{3}$. 77. 4. 78. $4\frac{1}{2}$. 79. $3\frac{1}{2}$. 80. 3. 81. $4\frac{1}{4}$. 82. 6.
83. 4. 84. 3. 85. $2\frac{1}{2}$. 86. $2\frac{1}{2}$. 87. 4. 88. 4. 89. 18.
90. $12\frac{1}{2}$. 91. 4 yrs. 92. $3\frac{1}{2}$ yrs. 93. 25 yrs. 94. 6 mo.
95. 16 yrs. 96. 89 yrs. 97. 5 mo. 98. 25 days. 99. 10 mo.
100. 219 days. 101. 3 yrs. 102. 2 yrs. 103. 5 yrs. 104. $8\frac{1}{2}$ yrs.
105. 1 yr. 106. $5\frac{2}{3}$ yrs. 107. $4\frac{1}{2}$ yrs. 108. 8 mo. 109. 5 mo.
110. 50 days. 111. £425. 112. £2375. 113. £3720. 114. £2325.
115. £525. 116. £1575. 117. £291, 17s. 6d. 118. £20,000.
119. £6666, 13s. 4d. 120. £164, 5s. 121. £750. 122. £225.
123. £425. 124. £555. 125. £375. 126. £314, 10s. 127. £710.
128. £365. 129. £148, 7s. 11d. 130. £1036. 131. £173, 5s.

132. £2, 1s. $9\frac{3}{5}d$. 133. £1, 6s. $3\frac{3}{4}d$. 134. £297, 1s. 8d. 135. £5, 8s. 9d.
136. £9, 2s. 1d. 137. £1, 12s. 6d. 138. £3, 12s. 2d. 139. £125, 2s. 6d.
140. £178, 14s. $1\frac{1}{2}d$. 141. 2 yrs. 7 mo. 142. 3 yrs. 143. £576, 18s. 9d.
144. $4\frac{1}{2}$%. 145. £375. 146. $2\frac{3}{8}$%. 147. $6\frac{1}{4}$ yrs. 148. $26\frac{2}{3}$ yrs.
149. Mar. 20, 1898. 150. $3\frac{1}{3}$%. 151. £4250. 152. £87, 10s.
153. 90%. 154. £30. 155. 5s. 3d. 156. £48. 157. 44 : 45.
158. A, £84; B, £21; C, £105. 159. £104932, 6s. 8d. 160. 4%; $3\frac{1}{2}$%.
161. $2\frac{1}{2}$ yrs. 162. 12 yrs. 163. £1500; £1800. 164. £2037; £1746.
165. $4\frac{3}{20}$%. 166. £63, 3s. 8d. 167. £8732, 18s. 9d. 168. £2, 8s. 6d.
169. £200, 17s. 5d. 170. £2, 13s. 4d.

LV. 1. £2756, 5s. 2. £4596, 16s. 3. £694, 11s. 6d.
4. £336, 4s. 5. £1098, 1s. 6d. 6. £38, 1s. 3d.
7. £289, 15s. $5\frac{1}{4}d$. 8. £615, 2s. 6d. 9. £106, 16s. 9d.
10. £409, 13s. $11\frac{1}{4}d$. 11. £572, 17s. 8d. 12. £1687, 5s. 11d.
13. £2677, 3s. 7d. 14. £188, 1s. 3d. 15. £106, 2s. 1d.
16. £3021, 8s. 17. £1129, 11s. 18. £476, 9s. 2d.
19. £1773, 14s. 8d. 20. £2672, 2s. 5d. 21. £16, 11s. 6d.
22. £631, 10s. 3d. 23. £29, 9s. 7d. 24. £51, 12s. 4d.
25. £25, 13s. 10d. 26. £144, 10s. 5d. 27. £56, 7s. 3d.
28. £54, 6s. 2d. 29. £43, 11s. 2d. 30. £339, 15s. 10d.
31. £875, 15s. 6d. 32. £468, 7s. 7d. 33. £810, 6s. 9d.
34. £4993, 10s. 2d. 35. £179, 11s. 6d. 36. £1187, 13s. 9d.
37. £5321, 8s. 4d. 38. £876, 13s. 2d. 39. £1345, 9s. 11d.
40. £258, 16s. 11d. 41. £463, 5s. 7d. 42. £811, 8s. 9d.
43. £2806, 0s. 6d. 44. £121, 16s. 3d. 45. ~~£3864,~~ 18s. 3d.
46. £1029, 14s. 4d. 47. £27, 12s. 3d. 48. £108, 10s. 3d.
49. £11, 14s. 9d. 50. £26, 17s. 11d. 51. £1711.
52. £1303. 53. £1363. 54. £1426. 55. £802. 56. £18573.
57. £1955. 58. £3403. 59. £2, 1s. 4d. 60. £20, 13s. 7d.
61. £9, 7s. 5d. 62. £1, 3s. 3d. 63. £7, 2s. 11d. 64. 3 yrs.
65. 3 yrs. 66. 3 yrs. 67. $3\frac{1}{2}$ yrs. 68. $2\frac{3}{4}$ yrs. 69. $2\frac{1}{3}$ yrs.
70. $1\frac{1}{2}$ yrs. 71. £1250. 72. £4000. 73. £6400. 74. £2430.
75. £64000. 76. £64, 15s. 77. £10, 3s. 4d. 78. £6, 13s. $7\frac{119}{250}d$.
79. £11, 2s. 80. £84. 81. 35909. 82. £875. 83. £7000.
84. £2000. 85. £563, 5s. 11d. 86. 7 yrs. 87. £441.
88. £901, ~~6s. 8d.~~ 89. £3225, 16s. 90. £9331, 4s.

LVI. 1. £4, 1s. 3d. 2. £2, 13s. 7d. 3. £6, 8s. 9d.
4. £12, 13s. 10d. 5. £3, 17s. 9d. 6. £3, 12s. 5d.
7. £3. 8. £2, 2s. 3d. 9. £276, 10s.
10. £407, 3s. 9d. 11. £322, 19s. 5d. 12. £250, 8s. 6d.
13. £4, 13s. 3d. 14. £1, 9s. 11d. 15. £10, 10s. 2d.
16. £8, 16s. 1d. 17. £6, 19s. 4d. 18. £296, 11s. 9d.
19. £271, 16s. 8d. 20. £1374, 13s. 8d. 21. £664, 12s. 6d.
22. £373, 8s. 6d. 23. £1111, 12s. 4d. 24. £560, 6s. 9d.
25. £2399, 9s. 10d. 26. £582, 10s. 27. £346, 19s. 7d.

28. £991, 4*s.* 7*d.* **29.** £296, 10*s.* 11*d.* **30.** £2100.
31. £515. **32.** £750. **33.** 5%. **34.** £561, 3*s.* 9*d.* **35.** May 30.
36. (i) £30; (ii) £750. **37.** (i) £200; (ii) £4000.
38. (i) £24; (ii) £640. **39.** (i) £7; (ii) £400.
40. (i) £30; (ii) £1000. **41.** (i) £4, 18*s.*; (ii) £280.
42. (i) £10, 8*s.* 4*d.*; (ii) £833, 6*s.* 8*d.* **43.** (i) £4, 14*s.*; (ii) £470.
44. (i) £1, 8*s.* 7*d.*; (ii) £149, 1*s.* 5*d.* nearly.
45. (i) £5, 3*s.* 3*d.*; (ii) £498, 6*s.* 9*d.* nearly. **46.** £1, 2*s.* $8\frac{1}{4}d.$
47. 5*s.* $2\frac{2}{9}d.$ **48.** $2\frac{1}{2}$. **49.** $3\frac{1}{3}$. **50.** 5 mo. **51.** 45 days.
52. £264, 17*s.* 3*d.* **53.** £312. **54.** £826, 0*s.* 10*d.* **55.** £1300.
56. £1717. **57.** £10. **58.** £50. **59.** £195; 16%. **60.** —.
61. £770, 19*s.* 6*d.* **62.** £222, 5*s.* **63.** £596, 17*s.* 5*d.*
64. £3525, 17*s.* 4*d.* **65.** £1351, 2*s.* 6*d.* **66.** £863, 16*s.* 9*d.*
67. £354, 11*s.* 11*d.* **68.** £1048, 8*s.* 6*d.* **69.** £293, 15*s.* nearly.
70. £449, 5*s.* nearly.

LVII.

1. £696. **2.** £2300. **3.** £507. **4.** £567. **5.** £1386.
6. £1281. **7.** £769, 10*s.* **8.** £2252, 10*s.* **9.** £433, 2*s.* 6*d.*
10. £7329. **11.** £1250 stock. **12.** £875 stock. **13.** £875 stock.
14. £832 stock. **15.** £1333, 6*s.* 8*d.* stock. **16.** £1600 stock.
17. £2000 stock. **18.** £533, 6*s.* 8*d.* stock. **19.** £800 stock.
20. £3200 stock. **21.** £102. **22.** £30. **23.** £72, 10*s.*
24. £21, 5*s.* **25.** £21, 15*s.* **26.** £391, 10*s.* **27.** £112, 9*s.*
28. £17, 12*s.* **29.** £46, 13*s.* 4*d.* **30.** £56, 10*s.*
31. £46, 17*s.* 6*d.* **32.** £70. **33.** £21, 17*s.* 6*d.* **34.** £30.
35. £34, 7*s.* 6*d.* **36.** £17, 7*s.* 6*d.* **37.** £37, 2*s.* 6*d.*
38. £26, 13*s.* 4*d.* **39.** £103, 10*s.* **40.** £10, 7*s.* 6*d.* **41.** £33, 8*s.*
42. £5, 5*s.* **43.** £10, 19*s.* **44.** £15. **45.** £10, 13*s.* 4*d.*
46. £3, 17*s.* **47.** £4, 17*s.* 6*d.* **48.** £20, 12*s.* 6*d.* **49.** £39, 1*s.* 3*d.*
50. £9, 3*s.* 4*d.* **51.** £2392. **52.** £581, 17*s.* 6*d.* **53.** £2609, 5*s.*
54. £679. **55.** £400, 12*s.* 6*d.* **56.** £342. **57.** £1401, 15*s.*
58. £980. **59.** £1889, 16*s.* **60.** £1834, 1*s.* 9*d.* **61.** £560 stock.
62. £1000 stock. **63.** £875 stock. **64.** £1500 stock.
65. £800 stock. **66.** £3200 stock. **67.** £140. **68.** £22, 10*s.*
69. £11, 5*s.* **70.** £43, 15*s.* **71.** £7, 3*s.* 9*d.* **72.** £4, 2*s.* 6*d.*
73. £40, 19*s.* **74.** £39. **75.** £48, 15*s.* **76.** £20, 9*s.* 6*d.*
77. £65. **78.** £23, 8*s.* 9*d.* **79.** £13, 1*s.* **80.** £21, 5*s.* 4*d.*
81. £12, 1*s.* 8*d.* **82.** £33, 16*s.* 8*d.* **83.** £58. **84.** £12, 13*s.* 9*d.*
85. £19, 5*s.* **86.** £52, 18*s.* 9*d.* **87.** £4, 2*s.* 6*d.* **88.** £2, 17*s.* 9*d.*
89. £6, 17*s.* 6*d.* **90.** £10, 11*s.* 9*d.* **91.** $2\frac{13}{56}$. **92.** $4\frac{4}{39}$.
93. $3\frac{19}{27}$. **94.** $3\frac{11}{23}$. **95.** 4. **96.** $5\frac{5}{7}$. **97.** The former.
98. The latter. **99.** The former. **100.** The latter. **101.** The latter.
102. The former. **103.** £20 increase. **104.** £34, 7*s.* 6*d.* increase.
105. £25 decrease. **106.** £33, 15*s.* increase. **107.** £75 increase.
108. £190 increase. **109.** £25 increase. **110.** £20, 11*s.* 3*d.* increase.
111. £3350, 0*s.* 3*d.* **112.** £34, 12*s.* 5*d.* **113.** £1852, 10*s.* 7*d.* stock.
114. £2110, 16*s.* 4*d.* stock. **115.** £691, 4*s.* **116.** £2606, 3*s.* 6*d.*
117. £18, 3*s.* 8*d.* **118.** £16, 1*s.* 2*d.* **119.** £7, 1*s.* 3*d.*

120. £5, 10*s.* 8*d.* **121.** £156. **122.** £245. **123.** £62, 10*s.*
124. £952. **125.** £3675. **126.** £75. **127.** £543, 15*s.* **128.** £18.
129. £45. **130.** £185, 18*s.* 9*d.* **131.** £28, 14*s.* **132.** £43, 4*s.*
133. £1, 11*s.* 6*d.* **134.** £1125. **135.** £24, 7*s.* 6*d.* **136.** 70.
137. 400. **138.** 60. **139.** 9. **140.** 70. **141.** 140. **142.** 54.
143. 80. **144.** 5. **145.** 7. **146.** £138, 10*s.* **147.** £847, 10*s.*
148. £315. **149.** £75, 12*s.* **150.** £73, 10*s.* **151.** £109, 16*s.*
152. £74, 8*s.* **153.** £1261, 10*s.* **154.** £416. **155.** £171, 5*s.*
156. $4\frac{1}{3}$. **157.** $3\frac{5}{7}$. **158.** $5\frac{1}{3}$. **159.** The former. **160.** The latter.
161. £1945 stock. **162.** £4000; £4250; £12000 stock. **163.** £3500.
164. £5950. **165.** £3480. **166.** £4875. **167.** £3600.
168. £3678, 6*s.* **169.** £436, 2*s.* 11*d.* **170.** £9000. **171.** £8700.
172. £22080. **173.** £21840. **174.** £14520. **175.** £148, 4*s.*
176. £6, 17*s.* 6*d.* **177.** No difference. **178.** £7750 stock.
179. £215. **180.** £139, 19*s.* **181.** $80\frac{2}{3}$. **182.** $93\frac{3}{4}$. **183.** $98\frac{3}{4}$.
184. £3240. **185.** At 90. **186.** £90. **187.** £196, 13*s.* 4*d.*
188. £1430. **189.** £54 increase. **190.** The former. **191.** 4.
192. $5\frac{1}{19}$. **193.** $3\frac{67}{191}$. **194.** 4. **195.** $4\frac{4}{29}$. **196.** $133\frac{1}{3}$.
197. 85. **198.** $96\frac{1}{4}$. **199.** £72, 10*s.* **200.** £$120\frac{3}{16}$. **201.** 6%.
202. $19\frac{9}{19}$% decrease. **203.** 96. **204.** £2000 stock. **205.** 120.
206. £10489, 10*s.* **207.** £19, 16*s.* 8*d.* less. **208.** 4*s.* less.
209. £1260. **210.** £3820. **211.** The latter. **212.** 128.
213. $114\frac{7}{8}$. **214.** $166\frac{2}{3}$. **215.** £5290 stock. **216.** £1600; £1706, stock.
217. $133\frac{1}{3}$. **218.** $121\frac{3}{4}$. **219.** £14, 1*s.* **220.** £100.
221. £10000 stock. **222.** £525 before; £625 after.
223. $18\frac{74}{377}$% decrease. **224.** B. **225.** 84. **226.** 80.
227. £2859, 10*s.* **228.** £14. **229.** At 98. **230.** £174735.
231. $4\frac{1}{4}$%. **232.** £24, 5*s.* 4*d.* **233.** £144, 10*s.*
234. The former is better. **235.** £7765, 6*s.* 8*d.* **236.** £40000.
237. £81577. **238.** £$133\frac{1}{3}$. **239.** £52, 16*s.* **240.** 10%.
241. £37, 10*s.* **242.** £1300 in $4\frac{1}{2}$ p.c.'s; £1350 in 4 p.c.'s.
243. £4207, 10*s.* in 5 p.c.'s; £5409 in $2\frac{3}{4}$ p.c.'s. **244.** £12967, 10*s.*
245. £3880 in 3 p.c.'s; £3120 in 4 p.c.'s.
246. £3894 in $2\frac{1}{2}$ p.c.'s; £3700 in $2\frac{3}{4}$ p.c.'s.
247. £958, 18*s.* 8*d.* **248.** £20416. **249.** £105 decrease.
250. The latter. **251.** £7500. **252.** 3%. **253.** £1728.
254. 80. **255.** £3990. **256.** £3900 stock at 93; £1100 stock at 88.
257. £2000 stock at $110\frac{1}{2}$; £3333, 6*s.* 8*d.* stock at 113.
258. £2000 stock; $114\frac{1}{2}$. **259.** 13 : 31. **260.** $8\frac{1}{2}$%.
261. £3, 6*s.* 9*d.* increase. **262.** £264, 12*s.* **263.** £10, 9*s.* $9\frac{3}{4}$*d.*
264. £936, 10*s.* 8*d.* **265.** £16, 3*s.* 7*d.* increase.
266. £4733, 6*s.* 8*d.* stock; £63, 11*s.* 8*d.* gain. **267.** £11, 18*s.* increase.
268. —. **269.** £236, 12*s.* 6*d.* **270.** £104, 19*s.* 2*d.*

LVIII.

1. 103 *fr.* 5 *c.* **2.** 1587 *fr.* 50 *c.* **3.** 236 *M.* 70 *pf.*
4. 76 *M.* $5\frac{1}{2}$ *pf.* **5.** 846 *l.* 75 *c.* **6.** 48 *fl.* 25 *kr.* **7.** 93 *R.* 45 *kop.*
8. \$7 45 *c.* **9.** \$154 70 *c.* **10.** \$6 $5\frac{3}{4}$ *c.* **11.** 74·25 *fr.* **12.** 8·05 *fr.*
13. 105·5 *M.* **14.** 16·055 *M.* **15.** 24·05 *l.* **16.** 6·25 *fl.* **17.** 62·05 *fl.*

18. 150·19 *R.* 19. $17·35. 20. $175·05. 21. 2297 *fr.* 88 *c.*
22. 1982 *M.* 60 *pf.* 23. $155 65 *c.* 24. 213 *fr.* 15 *c.* 25. 1053 *fr.*
26. 415 *M.* 80 *pf.* 27. $698 25 *c.* 28. 7 *fr.* 7 *c.* 29. $13·85.
30. 67 times. 31. £16, 18*s.* 2½*d.* 32. £98, 13*s.* 2*d.* 33. £3, 2*s.* 2*d.*
34. £18, 1*s.* 7*d.* 35. 4*s.* 9*d.* 36. £3, 2*s.* 2*d.* 37. £2, 5*s.* 3*d.*
38. £8, 13*s.* 9*d.* 39. 14*s.* 8*d.* 40. £54, 13*s.* 9*d.* 41. £7, 9*s.* 4*d.*
42. £2, 13*s.* 11*d.* 43. £908, 13*s.* 4*d.* 44. £892, 12*s.* 1*d.*
45. £8, 18*s.* 6*d.* 46. £9, 1*s.* 9*d.* 47. 192 *fr.* 63 *c.* 48. 953 *fr.* 26 *c.*
49. 534 *fr.* 84 *c.* 50. 10246 *fr.* 73 *c.* 51. 311 *M.* 23 *pf.* 52. 46 *M.* 12 *pf.*
53. 1915 *M.* 40 *pf.* 54. $17 60 *c.* 55. $1039 20 *c.* 56. 1291 *fl.* 65 *kr.*
57. 380 *fl.* 15 *kr.* 58. 65 *l.* 47 *c.* 59. 340 *R.* 48 *kop.* 60. 676 *R.* 72 *kop.*

LIX. 1. 4 *Km.* 5 *Hm.* 7 *Dm.* 3 *m.* 2. 4 *Mm.* 5 *Km.* 7 *Hm.* 3 *Dm.*
3. 4 *m.* 5 *dm.* 7 *cm.* 3 *mm.* 4. 4 *Hm.* 5 *Dm.* 7 *m.* 3 *dm.*
5. 4 *m.* 5 *dm.* 7 *cm.* 3 *mm.* 6. 4 *Km.* 573 *m.* 7. 45 *Km.* 730 *m.*
8. 457 *Km.* 300 *m.* 9. 20 *Km.* 534 *m.* 10. 2 *Km.* 53·4 *m.*
11. 3 *m.* 65 *cm.* 12. 36 *m.* 50 *cm.* 13. 1 *m.* 37·5 *cm.*
14. 8 *m.* 46 *cm.* 15. 7 *m.* 30·5 *cm.* 16. 2·347 *Km.*
17. 51·0702 *Km.* 18. 90·125 *Km.* 19. 13·025 *Km.*
20. 7·5 *Km.* 21. 4·005 *Km.* 22. 8·75 *Km.* 23. ·0437 *Km.*
24. ·04 *Km.* 25. ·0004 *Km.* 26. ·0002 *Km.* 27. 6000 *m.*
28. 13500 *m.* 29. 70 *m.* 30. ·7 *m.* 31. ·045 *m.*
32. 300 *cm.* 33. 750 *cm.* 34. 60 *cm.* 35. ·3 *cm.*
36. 300,000 *cm.* 37. 60·57 *Km.* 38. 718·1 *m.* 39. 1120·85 *cm.*
40. 9·775 *Km.* 41. 87·3 *m.* 42. 18·5 *cm.* 43. 50 *m.*
44. 24·54 *Km.* 45. 58·65 *Km.* 46. 8100 *m.* 47. 939·4 *m.*
48. 763·8 *Km.* 49. 4·535 *Km.* 50. 2·05 *Km.*
51. 59 *cm.* + 13 *cm.* over. 52. 64 *cm.* + 1 *cm.* over.
53. 5 *m.* + 500 *cm.* over. 54. 3·04 *m.* + 32 *cm.* over. 55. 47 times.
56. 18 times. 57. 702 times. 58. 305 times.
59. 242 times + 11·3 *m.* over. 60. 43 times + 2·5 *mm.* over.
61. 102 *fr.* 34 *c.* 62. 68 *fr.* 59 *c.* 63. 46143 *fr.* 75 *c.*
64. 195 *fr.* 91 *c.* 65. 133 *M.* 21 *pf.* 66. 16380 *M.*
67. 50 *fr.* 37½ *c.* 68. 32 *fl.* 62½ *c.* 69. 1880. 70. 167.
71. 18 *fr.* 24 *c.* 72. 61 *m.* 50 *cm.* 73. 51 *Km.* 562½ *m.*
74. 26 times + 16 *mm.* over. 75. 1006 *fr.* 75 *c.* 76. 1655 *fr.*
77. 54 *fr.* 90 *c.* 78. 17 *M.* 85 *pf.* 79. 23 *M.* 65 *pf.*
80. 237 *fl.* 60 *c.* 81. 8 *fr.* 10 *c.* 82. 34 *fr.* 20 *c.*
83. 48 *fr.* 79 *c.* 84. 56 *fr.* 5 *c.* 85. 282 *M.* 51 *pf.* 86. 21 *M.* 39 *pf.*
87. 561 *Kg.* 88. 16 *cg.* 89. 241 times + 1525 *g.* over.
90. 73 *fr.* 6 *c.* 91. 20 *M.* 52 *pf.* 92. 9 *fr.* 60 *c.* 93. 6 *M.* 7 *pf.*
94. 18 *fr.* 50 *c.* 95. 178 *fr.* 10 *c.* 96. 65 *c.* 97. 5 *fr.* 25 *c.*
98. 175 *fr.* 50 *c.* 99. 48 *c.* 100. 20886 *fr.* 25 *c.* 101. 300 *sq. m.*
102. ·03 *sq. m.* 103. 3000000 *sq. m.* 104. 2300 *sq. m.* 105. 230000 *sq. m.*
106. 23 *sq. m.* 107. 700 *sq. cm.* 108. 70000 *sq. cm.* 109. 4·85 *sq. cm.*
110. 3400000 *sq. cm.* 111. 5·8 *a.* 112. 43 *a.* 113. 1·98 *a.*
114. 630 *a.* 115. 166,304 *sq. cm.* 116. 21·4496 *Ha.* 117. 6·375 *Da.*
118. 23 times. 119. 5131 *fr.* 87½ *c.* 120. 61200 *fr.* 121. 24·15 *sq. m.*

122. 54·81 *a.* **123.** 136 *M.* 80 *pf.* **124.** 145 *fr.* 20 *c.* **125.** 333 *fr.* 35 *c.* **126.** 28 *fr.* 40 *c.* **127.** 146 *M.* 75, or 80, *pf.* **128.** 3000000 *cub. cm.* **129.** 4·85 *cub. m.* **130.** 150 *cub. m.* **131.** ·015 *steres.* **132.** 168 *cub. m.* **133.** 48037·5 *cub. dm.* **134.** 57 *fr.* 9 *c.* **135.** 30000 *Kg.* **136.** 6·25 *mm.* **137.** 105 *Hl.*; 10500 *Kg.* **138.** 55225 *tonn.* **139.** 1·1811 *in.* **140.** 1093·6 *yds.* **141.** 4·9710 *mi.* **142.** 134·52 *ft.* **143.** 1550·1 *sq. in.* **144.** 5·5802 *sq. in.* **145.** 2·4711 *ac.* **146.** 61027 *cub. in.* **147.** 1·3080 *cub. yds.* **148.** 1·7608 *pints.* **149.** 15·432 *grs.* **150.** 2·2046 *lbs.* **151.** 2·5400 *cm.* **152.** 15·240 *dm.* **153.** ·91438 *m.* **154.** 4·8279 *Km.* **155.** 6·4512 *sq. cm.* **156.** 2·0234 *Ha.* **157.** 36·420 *a.* **158.** 16·386 *cub. cm.* **159.** 5·3517 *cub. m.* **160.** 4·5435 *l.* **161.** 31·103 *gram.* **162.** 11·340 *Kg.* **163.** 4000 *sq. m.* **164.** 51 yds. nearly. **165.** 175 *cub. dm.* **166.** 1609 *m.* **167.** 160·9308 *Km.* **168.** 12 sq. yds. **169.** 784·54 *m.* **170.** More expensive. **171.** 12193 *sq. cm.* nearly. **172.** 78 *fr.* **173.** 5*s.* 4¼*d.* nearly. **174.** 1*d.* **175.** 25515 *Kg.* nearly. **176.** 144·7 *Kg.* nearly. **177.** 215·55 *M.* **178.** 18·7% nearly. **179.** 6·4 *c.* nearly. **180.** 1226 *g.* nearly.

LX. **1.** 13356 *fr.* **2.** 5799·75 *M.* **3.** 9882·60 *l.* **4.** 19774·80 *kron.* **5.** 7696·20 *gul.* **6.** 2250 *fl.* 12 *st.* **7.** $1185·30. **8.** 11691·09 *R.* **9.** 3008 *dol.* **10.** 2109·554 *mlr.* **11.** £340, 12*s.* 5*d.* **12.** £159, 12*s.* 7*d.* **13.** £61, 9*s.* 6*d.* **14.** £54, 5*s.* 2*d.* **15.** £181, 8*s.* 11*d.* **16.** £308, 19*s.* 4*d.* **17.** £345, 13*s.* 7*d.* **18.** £3816, 13*s.* 10*d.* **19.** £72, 12*s.* 6*d.* **20.** £59, 18*s.* 6*d.* **21.** 12197·44 *fr.* **22.** 3627·88 *M.* **23.** 19300·12 *kron.* **24.** 3051 *fl.* 4 *st.* **25.** 14097 *Rs.* 7 *a.* **26.** 3238·4 *mlr.* **27.** £824, 12*s.* 7*d.* **28.** £190, 2*s.* 9*d.* **29.** £590, 2*s.* 1*d.* **30.** £80, 9*s.* 7*d.* **31.** £169, 0*s.* 3*d.* **32.** £640, 11*s.* 5*d.* **33.** £198, 2*s.* **34.** £719, 9*s.* 3*d.* **35.** 25·07. **36.** 25·12. **37.** 49. **38.** 20·46. **39.** 94·64. **40.** 1*s.* 2⅛*d.* **41.** 25·40½. **42.** 12 2¾. **43.** 12·19. **44.** 40½*d.* **45.** 2½%. **46.** 4%. **47.** £157, 12*s.* 1*d.* **48.** £243, 6*s.* 10*d.* **49.** £140, 11*s.* 10*d.* **50.** £1730, 8*s.* 1*d.* **51.** 10667·09 *fr.* **52.** 8097·22 *R.* **53.** 6457·15 *l.* **54.** 3274 *Rs.* **55.** 24983·75 *fr.* **56.** Better to *remit.* **57.** Better to *draw.* **58.** 12·06 *fl.* for £1. **59.** 122·35 *fr.* for 100 *M.* **60.** $4·81½ for £1. **61.** Better through London; $151·5. **62.** 25·23 *fr.* for £1. **63.** £556, 15*s.* 3*d.* **64.** 25·25 *fr.* = £1. **65.** £1 = $16\cdot1\dot{4}\dot{5}$ rupees.

LXI. **1.** 1094·29. **2.** 2961·02. **3.** 6018·595. **4.** 17894·86. **5.** 231·732. **6.** 1491·77475. **7.** 18·4629. **8.** ·7902. **9.** ·1521. **10.** ·1464. **11.** £6, 9*s.* 4½*d.* **12.** £25, 3*s.* 3*d.* **13.** £35, 9*s.* 6*d.* **14.** £63, 10*s.* 6*d.* **15.** £407, 19*s.* 3*d.* **16.** 9*s.* 7*d.* **17.** 11*s.* 7*d.* **18.** 8*s.* 6*d.* **19.** £1, 4*s.* 7*d.* **20.** £1, 16*s.* 3*d.* **21.** £3, 13*s.* 10*d.* **22.** £1, 17*s.* 3*d.* **23.** £5, 2*s.* 4*d.* **24.** £19, 11*s.* **25.** 93 yds. **26.** 269 yds. **27.** 734·9 yds. **28.** 1862·5 yds. **29.** 103·7 yds. **30.** 335·2 yds. **31.** 95·7 yds. **32.** 649·2 yds. **33.** 190·19 *m.*

34. 454·45 *m.* **35.** 69·04 *m.* **36.** 1503·02 *m.* **37.** 383 lbs. 10 ozs.
38. 7645 lbs. 10 ozs. **39.** 866 lbs. **40.** 35 lbs. 7 ozs. **41.** 4599·90 *fr.*
42. 27760·80 *fr.* **43.** 42276·71 *fr.* **44.** 8811·43 *fr.* **45.** 775·48 *fr.*
46. 13185·70 *fr.* **47.** £400, 8*s.* 11*d.* **48.** £2114, 0*s.* 1*d.*
49. £17, 11*s.* 5*d.* **50.** £78, 9*s.* 11*d.*

LXII.

1. 30. **2.** 50. **3.** 80. **4.** 100. **5.** 600.
6. 1100. **7.** 15. **8.** 28. **9.** 99. **10.** 72. **11.** 16.
12. 63. **13.** 7. **14.** 9. **15.** 35. **16.** 96. **17.** 132.
18. 230. **19.** 91. **20.** 170. **21.** 290. **22.** 132. **23.** 48.
24. 570. **25.** 24. **26.** 32. **27.** 36. **28.** 63. **29.** 66.
30. 81. **31.** 96. **32.** 330. **33.** 128. **34.** 243. **35.** 325.
36. 625. **37.** 735. **38.** 6400. **39.** 1111. **40.** 9009. **41.** 43.
42. 58. **43.** 86. **44.** 107. **45.** 209. **46.** 237. **47.** 257.
48. 291. **49.** 321. **50.** 327. **51.** 409. **52.** 417. **53.** 527.
54. 743. **55.** 777. **56.** 1079. **57.** 1209. **58.** 1763. **59.** 1858.
60. 2307. **61.** 3313. **62.** 3526. **63.** 4379. **64.** 4722.
65. 5243. **66.** 6015. **67.** 9008. **68.** 9709. **69.** 13765.
70. 15215. **71.** 20103. **72.** 29075. **73.** 33104. **74.** 34002.
75. 37196. **76.** 92829. **77.** 93139. **78.** 104196. **79.** 3000071.
80. 7077077. **81.** $\frac{1}{100}$. **82.** $\frac{4}{5}$. **83.** $\frac{7}{20}$. **84.** $\frac{8}{11}$. **85.** $\frac{9}{25}$.
86. $\frac{2}{21}$. **87.** $\frac{6}{19}$. **88.** $\frac{18}{31}$. **89.** $\frac{5}{9}$. **90.** $\frac{35}{66}$. **91.** $\frac{32}{81}$.
92. $\frac{3}{121}$. **93.** $1\frac{1}{4}$. **94.** $1\frac{1}{12}$. **95.** $1\frac{2}{15}$. **96.** $1\frac{5}{7}$. **97.** $5\frac{2}{3}$.
98. $1\frac{1}{14}$. **99.** $1\frac{5}{14}$. **100.** $1\frac{2}{17}$. **101.** $1\frac{12}{23}$. **102.** $2\frac{1}{13}$. **103.** $5\frac{4}{5}$.
104. $8\frac{1}{9}$. **105.** $2\frac{25}{28}$. **106.** $140\frac{1}{2}$. **107.** $268\frac{1}{3}$. **108.** $549\frac{1}{4}$.
109. $1\frac{1}{111}$. **110.** $11\frac{8}{13}$. **111.** $4\frac{5}{7}$. **112.** $19\frac{2}{7}$. **113.** $1\frac{13}{42}$.
114. $18\frac{3}{8}$. **115.** 49. **116.** $\frac{2}{7}$. **117.** $\frac{10}{49}$. **118.** $1\frac{1}{5}$.
119. $\frac{24}{25}$. **120.** $\frac{2}{3}$. **121.** $\frac{5}{8}$. **122.** $1\frac{3}{5}$. **123.** 1·1.
124. 1·2. **125.** ·2. **126.** ·3. **127.** ·8. **128.** ·9.
129. ·01. **130.** ·06. **131.** ·07. **132.** ·002. **133.** ·005.
134. ·012. **135.** 2·1. **136.** ·17. **137.** ·73. **138.** ·00022.
139. 14·6. **140.** 3·13. **141.** ·303. **142.** 35·7. **143.** 4·57.
144. 5·05. **145.** 1·001. **146.** 11·01. **147.** 140·5. **148.** 1·683.
149. 30·03. **150.** 265·9. **151.** ·1263. **152.** ·0905. **153.** 35·04.
154. 35·79. **155.** 4·312. **156.** 562·1. **157.** 65·43. **158.** ·2907.
159. 79·05. **160.** 8·008. **161.** 11·211. **162.** 120·98. **163.** 1353·1.
164. ·1897. **165.** 1·8827. **166.** 241·03. **167.** 5·5555. **168.** 655·36.
169. 7790·5. **170.** 7·9901. **171.** 490·304. **172.** 65·4321.
173. 60010·06. **174.** 1·41421. **175.** 1·73205. **176.** 2·23606.
177. 8·36660. **178.** 1·58113. **179.** 6·95701. **180.** ·714142.
181. ·225831. **182.** ·547722. **183.** ·632455. **184.** 1·77245.
185. ·0236643. **186.** ·00316227. **187.** 63·4997. **188.** 8·13236.
189. 1·291. **190.** ·577. **191.** 2·661. **192.** ·852.
193. ·065. **194.** 4·518. **195.** ·120. **196.** ·804.
197. $\frac{2}{3}$. **198.** $\frac{1}{30}$. **199.** $\frac{1}{6}$. **200.** $\frac{11}{30}$. **201.** $1\frac{5}{6}$. **202.** $2\frac{7}{30}$.
203. $2\frac{5}{6}$. **204.** $\frac{17}{60}$. **205.** ·882. **206.** ·559. **207.** 1·058.
208. 2·035. **209.** ·775. **210.** ·302. **211.** ·645.
212. 1·061. **213.** 1·472. **214.** 1·099. **215.** 1·773.

216. ·848. **217.** ·594. **218.** 3·545. **219.** ·101.
220. ·083. **221.** 9·8995. **222.** 69·2820. **223.** 4·0557.
224. 2·1928. **225.** 9·7980. **226.** 4·4721. **227.** 1·9319.
228. 3·4912. **229.** 2·2361. **230.** 1·7321. **231.** 4·8990.
232. 2·6390. **233.** 5·0599. **234.** 1·2890. **235.** ·4142.
236. ·8090. **237.** ·6899. **238.** ·2743. **239.** 15·9373.
240. 15·2942. **241.** 5·2779. **242.** ·3482. **243.** ·2679.
244. 33·9705. **245.** ·0142. **246.** ·0455. **247.** ·45685.
248. 1·80278. **249.** ·33538. **250.** 1·875. **251.** 7 yds. 1 ft.
252. 16 ft. 7 in. **253.** 32 ft. 10 in. **254.** 86 ft. **255.** 110 yds.
256. 219 yds. **257.** 71 yds. **258.** 880 yds. **259.** 31 po.
260. 91 ft. **261.** $\frac{1}{4}$ hr. **262.** $5\frac{9}{10}$ min. **263.** £198.
264. £10, 5*s.* 6*d.* **265.** 137 ft. **266.** £96, 6*s.* **267.** £196, 10*s.*
268. £3, 11*s.* **269.** $61\frac{1}{2}$ yds.; 5*s.* $1\frac{1}{2}$*d.* **270.** 48 days. **271.** 31.
272. 186. **273.** 7 in. **274.** 10 in. **275.** 7 in.; 1 ft. 9 in.; 3 ft. 6 in.
276. 16·9. **277.** ·1367. **278.** 6·140033. **279.** ·0707106.
280. —. **281.** 13 ft. 11 in. **282.** $120\frac{1}{2}$ yds. **283.** 498·15 yds.
284. 14. **285.** (i) 438; (ii) 1. **286.** 2. **287.** 12; $2\frac{1}{5}$.
288. 103. **289.** 2·7. **290.** 2·0305. **291.** 1302. **292.** 96·8115.
293. 80 ft. **294.** 24 ft. **295.** 8 ch. 50 lks. **296.** 4·950 in.
297. 40 ft. **298.** 4%. **299.** $3\frac{1}{3}$%. **300.** $6\frac{1}{4}$%.

LXIII.

1. 20. **2.** 30. **3.** 40. **4.** 50. **5.** 6. **6.** 12.
7. 24. **8.** 63. **9.** 77. **10.** 26. **11.** 70. **12.** 88. **13.** 9.
14. 15. **15.** 24. **16.** 28. **17.** 33. **18.** 96. **19.** 125.
20. 567. **21.** 17. **22.** 23. **23.** 34. **24.** 37. **25.** 48.
26. 65. **27.** 79. **28.** 83. **29.** 131. **30.** 203. **31.** 950.
32. 1060. **33.** 234. **34.** 298. **35.** 658. **36.** 735. **37.** 1101.
38. 2901. **39.** 4372. **40.** 7436. **41.** 10090. **42.** $\frac{3}{20}$. **43.** $\frac{5}{12}$.
44. $\frac{4}{7}$. **45.** $1\frac{1}{6}$. **46.** $1\frac{1}{8}$. **47.** $2\frac{3}{4}$. **48.** $3\frac{5}{11}$. **49.** $13\frac{2}{7}$.
50. $75\frac{1}{8}$. **51.** $1\frac{1}{3}$. **52.** $\frac{1}{15}$. **53.** $\frac{1}{6}$. **54.** ·2. **55.** ·3.
56. 1·2. **57.** ·01. **58.** 1·3. **59.** 3·1. **60.** ·76. **61.** 18·6.
62. 3·27. **63.** 91·6. **64.** 31·07. **65.** 513·5. **66.** 127·65.
67. 1·26. **68.** 1·44. **69.** 1·91. **70.** ·86. **71.** ·53. **72.** 1·05.
73. 1·52. **74.** ·86. **75.** ·43. **76.** ·55. **77.** ·69. **78.** ·77.
79. 1·46. **80.** ·89. **81.** 1·82. **82.** 198. **83.** 10 ft. 11 in.
84. 171·61 sq. ft. **85.** $121\frac{1}{2}$ sq. ft. **86.** —. **87.** 11 ft. 11 in.
88. 3·2809 ft. **89.** $\frac{3}{4}$ in. **90.** 88 ft.; 176 ft. **91.** 18 ft.
92. 14; 21; 28. **93.** £8, 9*s.* **94.** 6. **95.** 2604. **96.** 11.
97. 16. **98.** 18. **99.** 25. **100.** 44. **101.** 59. **102.** 83.
103. 97. **104.** 44. **105.** $3\frac{1}{2}$. **106.** 1·122. **107.** 11.
108. 5·4 ft. **109.** 4%. **110.** 2%.

LXIV.

1. 11420. **2.** 43657. **3.** 163125. **4.** 20331. **5.** 38160.
6. 5514*t*. **7.** 12114. **8.** 101. **9.** 111515. **10.** 333478.
11. 34*t*539. **12.** 913*e*3. **13.** 1312304. **14.** 552716. **15.** 1425400.
16. 20452443. **17.** 1*t*3*t*5166. **18.** 58*e*164851. **19.** 803845.

20. 362t07 + 6. **21.** 1023. **22.** 104032 + 14. **23.** 14173.
24. 20t39. **25.** 22602. **26.** 122202220. **27.** 44111. **28.** 39e9e.
29. 496. **30.** 1331. **31.** 102332. **32.** 10172. **33.** 10020101222.
34. 762. **35.** 427011. **36.** 5752. **37.** 1010101111. **38.** 823.
39. 11211123. **40.** 16806. **41.** 214·043. **42.** 2t9·39. **43.** 46·$\dot{5}$.
44. 40·$\dot{8}5714\dot{2}$. **45.** 375·$\dot{3}48\dot{t}$. **46.** 11011100·0$\dot{0}\dot{1}$. **47.** 134.
48. 577. **49.** 3t7. **50.** 1613. **51.** e1. **52.** 1295; 216.
53. 65. **54.** 42. **55.** Scale 7. **56.** ·69. **57.** 1te·e3.
58. 143·69$\dot{4}$. **59.** 7t·t88. **60.** Scale 8. **61.** 2, 2, 3, 10, 14, 16.
62. ·$\dot{1}\dot{3}$. **63.** 43·34. **64.** 31·28$\dot{e}$.
65. When the den$^{r.}$ contains no factors except 2's and 3's; a *terminating* radix fraction. **66.** $\frac{61}{400}$. **67.** $\frac{251}{550}$. **68.** $\frac{51e}{ee0}$.
69. 1, 2^3, 2^5, 2^6 and 2^7, lbs. **70.** (i) 3^5, 3^4, 3^2, and 1, lbs.; (ii) 3^6, 3^4, 3^2, and 1, lbs. in one scale-pan, and 3^5 and 3, lbs. in the other.

LXV.

1. 52. **2.** £37, 10*s.* **3.** £18, 4*s.*
4. $2\times3\times7\times11\times13$; $2\times3\times7\times11\times17$; 462. **5.** $\frac{117}{153}$; $\frac{4}{6}$. **6.** $2\frac{1}{7}$.
7. ·163; ·163, ·036, ·005868. **8.** 6 st. 9 lbs. **9.** $5\frac{1}{3}$ min.
10. 19. **11.** 1770643201440. **12.** 12. **13.** 2772. **14.** —.
15. $\frac{29}{74}$; $\frac{33}{34}$. **16.** ·09347. **17.** £6, 2*s.* 3*d.* **18.** £1810, 13*s.* 6*d.*
19. 112. **20.** 165 days. **21.** 2146050. **22.** 150. **23.** $3\frac{2}{5}$.
24. 3250; 32·5. **25.** 6 cub. ft. 216 cub. in. **26.** £29, 2*s.* $11\frac{1}{2}$*d.*
27. 3 days. **28.** 178; 623. **29.** $1\frac{1}{2}$ hrs. **30.** 12.20 P.M.; $4\frac{1}{2}$ secs.
31. 9996; 1003. **32.** £12, 14*s.* $10\frac{1}{2}$*d.*; 17. **33.** 100. **34.** 19·9385; ·96.
35. A, £237, 19*s.* 7*d.*; B, £234, 12*s.* 6*d.*; C, £229, 0*s.* 8*d.* **36.** 1900.
37. £848, 12*s.* 4*d.* **38.** $2\frac{1}{2}$. **39.** 4 days, in all.
40. 12.30 P.M.; $31\frac{1}{2}$ miles. **41.** 192 oz. av. **42.** 288. **43.** 195.
44. The latter. **45.** 2*d.* **46.** £50, 15*s.* 10*d.* **47.** 4293.
48. 22 ft.; $37\frac{1}{2}$ secs. **49.** $11\frac{3}{29}$ days. **50.** £134, 8*s.*; £115, 4*s.*; £96.
51. 4215. **52.** £1, 12*s.* 9*d.* **53.** 1606 pieces; ·001 in. over.
54. $4\frac{1}{6}$. **55.** £1634, 8*s.* $7\frac{3}{4}$*d.* **56.** 3*s.* 4*d.* **57.** 1 man = 3 boys.
58. 24 secs.; 5 secs. **59.** 17*s.* 6*d.* **60.** 285. **61.** 65784.
62. 71716. **63.** —. **64.** 10065 times; $\frac{3}{8}$ pint. **65.** £1488.
66. £1555, 19*s.* $6\frac{3}{4}$*d.* **67.** In $51\frac{9}{11}$ min. **68.** 8480.
69. £2, 9*s.* $3\frac{1}{2}$*d.* **70.** 40. **71.** 0; 0. **72.** 19; 14; 11.
73. 7 st. 12 lb. **74.** $1\frac{3}{4}$*d.*; 6*s.* 8*d.* **75.** 106913. **76.** $1\frac{143}{144}$; ·08.
77. 233 fr. 82 c. **78.** £110, 15*s.* **79.** 18 days. **80.** A.
81. 32175. **82.** 17. **83.** 194. **84.** $3\frac{3}{20}$; 0. **85.** ·05; ·0157.
86. 142 qrs. 5 bush. $0\frac{3}{8}$ pk. **87.** 102 fr. 59 c. **88.** 12 min. to 6.
89. £7958, 15*s.* **90.** $1036\frac{4}{5}$ revs.; 3168 yds. **91.** 723.
92. 12*s.* $3\frac{3}{4}$*d.* **93.** ·208$\dot{3}$; ·$\dot{6}\dot{3}$; ·133. **94.** £1666, 10*s.*
95. £113, 3*s.* 4*d.* **96.** $2\frac{1}{2}$ yrs. **97.** 8%. **98.** $14\frac{2}{3}$ ft.; $13\frac{3}{4}$ ft.
99. 12800. **100.** $14\frac{2}{5}$ days. **101.** 11. **102.** $6\frac{3}{4}$*d.* **103.** 2*s.* 3*d.*
104. £1062, 19*s.* **105.** £4, 13*s.* **106.** £17, 15*s.* $10\frac{1}{2}$*d.*
107. 176 yds. **108.** $6\frac{1}{4}$%. **109.** £3, 5*s.* $10\frac{3}{4}$*d.* **110.** $14\frac{2}{7}$ days.
111. 37 ac. 3 ro. 8 po. **112.** $\frac{2}{3}$ **113.** ·00416671$\dot{6}$. **114.** 215 fr. $2\frac{1}{2}$ c.
115. (i) £12, 14*s.*; (ii) £2, 7*s.* $7\frac{1}{2}$*d.*, (iii) 3*d.* nearly. **116.** $14\frac{2}{5}$ days.

117. 28 min. 48 sec. to 9. **118.** $8\frac{8}{35}$ ft. **119.** 32 points.

120. 180 miles. **121.** 191. **122.** $\frac{1}{14641}$. **123.** 3125.

124. (i) 4·2, rem$^{r.}$ ·374; (ii) 4·28, rem$^{r.}$ ·0276; (iii) 4·286, rem$^{r.}$ ·00162.

125. $23\cdot8\dot{5}\dot{1}$. **126.** 1 sq. in. **127.** £12, 16*s.* 8*d.*

128. $6\frac{2}{3}$%. **129.** £1, 11*s.* 11*d.* **130.** 27 mi.

131. By the "Tests" 477576 is divisible by 4, by 9 and by 11, *i.e.* by 396.

132. 43923. **133.** (i) 108·09375; (ii) 5670·672; (iii) 10.

134. 9030·88 ozs. **135.** 16000 ac. **136.** $12\frac{3}{4}$ days.

137. £61, 6*s.* **138.** $43\frac{19}{42}$%. **139.** 13*s.* 2*d.*

140. $8\frac{7}{8}$ mi. **141.** £10416, 13*s.* 4*d.* **142.** 1.

143. $\frac{11}{625}$, four figs.; $\frac{7}{256}$, eight figs.; $\frac{9}{7500}$, four figs. **144.** 1*d.*; 5·2271.

145. 24800. **146.** 5%. **147.** 380. **148.** £515, 4*s.* 3*d.*

149. At 8 A.M. on June 14. **150.** £3, 14*s.* 3*d.* **151.** Tuesday.

152. 1. **153.** 9999, 3333, 1111, 101, 303, 909.

154. $19\cdot91\dot{6}$; 1 ton 3 cwt. 3 qrs. 25 lbs. **155.** $23\frac{1}{13}$ min. past 5.

156. 9. **157.** £17. **158.** 6%.

159. £6, 19*s.* 5*d.*; £8, 19*s.* 3*d.*; £12, 18*s.* 10*d.* **160.** 27.

161. 532500·829. **162.** $3\frac{272}{315}$. **163.** 46·864 c. dm.

164. 4·8; 1·1; £1. **165.** 98. **166.** 5 years.

167. 18*s.* 4*d.* **168.** £304, 10*s.* 7*d.* **169.** 13 : 5.

170. In $1\frac{1}{4}$ min.; $403\frac{1}{3}$ yds. from starting point; 30 mins. **171.** 844.

172. 57 crowns, 171 hf.-crs., 285 fls., 684*s.*, 1026 sixpences.

173. 1·114113. **174.** 12 times. **175.** 45. **176.** £17, 1*s.*

177. $2\frac{2951}{12500}$ lbs. **178.** £866, 13*s.* 4*d.* **179.** 4%. **180.** 132.

181. 5. **182.** Tea, 1*s.* 6*d.*; coffee, 1*s.*; cocoa, 10*d.* **183.** $\frac{39}{40}$; $\frac{10}{1221}$.

184. 11·875 hrs. **185.** 120 times. **186.** £11, 8*s.* **187.** £450.

188. £17, 18*s.* 5*d.* nearly. **189.** £203, 15*s.* 6*d.* **190.** 5 fur.

191. 339, 340, 341, 342, 343. **192.** 231; 308; 385. **193.** $12\frac{1}{2}$; 25.

194. (i) ·095; (ii) 1·907. **195.** 189 mi.; 28 mi.; 10.30 A.M. 5.15 P.M.

196. £2175 in each. **197.** $5\frac{1}{5}$ gal. **198.** 1100 yds.

199. £182, 8*s.* 9*d.* **200.** $5\frac{11}{12}$ days. **201.** 35134. **202.** 5.

203. 9801; 1024. **204.** 76·94... **205.** 720 times.

206. £23, 5*s.* $2\frac{1}{4}$*d.* **207.** (i) 2 : 1; (ii) 1 : 2. **208.** $74\frac{3}{8}$%.

209. (i) 780; (ii) 988. **210.** 3*s.*; 1*s.* 8*d.*; 6*d.* **211.** 1792.

212. 46 and 21. **213.** $\frac{31}{49}$. **214.** (i) 802; (ii) 2051. **215.** £750.

216. $14\frac{7}{8}$ mi. **217.** 40 days. **218.** 2514196. **219.** 90 lbs.

220. 6%. **221.** 500258; 6338. **222.** 332929.

223. 17·6. **224.** £1, 12*s.* 8*d.* **225.** £1, 10*s.* 10*d.*

226. A, 63·52%; B, 28·27%; C, 8·20%. **227.** $23\frac{11}{23}$ min. past 9.

228. £863, 16*s.* 9*d.* **229.** 1*s.* 2*d.* **230.** At 90. **231.** 63, 64, 65.

232. —. **233.** (i) $7\frac{3}{11}$ days: (ii), A, 16; B, 16; C, 80 days.

234. 300 grs. **235.** $9\frac{6}{11}$ min.; A, 4; B, 5; C, 6 rounds.

236. 49·5; 54·2; 50·6; 48·7 mi. per hr. **237.** £140. **238.** £53, 10*s.* nearly.

239. 366·63; $\frac{12}{35}$. **240.** £77, 12*s.* 1*d.* increase. **241.** (i) 2368; (ii) 1892.

242. $\frac{2}{3}$; 5. **243.** 25%. **244.** In 240 weeks. **245.** 7 : 5.

246. $21\frac{7}{8}$%. **247.** 13 : 31. **248.** £1836, 0*s.* 10*d.* nearly.

249. 5, 10, 15. **250.** $\cdot011\dot{0}\dot{1}$, ·23, ·5. **251.** Tuesday.

252. $214\frac{1}{2}$, $71\frac{1}{2}$. **253.** 123·275 grs. nearly. **254.** Four; six.

255. 40. **256.** 1*s.* **257.** 110 yds.; 99 yds.

258. £235, 10*s.* 10*d.* **259.** 13 ft. 5 in.; 11 ft. 6 in.
260. £38, 8*s.* $8\frac{1}{2}$*d.* **261.** 549320 hrs. **262.** £63; £50, 8*s.*
263. $1\frac{37}{553}$. **264.** $4\frac{164}{319}$ min. gain. **265.** £7, 8*s.* **266.** ·19739556.
267. $457\frac{1}{7}$, *i.e.* 458 leaps. **268.** 1*s.* 6*d.* **269.** £7, 1*s.* 3*d.*
270. $6\frac{2}{3}$%. **271.** 4, 2, 4. **272.** 347; 253. **273.** 48. **274.** 34.
275. 96. **276.** £17; 12 persons. **277.** £360. **278.** 3*s.*
279. 611; 799. **280.** 24; 36. **281.** 70.
282. The 5th stroke of the first with the 6th of the second, and the 10th of the first with the 12th of the second. **283.** 23 and 8.
284. 36 and 63. **285.** 65 and 35. **286.** 27 and 60. **287.** $\frac{4\frac{1}{5}}{11\frac{1}{5}}$.
288. 35. **289.** $\frac{3}{23}$. **290.** 98. **291.** 8 times. **292.** 60 times.
293. 8*s.* $6\frac{3}{4}$*d.* **294.** 24 men. **295.** 1 giant = 8 dwarfs.
296. 6 days. **297.** $10\frac{1}{2}$ days. **298.** $7\frac{3}{7}$ hours. **299.** £7, 9*s.*
300. $2\frac{2}{3}$ mi.; $\frac{2}{3}$ mi. per hour. **301.** 5 mi. **302.** 16 mi.
303. $7\frac{1}{2}$ min.; the starting point: (i) $\frac{1}{2}$ min.; $205\frac{1}{3}$ yds. from starting point: (ii) $2\frac{1}{2}$ min.; $146\frac{2}{3}$ yds. from starting point: (iii) 30 min.
304. 1 min. $7\frac{1}{2}$ secs.; 135 yds. **305.** 10 mi. per hr.
306. The steamer; 448 ft. **307.** A, £190, 12*s.* 6*d.*; B, £90, 12*s.* 6*d.*; C, £18, 15*s.* **308.** A, 8*s.* 9*d.*; B, 1*s.* 3*d.* **309.** 46.
310. $\frac{89}{1875}$ cub. ft. **311.** 14 a shilling. **312.** £1160. **313.** 73 days.
314. 1 lb. at $2\frac{1}{2}$*d.*, 1 lb. at 3*d.* and 3 lbs. at 4*d.*
315. 1 gal. at 18*s.*, 2 gals. at 15*s.* and 1 gal. of water.*
316. 15 lbs. at 2*s.* 8*d.*; 1 lb. at 3*s.*; and 2 lbs. at 3*s.* 6*d.** **317.** 6 gal.
318. 1 oz. of 18 carat, 2 ozs. of 15 carat, and 4 ozs. of alloy.* **319.** 17:53.
320. 97 : 15. **321.** $42\frac{7}{8}$. **322.** $\frac{1}{15}$. **323.** $\frac{853}{171}$; $\frac{171}{597}$.
324. 9·44 times as heavy. **325.** 102. **326.** 215821; 703.
327. 30029; 30031. **328.** $22\frac{1}{2}$ ft., 15 ft., 15 ft. **329.** $25\frac{1}{2}$ ft.
330. 256 sq. ft. **331.** 3 ft., $4\frac{1}{2}$ ft., 6 ft. **332.** £6250. **333.** £190.
334. $4\frac{5}{8}$ mo. **335.** £500. **336.** 27 and 21. **337.** 35 oxen.
338. 18 days. **339.** £7, 11*s.* 3*d.* **340.** $28\frac{91}{176}$ mi. per hour.
341. See Appendix F (V).
342. The product must be *less* than 10000×1000, and *not less* than 1000×100, &c.
343. Among any 5 consec. nos. there must be a multiple of 5, a mult. of 4 and a mult. of 3. (See Appendix C.)
344. Apply method of D (IV), p. 275.
345. By the method of D (V), p. 275, prove that a no. is divisible by 99, if the sum of the units', hundreds', &c., digits + *ten* times the sum of the tens', thousands', &c., digits, is so divisible.
346. Apply method of D (IV), p. 275; *e.g.*, $237000 = 237 \times 999 + 237$, &c.
347. A prime greater than 5 must end in 1, 3, 7, or 9, ∴ its square must end in either **1** or **9**, &c.
348, 349. Apply method of (I), p. 245.
350. The numbers must end in either 0 and 5, 1 and 4, 2 and 3, 6 and 9, or 7 and 8. ∴ their product must end in either 0, 4 or 6, &c.

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* Problems of this kind admit of more than one solution.

351. Apply method of (I), p. 245.
352. Apply method of (III), p. 246.
353. Prove Appendix F (viii). In the case of two *consec.* nos. their *diff.* is 1.
354. Apply method of Ex. 7, p. 277.
355. Any even no. = some odd no. $\times$ 2, $\therefore$ its cube = (some odd no.)$^3 \times 8$.
356. $5^{40} - 1 = (5^{20}+1)(5^{10}+1)(5^5+1)(5^5-1)$. See Ex. 2, p. 277. Now $5^5+1 = 3126$, which is divisible by 6; $5^5 - 1 = 3124$, which is divisible by 4 and 11, &c.
357. Apply method of Ex. 2, p. 277.
358. $0 + \frac{1}{1+}\frac{1}{1+}\frac{1}{1+}\frac{1}{2+}\frac{1}{3}$; 0, 1, $\frac{1}{2}$, $\frac{2}{3}$, $\frac{5}{8}$. (See Appendix H.)
359. $\frac{355}{113}$, *i.e.* the 3rd convergent to $\frac{314159}{100000}$. (See Appendix H.)
360. $2 + \frac{1}{1+}\frac{1}{4+}\frac{1}{1+}\frac{1}{4+}\cdots\cdots$ (See p. 279.)

EXAMINATION PAPERS.

Camb. Prelim. Local Exam. 1896.

1. 420,729,243. 2. 153503. 3. £5, 14*s.* $8\frac{1}{4}$*d.* 4. 20 tons 10 cwts. 2 qrs. 19 lbs. 14 ozs. 5. $23\frac{1}{2}$. 6. 8. 7. 6·4140625; 48 *fr.* 79 *c.* 8. £5, 17*s.* 10*d.* 9. £2660. 10. $3\frac{1}{3}$%. 11. 6779; 21·39. 12. £19, 14*s.* $0\frac{3}{4}$*d.* 13. £10, 8*s.* 3*d.* 14. £524, 5*s.* 15. £9, 15*s.* 10*d.*

Camb. Local Exams.

JUNIORS, 1896.—A 1. 13550 + 152 rem^{r}. A 2. 307. A 3. £194. A 4. 561 *Kg.* A 5. 13·814. A 6. £22, 1*s.* $1\frac{3}{8}$*d.* B 1. (1) 5; (2) 14*s.* B 2. (1) 51 pieces; ·00121 yd. over. (2) $5{\cdot}05\dot{7}$. B 3. 66 miles. B 4. $24\frac{4}{5}$%. B 5. £212, 6*s.* 6*d.* B 6. 99 times.

SENIORS, 1896.—A 1. £3, 1*s.* 9*d.* A 2. (i) 10; (ii) £45, 4*s.* 7*d.* A 3. ·000253; 2. A 4. 295·8 *Km.*; 382·8 *Km.* A 5. £40, 4*s.* 4*d.* A 6. 14 lbs. B 1. 209509; 221. B 2. $26\frac{7}{8}$ mi. B 3. (1) £8948, 10*s.* 11*d.*; (2) £61, 10*s.* 5*d.* B 4. £29500. B 6. $47\frac{1}{4}$ years.

Camb. Higher Local Exam., 1896.

1. $\frac{113}{468}$. 2. 21. 3. £106, 1*s.* $2\frac{6}{11}$*d.* 4. 32·5; 3250000. 5. £105, 16*s.* 6. £2045. 7. 8 : 3. 8. £30, 3*s.* 9*d.* 9. 84 min. 10. 23 : 27. 11. 8 days. 12. $844\frac{4}{5}$ yds.

Camb. Univ. Previous Exam., 1896.

1. £14, 19*s.* $2\frac{3}{4}$*d.* 2. 221; 677. 3. 10·1101; $5{\cdot}6\dot{9}8315\dot{1}$. 4. $90\frac{1}{2}$ yds. 5. £5266, 8*s.* 2*d.* 6. £56, 1*s.* 8*d.* 7. £10, 12*s.* 8*d.* 8. $48\frac{3}{44}$ mi. per hr. 9. £8268, 6*s.* 8*d.* 10. $4\frac{4}{17}$ min.

Oxford Local Exams.

JUNIORS, 1897.—1. (1) $\frac{17}{27}$; (2) 0. 2. ·05; ·012. 3. 17·$\dot{6}$; 1*s.* 2*d.*
4. £18. 5. £510. 6. 300 days. 7. £4, 8*s.*
8. 90 miles. 9. £5000. 10. 1718750 ozs. 11. 110$\frac{10}{19}$%.
12. A, £10000; B, £15000.

SENIORS, 1896.—1. (1) 2; (2) 9*s.* 7*d.*
2. ·521875; 4 cwts. 3 qrs. 8 lbs. 12 ozs. 3. £8, 9*s.* 2$\frac{1}{2}$*d.* 4. 2·2678.
5. £75. 6. £320. 6*. £90; £81; £108. 7. £10000. 7*. £410.
8. 64812$\frac{1}{2}$ ozs. 8*. 700 sq. ft. 9. 8000 eggs; £25. 9*. 560 lbs.
10. £227, 10*s.* 10*. £12, 9*s.*

Ox. and Camb. School Exams.

LOWER CERTIF., 1897.—1. 542. 2. £162, 6*s.* 9*d.*
3. 6 tons 9 cwts. 0 qr. 21 lbs. 3 ozs.; 343926 in. 4. 53; 1257054.
5. (1) $\frac{33}{164}$; (2) ·000375. 6. ·0390. 7. £186, 10*s.* 7$\frac{1}{2}$*d.*; 8302626 fr.
8. 20736. 9. £280, 2*s.* 6$\frac{1}{4}$*d.* 10. £18, 17*s.* 7$\frac{1}{2}$*d.* 11. 9 o'clock.

HIGHER CERTIF., 1897.—1. 492 tons 1$\frac{3}{7}$ cwts. 2. 11$\frac{1}{4}$ mi. 3. 3.
4. 63·143; 43·08868. 5. 247. 6. £187, 12*s.* 3$\frac{1}{4}$*d.* 7. A by £10.
8. 41·28 and 18·72 gals. 9. 45 days. 10. $\frac{275}{504}$ ton; £2, 9*s.* 3*d.*
11. £171429. 12. £116, 13*s.* 4*d.* A. £1113, 15*s.* 4$\frac{1}{2}$*d.* B. 2*s.* 8$\frac{3}{4}$*d.*
C. 3·011760. D. 56$\frac{12}{23}$%. E. £1, 19*s.* 1*d.* F. £4, 9*s.* 3*d.* increase.

Coll. of Preceptors Certificate.

3RD CLASS, 1896.—1. 57111104051. 2. 3 ton 4 cwt. 1 qr. 17 lbs.; 50000 ft.
3. £100, 2*s.* 6*d.*; £1, 2*s.* 6*d.* 4. 4*s.* 2*d.* gain. 5. £3740, 13*s.*
6. 45 weeks. 7. 2223; 22680. 8. 33$\frac{1}{20}$; 1$\frac{7}{8}$. 9. 14$\frac{73}{80}$; $\frac{1}{11}$.
10. 21 min. to 5.

2ND CLASS, 1896.—1. £3342, 1*s.* 6*d.* 2. 3917. 3. 8294400.
4. $\frac{1}{85}$. 5. 4·84. 6. 25·$\dot{6}\dot{6}8\dot{1}$. 7. £57, 19*s.* 11*d.*
8. £89511, 12*s.* 11*d.* 9. 123·454321. 10. 222·22. 11. $\frac{71}{80000}$.
12. 220,000,000 Hl.; 38,720,000,000 pints. 13. 30*s.*; 7*s.* 6*d.*; 3*s.*

1ST CLASS, 1896.—1. 2 hrs. 28 min. 2. 1. 3. ·0029.
4. £246, 10*s.*; 180 acres. 5. ·46132. 6. £58, 15*s.* 4*d.*; 2$\frac{1}{10}$%.
7. £37, 1*s.* 9*d.*; £2, 11*s.* 9*d.* 8. 108$\frac{1}{8}$. 9. 2916 sq. ft.; 49$\frac{17}{18}$ cub. yds.
10. 873529767. 11. 23 men. 12. 25*s.* 13. 12%.

Professional Prelim. Exam., 1896.

2ND CLASS.—1. 634394567. 2. 10032.
3. 14198399 farthings; £374, 10*s.* 11$\frac{3}{4}$*d.*
4. Each man, £25, 6*s.* 5$\frac{1}{4}$*d.*; each woman, £50, 12*s.* 10$\frac{1}{2}$*d.* 5. £20.
6. (i) 10$\frac{3}{8}$; (ii) 7$\frac{2}{9}$. 7. 5·8. 8. 365 days. 9. £231, 10*s.* nearly.
10. £13, 15*s.* 7$\frac{1}{2}$*d.* 11. 27·02; 2·132... 12. ·325 metre.

1st Class.—**1.** 21 yds. 1 ft. $11\frac{1}{6}$ in. **2.** £73, 17*s.* $10\frac{14}{45}$*d.*; £8, 12*s.* 1*d.*
3. (i) 1; (ii) 1. **4.** ·38048; 156250; $\cdot 0004236\dot{1}$. **5.** £246, 6*s.* $7\frac{5}{8}$*d.*
6. 84 days. **7.** Disc. ("True") £23, 6*s.* 8*d.*; Int. £71, 4*s.* 6*d.*
8. 1·77345969; 1·18321596. **9.** —. **10.** 120 miles.
11. English; £8000. **12.** £97826, 1*s.* 9*d.*; £459, 15*s.* 8*d.*; nearly. **13.** £2.

Scotch Leaving Certificate.

Lower Grade, 1896.—**1.** 579. **2.** £4, 0*s.* $7\frac{1}{2}$*d.* **3.** 16 tons.
4. 108290. **5.** 5040. **6.** 5 yds. **7.** (1) $2\frac{3}{10}$; (2) $\frac{3}{5}$.
8. (1) $\frac{13}{3960}$; (2) $\cdot 21\dot{3}$. **9.** (1) 126·72; (2) $\frac{5}{39}$. **10.** £57699.
11. £5, 11*s.* 6*d.* **12.** $4\frac{3}{4}$%.

Higher Grade, 1896.—**1.** (1) 2 fur. 9 ch. 13 yds. 1 ft. 4 in.; (2) 19 c. yd. 10 c. ft. 216 c. in. **2.** $6\frac{3}{10}$ days.
3. See p. 26; May 15, 1908. **4.** 11364896. **5.** (1) 1; (2) 125.
6. £11, 2*s.* 9*d.*, man; £6, 9*s.* $11\frac{1}{4}$*d.*, woman; £4, 12*s.* $9\frac{3}{4}$*d.*, boy; £2, 15*s.* $8\frac{1}{4}$*d.*, girl. **7.** £7200. **8.** 429 ft. **9.** $107\frac{1}{4}$.

Irish Intermed. Exams.

Junior Grade, 1896.—**1.** 102563 tons 19 cwt. 0 qr. 4 lb. **2.** £11, 18*s.* 7*d.*
3. See pp. 56, 57. **4.** Showing $5\frac{1}{2}$ min. to 11. **5.** 10 hrs.
6. 69 mi. 1226 yds. 2 ft. **7.** £3, 5*s.* **8.** $2719\frac{23}{49}$. **9.** £417, 17*s.* 6*d.*
10. $2\frac{1}{4}$%. **11.** June 15, 1895. **12.** 530 yds. **13.** 1.
14. 145·6849 sq. in.

Middle Grade, 1897.—**1.** 16. **2.** $\frac{121}{400}$. **3.** £880; 1200. **4.** 10 hrs.
5. $3\frac{7}{16}$ miles. **6.** 14 ft.; $5\frac{3}{5}$ ft.; $2\frac{4}{5}$ ft. **7.** £1320, 12*s.* **8.** £710.
9. $5\frac{1}{2}$%. **10.** In the ratio 25 : 26; each receives £3515, 4*s.*
11. 70 apples; $23\frac{1}{3}$% gain. **12.** $95\frac{3}{8}$. **13.** £10 increase. **14.** £21537, 6*s.*

London Univ. Matric. Exam.

June, 1896.—**1.** 21·€. **2.** 1880, 1852, 1824.
3. 5 gals. water to 28 of milk. **4.** £1865, 14*s.* 10*d.*

January, 1897.—**1.** 3955·76...miles. **2.** (i) £631, 11*s.* 7*d.* nearly; (ii) $21\frac{3}{7}$%.

Science and Art Depart., 1896.

1. (*a*) 6·53809; (*b*) $2\frac{37}{40}$. **2.** (*a*) 5; (*b*) $59\frac{1}{2}$, 68, $76\frac{1}{2}$. **3.** 179 yds.
4. £213, 10*s.* $0\frac{109}{112}$*d.* **5.** (*a*) £3, 15*s.*; (*b*) £78, 2*s.* 6*d.*
6. £1, 14*s.* 2*d.*; $30\frac{10}{27}$%.

Civil Service.

Boy Copyists, 1897.—**1.** See pp. 118; 113, *Note*; 116. **2.** 70%.
3. 28 yds. **4.** £1000. **5.** $32\frac{33}{40}$%. **6.** 1*s.*

Engineer Students, 1896.—**1.** 4050. **2.** $12\frac{4}{5}$ hrs.
3. 6565; 362254; 402152. **4.** $\frac{3}{340}$; $18\frac{9}{17}$ (see Ex. D, p. 95).
5. 5 hrs. 30 min. **6.** See p. 246, and *Note*, p. 245.

FEMALE CLERKS, 1896.—**1.** 119; either 119 or 595. **2.** 17*s.* 9*d.*
3. 375 hours; $22\frac{1}{2}$ min. past 2 A.M. and $22\frac{1}{2}$ min. past 3 A.M.
5. (1) There must be an odd number of odd digits. (2) There must be an even number of odd digits. (See Appendix F.) **6.** 3·141592.
7. 4·88461862. **8.** £55; 5 years and 5% (*simple* int.). **9.** The former.

EXCISE, 1896.—**1.** 5 lbs. 5 ozs. **2.** 58 gal. 2 qts. 1 pt. 1 gill.
3. 3 mi. 1 fur. 36 po. **4.** 153549 sq. ft. **5.** 47. **6.** 7560.
7. $\frac{9}{10}$. **8.** $8\frac{37}{112}$. **9.** $2\frac{47}{48}$. **10.** 16. **11.** $\frac{3}{10}$. **12.** 44·793.
13. 5·94468. **14.** 14·636934. **15.** 35·004. **16.** ·8144. **17.** $\frac{3}{5}$.
18. 6 lbs. 5·01504 ozs. **19.** £52, 9*s.* $4\frac{1}{2}d.$ **20.** £13, 6*s.* **21.** 15 days.
22. 100. **23.** £1, 2*s.* 6*d.* **24.** $3\frac{1}{3}$%. **25.** $2\frac{5}{8}$ sq. ft.

EXCISE, 1896, HIGHER ARITH.—**1.** Apply method of (I) p. 245.
2. $43\frac{3}{19}$%. **3.** 55·86591; 3025; 31203396; 3120999. See Appendix H.
4. £52083, 6*s.* 8*d.* **7.** 1897; 1904, 09, 15, 26, 32, 37, 43, 54, 60, 65, 71, 82, 88, 93, 99. **8.** Fully due on Sep. 29. ("True" disc. was intended.) **9.** 3960 π secs.; 7425, 7680, 8096.

2ND DIVISION CLERKS, 1896.—**1.** 10; $7+\frac{1}{14+}\frac{1}{14+}\cdots$ **2.** See p. 56; 975.
3. See Appendix E. **4.** 211·025. **5.** $7\frac{1}{2}$; $77\frac{1}{2}$ min.
6. See Appendix F. £1666, 13*s.* 4*d.* The increase depends upon the *difference* in the rates, which is independent of the original rate.
7. See p. 105; 13; 44. **8.** 52·486.
9. $3050\cdot7\dot{2}2755\dot{0}$. That the result would be a *recurring* radix fraction.
10. Greater on the *outward* journey, by $1\frac{23}{64}$ miles. In the 2nd case, greater on the *return*, by $\frac{29}{420}$ mile.

Military Entrance Exam., 1896.

1. $13\frac{1}{5}$. **2.** 28920. **3.** 3570; $\frac{1}{5}$. **4.** 10*s.* 7*d.* **5.** £24, 9*s.* $6\frac{5}{16}d.$
6. 96 yds.; 29 yds. 1 ft.; $\frac{32}{55}$. **7.** $3\frac{1}{3}$%. **8.** 33 ft. 6 in.
9. £7, 10*s.*; £6, 15*s.*; £10, 10*s.* **10.** See p. 52, and Appendix D (1) and F.
11. 91 cub. yds. 7 c. ft. **12.** 7 lbs. coffee to 5 lbs. chicory.
13. £41, 13*s.* 4*d.* **14.** A, 18 mi.; B, 10 mi. per day.

Queen's Scholarship.

ENGLAND, 1896.—**5.** £9, 5*s.* $2\frac{1}{2}d$; £37, 0*s.* 10*d.* **6.** £767, 2*s.* $5\frac{1}{2}d.$
7. $\frac{1}{2}d.$ too much. **8.** $\frac{82}{151}$. **9.** (*a*) 514720; (*b*) ·0051472; (*c*) $20\cdot\dot{0}2\dot{0}$.
10. 1*d.* **11.** Banker's disc. £339, 7*s.* 2*d.*, "True", £303.
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16. 9 min. 40 secs.; 4 min. 50 secs.; 2 min. 25 secs. **17.** —.
18. $5\frac{5}{6}$%. **19.** £14400 stock; $83\frac{1}{3}$.

SCOTLAND, 1896.—MALES.—**1.** $\frac{1}{4}$. **2.** ·625. **3.** 357. **4.** 80.
5. £620. **6.** $32\frac{8}{11}$ min. past 6. **7.** £4640. **8.** 60% gain.
9. £6250. **10.** $1\frac{1}{2}$ hrs. **11.** 930. **12.** 24, 72, 80.
13. A, 864 ac.; B, 1440 ac.; C, 2304 ac. **14.** 176 yds.; 45 mi. per hr.

SCOTLAND, 1896.—FEMALES.—**1.** $\frac{1}{8}$. **2.** ·6. **3.** 180. **4.** 4092. **5.** £1200. **6.** £200, 8*s.* 4*d.* **7.** 480. **8.** $\frac{2}{5}$. **9.** £11760. **10.** 90 half-sovs.; 120 half-cr., 210 sixpences. **11.** 72 lbs.

Certificate, England, 1896.

MALES, 1ST YEAR.—**2.** (i) $\frac{1}{51} + \frac{1}{85} = \frac{1}{17}$ of $(\frac{1}{3} + \frac{1}{5}) = \frac{1}{17}$ of $\frac{8}{15}$ and $\frac{1}{17}$ of $\frac{8}{15}$ of $15 \times \frac{17}{4}d. = 2d.$
(ii) £672 is 8 times £84; and £84 buys a net inc. of £3 × $\frac{230}{240}$ = £$\frac{23}{8}$; ∴ £672 buys a net inc. of **£23**. (iii) The integral part of the sq. root of 79 is 8, and the figure whose square ends in 1 is 9; ∴, *assuming that the given number is a perfect square*, its sq. root is 89.
3. 4·7688 grs. **4.** £540. **5.** £7560. **6.** £105, 8*s.* **7.** $\frac{6}{7}$. **8.** 880 ft.; 352 times. **9.** B by $8\frac{8}{9}$ yds. **10.** £62, 10*s.* increase. **11.** $22\frac{8}{11}$ years. **12.** $33\frac{1}{3}$ in. **13.** 5 min. fast.

FEMALES, 1ST YEAR.—**2.** (i) 1*s.* 4*d.* is £$\frac{1}{15}$, ∴ 150 men earn £10 per hour; and £10 × 8 × 18 = £1440. (ii) 146 days = $\frac{2}{5}$ year; $\frac{2}{5} \times \frac{5}{4} = \frac{1}{2}$; $\frac{1}{2}$ of £360 = £180, and £$\frac{180}{100}$ = £$1\frac{8}{10}$ = £1, 16*s.* (iii) Area of floor is 48 sq. yds.; width of carpet, $\frac{3}{4}$ yd.; ∴ length of carpet required is $48 \div \frac{3}{4} = 64$ yds., and 1*s.* 8*d.* is £$\frac{1}{12}$, ∴ cost = £$\frac{64}{12}$ = £5, 6*s.* 8*d.*
3. 5 tons 13 cwts. 2 qrs. 9 lbs. 13 oz. 10 dr. **4.** 540. **5.** £27, 5*s.* $9\frac{3}{4}d.$ **6.** $\frac{100}{231}$; £2, 0*s.* $10\frac{1}{2}d.$ **7.** £114. **8.** —. **9.** 360. **10.** 112; £41, 3*s.* 4*d.* **11.** 8 min. **12.** 5*s.* 6*d.* **13.** 88 yds.

FEMALES, 2ND YEAR.—**2.** £711, 11*s.* 3*d.* **3.** $\cdot 52\dot{7}$. **4.** Twice. **5.** 38 ac. **6.** 400. **7.** 40 min. 46 secs. past 10. **8.** 56 lbs. **9.** 125 lbs. **10.** £100. **11.** $\frac{1}{2} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} = \frac{67}{60}$; £240, 8*s.* $1\frac{1}{2}d.$; £120, 4*s.* $0\frac{3}{4}d.$; £96, 3*s.* 3*d.*; £80, 2*s.* $8\frac{1}{2}d.$

Certificate, Scotland, 1896.

MALES, 1ST YEAR.—**1.** (*a*) 1·43648; $2\frac{1777}{3700}$. (*b*) $2\frac{97}{189}$. **2.** (*a*) $5\frac{1}{5}$; 300·03. (*b*) 14. **3.** 12 days. **4.** 5*s.* $0\frac{1}{2}d.$ **5.** $\frac{3}{4}$. **6.** Banker's disc. £12, 1*s.* $7\frac{1}{5}d.$; "True" disc. £12. **7.** £1155. **8.** $166\frac{2}{3}$. **9.** $6\frac{2}{33}$%. **10.** $25\frac{1}{8}$%.

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